

CASE STUDY

Students being set up to make mistakes in class through an error-eliciting task

Claire Cornock, Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield. Email: c.cornock@shu.ac.uk

Alex Shukie, Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield. Email: a.shukie@shu.ac.uk

Ros Porter, Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield. Email: r.porter@shu.ac.uk

David O'Sullivan, Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield. Email: d.osullivan@shu.ac.uk

Abstract

In order for students to develop a better understanding and the skills to question future work, a session was introduced into a teaching workshop which sets students up to make common mistakes. These mistakes are often made by blindly following procedural methods. The students' views on these mistakes and how they found the error-eliciting task were gained through focus groups on the day of the activity. Factors such as knowing whether they had the right answer and the amount of staff involvement were discussed. In a follow-on focus group two weeks after the session, there were indications that the session had an impact on how they worked generally as there was more discussion within class and a shift in views about making mistakes.

Keywords: Making mistakes, error-eliciting problems, tutor reliance, willingness to try, routine approaches.

1. Introduction

As presented by Fischbein (1994), the 'blind learning of algorithms' leads to problems as lack of understanding can lead to the wrong application of processes. In this paper, we consider the teaching of a topic where this is common. It is situated in a level 6 elective called 'Abstract Algebra' which is based on applications of group theory and monoid theory. More information about this module can be found in Cornock (2021) and Cornock (2015). The students are introduced to the pumping lemma (as part of formal languages) in a usual workshop style session. This requires picking a word and a value that leads to a contradiction to show that a language is not recognisable. Some words work better than others and some cannot be used for various reasons (e.g. you must have a word that is in the language). The students are shown two proofs in which the starting examples are provided. They are also given some exercises to work through without the starting examples or values, but the most sensible guesses lead them to the correct answer. It is the type of proof that can be reproduced with little understanding. More information about the mathematical topic can be found in Lawson (2004).

Error-eliciting problems are 'designed to specifically bring forth among students common mistakes pertaining to a particular mathematical concept' (Lim, 2014). Eggleton and Moldavan (2001) provide examples of teaching practice within middle school where pupils consider mistakes. The idea is that students are left to investigate, with very little guidance. On the basis of questions asked by the teacher, pupils can explore further and ideally reach conclusions. One of their activities was an exercise where the pupils had to decide the order of the numbers. Common mistakes were factored in. For example, numbers such as 4% and 4/10 were included. They state that 'some teachers have

become so accustomed to the common mistakes made by students that their strategy for teaching precision is to warn students about possible mistakes in advance'. Instead, the pupils are required to 'confront a possible error'. This difference means that pupils build the skills to question future work and resolve issues themselves. These concepts, presented by Eggleton and Moldavan (2001), form the premise for a teaching session within Higher Education. More examples of error-eliciting problems can be found within Lim (2014).

In this paper, we present information and evaluation on a teaching session on the pumping lemma in which students are set up to make common mistakes through an error-eliciting exercise. The activity takes place a week after the initial introduction session that is described above. The students are provided with a different language to work with. Instead of letting the students choose their starting word and value, they are given six words and three values. Some of these lead to contradictions, but most of them do not and are based on common mistakes seen in work done by previous cohorts when the students have just followed the process with little thought. With all the examples, the students can work through the method as given in the lecture, but with many of the provided examples there is a reason why they do not lead to a contradiction. The session takes place in a room with many whiteboards, where the students work on the problems in small groups of around four students. There are two lecturers present, who are on hand to answer questions and circle the room. They ask the students questions about why they have taken the approaches they have, and discuss any problems and incorrect answers (especially if they have not been spotted by the group). This approach is very similar to the example presented by Eggleton and Moldavan (2001), just with material for a higher level.

2. Methodology

Two focus groups took place on the same day as the session being considered in this study in 2018/19, containing eight students in total. Another focus group took place two weeks later and contained six of the students from the first two focus groups together and no additional students.

At the start of all three focus groups, it was pointed out that there are different types of mistakes and that not paying enough attention to the technicalities of the method was of interest within the focus group. The students were reminded that in the 'Abstract Algebra' session that they did an exercise that required trying the given examples. It was pointed out how the activity was created based on the tutors' knowledge of why certain mistakes were made and carefully designed so that the students were likely to make common mistakes.

2.1. Focus groups on the same day as the activity

After being presented with the information above in the focus group on the same day, the main question was regarding how the students found the experience in which they were led to make mistakes. In particular, they were asked how they felt about making a mistake and how they respond to one, both generally and in the session. There were questions about whether they felt differently about making mistakes now that they have been led to make one, if anything had changed in their view of mistakes and what they had learnt from the experience. They were asked whether they view being led to make mistakes as an opportunity to learn, both in general and when they were led to make a mistake in the session.

2.2. Focus group two weeks after the activity

Two weeks later, the students were reminded of the same information at the start of the focus group that had been presented at the start of their first focus group. The main question in this focus group was whether the experience of being led to make mistakes had impacted or changed their approach when trying questions since the last focus group. They were also asked again about how they found

the experience of the session and what they learnt from the exercise, and whether their opinion of the session had changed. There were questions about mistakes they had made in the last two weeks and how they had approached new problems, particularly ones they were not confident about, and for any differences. The students were also asked whether they thought the experience had affected the way they felt about making a mistake.

3. Results and discussion

There were mixed opinions about making mistakes, with some of the students in the focus groups being negative about it. They felt that *"if you get stuck with Maths, you get stuck"* and there is *"no sort of leeway"*. When talking about the exercise done in class that day, a student mentioned that they are *"always quite apprehensive"* about starting a question if they cannot see the end point. There were some themes that emerged in these focus groups that had appeared elsewhere (Cornock et al., submitted). For example, there was a willingness to try on a whiteboard; one student liked how they *"could have done it completely wrong and it's been rubbed off a board and no-one's going to know"*. There were also comments from some of the students, around the same themes in the other study, about how they do not want mistakes in their notes and there was concern about saying *"something stupid"* in front of people they do not know.

However, there was the acknowledgement that mistakes are going to happen and you can learn from them, especially within 'Abstract Algebra'. They said that they have to try different approaches and *"need to make mistakes"* in order to go down a different path and get an answer. One student said that they can compare work containing errors with correct work they produce.

3.1. Thoughts on the session

When asked their thoughts on the exercise they worked with during the 'Abstract Algebra' session, the students thought that it was *"well tailored to actually making a mistake"* and resulted in more errors being made than usual. They also acknowledged that it gave them practice of correcting the errors. One student said that they did not like being led towards making a mistake, whereas another described the session as a *"good change"* as they were actively engaged. Another found the questions *"challenging"* and got *"frustrated"* when they got stuck with the work.

The students said they discovered that making mistakes is an essential part of the learning process, specifically that they can learn from those mistakes to head *"in the right direction"*, although there were indications that they already thought they can learn from mistakes. Comments included that they see the importance of trying different approaches and how that this is better than not doing anything. Comments included that *"it doesn't matter if it's wrong"* and it allows them to *"rule something out"*. One student said that before the class they *"would immediately panic and try and go for it"*, but now they recognise that they do not need to panic. Another student felt that they still approached the work in the same way as before the session, which involved attempting the question to *"see what happens"*. The session seemed to make little difference to some students, but other students were specifically helped by seeing that an incorrect example would help them find one that would work.

There were concerns about having a similar question in an exam. In particular, one student was worried about whether they would keep trying the question. Another student acknowledged that there were in a better place for having done the session as they gained more awareness of what could be done and what to look out for. That student said that they would have more confidence to try a question as they now have a *"better understanding"*. The students said that it allowed them to reflect about what they could try and why they were taking the approaches. They acknowledged that it helped them with understanding, even if this did not lead to them getting a correct answer. This is a

large step away from students blindly following examples. The students *"think some people need to be more encouraged to get up and go for it"* as some students do not. They recognise that not all students will carry out the same reflection and so *"some people might have learned more from today than others"*.

There was an acknowledgement that the students would have liked more lecturer support in the session as they wanted a member of staff to correct their errors and said that they would start talking about something else without frequent input.

3.2. The group work aspect

The students thought that the group element helped with the exercise. One mentioned that *"if you were making a mistake by yourself, it's hard to actually get around it and start working on the actual process"* and how *"if you had someone to talk to...I think together you could probably get past most things"*. One of the students gave an example of working for 14 hours on an assignment question as it was individual work and hence they were unable to discuss it with others on the course. They said that they had just been *"sitting there and sitting there and sitting there"*.

There was a discussion in one of the focus groups about being with a familiar group. They admitted that *"it is very easy to sit back"* and have conversations about other things when they know everyone in their group and are more actively involved when working with people they do not know. However, there were concerns that they would *"say something stupid"* if they were with a group of people they did not know and therefore would be reluctant to say anything. They felt that the right balance would be a mixture of people they did and did not know in their group. The students suggested that this would give them enough motivation to contribute but alleviate some of their fears as they would have people there to defend them.

3.3. Concerns about not knowing if they were incorrect

Within the two initial focus groups, there was concern about not knowing whether their work was correct. One student said that *"I could think I've done really well today and actually did it all wrong and didn't realise"*. The students were concerned about using incorrect information from the session later in an assignment. One student pointed out that they *"tend to stick quite religiously to what we wrote down in the lectures"* and that they need correct answers to exercises.

There were some mixed comments about the lecturers not going through the answers afterwards. One student said that *"it's only safe to make mistakes if you're going to have the correct answers given to you at the end"* and another said that they would not be as reluctant to try if they knew they were going to be provided with the correct answers. However, there were some concerns that if they knew they would get the answers, then groups may not have been as motivated and would have waited for the answers. A theme that emerged was that there was a reliance on lecturers and a need for receiving correct answers from them.

There was an interesting comment made by a student about how they did not know the reasons for the errors in the session. They did not know whether it was because they were led to make the mistake or that they were making their own errors that were not intended. One student said that they would automatically think they had done something incorrectly rather than think it is an example that could not be done.

The students thought that there is *"more exploration"* in Mathematics at university, that there is a lot more choice within work and possibly lots of ways to get to an answer. Also, at university level, they do not necessarily know when they have made a mistake and can do a lot of work without realising

there is an error. The students also did not like how they may not be able to see a mistake that has been pointed out to them. One student said that this makes them feel *"a bit discouraged"*. A student said that they may get asked why they have done something in their working (for example, by a lecturer), but they do not necessarily know as they have been blindly following the book. There is a tendency to follow methods provided by lecturers without understanding, and there is dislike amongst students that this may lead to an error and they may not necessarily be aware of the mistake.

3.4. Reflection two weeks after the task

In the focus group two weeks after the task, the discussions suggested that there had been changes. Some students said they now use different resources; for example, one student said they now use a pencil and another pointed out how they started to use the individual whiteboards to show their thoughts to other people. The students suggested that a large change is that more people are working together. They are checking their answers with someone else, pointing out errors to each other and explaining things more. They noticed that the whole class is a lot louder, there is more interaction and *"it doesn't sound like people are just sat working by themselves"*. One student acknowledged that it is *"easier to work with people a little bit more"* as a consequence of the activity.

The students said that there is now more of a willingness to try, including when they are stuck. One student said they now just try anything. Another comment was that they try various approaches as they can rule them out if they are incorrect and that could lead on to another idea which results in a correct answer. They said that if ideas *"don't work, then you can just throw it away"* and *"it doesn't matter"*. The students felt that the session re-enforced how they *"will make mistakes"*, that is unlikely that they will be able to get the answer first time and they will have to try different attempts. They feel that they do not have to be concerned about making a mistake. They see making mistakes as progress as they are *"one step closer to finding the right answer"*.

The students found the class *"useful"* when starting preparation for the subsequent assignment, in which the students create their own examples in groups (see Cornock, 2021). They are now aware that when an example does not work, they can go back and reflect about the starting point. They would consider whether the example is adequate rather than just thinking that they had just made an error in their working.

Before the activity, the students generally viewed mistakes as negative, but could still see how they could learn from mistakes. The activity re-enforced this benefit and has resulted in a change of behaviour and attitudes within class. There is now more of a willingness to try and more discussions taking place.

4. Conclusions

Some students already had a very healthy view of making mistakes, but specific realisations following the error-eliciting exercise included that they could use an incorrect example to help them understand and use that to find one that would work. Rather than blindly following examples that they had already, the students are now questioning starting points and examples.

There was concern about making mistakes at university and not knowing they had made an error. Even at Level 6, there was a reliance on the lecturer, particularly in providing correct answers to the students. However, the group work element of the activity helped with their progress. After the session containing the error-eliciting task, students were talking to each other more and there was an increased willingness to try exercises. The activity will continue within the module. In the response to the comments made, the students will be provided with full solutions at the end of the class without advance warning that they will be available.

5. References

- Cornock, C., 2015. Teaching group theory using Rubik's cubes. *International Journal of Mathematical Education in Science and Technology* 46 (7).
- Cornock, C., 2021. Student-generated examples and group work in Mathematics, *MSOR Connections*, 19(1), pp 31-39.
- Cornock, C., Majin, W., Robinson, M. and Shukie, A. (submitted) Student use of whiteboards: experimentation, confidence and mistakes.
- Eggleton, P.J. and Moldavan, C.C., 2001. The value of mistakes *Mathematics Teaching in the Middle School*, Reston 7(1), pp. 42-47. Available at <https://search.proquest.com/docview/231069603?pq-origsite=gscholar&fromopenview=true> [Accessed 25 October 2020].
- Fischbein, E., 1994. The interaction between the formal, the algorithmic, and the intuitive components in a Mathematical activity. In R. Biehler, R. W. Scholz, R. Sträßler, B. Winkelmann, eds. *Didactics of Mathematics as a Scientific Discipline*, pp. 231-245. Available at https://www.researchgate.net/profile/Rudolf_Straesser/publication/227113904_Cultural_Framing_of_Teaching_and_Learning_Mathematics/links/0deec5231ab119d511000000.pdf#page=242 [Accessed 25 October 2020].
- Lawson, M.V., 2004. *Finite Automata*. Chapman & Hall / CRC.
- Lim, K.H., 2014. Error-Eliciting problems: fostering understanding and thinking, *Mathematics Teaching in the Middle School* 20(2), pp. 106-114. DOI: 10.5951/mathteachmidscho.20.2.0106.