RESEARCH ARTICLE

Changes made to the teaching of linear algebra and calculus courses in the UK in response to the COVID-19 pandemic

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Abstract

In response to the COVID-19 pandemic, university mathematics departments in the UK adapted their teaching for 2020-21, with some courses being delivered digitally and others through a mixture of on-campus and digital delivery. A survey of linear algebra and calculus lecturers was carried out in the spring of 2021 to investigate what changes were made to courses, as well as lecturers' perceptions of institutional decision making and support. This survey found that a majority of the 41 participants were satisfied that the choice of delivery mode was correct, although views about the importance of offering on-campus classes were mixed. There was a significant increase in the use of video clips made by the lecturer, video-conferencing software, discussion forums, electronic submission of written work and on-screen marking tools. Most lecturers reported a reduction in the amount of time that students were expected to be taking part in live teaching activities and an increase in the amount of time they were expected to be working on asynchronous activities. While some were keen to return to their previous practice, others were enthusiastic about retaining features introduced in response to the COVID-19 pandemic.

Keywords: university mathematics teaching, COVID-19.

1. Introduction

1.1. Background

The COVID-19 pandemic forced university mathematics departments to adapt at speed to new constraints on the way they teach. On 18 March 2020, the UK government, the Scottish government, the Welsh government and the Northern Ireland Executive announced that state schools would close to most pupils. Around the same time, university departments across the UK began to pause or cancel on-campus classes and introduce emergency remote teaching. On 23 March 2020 governments across the UK announced stay-at-home measures to slow the spread of COVID-19 that prevented on-campus teaching.

Measures were put in place to complete the 2019-20 academic year and universities began to plan teaching for 2020-21 under the assumption that on-campus teaching would be limited or impossible. Throughout the summer of 2020 discussions took place within university mathematics departments and across the academic community, facilitated by initiatives such as the newly-formed Teaching and Learning Mathematics Online (TALMO) conference, about how mathematics courses could be taught digitally in 2020-21. This article aims to record what actually happened in university mathematics courses, specifically calculus and linear algebra courses, in 2020-21 and how any changes made might influence undergraduate mathematics teaching in the future.

1.2. Terminology

Many new terms were introduced during 2020-21 and the following will be used here to avoid ambiguity.

- *On-campus* teaching occurs with all participants in a university building; we use this term rather than *face-to-face* because digital teaching could also legitimately be described as face-to-face if cameras are used.
- *Digital* teaching uses platforms that can be accessed no matter where the participants are physically located; we use this term rather than *online* to avoid confusion with entirely online courses or distance-learning programmes.
- *Live* teaching is conducted in real-time, synchronously; *asynchronous* teaching is not live.

1.3. Research questions

The following research questions were considered.

(RQ1) What modes of delivery were used in 2020-21 for undergraduate mathematics teaching in the UK and did lecturers feel these were appropriate?
(RQ2) Did those lecturing undergraduate mathematics courses in the UK feel that their institutions were prepared for digital delivery in 2020-21?
(RQ3) How did teaching tools used change as a result of the COVID-19 pandemic?
(RQ4) How did expectations made of students change as a result of the COVID-19 pandemic?
(RQ5) Do lecturers intend to keep any changes to teaching made as a result of the COVID-19 pandemic in the longer term?

2. Method

2.1. Design and administration of survey

An online survey was designed with a mixture of open-response, multiple-choice and Likert scale questions. Survey items were inspired by the study of Drijvers et al. (2021). Participation was sought only from those teaching first courses in calculus or linear algebra as these are the only two subjects named explicitly as being common to mathematics degrees in the QAA's Subject Benchmark Statement for Mathematics, Statistics and Operational Research (2019). By focussing on calculus and linear algebra the responses can be compared directly, which would be difficult when including a broader range of subjects. This restriction does skew the results to courses which typically have larger classes and are taught in the earlier years.

The survey was administered electronically using JISC Online Surveys and responses were invited using the authors' professional networks, the Heads of Departments of Mathematical Sciences (HoDoMS) mailing list and a call in an email announcement by Teaching and Learning Mathematics Online (TALMO). The study was conducted according to the University of Edinburgh ethics procedures. The survey questions are given in the Appendix. The anonymised data, along with notes and code used during analysis, are available at https://osf.io/6pujb.

2.2. Analysis of responses

In Question 7 of the survey, lecturers were asked to indicate, from a list, which digital tools they had used before the COVID-19 pandemic and which they had used in the 2020-21 academic year. The data for each tool were subjected to the mid-*p* version of the McNemar test. This particular version of the McNemar test was chosen over the more common asymptotic McNemar test, following Fagerland

et al. (2013), due to the modest sample size. To correct for the fact that multiple hypotheses were being tested simultaneously, and so rare occurrences were more likely to be observed in the results, the so-called multiple testing problem, the Holm-Bonferroni correction (Holm, 1979) was used to test for significance at the 5% level. The Holm-Bonferroni method was preferred to the more conservative Bonferroni correction (where an adjusted $\alpha = 0.05/15 \approx 0.003$ would have been used for each hypothesis test) to reduce the risk of not detecting a change (i.e., to reduce the risk of type II errors). However, for the collected data, the standard Bonferroni correction would have resulted in the same conclusion; that is, the same tools would have been identified as having seen a change in their use.

In Question 9, participants were invited to describe any other tools that had been used in 2020-21 and whether these tools had been used before the COVID-19 pandemic. On inspection of the responses to Question 9, some of the tools fell into the categories listed in Question 7. In each of these cases, the response given in Question 7 was checked for consistency with that given in Question 9; in one case, a response to Question 7 was modified accordingly. Notes were kept in the spreadsheet of survey results.

The open-response questions (5b, 8, 9, 11, 12 and 14-18) were analysed to identify common themes. Initially responses were coded according to the themes that they contained by one of the authors. All three authors then met together, looked through the themes and coding, and resolved disagreements by consensus.

3. Results

Between 22 March and 2 June 2021, 44 participants completed the survey. Question 2 was used to screen out three participants who had not taught linear algebra or calculus in 2020-21. The sample consists of 41 lecturers; 20 had been responsible for delivering a first course in linear algebra, 20 a first course in calculus, and one taught a course covering both subjects. Of these, 34 chose to identify their institution. There were participants from at least 21 distinct UK institutions; 20 in England and one in Scotland. Six of these institutions were members of the Russell Group. There were seven participants in the sample who did not identify themselves or their institution. For context, the Complete University Guide lists 72 departments in its UK Mathematics Subject League Table 2022, suggesting that the survey had a reasonably high response rate.

3.1. Modes of delivery (RQ1; survey Q3, Q4, Q5)

Figure 1. Summary of responses to Question 4: In 2020-21, what was the mode of delivery of your course?

Table 1: The number of courses delivered through on-campus and digital activities in different terms.

	A mixture of on campus and digital	Fully digital (with no on-campus activities)	Total
Before Christmas 2020	10	11	**21**
AAfter New Year 2021	1	9	**10**
Whole academic year 2020-21	6	4	**10**
Total	**17**	**24**	**41**

A small majority (24/41; 58.5%) reported no on-campus activities in 2020-21 (see Figure 1); the remainder reported teaching using a mixture of digital and on-campus activities. During October and November 2020 all areas of the UK introduced tiered systems designed to control the spread of COVID-19. On 19 December 2020 more restrictive measures were introduced in parts of England, in Scotland and in Wales. These measures constrained on-campus teaching and explain the marked shift in delivery mode after New Year 2021, with 9 of the 10 courses being fully digital (see Table 1). The responses to the question "Who made the decision about the mode of delivery?" are summarised in Figure 2. The two lecturers who responded "Other" both felt there was no decision to be made since government restrictions had forced them into digital delivery, and indeed these courses were both delivered after New Year 2021.

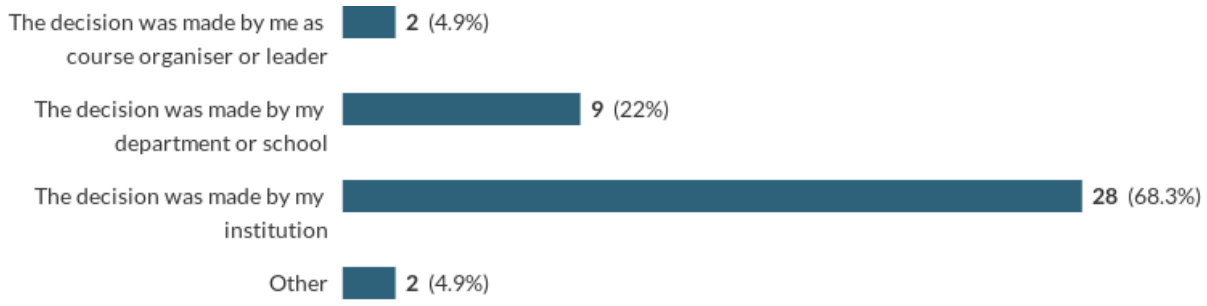

Figure 2. Responses to Question 5: Who made the decision about the mode of delivery?

Of those who expressed an opinion, there was near unanimity that the mode of delivery used had been the correct one given the circumstances. Some lecturers who had offered on-campus classes felt that these had been worthwhile, even under restrictions.

"I think it was correct, fully online would be worse, even the chance of having some in-situ examples classes were useful."

Several lecturers were concerned that students had lacked the opportunity for social interaction and that on-campus teaching activities would have been beneficial.

"The first year students would have benefitted more from more on-campus activity. The social cohesion of the students hasn't been great."

Others who had offered on-campus classes commented that these were poorly attended by students.

"Some element of in-person teaching was allowed, and reasonable to provide; however, it was hard to do, and attendance was very poor."

One lecturer commented that, in their experience, while some teachers preferred to give on-campus sessions, the physical-distancing restrictions impinged on the operation of classes so much that offering fully digital classes was preferable for learning. However, they did concede that students may have missed out on the social aspect of on-campus teaching.

"Some of the problem class leaders preferred the on-campus versions, but the group work we usually do in those sessions made the safety very difficult. And on campus sessions were cut much shorter due to cleaning time. So educationally and from a health perspective, (though perhaps not socially), I think it was better online."

Only two lecturers said explicitly that they felt the choice to have on-campus classes had been wrong. One cited the increased workload involved in delivering physically-distanced classes as the reason against offering on-campus activities; the other felt that having on-campus classes during a pandemic had put staff at risk.

Almost all lecturers who had taught fully digitally supported this decision, suggesting there was no other option (29/38; 76.3%). Three lecturers (3/38; 7.9%) raised concern about ensuring an equitable experience for students unable to travel to the campus. One felt students would have benefitted from face-to-face contact on campus while another suggested fully digital delivery provided a saving in workload.

3.2. Perceptions of institutional preparedness for digital delivery (RQ2; survey Q6)

A summary of the responses to Question 6 are shown in Figure 3. The vast majority (36/41; 87.8%) of lecturers agreed or strongly agreed with the statement "My institution as a whole has the technical infrastructure to deliver courses digitally", with only three participants disagreeing; two did not know or were neutral. Confidence in technical infrastructure at a departmental level was similarly high; 35/41 (85.4%) agreed or strongly agreed that "My department/school has the technical infrastructure to deliver courses digitally".

However, there was less confidence that institutions and departments had the expertise to provide excellent teaching digitally. The majority of those who expressed a view felt that their institution as a whole had the expertise to deliver excellent teaching digitally, but the most common response was "Neutral/Don't know" (14/41; 34.1%). More participants (25/41; 61.0%) agreed or strongly agreed that their department had such expertise, but still a substantial number said they were neutral or did not know (11/41; 26.8%). There are many possible reasons for these results: it may be that a shared sense of what constitutes excellent digital teaching had not developed in UK mathematics departments or that communication within departments during the pandemic had been difficult.

The majority of participants (32/41; 78.0%) agreed or strongly agreed with the statement "My department/school supports me to deliver courses digitally"; seven did not know or were neutral and two disagreed or strongly disagreed. Inspection of Figure 3 suggests that participants felt more supported by their departments than by their institutions; this is perhaps unsurprising. Another possible explanation is that centralised support provided by universities may not have addressed the specific needs of mathematics lecturers, such as how to share writing containing mathematical notation. Similarly, participants had more confidence that students were supported by their department, rather than their institution as a whole, although most respondents agreed or strongly agreed that students

had been supported to engage with digital activities.

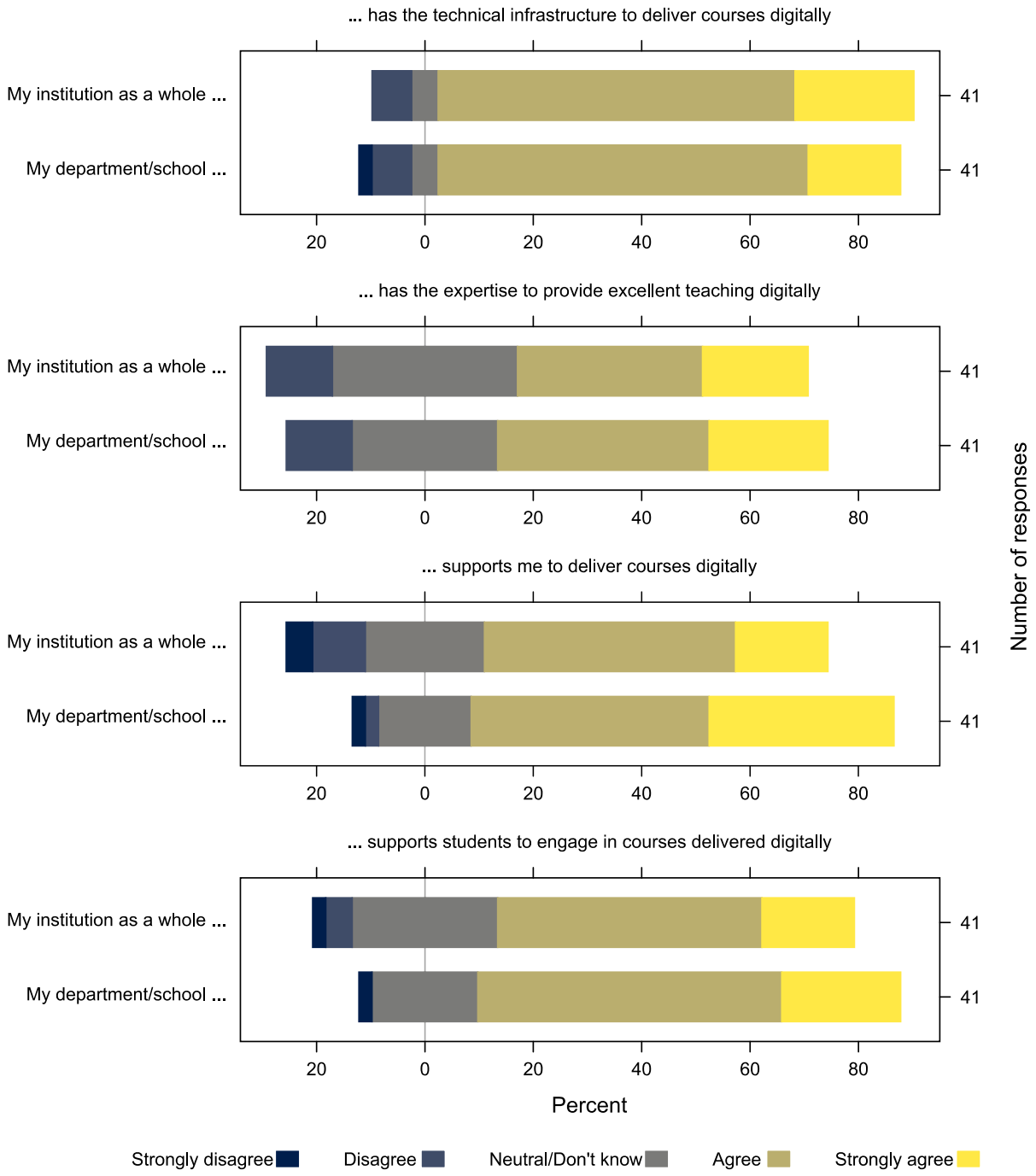

Figure 3. Summary of responses to Question 6: For each of the following statements, please choose the option which most closely aligns with your view.

3.3. Tools used in teaching (RQ3; survey Q7, Q8, Q9)

Responses to Question 7 of the survey are summarised in the alluvial plots in Figure 4. Each of these plots shows the proportion of participants using a given tool before COVID-19 and during 2020-21, and indicates those whose use changed in the interim. The results of the statistical analysis described in Section 2.2 are shown in Table 2.

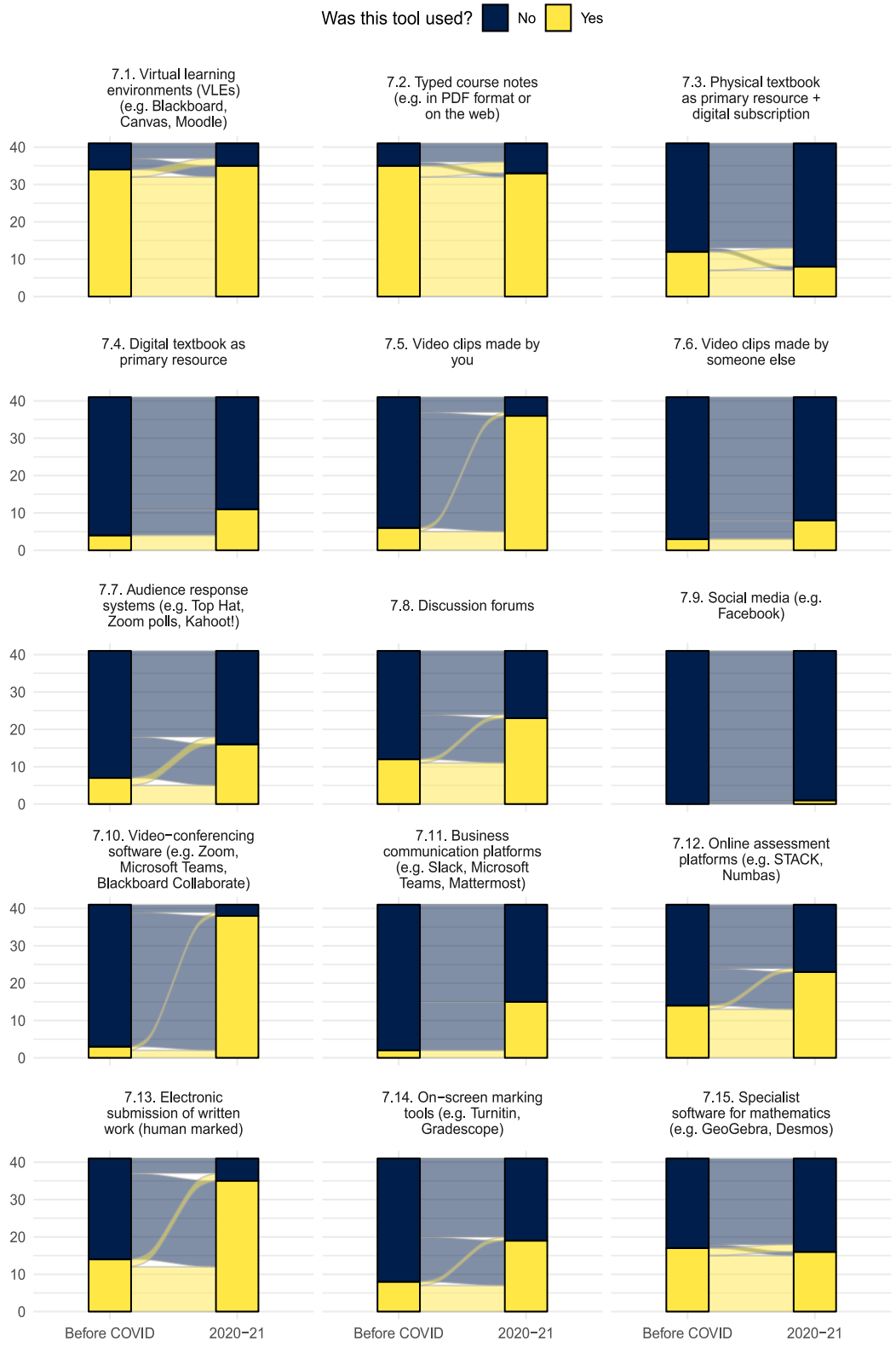

Figure 4. Summary of responses to Question 7: Thinking about how you would have delivered (or did deliver) the same course before the COVID pandemic, which digital tools would you have used (or did you use)? And which of these tools have you used to deliver the course in 2020-21?

Table 2: Results of the statistical analysis of responses to Question 7. Six tools saw a significant change in use in 2020-21 as compared to before the COVID-19 pandemic.

Tool	*p* value	Rank	Adjusted α	Significance
Video clips made by you	< 0.0001	1	0.003	sig
Video-conferencing software (e.g., Zoom, MS Teams, BB Collaborate)	< 0.0001	2	0.004	sig
Electronic submission of written work (human marked)	< 0.0001	3	0.004	sig
Business communication platforms (e.g., Slack, Microsoft Teams, Mattermost)	0.0001	4	0.004	sig
Discussion forums	0.0018	5	0.005	sig
On-screen marking tools (e.g., Turnitin, Gradescope)	0.0018	6	0.005	sig
Online assessment platforms (e.g., STACK, Numbas)	0.0063	7	0.006	non-sig
Digital textbook as primary resource	0.0078	8	0.006	non-sig
Audience response systems (e.g., Top Hat, Zoom polls, Kahoot!)	0.0129	9	0.007	non-sig
Video clips made by someone else	0.0312	10	0.008	non-sig
Physical textbook as primary resource + digital subscription	0.1250	11	0.010	non-sig
Typed course notes (e.g., in PDF format or on the web)	0.3750	12	0.013	non-sig
Social media (e.g., Facebook)	0.5000	13	0.017	non-sig
Specialist software for mathematics (e.g., GeoGebra, Desmos)	0.6250	14	0.025	non-sig
Virtual learning environments (VLEs) (e.g., Blackboard, Canvas, Moodle)	0.6875	15	0.050	non-sig

Six tools saw an increase in their use, significant at the 5% level. These were: (i) video clips made by the lecturer, (ii) video-conferencing software, (iii) business communication platforms, (iv) discussion forums, (v) electronic submission of written work (human marked) and (vi) on-screen marking tools. None of these results are surprising given the shift towards greater digital delivery, but these results do provide a record of changes that have occurred within this sample of mathematics departments across the UK.

Seven of the tools were used by more than half of respondents in their teaching in 2020-21: (i) virtual learning environments (VLEs), (ii) typed course notes, (iii) discussion forums*, (iv) video clips made by the lecturer*, (v) video conferencing software*, (vi) online assessment platforms, and (vii) electronic submission of work (human marked)*. Taken together, these appear to make up a common toolkit used to teach mathematics digitally. The four tools marked * also saw their use increase from pre-COVID times, according to the analysis in Table 2, suggesting that there was a major change in the tools used to teach first-year undergraduate mathematics.

In Question 8 participants were invited to describe any other tools that they used before the COVID-19 pandemic that they did not use in 2020-21. There were responses from 20 of the 41 participants. All respondents described tools or activities associated with traditional on-campus mathematics classrooms such as “Live in person lectures!”, “Chalk, markers and boards”, “A proper blackboard” and “Physical whiteboard and marker pens. Overhead projector”.

In Question 9 participants were asked to describe any other tools that they had used in 2020-21 and whether they had used them before the COVID-19 pandemic. There were two tools in the responses that were mentioned by more than one participant, namely: (i) writing tablets, mentioned by five participants; and (ii) online collaborative whiteboards (those named were Miro and OneNote), mentioned by two participants.

3.4. Expectations made of students (RQ4; survey Q10, Q11, Q12, Q13, Q14)

In Question 10 of the survey, participants were asked how much time students had been expected to participate in live teaching activities during 2020-21 compared with before the COVID-19 pandemic. In Question 11, they were asked to give reasons. Responses to Question 10 are summarised in Figure 5. A large majority of respondents (30/41; 73.2%) said that students had been expected to spend less time taking part in live activities than in previous years; only four (9.8%) said students were expected to spend more time, and seven just the same as before (17.1%).

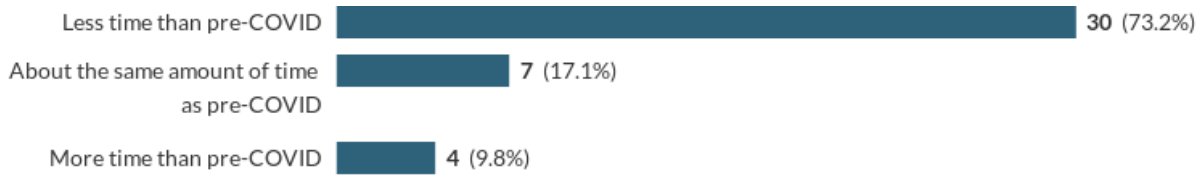

Figure 5. Responses to Question 10: In a typical week of your course in 2020-21, how much time did you intend for students to spend taking part in live activities compared with what you would have intended pre-COVID?

Continuity with previous years was mentioned by all five of those who gave reasons for expecting the same amount of time on live activities as before the pandemic. One lecturer suggested two arguments in favour of running sessions live, even if done so digitally: to cater for student preference and to avoid the workload associated with preparing video clips.

“[The amount of time is the same] Because I've continued to do essentially everything live. The students seem to prefer it like this, and saves me time as I don't need to spend lots of time preparing recordings.”

Of those who said that expected time participating in live sessions had increased, a common reason was the introduction of additional support sessions for students.

Among those who expected less live participation from students, the vast majority (27/29; 93.1%) referred to a reduction in the use of live lectures. Most (22/29; 75.8%) said that these had been replaced by pre-recorded video clips. The Teaching and Learning Mathematics Online (TALMO) meetings were mentioned by one participant as having influenced choices about how to run courses.

“We decided, given lots of advice (including TALMO), that a mix of live lectures and some pre-recorded videos would work best.”

Some of the participants (4/29; 13.8%) had concerns about the access to and the reliability of digital technology for giving live digital classes. There were concerns about students not having adequate software, hardware, a suitable working environment, or internet connection to participate in live digital classes. There were also concerns about reliability of software at the lecturer's end.

When asked, in Question 12, if the nature of live activities had changed compared to before the COVID-19 pandemic, all respondents suggested that changes had occurred. A sizeable number of participants suggested they had reduced the use of traditional lectures in place of other activities. Most common was replacing lectures with video clips and introducing office hours, in-class polling for more interactive sessions, Q&A sessions or review lectures.

Just under one third of respondents (13/41; 31.7%) said they felt they had less interaction with students during live classes. Reasons given included the apparent reluctance of students to appear on camera and speak out in digital classes, as well as difficulties seeing the reactions of the audience. More than a third of respondents (15/41; 36.6%) reported an increase in interaction with students during live activities, though sometimes through non-traditional means. Those who had designed live classes to encourage participation from students (for example through the use of a flipped classroom, in-class polling or Q&A sessions) seemed to report an increase in interaction.

"I did flipped classroom for the first time, encouraged by the Covid online situation. So the live sessions were entirely different to what I used to do before. Much much more interactive, with the lecturing of new material done in pre-recorded videos instead."

Several participants (5/41; 12.2%) mentioned that students seemed to prefer to make contributions using the text chat instead of speaking out during live classes. A few felt that this was a disadvantage.

"live seminars were less interactive due to students not switching on cameras and using time-consuming chat instead of speaking up"

Others saw the increased student participation as an advantage.

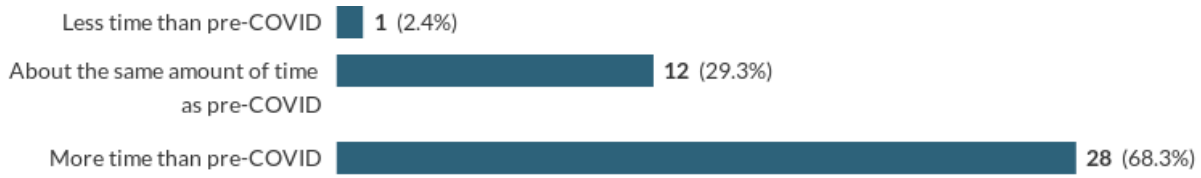

Figure 6. Responses to Question 13: In a typical week of your course in 2020-21, how much time did you intend for students to spend working on asynchronous activities compared with what you would have intended pre-COVID?

The responses to Question 13 are summarised in Figure 6. A large majority of respondents (28/41; 68.3%) said that students had been expected to spend more time taking part in asynchronous activities than in previous years; only four (2.4%) said students were expected to spend less time, and (29.3%) just the same as before. These results broadly mirror the changes in expectations of student participation in live activities discussed above.

In Question 14, participants were asked if students had acted as the lecturer had intended when designing the course, and on what evidence this judgement was based. More than a third (13/37; 35.1%) of the respondents said they thought students had broadly acted as intended, around a tenth

(4/37; 10.8%) suggested students had not acted as intended, around one fifth (8/39; 21.6%) said there had been mixed behaviour, and the same number (8/39; 21.6%) indicated that they did not know how students taking their course had behaved. Among the respondents who described how they had gauged student activity, most common was the use of some form of online activity tracking (14/37; 37.8%), for example the number of times a video clip had been viewed or reports from virtual learning environments. The next most common measure of how students had behaved was their engagement in live classes, which may have been digital or on campus (7/37; 18.9%). These respondents largely described a mixture of behaviour.

"I think the engaged students did work well and interacted with the material as intended. But there was a huge drop-off of engagement in the second semester […] there was reasonable engagement in the first semester (higher lecture attendance than in previous year for longer), but much lower in the second semester (much less than in previous years)."

A couple of respondents reported that students had not been keen to speak out in live digital classes. Several responses (5/37; 13.5%) mentioned that student attainment in coursework had been in line with expectations.

"The performance in assessments so far seem as good as in previous years, but we need to wait and see how the final exam goes."

Three of the responses (8.1%) suggested that lecturers felt some students had struggled with motivation and keeping pace with the course, as well as missing out on social interaction.

"There has been a bifurcation, with around half the students fully engaging as intended, but another half struggling to maintain work levels without the discipline of physical attendance and interaction with coursemates."

3.5. Looking to the future (RQ5; Q15, Q16, Q17, Q18)

In Question 15 of the survey, participants were asked to think about the digital tools they had used in 2020-21, and whether they intended to keep any changes to teaching made as a result of the COVID-19 pandemic. Only 4/40 lecturers suggested that they would like to return to the tools used before the COVID-19 pandemic while the rest expressed an intention to retain at least some of the changes that they had made for 2020-21, notably video clips (17/40; 42.5%). Five of these suggested that recorded lectures would be helpful resources for students.

"I may use the recorded video lectures as additional revision materials in the future. Students always appreciate having more help with their revision."

One suggested keeping the video recordings to help cater for students with varying prior attainment.

"Use of video material. There are parts of the course where students have varied background. Students who haven't seen the material at A-level can access the material at their own pace."

Of those who responded, 14 (35%) suggested that they would like to keep online assessment platforms. The reasons given included allowing students to practice with immediate feedback and preventing cheating on tests through question randomisation.

“I am planning to continue using STACK tests, since they worked very well, and are good both as a form of formative assessment and for providing immediate feedback. Finally, they make copying work from other students nearly impossible (although other forms of cheating may still be taking place).”

Nine lecturers (22.5%) said they would consider some form of flipped classroom, continue to use in-class polling or expressed a desire to have more interactive live sessions.

“I think polling is good because it gives the shy students the opportunity to still engage anonymously.”

When asked to think about the mix of live and asynchronous activities (Question 16) half (18/36) suggested that they would like to go back to delivering courses largely as they had done before the pandemic, but with some indicating that they would like to retain video clips for use as an additional resource.

“I prefer to go back to the balance of classes that we had before the pandemic. Many students benefit from the face-to-face interaction. But I would like to make the recordings of lectures available for students that can't attend or need to watch twice.”

Most of the remaining respondents (11/36; 30.6%) said that they would consider running their courses with a different mix of live and asynchronous activities than they would have traditionally. Reasons for this included giving students more time to learn at their own pace and having live classes focus on discussion or addressing students’ difficulties.

“Yes. I think that the asynchronous delivery of more straightforward parts of the course was really helpful to allow students to go at their own pace. I would not return to the way that the course was delivered pre-COVID, but instead introduce some live delivery of the more tricky concepts.”

In Question 17 participants were asked to suggest three things they had changed that they felt had helped students to learn and that they would recommend to colleagues for the longer term. Of the 35 responses, 12 (34.3%) mentioned online assessment, 6 (17.1%) suggested the importance of clear messaging about what is expected of students, and when. Five respondents suggested recording live sessions so that they could be accessed by “students that can't attend or need to watch twice”. Four recommended using in-class polling to encourage active participation in live sessions.

Lastly, participants were asked for any recommendations of innovations that should be avoided (Question 18). Eighteen participants gave no response and the most common answer, given by 8/23 (34.8%), was that there was nothing they would recommend avoiding. Three participants recommended avoiding the use of breakout rooms in digital classes. All of these mentioned students’ dislike of or lack of participation in discussions. However, this experience was not universal; other respondents elsewhere in the survey commented that they felt digital tutorials had been successful “[s]mall group synchronous maths tutorials worked reasonably well by electronic contact” and “[t]hey really enjoyed doing groupwork”. Two responses recommended avoiding take-home assessments, with one participant commenting on the difficulty of ensuring academic integrity. However, the same response did suggest that open-book exams were new to the students; this may partly explain why this style of assessment had not worked well in this case.

4. Discussion and conclusions

4.1. Answers to research questions

(RQ1) What modes of delivery were used in 2020-21 for undergraduate mathematics teaching in the UK and did lecturers feel these were appropriate?

During 2020-21 more than half of courses in calculus and linear algebra were taught digitally with no on-campus activities; the remainder used a mixture of digital and on-campus delivery. On-campus activities were more common before New Year 2021 when COVID-19 restrictions were looser. Almost all respondents felt that their mode of delivery had been correct given the pandemic. Some of those who had offered on-campus teaching felt that it gave students valuable opportunities for social interaction, though it was reported that such activities had been poorly attended. Some institutions who had intended to have on-campus activities were forced to switch to digital delivery as a result of lockdowns and low student attendance. A small minority felt that offering on-campus classes at all during the pandemic had been the wrong decision. The responses suggest that fully-digital delivery was easier to implement than offering a mix of on-campus and digital teaching, and that doing so helped to make students' experiences more uniform.

(RQ2) Did those lecturing undergraduate mathematics courses in the UK feel that their institutions were prepared for digital delivery in 2020-21?

The vast majority of participants felt that both their institution as a whole and their department had the necessary technical infrastructure to deliver teaching digitally but only around half were confident that these also had the correct expertise to provide excellent teaching digitally. Perhaps more work is needed to develop a shared sense of what it means to provide excellent digital teaching in mathematics. The vast majority of participants felt supported to deliver courses digitally by their institutions as a whole and by their departments. Even more felt that their students had been supported to engage in digital activities, though it should be noted that the views of students were not sought in this research.

(RQ3) How did teaching tools used change as a result of the COVID-19 pandemic?

There was a change in the tools commonly used by lecturers to teach calculus and linear algebra courses. Seven tools were used by majorities of respondents in their teaching in 2020-21: (i) virtual learning environments (VLEs), (ii) typed course notes, (iii) discussion forums*, (iv) video clips made by the lecturer*, (v) video conferencing software*, (vi) online assessment platforms, and (vii) electronic submission of work (human marked)*. Four of these tools (marked *) saw increases in use in 2020-21 compared with before the COVID-19 pandemic, significant at the 5% level. A further two (on-screen marking tools and business communication platforms) saw increases in their use since before the COVID-19 pandemic, even though they had not been used by a majority of respondents during 2020-21.

These findings expand on work done to understand changes made during the period of emergency remote teaching (ERT) at the end of the 2019-20 academic year. For example, Ní Fhloinn & Fitzmaurice (2021) found large increases in the use of digital hardware and software for teaching. In particular, they report that video-conferencing software was used by almost all lecturers in their study. Unsurprisingly, this practice appears to have continued into 2020-21. Ní Fhloinn & Fitzmaurice found that only a very small number of lecturers had used pre-recorded video clips prior to ERT; our results similarly show that the use of video clips increased significantly during 2020-21. However, Ní Fhloinn

& Fitzmaurice found that delivering live digital sessions was the most common approach during the period of ERT; by contrast, we found that lecturers typically changed their approaches to introduce more asynchronous activity in place of some live digital classes for the 2020-21 academic year (see also (RQ4) below).

(RQ4) How did expectations made of students change as a result of the COVID-19 pandemic?

There was typically a decrease in the amount of time students were expected to participate in live teaching activities, with an increase in the amount of time students were expected to spend working on activities asynchronously. Almost all of those who expected less live participation from students had replaced live lectures with pre-recorded video clips to be studied asynchronously. Reasons given for this decision included concerns about the reliability of the technology needed to provide live digital classes and concerns that students may not have been able to access live sessions digitally.

Lecturers whose courses still had live classes (whether online or on campus) reported changes in the nature of those classes: more than a third said that they felt classes were more interactive than in the past, while around a third said they felt they had less interaction with students. It appears that lecturers who designed their classes with digital interaction in mind, and made full use of the tools available, saw more interaction than those who attempted to translate traditional lectures to a digital setting. Lecturers felt that students seemed reluctant to use video and contribute orally but were more willing to use text chat to interact during activities.

Lecturers gauged student participation using online activity tracking, their perception of student engagement during live classes, and student performance in coursework. Just over a third of lecturers felt that students had engaged with asynchronous material as intended and around a fifth said that they had seen a mixed response. Around a fifth of lecturers responded that they did not know how students had acted on their course. A few lecturers felt that, as well as missing out on social interaction, some students had struggled with motivation to study and keeping pace with the course.

(RQ5) Do lecturers intend to keep any changes to teaching made as a result of the COVID-19 pandemic in the longer term?

Around 90% of lecturers suggested they might keep some of the changes made in response to the pandemic; around 10% suggested they would like to return to their pre-pandemic teaching methods. More than a third of lecturers said they would keep using recorded video clips, either as an additional resource or to replace traditional on-campus lectures and free up class time for different activities. Just over a third said they would like to keep using online assessment platforms due to them providing students with opportunities to practice, the ability to give immediate feedback, and the ability to randomise questions. Around a fifth of lecturers said they would like to use some form of flipped classroom, in-class polling or expressed a desire to have more interactive classes than traditionally.

Half of the lecturers suggested they would like to return to their traditional mix of live and asynchronous activities in the longer term. Around a third said they would consider a different mix after their experiences in 2020-21, perhaps having students read or watch video clips asynchronously ahead of more interactive live on-campus classes involving polling.

When asked to choose three practices to recommend to colleagues for the future, over a third of lecturers again suggested keeping online assessment platforms. A sixth said that they would recommend clearly communicating expectations to students, perhaps using a to-do list or a week-by-week schedule.

Over a third of lecturers said explicitly that there was nothing they had tried that they would recommend colleagues avoid in the future. Two suggested avoiding take-home examinations in mathematics, with one mentioning concerns about cheating.

4.2. Concluding remarks

The intention of this research was to collect evidence of mathematics teaching practices in UK higher education during 2020-21 and to record the views of lecturers about their practice in the future. Lecturers responded from at least 21 distinct UK institutions, around 30% of all institutions offering BSc Mathematics. While this is a high response rate, it must be acknowledged that there was no response from most UK mathematics departments and so the results should be understood in that context.

The study only considered the responses of those who had taught calculus or linear algebra courses in the UK: one might wonder if the picture would be different for other courses and in other countries. No doubt further research will consider the wider picture.

7. Appendix

The survey questions are given below.

Page 2: Context and decision making in your institution

2. Which course or module were you responsible for delivering in 2020-21?
    - A first course in linear algebra
    - A first course in calculus
    - Other
3. When did your course take place?
    - Semester 1, 2020-21 (i.e. before Christmas 2020)
    - Semester 2, 2020-21 (i.e. after New Year 2021)
    - This course covers the full academic year 2020-21
    - Other

Mode of delivery

These next two questions are about the mode of the delivery of the course, by which we mean either fully digital (with no on-campus activities), fully on campus or a mixture of the two.

4. In 2020-21, what was the mode of delivery of your course?
    - Fully on campus
    - Fully digital (with no on-campus activities)
    - A mixture of on campus and digital

5a. Who made the decision about the mode of delivery?

- The decision was made by me as course organiser or leader
- The decision was made by my department or school
- The decision was made by my institution
- Other

5b. With the benefit of hindsight, do you think the choice of mode of delivery was the correct one? Why?

Page 3: Readiness and support for digital teaching

6. For each of the following statements, please choose the option which most closely aligns with your view.

   Options: Strongly disagree, Disagree, Neutral/Don't know, Agree, Strongly agree

6.1 My institution as a whole has the technical infrastructure to deliver courses digitally.
6.2 My department/school has the technical infrastructure to deliver courses digitally.
6.3 My institution as a whole has the expertise to provide excellent teaching digitally.
6.4 My department/school has the expertise to provide excellent teaching digitally.
6.5 My institution as a whole supports me to deliver courses digitally.
6.6 My department/school supports me to deliver courses digitally.
6.7 My institution as a whole supports students to engage in courses delivered digitally.
6.8 My department/school supports students to engage in courses delivered digitally.

Page 4: Tools

7. Thinking about how you would have delivered (or did deliver) the same course before the COVID pandemic, which digital tools would you have used (or did you use)? And which of these tools have you used to deliver the course in 2020-21?

   Options: Before COVID, 2020-21

7.1 Virtual learning environments (VLEs) (e.g. Blackboard, Canvas, Moodle)
7.2 Typed course notes (e.g. in PDF format or on the web)
7.3 Physical textbook as primary resource + digital subscription
7.4 Digital textbook as primary resource
7.5 Video clips made by you
7.6 Video clips made by someone else
7.7 Audience response systems (e.g. Top Hat, Zoom polls, Kahoot!)
7.8 Discussion forums
7.9 Social media (e.g. Facebook)
7.10 Video-conferencing software (e.g. Zoom, Microsoft Teams, Blackboard Collaborate)
7.11 Business communication platforms (e.g. Slack, Microsoft Teams, Mattermost)
7.12 Online assessment platforms (e.g. STACK, Numbas)
7.13 Electronic submission of written work (human marked)
7.14 On-screen marking tools (e.g. Turnitin, Gradescope)
7.15 Specialist software for mathematics (e.g. GeoGebra, Desmos)

8. Please describe any other tools that you would have used to deliver this course before the COVID pandemic but that you did not use in 2020-21.

9. Please describe any other tools that you used in 2020-21 and specify whether you had used them before the COVID pandemic.

Page 5: Live and asynchronous

Live activities

10. In a typical week of your course in 2020-21, how much time did you intend for students to spend taking part in live activities compared with what you would have intended pre-COVID?

    - Less time than pre-COVID
    - About the same amount of time as pre-COVID
    - More time than pre-COVID

11. Why is this the case?

12. In 2020-21, did the nature of your live activities change compared to pre-COVID? For example, if you gave live lectures were these more or less interactive than in the past?

Asynchronous activities

13. In a typical week of your course in 2020-21, how much time did you intend for students to spend working on asynchronous activities compared with what you would have intended pre-COVID?
    - Less time than pre-COVID
    - About the same amount of time as pre-COVID
    - More time than pre-COVID

14. Did students act as you had intended? On what evidence are you basing this judgement?

Page 6: Looking to the future

15. Thinking again about the digital tools you have used in your course delivery in 2020-21, are there any that you would want to keep for the longer term once the COVID pandemic is over? Why?

16. Thinking about the balance of live and asynchronous activities in your course in 2020-21, would you now consider a different live/asynchronous balance when delivering the same course once the COVID pandemic is over? Why?

17. Can you suggest three things which you changed that you believe were effective in helping students learn, and which you would recommend to colleagues for the longer term?

18. Is there anything which you changed that you would recommend colleagues avoid?

8. References

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