CASE STUDY

Writing mathematics collaboratively in online workshops

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Abstract

This case study examines the changes that were made to workshops for first year mathematics students when moving from in-person to online in the 2020/21 academic year. In the workshops, students tackle unfamiliar problems in small groups, with a focus on group work and mathematical communication skills. Transitioning to online workshops presented several difficulties around how best to enable students to have meaningful mathematical discussions and collaborate in writing their solutions when working online. We discuss the changes and mitigations we implemented in order to move the workshops online and how this will inform future in-person workshops.

Keywords: Mathematical writing, group work, study skills, online learning.

1. Introduction

Mathematical Investigations is a first-year unit at the University of Bristol which is compulsory for single-honours mathematics students. The unit focuses on developing teamwork and study skills through a combination of weekly two-hour workshops alongside multi-week group projects. In the workshops, students work in groups of around four on worksheets which lead them through a problem in steps. The topics are chosen to reinforce concepts from other units that they have typically found challenging. Emphasis is placed on communication skills: discussing the problems and learning how to write mathematics well. We discuss the goals and structure of the workshops in Gunns, et al. (2020). At the end of each workshop each group submits a collaborative solution which is marked for feedback. Students must attend and engage with the workshops but the work is formative only; the grade for the unit comes from the project component.

2. Aims of the workshops

2.1. Mathematical writing

Precise mathematical writing, the ability to clearly articulate complex ideas and an ability to work in teams are important competencies for UK mathematics graduates (QAA, 2019). Writing mathematics well is frequently challenging for students; an overview can be found in Schleppegrell (2007). Part of the difficulty is that it is not necessarily clear to students what is meant by "good" writing. In Lew & Mejía-Ramos (2019) fourteen common types of errors in undergraduate students' mathematical writing are identified and a following study (Lew & Mejía-Ramos, 2020) finds differences of opinion among mathematics teachers as to which of these are reasonable expectations of students writing. Common types of improvements we focused on in workshops included having students write grammatically in full sentences; that they ensure notation was defined when introduced and used correctly; and that equations were connected with an explanation of how to get from one step to the next, where all assumptions were clearly stated. These correspond to the three main themes identified in Lew & Mejía-Ramos (2019), and were also informed by tutors' experiences of marking student work.

2.2. Group work

The workshops were designed as a group task. In part this was to help students build teamwork and communication skills, with students working together both to solve the mathematical problems and to decide how to clearly express their arguments in writing. Group work has well established benefits including allowing students to become more actively engaged in learning, share ideas and build supportive learning communities (e.g., Healey, et al., 1996). For mathematics, the introduction of communication skills can aid students in the development of mathematical problem-solving skills (Taylor & McDonald, 2007). An aspect of this is that students' roles in their learning are shifted, as summarised in Nilson (2016), to include students as active participants in problem solving and as sources of feedback for their peers. The collaborative nature of the workshops allows these benefits to be realised in this unit as the collaborative writing process allows students to check that their arguments are understood by peers as they go. Group work also permits students to be able to tackle more challenging tasks that are difficult to do alone. We believe that learning to write mathematics well is the type of challenging task that becomes more approachable in a group setting.

3. Moving the workshops online

When planning for the 2020/21 academic year during summer 2020, it was decided that the Mathematical Investigations workshops would be held online. The requirements on social distancing and mask wearing meant that group work would have been difficult, verging on impossible, in person, and so it was necessary to explore options for online platforms and formats that would best allow us to transfer the workshop experience online.

The workshop team began by considering the key functions that online platforms would need to be able to effectively host workshops. These included breakout room functionality, so that students could work together in small groups; the ability for an instructor to easily move between breakout rooms to check on and help students; the ability for students to signal to the instructor if they required help; screenshare functionality that could be annotated by all students in the room to aid discussion of the worksheet; the ability to save annotations to revisit and submit work; and considerations of the user-friendliness of the platform. After testing platforms including Zoom, Bluejeans, Blackboard Collaborate and Microsoft Teams, it was decided that Zoom was the best option for satisfying these requirements. While testing the various platforms, members of the workshop team also accessed test sessions using a range of different devices.

While testing the different platforms, it became clear that moving the workshops online was introducing a conflict between two of the key goals: working collaboratively and writing well. In person, these goals were often aligned: by working in groups students would have to communicate their arguments to their teammates clearly, and when writing their solutions could discuss the wording and get feedback from their group easily. Online, this became much more difficult. The most effective way for students to write their solutions clearly would be for them to write individually using pen and paper, then scan and upload their solutions. However, this would make it difficult for students to discuss the wording of their solutions, as it would be time-consuming to upload and share their solutions with other students, especially if multiple iterations were required. Additionally, if students were not working on writing their solution together it seemed likely that many would not discuss their work either, especially as they would not necessarily know the other students in the breakout room. On the other hand, prioritising collaboration and requiring students to use online annotations would encourage discussion and allow all students to contribute more easily to each question, but using annotations is more difficult than writing on paper and so adds an additional barrier to the writing process. This might lead to students taking shortcuts or including fewer details because of the additional difficulty, or to unequal contributions depending on different equipment. After weighing up these contrasting options, we ultimately decided to prioritise the collaborative aspect of the workshop and encourage students to use

the annotation function to submit their work. This was because there were few opportunities for students to talk to each other in general, so this was a good opportunity for them to both learn to discuss mathematics and to get to know each other.

4. Changes made to online workshops

In order to effectively move the workshops to an online setting, we implemented a number of changes, some of which have led to longer-term changes that will be carried forward to future in-person workshops.

4.1. Structural changes

The initial change that was made to all workshops was a practical one to make the worksheets easier to use in the online setting. To facilitate the process of using annotations, we altered the format of the worksheets provided to students. Instead of a question document to be answered on separate paper, we created slides from each worksheet containing one question per slide, and lots of white space for the students to annotate. Each group then screenshared so they had a shared workspace to discuss and answer the question. Students could then take a screenshot of each slide, and then submit these to be marked. This avoided the need to keep switching between a whiteboard and the question document. Notation was altered in several workshops (for example, changing Greek characters to Roman characters where possible) to reduce the amount of drawn (rather than typed) input required, and some questions were altered to make annotated answers easier. In workshops where students were asked to read proofs, we ensured that the entire proof could be displayed on one slide. This helped students focus on seeing the big picture and write notes about the proof without needing to delete the notes to change slide. We repeated previously calculated answers and pictures from previous slides so the students didn't have to make a note of all their answers outside their submission. These changes, though useful in the online setting, were no longer needed when the workshops returned to taking place in-person.

4.2. Induction activity

To help students get used to the online annotation tools and Blackboard submission process, we replaced the first workshop with an induction session. This consisted of several short activities designed to introduce students to the various Zoom tools and also to promote some initial discussions. For example, one activity asked students to consider some common misconceptions about mathematics and annotate a scale on the screen to indicate how much they agreed with the statement. These misconceptions were based on a list from Alcock et al. (2015), building on the work of Schoenfeld (1992). Another activity separated students into breakout rooms and asked them to produce some simple annotations, screenshot these, and practice uploading them. A short guide about using Zoom (including how to annotate, screenshare, save screens and use the 'raise hand' function to get the instructor's attention) and how to submit work on Blackboard was also provided to students.

The induction activity provided a good opportunity to provide a clearer introduction to the workshops, stimulate some discussions regarding students' expectations of university mathematics, and outline our expectations of good mathematical writing in advance, as well as getting students to practice the technical skills they would need. We have retained the discussion-based elements of this activity now that in-person workshops have resumed.

4.3. Different question types

Even with the changes to the worksheet structures discussed above, it was clear that some questions would not translate well to the online setting, as it was likely that students would have to spend too

much time thinking about how to deal with complicated notation online rather than focusing on the content of their answers. To deal with this, we implemented new types of question.

Some questions which involved long but standard calculations we changed to 'fill in the blank' style questions, where we gave students a large part of a sentence or calculation, but asked them to fill in the justification. By giving them the framework and asking for the details, the students still had to work through the problem, but not write down all the awkward notation. This also allowed us to model the format of solutions we were looking for, by indicating where we would expect students to provide links and justifications.

For some questions where we had previously asked students to prove a result but where notation may have been difficult to write online, we instead provided examples of poorly constructed proofs, and asked students to identify the problems. As discussed in Selden & Selden (2003), students often struggle to determine the validity of a proof. The examples we gave students contained errors that we had commonly seen students make in previous years' workshops. By discussing these errors, we hope to more easily address the misconceptions and help students to improve their proof validation skills, and these questions can be used at the end of the session for a whole class conversation about what 'good writing' means.

4.4. Markschemes

Another change was to introduce marking criteria for each week in the second term. Online, it was harder to informally discuss and direct students than it had been in person, so it became more important to clarify the goals of each workshop. These generally involved 4 elements, each marked out of 2, with an aim to reinforce to students the writing aims for that week. They also served as reminders that students should continue to do certain things, e.g. "All answers are written in complete sentences." was a criterion used every week. At least one criterion usually focused on the mathematical content for that week e.g., "Demonstrates understanding of conservation laws". These marking criteria provided a simple framework for students to check that they had completed the key aims for each week, reminded them of the writing focus for that week, and allowed for more focused feedback. These marks did not contribute towards the assessment for the unit, and if full marks were not awarded for these criteria, then some comment on the script would be made by the tutor. The role of assessment on student learning has had long-standing interest (e.g., Gibbs & Simpson, 2004). Here they highlight conditions which assessment should aim to achieve, such as "Feedback is acted upon by the student" (this is their Condition 10). Our marking criteria seemed to achieve this, by focusing students on the aims for that week. Relevant for the pandemic, Gibbs & Simpson (2004) also say that "[students] can cope without much, or even any, face-to-face teaching, but they cannot cope without regular feedback on assignments." This year we have applied marking criteria to every assignment, carefully considering how these requirements build on those from the previous weeks.

4.5. Online submissions

The way that students received their feedback after online sessions also changed. Originally, submissions were handed in and marked on paper and returned to the students in the next workshop. Within this setting, students could see their feedback immediately before starting their next assignment, and hopefully this encouraged them to act on their feedback. However, if they missed the session, were late, or otherwise did not get a chance to see the comments on their written work, then this could lead to them missing their feedback. Submissions were made as a group in 2020/21, as in previous years, but the submission was done online through Blackboard. Students were then notified of their feedback and score by emailing all students in each group and including the feedback for their work either through annotations to their electronic submission or by typing it into the email. This had the advantage that students could get their work back soon after the session rather than having to wait

until the next week. This has the benefit that the feedback is more timely (see Condition 6 of Gibbs & Simpson, 2004). It also meant that there were no issues with legibility of comments, and they could be done in relation to the specific marking criteria (which were discussed in the previous paragraph). Of course, in the online setting, it was unclear whether students even read the comments! Student submissions did appear to satisfy the criteria relating to earlier assignments, but whether this was due to the feedback would be difficult to judge accurately. Sending the comments by email could be seen as a way to start a direct dialogue with a student, since replies or clarifications could be asked for, but no instances of this occurred in 2020/21. Use of discussion boards where the groups and the tutor could comment on work might be one way to encourage such a dialogue, and to check that students are actively engaging with their feedback. In the future we will continue to provide feedback via email, due to the advantages we saw whilst the workshops ran online.

5. Conclusions

The move to online workshops for the 2020/21 academic year necessitated a variety of changes to be made to the format in order to run the sessions smoothly. Some of these changes, such as reformatting the worksheets into slides, were purely practical changes, and will not be retained for in-person workshops. However, many of the other changes (such as the induction session, some new question types, mark schemes and the feedback procedure) arose from the move online but appear to be generally beneficial to students' learning, and so will be continued now that in-person workshops have resumed. In addition, we have extended some of these ideas further in the in-person workshops. As well as marking criteria, we begin each workshop with a more in-depth reminder of pitfalls to be aware of in the current workshops, and highlight particular writing features that we want students to focus on each week. Building on the induction activity and variety of question types, we also end each workshop with a whole-class discussion, asking students to reflect on the work they have just done, discuss examples of proofs and look for flaws, and consider extensions of the topic.

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