CASE STUDY

Using online STACK assessment to teach complex analysis: a prototype course design?

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Abstract

We describe a new course design, informed by our experience of the pandemic, that we think could be used in other high-level mathematics courses. The course's main resource was a set of interactive STACK workbooks containing the course notes, automatically-marked comprehension and practice questions for self-assessment, and short videos of examples, calculations, and high-level motivation. This freed up synchronous class time to address conceptual understanding using interactive polling. We describe the course and discuss how it worked in practice.

Keywords: university mathematics teaching, blended learning.

1. Introduction

Some courses in the School of Mathematics at the University of Edinburgh have used blended learning techniques for many years (see, for example, Sangwin & Kinnear (2021)). Here, we use the term *blended learning* to mean approaches to teaching that "use multiple methods to deliver learning by combining face-to-face interactions with online activities" (the definition adopted by Advance HE). A blended design offers a variety of approaches to teaching and allows for a range of learning activities to be used throughout a course.

Here, we describe a course redesign which sought to optimize the choice of delivery method for each individual element of the material, exploiting the selection of technology now available, and building on both experiences of teaching during the Covid-19 pandemic and existing practice at Edinburgh. Blended learning can offer students some flexibility and agency in how they engage with a course, promoting independence and self-guided study, and opening up the provision to students for whom solely in-person delivery proves logistically difficult to attend satisfactorily (e.g. owing to caring responsibilities or health conditions). It enables instructors to encourage more active learning, by embedding in the course design regular activities and exercises with which students can engage. Certain efficiencies and better investments of time are gained by re-allocating particular content typically taught in person to recorded asynchronous enabling automatic assessment of student understanding.

2. Course description and design

We describe innovations made to the course *Honours Complex Variables* in preparation for the academic year 2021-22. This is a one-semester, 20-credit, SCQF level 10 course typically taken by students in Year 3 of a mathematics programme. The content is typical of a first course in complex analysis and includes, for example, the definition of holomorphic functions, complex integration and Cauchy's Integral Theorem, Liouville's Theorem, Taylor and Laurent series, analytic continuation, and residue calculus. 239 students took the course in 2021-22; the course was organized and lectured by Richard Gratwick, supported by a course administrator and a team of nine tutors.

In the recent past, the course was run in a fairly traditional way, having three 50-minute whole-class lectures and one 50-minute workshop (tutorial) each week. The content was based on an established set of PDF course notes, developed by Richard Gratwick from materials used by previous lecturers. In addition to the lecture notes, there were weekly problem sheets (each containing between five and ten questions) for discussion during the workshops.

Having taught the course in three previous academic years, the lecturer had identified two possible areas for improvement. First, a tendency of students to attempt only assessed problems from the problem sheets and not to engage with self-directed study expected of students at this level. Second, and related, that students seemed not to do the assigned reading ahead of classes. With this motivation, we aimed to redesign the course for long-term blended delivery in a way that (i) built on established practice within the School, (ii) maintained some positive features introduced in response to Covid-19, (iii) minimized the work needed to create new course materials, and (iv) developed a prototype design that could be used in other courses.

In the redesigned course, the main resource was a collection of Moodle workbooks based on the existing course notes. These followed the model of coherently organized digital exercises and expositions, as discussed by Sangwin and Kinnear (2021). Each section of the existing course notes became an online workbook, making it possible to use different media for different parts of the material, and creating a more active learning resource for students by including multiple-choice and STACK questions. Examples of workbook content are given in the next section.

Course activities for the first four weeks of the course are shown in Figure 1.

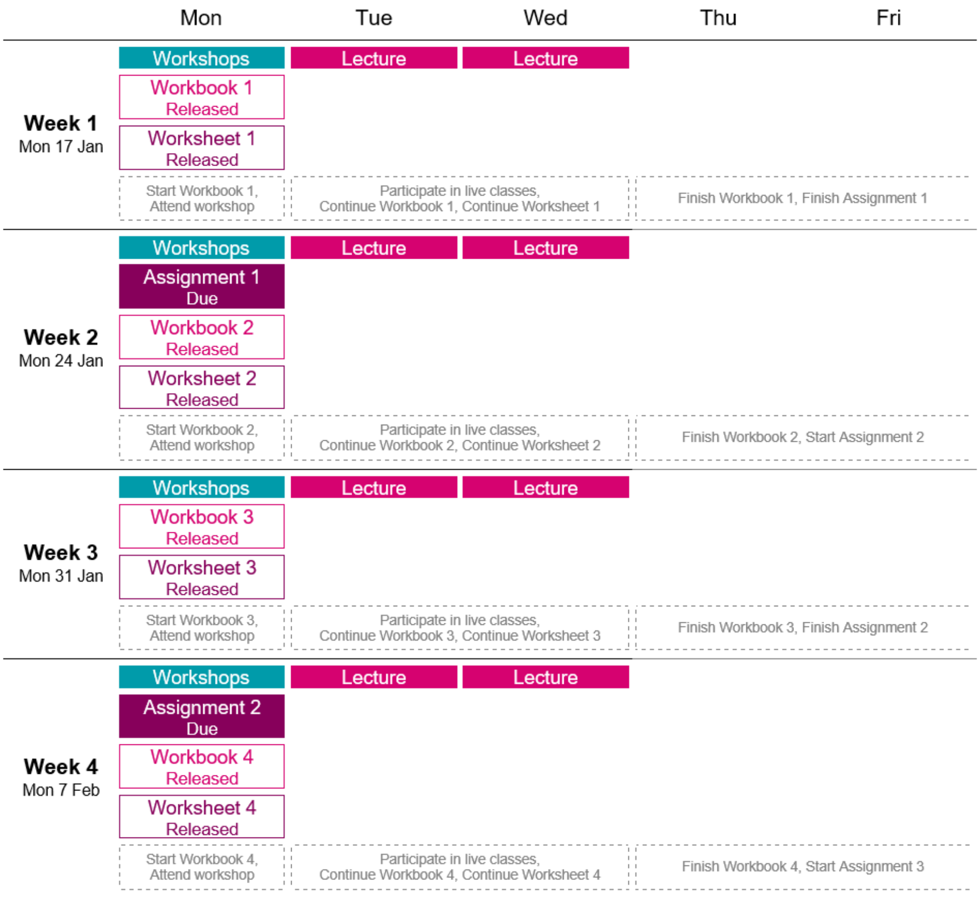

Figure 1: The structure of course activities for the first four weeks, including an illustration of what students were expected to be doing each day.

The course timetable remained largely unchanged. As before, there were weekly worksheets of problems to be discussed in weekly 50-minute workshops, and bi-weekly assignments based on these worksheets. To account for the fact that the workbooks contained computer-assessed exercises, and so students should be spending more time working through these, the number of whole-class lectures each week was reduced from three to two. In 2021-22, these lectures took place online due to pandemic restrictions. It is worth noting that students were simply encouraged to work through the workbooks, and no course credit was given for them doing so.

3. Task design

3.1 Lectures

Two 50-minute lectures were delivered each week. Institutional guidance determined that, given the size of the class and the uncertainty around the Covid-19 pandemic at the point of planning, these were delivered online. This was not a design decision and future iterations of the course will use in-person lectures (which will, as has long been standard, be recorded for students to review later if they wish).

The lectures were timetabled early in the week, so students were not expected to engage with the workbooks substantially before attending (although some did choose to do so, see below). Some content that would in previous years have been presented in lectures was moved to asynchronous content in the workbooks, for example some more routine examples, calculations, and proofs. The content of the lectures could therefore be more conceptual in nature and less involved with technical detail. The lecturer was able to spend more time motivating the subject, and highlighting connections between parts of the material both internally within the course, and beyond to other courses that many students enrolled would likely also be taking. The lecturer felt that students previously did not have much opportunity to appreciate the context of the subject within the wider discipline.

The lectures were not delivered as part of a fully flipped classroom, but rather a tilted one (Alcock, 2018). That is, that the lecturer would indeed spend substantial periods of time presenting content, albeit in limited technical detail, but also some polling was conducted during lectures to encourage active learning. Typically, one or two questions were asked of the students in each lecture. Had the lectures been in-person, these would have been accompanied by appropriate rounds of peer instruction, but we decided not to attempt this online. The chat function was used by students to ask live questions of the lecturer, which were answered either by them or, sometimes, by other students.

We would like to note one element of interactivity that arose under the students' initiative: an ad hoc "watch party" was formed which benignly (we believe) took over some of the social space available to students in the building to watch the lectures in a group of something of the order of twenty students. In this situation students did indeed discuss the polling questions with each other and engage in quite unprompted peer instruction.

3.2 Workbooks

We now discuss the content of the workbooks, which were the main resource of the course. Material was arranged in order to encourage students to be active while studying, and to support them to behave like good students would when reading traditional PDF notes. A typical pattern of content is shown in Figure 2. We see a definition followed a short discussion and video clip of a worked example by the lecturer. An automated and randomized STACK question then gives the student an opportunity to check their understanding of the material.

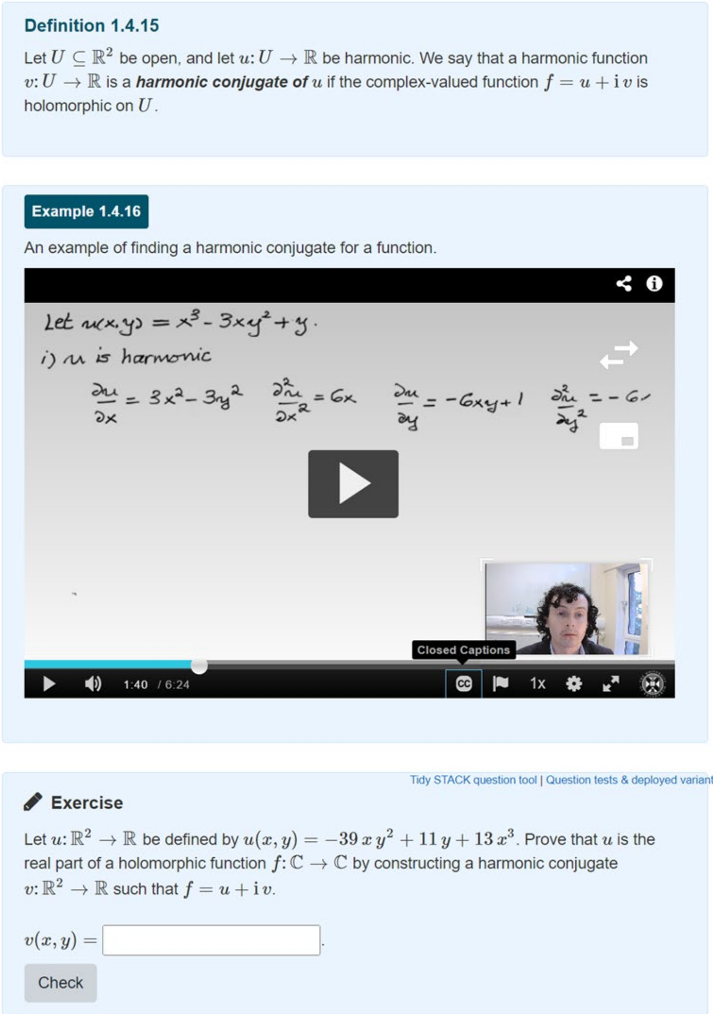

Figure 2: A typical pattern of content from an online workbook, showing a text definition, a video clip demonstrating a standard technique, and an automated STACK question to allow students to practise the technique.

This example demonstrates how an online workbook allowed us to use different media appropriately in the course materials, unlike in a static PDF file. We felt it was important that mathematics students should be expected to read definitions and results, and these were presented as straightforward text, as in traditional lecture notes. The video clip allowed the lecturer to discuss the statement and to demonstrate a method or computation, as would typically happen in a traditional lecture. A more dynamic delivery than text suits explanations of methods and spending live contact time between lecturer and students demonstrating routine computation is not necessarily the best use of that time. Pre-recorded video thus enables class time to be spent in richer and deeper discussion. The automated STACK question encouraged the student to stop and practise working with the new concept immediately. Since the question was randomized, the student could generate another question if they wanted to practise more. While the previous lecture notes regularly included printed exercises, the automatic assessment and immediate feedback of STACK clearly offer a more rewarding engagement with such exercises.

STACK is most commonly used to ask questions where the answer is a number or a mathematical expression, usually resulting from the student carrying out a computation. In the course Honours

Complex Variables, we also wanted to test the student's understanding of concepts and edge cases. Figure 3 shows an example of one way such questions were asked.

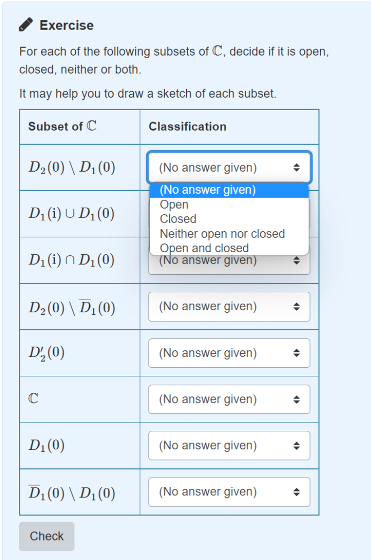

Figure 3: A randomized question designed to help students test their understanding of a new concept.

Another important feature of STACK questions is the ability to generate worked solutions tailored to the question so that students can check their method of solution or remind themselves of standard techniques. An example of a standard question and its worked solution is shown in Figure 4. Again, students had the opportunity to generate another question if they wanted further practice.

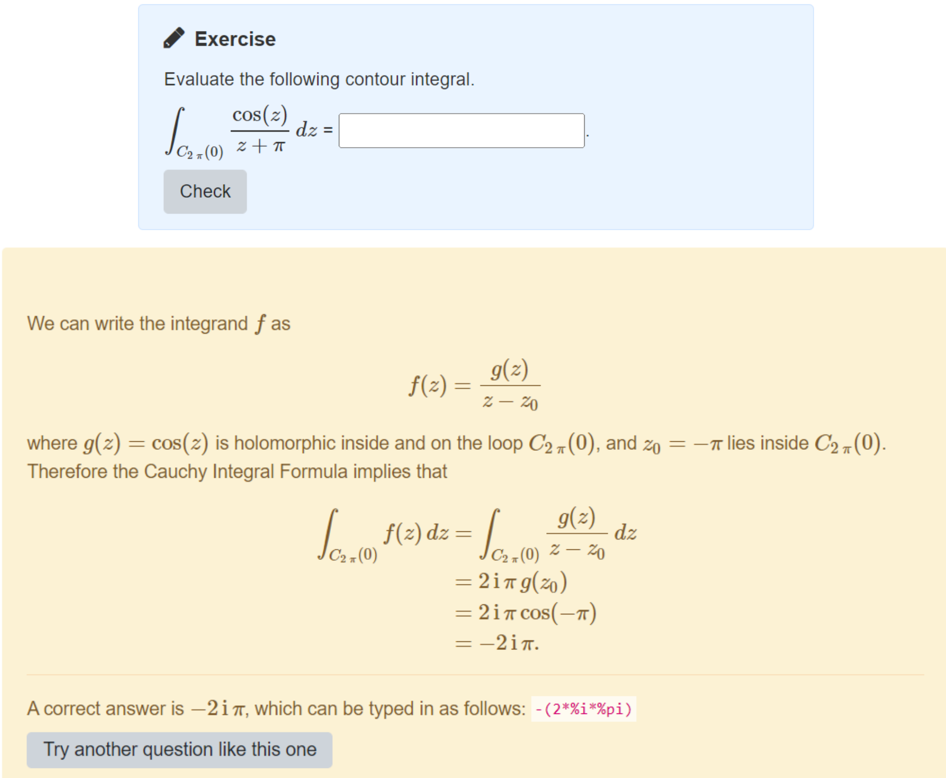

Figure 4: An example of a question and its tailored worked solution.

4. Discussion and conclusions

4.1 Student feedback and behaviour

Student feedback on the course was overwhelmingly positive, with one student responding to the end-of-course survey as follows.

"*Genuinely this course has been the perfect mix of activities for my learning, I'd go as far say to the best organised course I've taken in [the School of Mathematics], certainly this year anyway. The notes being delivered in stack are great and much more engaging than a pdf (the supplementary pdf is much easier to navigate for finding Theorems etc. however), which actually makes me do all the reading before lectures, so I gain so much more from them. Stack is good in part because of the instant feedback on most exercises which are immediately relevant to what you're learning, but also because it breaks the material up well. Stack being the main resource works perfectly with the 2 lectures delivered a week and the tutorial. [The School] should considering delivering all courses in this fashion.*"

Figure 5 shows the average percentage of students in the class attempting questions from each week of the semester. Given that no course credit was awarded for completing the workbooks, we were encouraged by how much they were used by students. Several students also made multiple attempts at a given question, taking advantage of the randomization for extra practice.

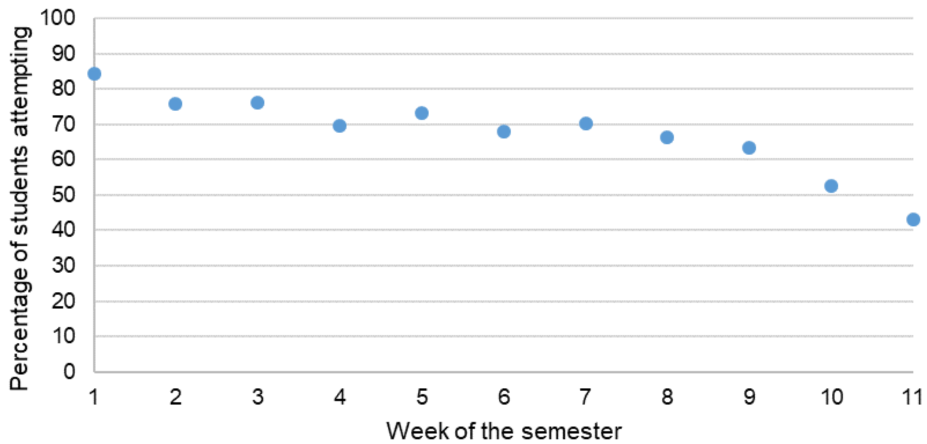

Figure 5: The average percentage of students in the class attempting questions from each week of the semester.

As a remark, we note that Moodle stores detailed data about student interactions with quizzes. It was therefore possible to track student engagement with workbooks, and to produce the plot in Figure 5. Other information beneficial to teaching could also be extracted, such as areas of common misunderstanding in the class or even a personalized report for each student.

4.2 Workload involved in creating the workbooks

We were fortunate to have the assistance of student interns Ivona Gjeroska, Maddy Baron, and Jie Xin Ng to help convert the existing course notes to the new online workbooks. They were employed for some weeks of summer 2021 on this course and other projects. They had the tasks of copying the text from the LaTeX source to the Moodle quiz platform and writing quiz questions as specified by the lecturer. The authors are grateful for the significant amount of time which this saved them on the more mundane tasks involved in the implementation of this redevelopment. This allowed us to invest more time in consideration of the structure and design of the workbooks, recording of the video clips, authoring of new questions or more sophisticated adaptations of existing questions, and rewriting of the live lecture material. The workload involved overall was substantial, but largely it was one up-front investment, and we believe the course to be in a robust position for future delivery.

In order to assist colleagues in other institutions who are interested in making similar changes to their course designs, we intend to publish workbook content online as an open educational resource. This has not happened at the time of writing but readers who would like a copy of the materials may contact the authors directly.

We are grateful to Giampaolo D'Alessandro for sharing pre-existing STACK questions on complex analysis that were created at the University of Southampton.

4.3 Concluding remarks

For the delivery in 2022-23 the online lectures shall move easily to on-campus activities using interactive polling and peer instruction, which had been an established practice in the School before the pandemic. With that modification we believe the redesign of the course to be highly successful and would like to consider the mode of delivery as a prototype for courses of the future.

5. References

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