CASE STUDY

Student Co-Creation of Resources in a Second Year Linear Algebra Course

Morgiane Richard, Centre for Academic Development, University of Aberdeen, Aberdeen, United Kingdom. Email: m.richard@abdn.ac.uk
Mirjam Brady-Van den Bos, School of Psychology, University of Aberdeen, Aberdeen, United Kingdom. Email: mirjam.brady@abdn.ac.uk

Abstract

In this project, we investigated how designing their own tutorial questions could help students negotiate the often-challenging transition to university mathematics. One student from the first linear algebra course at the University of Aberdeen volunteered to create practice questions on topics of linear algebra they selected. Through analysis of the participant's interview, we showed, in accordance with the literature, that the activity benefited them in terms of study skills (motivation, focus, independence), and in terms of mathematics learning (deep learning and mathematical knowledge construction). We also found that the activity had contributed to improving the volunteer's resilience to learning mathematics, and their sense of legitimate participation in the mathematics community. This has not been discussed in the literature and is a significant finding considering the known difficulty for mathematics students, in particular women, to feel they 'belong'. This research further suggests that if the activity took place in groups, it could also help develop a peer-support community which students could rely on to negotiate the transition to university mathematics.

Keywords: mathematics education; co-creation pedagogy; community of practice; zone of proximal development.

1 Pedagogic Context

The transition from level 1 to level 2 mathematics in Scottish University mathematics and natural sciences degrees is known to be challenging. In the literature, this corresponds to the transition from school to university mathematics (Martin, 2016a) due to the difference between the English and Scottish University curricula (level 2 Scottish university courses are SCQF level 8 corresponding to RQF/CQFW/EQF level 5). The consequences of this transition are measured in terms of retention and progression in mathematics and mathematics-heavy degrees (Solomon, 2007; Martin, 2016a).

There is a wide range of reasons why students may find this transition difficult, some will be true for any disciplines, such as the difference in culture between school and university, personal and financial difficulties, or lack of feeling of belonging to the university community (Martin, 2016a). There are some specificities to mathematics courses though (Martin, 2016a; Martin, 2016b): extensive use of formal notation, and a high number of definitions and systematic use of proofs (Solomon, 2007; Alcock and Simpson, 2009; Iaonnone and Simpson, 2020). Students find proof particularly disconcerting (Cronin and Stewart, 2022), as they must prove properties they feel are 'obvious' (Alcock and Simpson, 2009), and also because proofs often rely on one own's intuition and the use of tricks (Alcock and Simpson, 2009; Iaonnone and Simpson, 2020; Cronin and Stewart, 2022).

At the University of Aberdeen, we are also observing that students find level 2 mathematics courses difficult. Staff in the Mathematics department as well as the Maths Support Adviser are aware that students in level 2 can feel overwhelmed by the course content. Focusing on the level 2 linear algebra course, we decided to investigate whether creation by students of linear algebra examples and tutorial-style questions could increase the authors' understanding of, motivation for, and commitment to, linear algebra, and more generally mathematics.

The linear algebra course is a pure mathematics course, but with applications relevant to a wide range of sciences. The details of the course curriculum can be seen in Appendix 5.1. In 2022, 74% of students registered on the course were on Natural & Computing Science degrees (Maths, Applied Maths, Maths-Physics and Computing-Maths), but a wide range of disciplines were present, including humanities (see Figure 1). It is important to highlight this, as the present project investigates how to support the transition from level 1 to level 2 mathematics, and the discussion which follows will examine this in terms of a community of leaners of mathematics, resilience in learning mathematics, and legitimacy of participation. Readers must keep in mind that the group of students that is concerned is very diverse, and not all intending to be pure mathematicians, but they have in common a certain will to do, and an interest for, advanced mathematics. This is a consequence of mathematics being an academic discipline as well as a tool for many others.

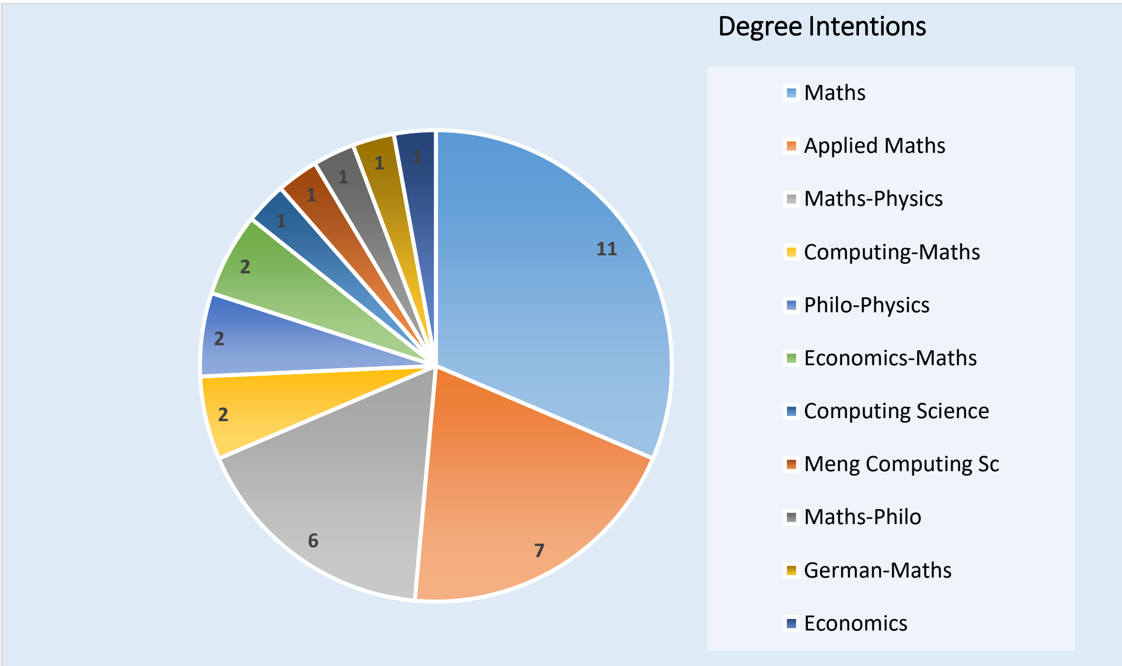

Figure 1: Degree intention breakdown for the MA2008 class register, academic year 2022-2023. 'Maths' and 'Applied Maths' combines BSc and MA degrees. Philo=Philosophy, Sc=Science. 74% of students were studying Maths, Applied Maths, joined Maths-Physics and joined Computing-Maths degrees.

The creation of mathematical examples by students is a very powerful tool for deep learning in mathematics (Watson and Mason, 2002; Bills, et al., 2006; Cornock, 2021). It is a form of co-creation pedagogy (Mercer-Mapstone, et al., 2017; Kukulska-Hulme, et al., 2021) and has been shown to enhance engagement and motivation for learning, and to encourage students to become independent and self-regulated in their studies (Bovill and Bulley, 2011; Bovill, et al., 2016; Weller, 2019). In the context of mathematical education, the creation of mathematical examples by the students will help strengthen the concept definition and concept image of mathematical objects (Alcock and Simpson, 2009; Martin, 2016b; Iaonnone and Simpson, 2020). The concept definition is the formal definition of a particular mathematical concept, and the concept image is the set of applications, examples, and imagery that a mathematician will associate with the concept definition. When doing mathematics, expert mathematicians will constantly draw from both their concept definitions and concept images, possibly unconsciously (Alcock and Simpson, 2009; Iaonnone and Simpson, 2020). The difficulty for novice mathematicians, including students, is that their concept images may be limited to non-existent, or skewed because some mathematical objects have a meaning in common language, which can imply a wrong mathematical interpretation (Alcock and Simpson, 2009). It is therefore essential to help students develop sound and rich concept images of the mathematical concepts we are teaching them.

The plan was to recruit volunteers amongst students on the first semester linear algebra course of the University of Aberdeen and form pairs of students to discuss and create examples, and code them in the e-assessment tool NUMBAS (Lawson-Perfect, et al., 2002). The significance of group work could then have been assessed. However, only one student volunteered, so the benefits of peer-support could not be investigated. Through analysis of the interview of the participant, however, using Interpretative Phenomenological Analysis (Eatough and Smith, 2017), we show that the activity has contributed to the participant's motivation, confidence, and resilience. Furthermore, the participant's interview suggests that working in groups would have given them the opportunity to be part of a community of newcomers to the community of practice of mathematicians. Consequently, student creation of mathematical examples in groups may be an answer to the problem raised by Solomon who showed that in mathematics, students, and in particular female students, commonly feel they do not 'belong' (Solomon, 2007).

2 Methodology

2.1 Ethics Considerations

Ethics approval was obtained from the School of Education of the University of Aberdeen Ethics board 28/11/ 2022.

2.2 Participants and recruitment

Participants were recruited during the first semester, in the level 2 linear algebra course MA2008, which had 35 registered students. Working groups would work on designing and coding questions in NUMBAS (Lawson-Perfect, et al., 2002) during January and February 2023.

We designed a video using the software Panopto to explain the project to students and call for participants. It was made available on the University of Aberdeen VLE course site on 28/11/2022 and 3 emails were sent to students to call for volunteers (on 29/11/2022, 20/12/2022 and 16/01/2023). Only one student volunteered to participate, and on the 24th of January 2023, the participant and I decided to start the activity even though no other student had expressed interest.

2.3 Protocol design

Two short individual questionnaires of 5 questions each and a semi-structure interview protocol of 15 questions were designed using Microsoft Forms (see Appendices for a link to the three questionnaires). The choice of questions was loosely based on Cornock's study on student-generated examples as a mode of assessment (Cornock, 2021).

The individual questionnaires were aimed at gathering feedback on participants' perceived confidence in mathematics in general, and in linear algebra specifically, and expectations from the activity prior to the activity and feedback on activity and perceived benefits after the activity. We used Likert scales and open text questions.

The semi-structured interview aimed at generating discussion amongst participants about what they found enjoyable or challenging in the activity, how participating in the activity had changed their motivation for learning of linear algebra and mathematics, and how they saw their roles as students.

2.4 Analytical Strategy

Because we had only one volunteer the project results are therefore qualitative. We held the interview on the 15th of March 2023, and it lasted 40 minutes. The interview was recorded using the software Audacity and we transcribed the interview recording on Microsoft Word and analysed it in paper copy using Interpretative Phenomenological Analysis (IPA) (Eatough and Smith, 2017), which is well-suited to analyse case studies. IPA consists of characterising a participant's experience by generating themes and subthemes in their interview transcript.

3 Results

The participant's answers to the pre-activity questionnaire are summarised below (Table 1). The post-activity questionnaire was not completed, because the same information was gathered from the semi-structured interview given there was only one participant.

Question	**Answer**
Degree intention	Mathematics
How confident did you feel you were with Mathematics at the end of last academic year?	Not very confident
How confident do you feel you are with linear algebra?	Not very confident
I look forward to attending linear algebra lectures and tutorials.	Agree
What do you expect to gain from engaging with this activity?	*"I'd like to focus on things I found difficult and improve on them."*

Table 1 Participant's answers to the pre-activity questionnaire

Table 2 provides a summary of themes and subthemes which we generated from the interview transcript. We detailed in the following sections the main themes and subthemes that we have extracted from the participant's interview.

Themes	**Subthemes**
They would have liked a group work experience	Exchanging knowledge with peers Feeling safe amongst other novices Belonging to a community
How they see teaching and learning	Wishing to revisit difficulties to consolidate knowledge Learning something unexpected Becoming a critical learner
How they see themself	They can do mathematics They are a legitimate student of mathematics They feel well

Table 2 List of themes and associated subthemes identified from the participant's interview transcript.

3.1 Theme 1: 'They would have liked a group work experience'

This seems to be very important for the participant and they mentioned it first.

- **Exchanging knowledge with peers**: the participant recognises that everyone is good at different tasks, and that learners can gain from each other *"we have like different gaps in our knowledge"*, *"we do actually help each other or teach each other".* It is interesting in the context of maths, as the belief that some would be naturally good at maths and others bad at it is common.
- **Feeling safe learning amongst other novices**: the participant explains that *"it is easier [ ] asking maybe like stupid questions"*, *"easier to ask sometimes a fellow student [ ] to going to a professor and ask something which you probably should have known".* The participant makes a distinction between the staff and the other students, with whom informality and imperfection are allowed. This is indeed quite common amongst students.
- **Belonging to a community**: *"it would be great [ ] to actually do a group project [ ] be with other people"* and expect *"it would have been quite fun [ ] you know talking about maths with other people who are interested in it [ ] to … have a chat yeah of the subject".* The project would have been an opportunity to create a connection amongst participants, a sense of group, which is more difficult to create at university.

3.2 Theme 2: 'How they see teaching and learning'

- **Wishing to revisit difficulties to consolidate knowledge**: the participant mentioned this as one of the main reason to participate *"to actually go deep into the areas which I am actually not really good", "to go back and identify those gaps", "there are some gaps in my knowledge and hopefully that will help me learn a bit which it did meet hum and it was fantastic", "I think it is really beneficial to go back and identify mistake and so on".* This is probably quite uncommon, and indeed the participant reports that other people in the class *"weren't really keen to go back and do more of something that is already kinda past"*, which would suggest that most students see courses as compartments of their degree rather than a progression in acquiring expert skills.
- **Learning something unexpected**: the participant says that they did not expect NUMBAS to require so much programming *"I just actually wasn't quite aware [ ] to the extend it actually going to involve a bit of coding in NUMBAS"*, however, they do not see this as a *"bad thing"*, but something *"actually gonna be really beneficial for me in the future"*, and something they actually enjoyed *"I just got hooked, and I feel like, like yeah that's something I am quite interested in".*
- **Becoming a critical learner**: *"[the project] has given me a little bit of a different eye for tutorial questions" "I am actually thinking a bit more about behind like OK what this question actually teaching me"* rather than *"just like trying to get through them".* The participant has re-gained control over the tutorial, which they are no longer seeing as a painful experience, which needs to be done. They are taking ownership of the tutorials – this is what we would want for all students, but that they are indeed finding difficult in this level 2 transition.

3.3 Theme 3: 'How they see themselves'

- **They can do mathematics**: when asked about how their confidence may have changed, the participant answers that *"definitely give me a little bit, yeah, a bit of a confidence yes"*, and that they realised *"like actually yeah I am doing this so definitely*" and that they *"can actually learn new things".* The project has given the participant a sense of ability.
- **They are a legitimate student of mathematics**: The participant does not see themself as a mathematician yet *"I would not introduce myself as a mathematician, I feel like I am a student"*, but is becoming one *"hopefully like maybe from next year"* and the *"project has helped me to gain a little bit more confidence this why I am doing it and I am enjoying it so I should"*. The participant weights this against them having *"this imposter syndrome"* and how *"sometimes I have like nooo idea what I am doing here"*. This is possibly common in mathematics, where, as mentioned above, there is a widespread view that some are good at maths and others bad. There seems to be two conflicting views for the participant: on the one hand, everyone has different mathematical strengths and has something to contribute to the community of learners, and on the other hand, there are some imposter students of mathematics.
- **They feel well**: throughout the interview, the participant describes the project experience in 'good-feeling' and well-being terms: they state that *"I am really happy actually I did go"*, this was "*overall definitely positive*" and that they "*did enjoy that*", with a strong emphasis on motivation "*my attitude definitely it helped me let's say hum the project to kind of keep me focused and motivate me to do these things, so that's like a big plus*".

4 Discussion and Conclusions

The IPA (Interpretative Phenomenological Analysis, see section 2.4) of the interview with the participant shows that they found clear benefits to having taken part in the activity: improved motivation, focus, and independence. These benefits have been noted in the SaP (Student as Partners) literature across a range of co-creation activities (Bovill and Bulley, 2011; Bovill, et al., 2016; Weller, 2019). We will categorise these as *benefits for study skills,* and they would be transferable to any course or discipline.

Furthermore, it appears that the project has been a space to continue improving and consolidating the participant's understanding of linear algebra, through revisiting difficulties and testing what they thought they understood. This agrees with findings from the literature that the creation of mathematical examples by learners provides an opportunity for knowledge construction and consolidation (Bills, et al., 2006; Cornock, 2021). Consequently, these benefits are particular to the exercise of creating mathematical examples and applicable to the learning of mathematics. We will call this: *benefits for learning mathematics*.

We have identified an additional category of benefits that transpires in the discussion with the participant, which, to our knowledge, has not been discussed on the literature of SaP or exemplification in mathematics education. We call these *benefits for the student resilience and legitimate participation*. In this category, we include the (re)gained feeling of being capable, feeling a legitimate student of mathematics, enjoying learning new concepts, increased confidence, sense of being in transformation and becoming a mathematician, having a space to grow and learn from mistakes and failures. In other words, the activity was an opportunity for the participant to be a *legitimate peripheral participant* in the *zone of proximal development* (ZDP), as defined by Lave and Wenger (Lave and Wenger, 1994; Lave, 2008). Solomon shows that it is particularly difficult for girls to feel rightful mathematicians (Solomon, 2007), so given that the participant is a female, the finding that co-creation of mathematical examples has benefits for resilience and legitimate participation is significant.

The range of benefits that were identified suggests that implementing creation of examples by level 2 students in mathematics would be a powerful strategy to help the level 1 – level 2 transition. What the project failed to implement, however, is group work in co-creation, because only one student volunteered to take part. The main reason for low interest amongst students seems to be that the activity was not synchronised with the course but was taking place after the course had finished. This suggests that students do not see their degrees as a progression towards becoming an expert through inter-dependant courses, but as a linear set of independent modules to validate. Yet, in the case of linear algebra, there is an obvious progression, from the level 1 algebra course to the linear algebra 1 and 2 courses in level 2.

According to the participant, the group experience would have been an opportunity to discuss mathematics with peers in an informal, safe, and friendly environment: it would have been a way to create a *community of practice of newcomers in the community of mathematics* (Lave and Wenger, 1994; Wenger, 2008). Solomon showed that students in mathematics have difficulties in developing a feeling of belonging to a community of practice, and this research implies that group co-creation of mathematical examples would help developing this feeling, however, this would need further investigations. In collaboration with the lecturer of the level 2 linear algebra courses, the creation of exercises has been implemented as a core, mandatory, group activity of the course in 2023-2024, and we are collecting students' feedback. The analysis of the results will continue to inform the benefits of this pedagic approach.

5 Appendix

5.1 MA2008 linear algebra course curriculum

The MA2008 linear algebra course is coordinated and tought by the Maths Department, School of Natural and Computing Science. It is a mandatory course on the single and joint honours maths degrees, and open to all other degrees. The course curriculum covers:

1. Sets (basic notions, relations, maps and principle of induction)
2. Groups, Rings and Fields (groups, rings and fields and integers mod n and the field of prime numbers)
3. Linear equations
4. Multiplication and addition of matrices
5. Vectors spaces (definition of a vector space and examples, subspaces, spanning, linear independence, bases)
6. Linear transformations (definitions, examples and elementary properties, kernels and images, linear transformations defined on basis elements)
7. Matrices and linear transformations (matrix of a linear transformation, invertible matrices, Gauss-Jacobi method, change of basis, rank of a matrix)
8. The determinant

5.2 Links to questionnaires

The pre-activity questionnaire can be found at:

https://forms.office.com/Pages/ResponsePage.aspx?id=rRkrjJxf1EmQdz7Dz8UrP-jz8LfXu3JLgXr8w7vGpf1UNlRQNE1XVFRCN0pQVTYyTVJKRUJJMVQ2WC4u

The post-activity questionnaire can be found at:

https://forms.office.com/Pages/ResponsePage.aspx?id=rRkrjJxf1EmQdz7Dz8UrP-jz8LfXu3JLgXr8w7vGpf1UNFJXSlhBWEkzUk5BWTNQRzdONUZYOTVVTi4u

The interview questions can be found at:

https://forms.office.com/Pages/ResponsePage.aspx?id=rRkrjJxf1EmQdz7Dz8UrP-jz8LfXu3JLgXr8w7vGpf1UNTRFRlM3M0dPNzZYN1dTMEJUU1lUSE9OTi4u

5.3 Reflexivity on IPA method

IPA is an interpretative method of analysis of interview conversation, well suited for the analysis of interviews of single participants. However, because it is an interpretative method, the personality, history, and experience of the researcher necessarily influences the coding of themes.

As a woman who has studied and worked as a researcher in STEM disciplines, I (Morgiane Richard, Academic Skills Adviser in Maths and lead researcher in this project) am probably particularly sensitive to the gender imbalance in STEM subjects, and to the difficulty for girls and women to feel as capable as boys and men to do well in those subjects. I feel quite strongly about this, and it would be fair to say that another researcher might not have picked up this thematic in the same way in the interview transcript.

To help decide whether the themes I coded were reasonable, my colleague on this project, Dr Mirjam Brady-Van den Bos, who is a lecturer in the School of Psychology, also read the transcript and generated some themes. As the themes were consistent between our two codings, and particularly since Mirjam does not have a background in either mathematics or mathematics education, this comforted me with the idea that the themes were compatible with both our backgrounds, and not solely linked to my background as maths support adviser and physical scientist.

6 References

Alcock, L. and Simpson, A., 2009. *Ideas from mathematics education: an introduction for mathematicians.* The Higher Education Academy. Maths Stats and OR Network (MSOR Network).

Bills, L., Dreyfus, T., Mason, J., Tsamir, P., Watson, A. and Zaslavsky, O., eds. 2006. *Proceedings of the 30th International Group of the Psychology of Mathematics Education*.

Bovill, C. and Bulley, C.J., 2011. A model of active student participation in curriculum design: exploring desirability and possibility. Rust, C., ed. 2011. *Improving Student Learning (ISL) 18: Global Theories and Local Practices: Institutional, Disciplinary and Cultural Variations.* Oxford Brookes University: Oxford Centre for Staff and Learning Development, pp.176–188.

Bovill, C., Cook-Sather, A., Felten, P., Millard, L. and Moore-Cherry, N., 2016. Addressing potential challenges in co-creating learning and teaching: overcoming resistance, navigating institutional norms and ensuring inclusivity in student–staff partnerships. *Higher Education,* [e-journal] (71), pp.195–208. https://doi.org/10.1007/s10734-015-9896-4. <http://eprints.gla.ac.uk/107084>.

Cornock, C., 2021. Student-Generated Examples and Group Work in Mathematics. *MSOR Connections,* 19 (2), pp.31–39.

Cronin, A. and Stewart, S., 2022. Analysis of tutors' responses to students' queries in a second linear algebra course at a mathematics support center. *The Journal of Mathematical Behavior,* [e-journal] 67, pp.100987. https://doi.org/10.1016/j.jmathb.2022.100987 <https://www.sciencedirect.com/science/article/pii/S0732312322000554>.

Eatough, V. and Smith, J.A., 2017. Interpretative phenomenological analysis. Willig, C. and Stainton-Rogers, W., eds. 2017. *Handbook of Qualitative Psychology.* Second ed. London, UK:Sage. , pp.193–211.

Iaonnone, P. and Simpson, A., 2020. Mathematics and statistics. Marshall, S., ed. 2020. *A handbook for teaching and learning in higher education: enhancing academic practice.* Fifth ed. London:Routledge. , pp.206–217.

Kukulska-Hulme, A., Bossu, C., Coughlan, T., Ferguson, R., FitzGerald, E., Gaved, M., Herodotou, C., Rienties, B., Sargent, J., Scanlon, E., Tang, J., Wang, Q., Whitelock, D. and Zhang, S., 2021. *Innovating Pedagogy 2021: Open University Innovation Report 9.* Milton Keynes: The Open University.

Lave, J., 2008. The practice of learning. Illeris, K., ed. 2008. *Contemporary Theories of Learning: Learning Theorists … In Their Own Words.* London:Routledge. , pp.200–208.

Lave, J. and Wenger, E., 1994. *Situated learning: Legitimate Peripheral Participation.* Cambridge: Cambridge Univ. Press.

Lawson-Perfect, C., Foster, W., Youd, A., Graham, C., and Stagg, G., 2022. *Numbas* (Version 6.2) [Software]. Apache-2.0. https://github.com/numbas/Numbas/

Martin, G., 2016a. Transition to University Mathematics 1: Transition. Available at: <https://blogs.bath.ac.uk/exchange/2016/transition-to-university-mathematics-1/> [Accessed 15 March 2024].

Martin, G., 2016b. Transition to University Mathematics 2: Maths at University. Available at: <https://blogs.bath.ac.uk/exchange/2016/transition-to-university-mathematics-2/> [Accessed 15 March 2024].

Mercer-Mapstone, L., Dvorakova, S., Matthews, K., Abbot, S., Cheng, B., Felten, P., Knorr, K., Marquis, E., Shammas, R. and Swaim, K., 2017. A Systematic Literature Review of Students as Partners in Higher Education. *International Journal for Students as Partners,* [e-journal] 1 (1), pp.1–23. https://doi.org/10.15173/ijsap.v1i1.3119.

Solomon, Y., 2007. Not belonging? What makes a functional learner identity in undergraduate mathematics? *Studies in Higher Education,* 32 (1), pp.79–96.

Watson, A. and Mason, J., eds. 2002. *Proceedings of the 26th Conference of the International Group of the Psychology of Mathematics Education.*

Weller, S., 2019. Working with students: from engagement to partnership. 2019. *Academic practice: developing as a professional in higher education.* London:Sage, pp.77–100.

Wenger, E., 2008. A social theory of learning. Illeris, K., ed. 2008. *Contemporary Theories of Learning: Learning Theorists … In Their Own Words.* Routledge, pp.209–218.