

CASE STUDY

Affections in the transition to undergraduate mathematics

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Abstract

The module 'Introduction to Study and Research in Mathematics' is a credit-bearing unit of teaching designed particularly with the aim of supporting students in the transition from school to studying mathematics at undergraduate level in the UK. This case study discusses how the design of the module was impacted by consideration of the affective domain, aiming to build both students' understanding of and interest in mathematics as an academic discipline and their confidence in tackling mathematics questions they do not initially know how to answer.

Keywords: secondary-tertiary transition, affections, space to fail, comfort zone.

1. The importance of considering affections

Since I first read it, the following statement has had an ongoing impact on how I think about teaching maths.

*“Three factors influence how effective your mathematical thinking is:
your competence in the use of the processes of mathematical enquiry;
your confidence in handling emotional and psychological states and turning them
to your advantage;
your understanding of the content of mathematics and, if necessary, the area to
which it is being applied.
... a knowledge of mathematical content ... usually hogs the space. Often it is
presented as the **only** important factor ...” (Mason, et al., 1985, pp. 146-147).*

This resonated with my personal experience. I cannot recall any instance in my mathematical education that addressed the idea that I, as the one doing the maths, am a human being. Meanwhile, I had seen the impact of emotions and attitudes on students' performance. One particular group I taught told me repeatedly “*you have to be (the top student in the class) to do this*”. Assuming a task is beyond you can become a self-fulfilling prophecy.

Di Martino, Gregorio and Iannone (2023) surveyed mathematics education literature between 2008 and 2021 related to the secondary-tertiary transition. They found that the discussion has become increasingly holistic, rather than focusing solely on cognitive aspects, but with consideration of affective elements still under-represented.

More recently, Geisler, Rolka and Rach (2023) modelled the extent to which measures of affect in mathematics undergraduates at the start of and later in their first semester correlated with those who had left their programme by the start of second year, whether voluntarily or not. They concluded that:

“students’ interest in university mathematics as well as their mathematical self-concept was associated with less risk to drop out ... [Also,] it seems plausible that increasing interest in university mathematics as well as increasing self-concept [over time] go along with less risk to drop out” (pp. 47-48).

Here the term ‘self-concept’ denotes how the students view their mathematical abilities. Next, the authors used qualitative methods to pinpoint incidents that impacted students’ self-concept (p. 48). They found that:

“activities ... like conducting proofs or working with definitions were mentioned more often than concrete content ... It seems that these activities are a hurdle for students, which influences their self-concept independent from the specific mathematical content” (pp. 48-49).

They also highlight a comment from one student who *“indicated that he felt uncomfortable to have to ask the teaching assistant for help”* (p. 51).

The transition from school to university mathematics brings numerous changes for students; a good summary is given in (Geisler & Rolka, 2021). In the UK, by the start of university many students have experienced an extended duration in which their expected future identity is as a mathematics graduate. Consciously or otherwise, they are likely to have invested emotionally in this future self. Discovering a new style of mathematics brings into question their earlier decision to study maths. Those who continue their programme must adjust intellectually to the new approach required from them but should also adapt their self-image to account for the change. This can be an emotionally painful process.

My thinking took a step further while reading about teacher education. One thing a trainee teacher needs to learn is how to cope with being responsible for a class of children. Lecturing is not a suitable teaching approach for this skill. The learning required is affective, not cognitive, and can’t be acquired from third-hand or even second-hand experiences; first-hand lived experience is needed. The same is true for the affective aspects of studying mathematics. This mindset was one of the cornerstones when I designed the module ‘Introduction to Study and Research in Mathematics’ (denoted in this case study by ISRM).

2. The module

ISRM is a module taken by the first-year students on our main mathematics undergraduate programmes in semester 1. That is, the module usually forms part of the transition from school to university mathematics; most students have recently completed UK secondary education. They take compulsory modules in a range of areas of maths in their first year, with an increasing selection of optional modules later allowing them to focus on their area of interest.

The syllabus has multiple foci. One part covers introductory aspects of the foundations of maths: informal logic, naïve set theory, numbers, functions and relations. Another part discusses the processes of engaging with university mathematics: reading mathematical texts; the role of definitions, theorems, proofs and examples; methods of proof; and ‘writing up’ mathematical arguments. A third strand aims to build the students’ awareness of mathematics research and the wider academic community, while the final strand relates to careers and employability, particularly exploring the variety of roles open to mathematics graduates.

The class has in the order of 100 students and is taught over 12 semester weeks. In the initial implementation, each student had four weekly contact hours. The first hour took place in a lecture theatre and was fairly traditional in style. In 2020, all teaching activities moved online. This lecture was replaced by asynchronous teaching in the form of videos with 'quiz' questions built into them. It remained in this format after the return to in-person teaching.

The second hour was focused on active learning; the pace was controlled by the lecturer, but most class time was devoted to students attempting questions. The two-hour session at the end of the week was taught in smaller groups. Students were given a problem sheet to work through at their own pace, with lecturers, TAs and classmates available for discussion. The intention was for all these classes to take place in rooms where students could work together around tables. In practice, this was limited by the availability of suitable rooms on campus.

Additional 'background reading' resources were distributed through the VLE, comprising articles, videos and graphics on topics such as the nature of mathematics, biographies of high-profile mathematicians, and the axiom of choice.

There are two summative assessments for ISRM. One is an auto-graded assignment formed of short-answer questions completed using the Möbius online platform. The second assessment is a 'portfolio' of tasks completed over the course of the semester, with each task typically taking 1-2 sides of A4 paper. There is no time-controlled assessment for the module. Passing the module is a pre-requisite to progressing into the second year of study, but the grades do not impact the students' degree classifications.

3. The module survey

Late in academic year 2023-24, an online survey about ISRM was sent to all current students who had previously taken the module (between 0.5 and 3.5 years earlier). The objective was to learn about students' perceptions of the module following time to reflect on it in light of their later experiences. The flip side of this is that students might not recall details of the module accurately. Responses from some relevant survey questions are included below.

4. Ask A Tutor

One task the students must complete as part of their portfolio is named 'Ask A Tutor'. The instructions are as follows.

1. Read (Alcock, 2013) sections 10.1 – 10.5.
2. Talk to one of your university tutors/lecturers/advisor about some maths. This can be about any of your modules, or (if they are willing) about another area of maths, but it must not be solely related to problems sheets.
3. Write down details of the conversation:
 - who you spoke to;
 - when you spoke to them (give the date);
 - where/how you spoke to them;

along with a short description (1–2 paragraphs) of the conversation and its context.

The book (Alcock, 2013) is essentially a handbook for mathematics undergraduate students. Sections 10.1 – 10.5 discuss the process of getting support from a lecturer, including how the student–lecturer relationship differs from the pupil–teacher relationship of secondary school. To these instructions, I added the following explanation of the purpose of the task. Anecdotally, adding this reduced the level of resistance from the students to doing it.

FAQ: Why do I have to do this? I don't want to!

The fact that students don't want to do this task is the reason we require it. Going to office hours is one of the best options for getting help with your studies. However, there is an emotional barrier to doing so. We want you to get over that barrier now, so that it's easier to go to office hours again later.

This task is not difficult in an intellectual sense. For many students, the hardest part is thinking of a suitable question to ask; some students ask to be told what to ask. However, for some students, such as those who experience social anxiety, completing the task as we would prefer it to be done is much more challenging. For inclusivity purposes, therefore, quite a lot of flexibility was given in completing the task, for example by contacting the lecturer via email. Allowing remote methods is also useful in the case of students resitting the module in the summer while off-campus. In practice many students opted to put minimal effort into completing the task. Tightening the requirements, with reasonable adjustments when needed, might be more beneficial for the cohort overall.

5. Research talks

In my experience, on telling someone “*I do maths research*”, the general public are often surprised that research in mathematics is possible. Some seem to conclude that I do sums with very big numbers. Those who studied more maths ask “*do you turn your paper sideways to fit your equations on?*” (Answer: occasionally).

Expecting undergraduates to understand the content of research mathematics is usually unrealistic. Nevertheless, we can familiarize them with the breadth, nature, aims, processes and environment of mathematics research. By default, mathematics graduates will function as ambassadors of the discipline to wider society. Doing so also enables them to make an informed decision about whether to join the research community themselves.

The main teaching activity in ISRM on this takes the form of ‘research talks’, where departmental lecturers present their research like in a research seminar but at a level suitable for the students. The talks are around 20 minutes in length, allowing two talks to fit within a one-hour class with time for questions. One purpose was to give the students more awareness of what the academics in the department are doing, to help them to feel part of the departmental community.

Two sets of two talks were scheduled a couple of weeks apart. The original reason for having the first class of the week in a lecture theatre was to provide a setting for these talks that mirrored that of academic research seminars. An alternative idea was to group the talks together to form a mini ‘conference’; unfortunately, this did not fit well around other modules. More recently, we have used a mixture of in-person and online talks, reflecting the increased diversity in how research seminars now operate.

Besides considering the subject area of potential speakers, I invited lecturers to speak whom students would not have lectures with in their first year, incorporating diversity in career stage, gender and cultural background, to provide a range of different role models. Alongside describing their research, speakers were asked to discuss their day-to-day experiences as an academic and their career journeys.

Additionally, some of the 'background reading' resources were selected to provide insight into the research community. These include varied papers from undergraduate mathematics journals presented like research journals but with content accessible to first-year students. Other texts address the human side of mathematical community such as how our understanding of maths is impacted by gender, cultural background or disability.

The survey responses in Figure 1 suggest that a large minority of the students were conscious of these activities impacting their thinking.

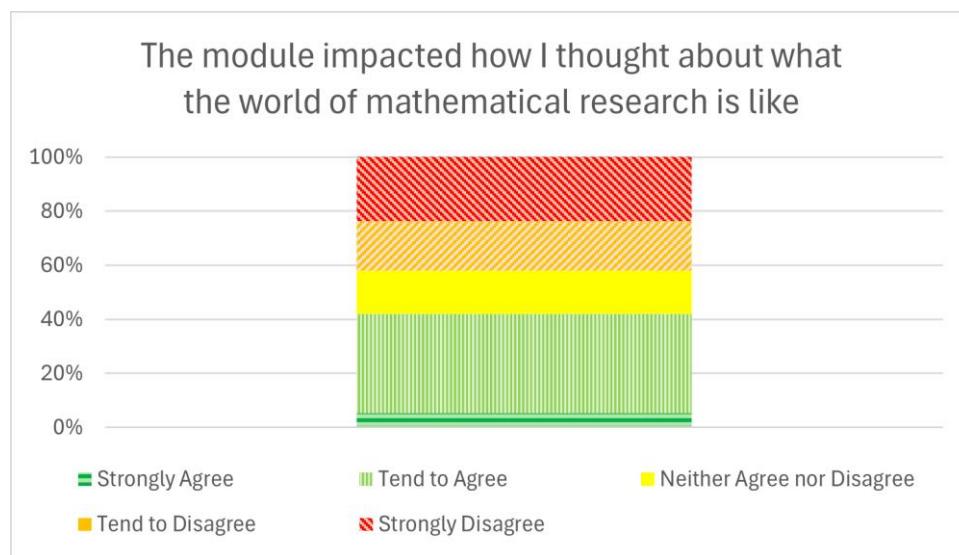


Figure 1: Survey responses on a 5-point Likert scale showing agreement with the statement 'The module impacted how I thought about what the world of mathematical research is like'.

This element of the module is assessed through a piece of 'personal writing' as part of the portfolio. This is a short piece of reflective writing (the instructions suggest $\frac{2}{3}$ of a page), with the marking criteria set to reflect that this is a non-standard task for these students. The intention is for students to demonstrate that they have engaged with the learning materials and developed their thinking in some way, while also practicing their writing skills (other formats would be accepted, but I have not had any student choose to use one). This often causes contention; a number of students have asserted that "*I didn't choose maths to write essays*". Virtually all students will need to complete some similar form of writing later in life, for example when applying for jobs. Here they can practice a skill they are uncomfortable with in a low-stakes environment, rather than waiting until a time when succeeding matters more.

6. What does it mean to be a graduate mathematician?

When designing ISRM, I decided to encapsulate the syllabus using the question ‘what does it mean to be a graduate mathematician?’ That is, what does the process of becoming one look like during university, and what does being one look like after graduation, either in academia or in industry? It is important to have a coherent narrative, rather than giving the impression of the module being made up of ‘loose ends’.

The choice of the phrase ‘graduate mathematician’ was deliberate, taking into consideration the potential impact on the students’ sense of mathematical identity. There were two ideas I wanted to avoid communicating to students. One was the idea that those who stay in academia are ‘proper mathematicians’ while those who work in industry have in some sense ‘failed’. Mathematics has value in both settings, as shown for example by the requirements for the designation Chartered Mathematician (Institute of Mathematics & its Applications), and we want to encourage students to choose whatever career path is most suitable for them.

Consequently, I initially wanted the module careers activities to include a third set of ‘research talks’ from ‘mathematicians in industry’. In practice it proved too difficult to secure suitable speakers for this. Instead, we have had speakers a few years ahead of the current students, either those who have recently graduated from a mathematics degree or those who have completed a year in industry. This offers a different form of support for the students. Many students have chosen to study mathematics knowing that it keeps their career options open but with little idea of what direction they want to go. They are often consciously ignoring the looming threat of needing to begin career preparations. People who have recently been through similar experiences themselves can provide reassurance that, firstly, these students are not alone in feeling this way and haven’t shut off their options through inaction, and that, secondly, the process of finding suitable employment is not as overwhelming as the students fear it to be.

The second idea I wanted to avoid communicating to students was that they are not ‘mathematicians’. Many mathematics undergraduates will think of themselves as mathematicians. Academics may not agree, but communicating that disagreement to students will only serve to alienate them. Meanwhile, it is clear that mathematics undergraduates are not yet *graduate mathematicians* (except in very rare circumstances) but by joining the programme they have expressed a desire to become one.

7. Space to ‘fail’

The teaching approach used for the rest of the module syllabus mirrors that described in (Epp, 2003), except without formal logic. Aside from the ‘personal writing’, all module activities are ones we normally expect of students: reading mathematics; reading about mathematics; watching talks about mathematics; discussing with other students and lecturers; working with definitions and theorems; breaking down proofs to understand them; trying to prove/disprove mathematical statements; and writing up solutions. The primary differences from a ‘traditional’ maths course are that the questions presented to students require problem solving skills (rather than being exercises in following a taught method) and that the focus is on understanding the underlying ideas rather than relying on algorithmic approaches. For example, proof by induction is introduced through questions that use different base cases and step sizes, meaning the principle must be adapted to the context.

I describe the process of doing much school maths as like a train on a railway. At pre-defined points there are a few different directions you might select, but mostly you just keep chugging along doing

the next step until you reach your answer. In contrast, tasks at university can be more like exploring a maze. You have a goal to reach, but you don't necessarily know how to reach it. Trial-and-error, arriving at dead ends, back-tracking and taking a new path to see where it leads are normal parts of the process; indeed, for research mathematicians they make up the bulk of the process.

The teaching of the module is intended to provide lots of time for students to practice this unfamiliar approach to maths problems, with lots of support available in real-time from classmates and teaching staff. Teaching staff aim to communicate that the value lies in developing skills rather than necessarily reaching 'the correct answer'. The 'Module Philosophy', which students are asked to read at the beginning of the module, finishes with the following.

You should not expect to necessarily be able to complete all the activities for this module first time. Learning how to cope with that situation, and what to do when it happens, is a deliberate part of these activities. The assessment for the module is designed so that most of the activities do not directly count towards your mark, so that you have time to try, fail, and try again, without losing marks as a result. An important question to keep asking yourself is '**did I manage more this time that I did before?**'

Nevertheless, the survey results shown in Figures 2 and 3 suggest that students generally find the module challenging both intellectually and emotionally. Kiknadze and Leary state that *"people invoke the notion of comfort zone only when a behavior they desire to perform evokes anxiety because of potentially negative outcomes"* (2021, p. 2). Of the major categories of negative outcomes they identify, the only ones seemingly relevant here are unfavourable evaluations from either students themselves or those around them (module teaching staff and the other students sat with them in class).

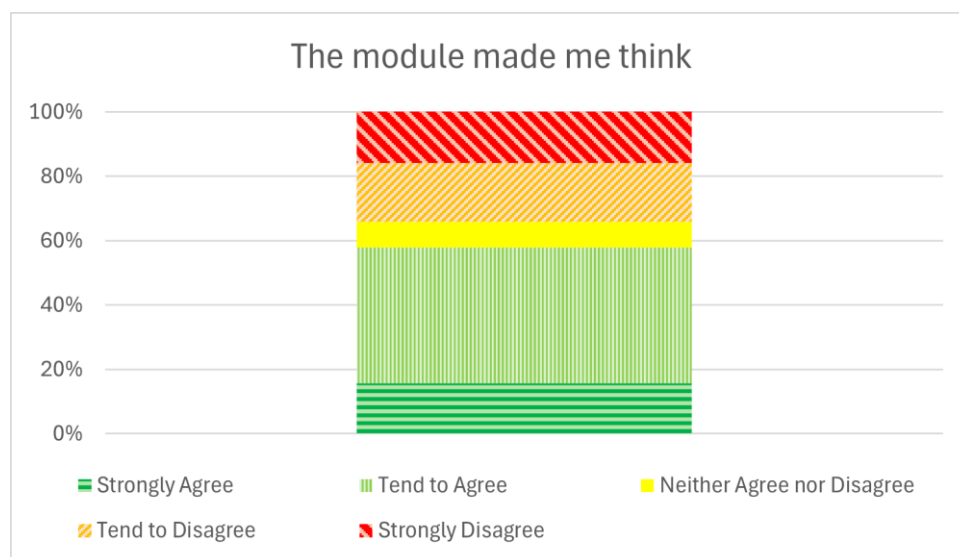


Figure 2: Survey responses on a 5-point Likert scale showing agreement with the statement 'The module made me think'.

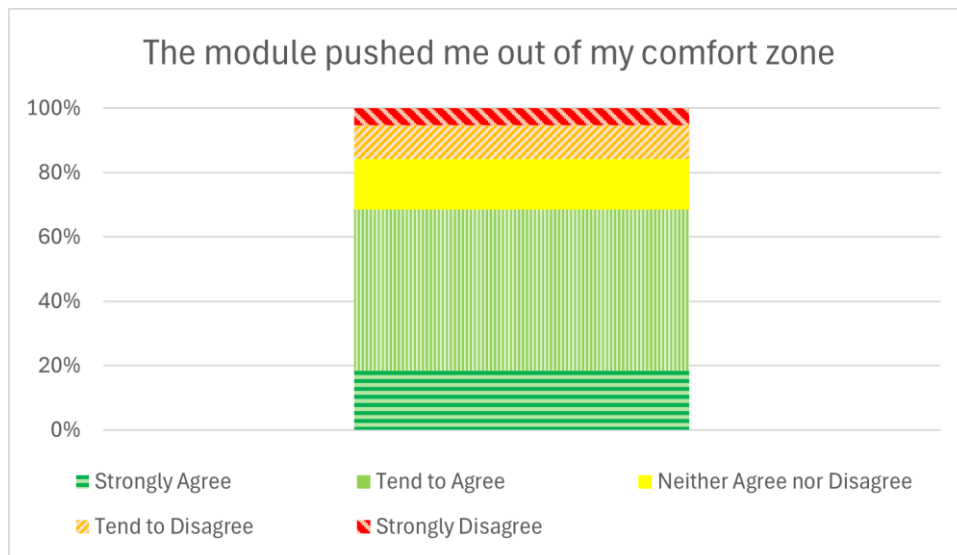


Figure 3: Survey responses on a 5-point Likert scale showing agreement with the statement 'The module pushed me out of my comfort zone'.

This perhaps mirrors the finding that the main reason surveyed students gave for not completing formative assessments was *“fear of failure”* (Finch, 2024). In contrast, the MSOR Subject Benchmark Statement says that, by the time they graduate, students are expected to have *“the ability to work independently with patience and persistence, pursuing the solution of a problem to its conclusion”* (QAA, 2023, p. 22).

8. Concluding reflections

ISRM was designed to provide students with a well-supported, low-stakes environment in which to practice skills that are central to their chosen field of study. Students who take advantage of the support available seem to value the rewarding sensation that results from hard-won insight.

However, a noticeable proportion of the students dis-engage from the module. It is unlikely that any mathematics student would have neither need nor desire to discuss at least part of the module content. On the face of it, these students therefore either:

- do not see developing these skills as sufficiently valuable to justify the effort required;
- are too afraid even with the level of support offered here;
- or are under too much time-pressure to pursue their studies properly.

In the first two cases, what can we do to mitigate these issues? To what extent is it in our power to do so, given that affections are not always rational? Should we accept the status quo?

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