

CASE STUDY

Implementing Active Learning in Undergraduate Mathematics Using Tarsia Puzzles

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Abstract

Active learning is a pedagogical approach that emphasizes student engagement and participation in the learning process. One innovative tool that has shown promise in facilitating active learning in mathematics is Tarsia, a software for creating jigsaw-style puzzles. This paper describes the design and implementation of Tarsia puzzles used in a tutorial for a first-year calculus course for engineering students, where participation was optional. It also highlights their potential to enhance conceptual understanding, foster collaboration, and increase student motivation.

Keywords: active learning, puzzles, calculus, collaboration, student engagement.

1. Introduction

Undergraduate mathematics education has long grappled with the challenge of engaging students in meaningful learning activities. Despite the pervasive presence of technology in all walks of life, mathematics teaching methods have not changed significantly within the last half-century. Although traditional mathematics pedagogy is often characterized by passive reception of information and may fail to stimulate student interest or promote deep understanding, it remains ubiquitous across all levels of education, from K-12 to higher education. Mathematics education research consistently advocates for progressive, evidence-based teaching methods that emphasize active learning strategies.

The term “active learning” is used to describe “classroom [teaching] practices that engage students in activities such as reading, writing, discussion, or problem solving, that promote higher-order thinking” (Conference Board of the Mathematical Sciences, 2016) (p. 1). This definition focuses on what students do rather than what the teacher does. Thus, active learning is student-centred rather than teacher centred.

In their paper entitled, “What does Active Learning Mean for Mathematicians?”, Braun et al. (2017) suggested some strategies for implementing active learning in the classroom. Some of these strategies relate to the learning environment, while others focus on the design of learning tasks. Some examples of active learning strategies in the literature are think-pair-share, use of classroom response systems, puzzles and games, flipped classroom, inquiry-based learning, modelling in computer laboratories, and writing (Braun et al., 2017; Rosenthal, 1995). TACTivities (tactile activities) are group-based, hands-on learning tasks that promote engagement, collaboration, communication, and critical thinking. They are low-tech manually created resources such as card matching, dominoes, and placement cards as described by Hodge et al. (2020). As such, TACTivities can be beneficial in promoting both mathematical thinking and communication skills.

Active learning strategies which engage students in critical thinking appear to offer a promising alternative to the traditional mathematics pedagogy. Among these strategies, the use of Tarsia

puzzles has emerged as an effective tool for active learning in mathematics education. Tarsia puzzles are generalized domino-like activities, created using specialized software. The puzzles range from traditional dominoes to more complicated jigsaw puzzles, as well as follow me and matching cards. Digital format allows for quick updates and re-purposing of puzzle content, producing outcomes that are generally more challenging than manually created TACTivities. While TACTivities, as described by Hodge et al. (2020) rely on physical material and manual creation, Tarsia puzzles offer a digitally enhanced alternative that preserves tactile engagement while enabling greater scalability and content complexity.

This paper describes the design and implementation of Tarsia puzzles and their use at Toronto Metropolitan University (TMU) in a tutorial for a first-year calculus course for engineering students, where participation was optional. It also highlights the potential of Tarsia puzzles to enhance conceptual understanding, foster collaboration, and increase student motivation.

In the next section, we provide a brief description of the extant literature on the efficacy of active learning in mathematics education.

2. Research on Active Learning

Research on active learning in undergraduate mathematics has grown over the last two decades (Duran et al., 2024; Freeman et al., 2014; Lugosi & Uribe, 2022; Stanberry, 2018; Theobald et al., 2020). Freeman et al. (2014) conducted a meta-analysis of 225 studies on active learning. They found evidence that active learning was more effective when coupled with course examinations that “are designed to diagnose known misconceptions, in contrast to [those] that emphasize content mastery or the ability to solve quantitative problems” (p. 8411). Establishing effectiveness of active learning is difficult because the effect could be attributed to either student gaining better understanding of the material due to the active learning strategies used, or instructors teaching to the test. Examinations in mathematics classes continue to reward speed and accuracy and are typically insensitive to the quality of a student’s understanding.

Some of the research on active learning has explored the efficacy of active learning on achievement gaps experienced by underrepresented and marginalized students in STEM disciplines. For example, Theobald et al. (2020) tested the hypothesis that underrepresented and marginalized students in active-learning classes experience narrower achievement gaps than those in traditional lecturing classrooms, averaged across all STEM fields and courses. They found that on average active learning worked to narrow such achievement gaps. However, they cautioned that the effect size was highly dependent on the extent to which students were engrossed in active learning activities and that in some cases “...active learning increased achievement gaps instead of ameliorating them,” (p. 6479). The prevalence of group interactions in active learning activities makes it incumbent upon instructors to be mindful of the impact of social dynamics on students’ experiences of those activities, especially when those students belong to groups that have historically been underrepresented or marginalized in the field of mathematics.

Active learning, when carefully designed, can increase students’ grades, attendance and engagement in classes. Duran et al. (2024) found that students in calculus classes that employed active learning had higher grades than those in lecture-based calculus classes, after controlling for attitudes towards mathematics and various demographics. Stanberry (2018), reflecting on a redesign of a calculus course to employ active learning as a new pedagogical strategy, observed increased student attendance and engagement in classes.

3. Active Learning Classroom at TMU

An active learning classroom (ALC) at TMU was designed for instructors to try new learning strategies with their students and share the experiences with the TMU community. Figures 1 and 2 depict two different configurations of the ALC space.

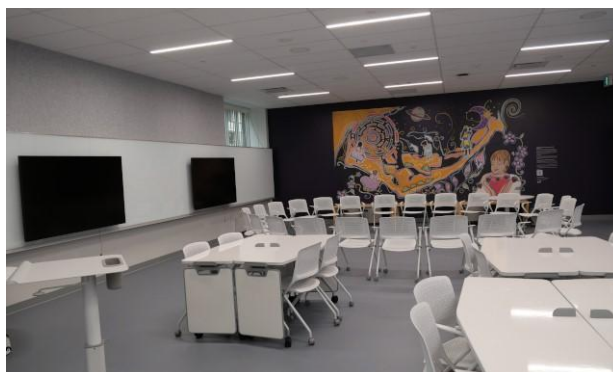


Figure 1: Active learning classroom at Toronto Metropolitan University



Figure 2: Active learning classroom at Toronto Metropolitan University

During the Winter 2024 semester, the first author used this classroom for 8 weeks to pilot two active learning strategies in a first-year calculus course for 1400 engineering students divided into several sections. This course has scheduled weekly lectures and labs which students are expected to attend. In addition to these, a tutorial with optional attendance was introduced to promote active learning using Board Rounds and Tarsia puzzles. These tutorial sessions were run on Fridays from 4:00 – 6:00 pm and considering that they were not on students' schedules we did not expect a high turnout. On average 15 students attended these sessions, with the maximum attendance of 30 and the minimum of 10 students. There was a core group of 10 students who never missed these sessions.

Board Rounds involved groups of 3 to 5 students who worked on solving problems written on boards on the walls in the classroom or on the tops of desks which were whiteboards.

Tarsia puzzles are akin to dominoes. These puzzles were created using a software known as Tarsia Formulator. In the next section we describe the software and how the puzzles were designed and implemented.

4. Implementing Tarsia Puzzles in Calculus

4.1 Tarsia Puzzle Software

Tarsia Formulator is a puzzle generating software that was created by Hermitech Laboratory (<http://www.mmlsoft.com/index.php/products/tarsia>). The software has a free license and requires Windows operating system to work. The software, while it could be used in any discipline, was originally created to support mathematics teachers. Although the software has been extensively used for primary to post-16 mathematics, particularly in the UK, it has rarely been used to create puzzles for university mathematics teaching.

The Tarsia software enables teachers to create mathematics puzzles that require students to identify where logical links exist between the teacher-defined mathematical objects or terms. Students match pieces together to establish these logical links in a way akin to dominoes. In addition, it also enables instructors to customize their puzzles to the needs of their students and to their teaching style.

To create effective puzzles, the mathematical objects or terms used need to be clear and simple to enable establishing the logical links. In addition, the following factors need to be considered:

- *Alignment of Puzzle Activities with Learning Outcomes:* Puzzles should be designed to align with the specific learning outcomes of the topic the puzzles are based on. This ensures that the activity reinforces key concepts and skills that students need to understand and acquire.
- *Clear Instructions and Support:* Clear instructions and appropriate scaffolding are essential for successful implementation. Instructors should include pieces with logical links, starting with concepts most familiar to students and progressing to concepts that have been introduced to them more recently. Instructors should explain the purpose of the puzzle and provide guidance as needed to help students stay on track.
- *Allow Time for Reflection and Discussion:* When students complete solving the puzzle instructors should bring students together to reflect on the activity. This can include reviewing the solutions, discussing strategies used, and addressing any misconceptions that arose during the activity.

The software has a user-friendly interface that enables point and click. Creating a single puzzle takes as little time as 30 minutes, even for someone new to the software.

One thing we learned about the use of the software is that the formatting of equations needs to be checked before printing. For example, we know trig functions need not be italicized. However, even with the best of efforts, sometimes, the formatting reverts to italicization.

The puzzles that were used during the tutorial fostered mathematical discourse. The reason for this is because of the deliberate choice of elementary integrals and series as puzzle theme. Our idea was to start with simple examples that can be solved without paper and progress towards more complicated ones. The first author who facilitated the tutorial sessions used question and answer approach to provide a scaffold to the students to enable them to make links between puzzle pieces. In this sense, the first author acted as the “knowledgeable other” in facilitating learning (Vygotsky, 1978). The discussions that ensued enabled the students to address misconceptions they might have had.

4.2 A sample of Tarsia Puzzles used in class

Three examples of Tarsia puzzles that we used during the tutorial sessions are shown in Figures 3, 4 and 5.

Figure 3 depicts the Tarsia puzzle based on the topic Integration. The chosen integrals were elementary and could be solved by inspection only. Students were expected not to write on paper but rather to engage in a discussion that will eventually lead them to match pieces of triangles with logical links between them.

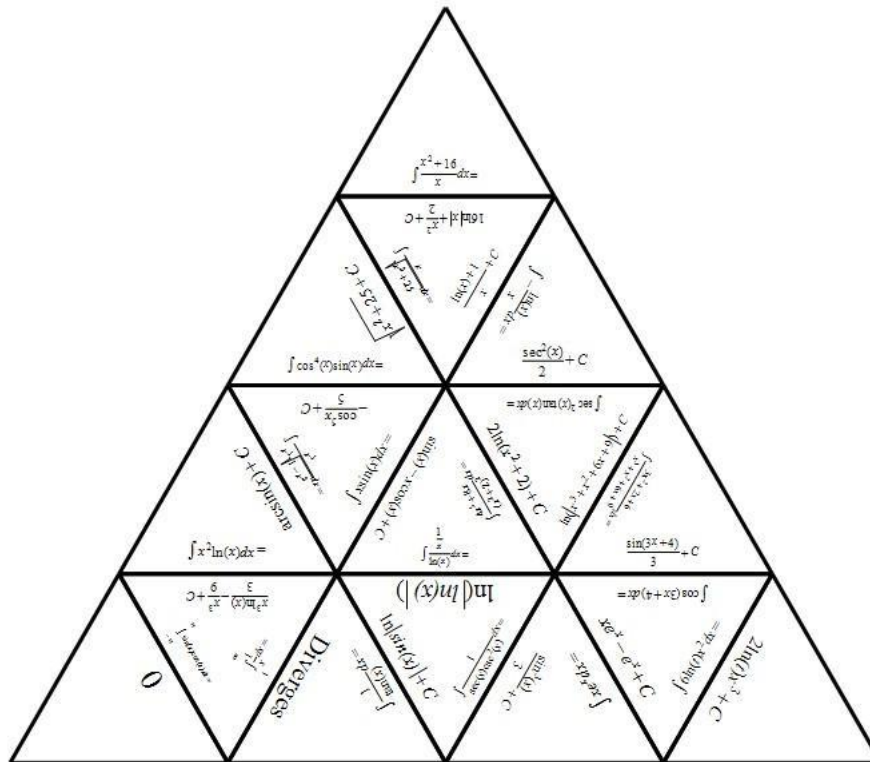


Figure 3: Integration techniques puzzle

The classical domino-looking Tarsia puzzle depicted in Figure 4 was based on the topic Series. The mathematics objects and/or terminologies with logical links were chosen to enable the students to consolidate their knowledge of series and their behaviour.

					Start
					e^x
$a_n = \frac{n^3 + 5}{n + 2}$	Convergent	$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt[3]{n^4}} \right)$	$a_n = (-1)^n n$	Not Monotonic	$\sum_{n=0}^{\infty} \frac{x^n}{n!} =$
Increasing Sequence					
$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) =$					
1	$a_n = \frac{1}{2n^2 + 1}$	Decreasing Sequence	$\sum_{n=1}^{\infty} \left(\frac{3}{2} \right)^{n-1}$	Divergent	Finish

Figure 4: Series puzzle

Figure 4 depicts the Tarsia puzzle on a mix of topics: series, functions of several variables, and partial derivatives. The puzzle was used in a session just before the final exam and was created for students to consolidate their knowledge of the material we believed students found difficult.

The following is the link to [Tarsia puzzles](#).

We observed that the puzzle in Figure , which is a classical domino-style puzzle on series, was the least challenging one for the students. This may have to do with familiarity with classical dominoes. The next challenging puzzle is the one in Figure on integration. We intentionally picked up Elementary Integrals as a theme for this puzzle so that students could engage in completing the puzzle by discussing the links rather than having to compute integrals on paper. Finally, the puzzle in Figure was the most challenging one because it was on mixed topics and included two different geometric shapes of puzzles.

In the next section, we reflect on the learning experience from our point of view as instructors and suggest future directions for research on active learning in undergraduate mathematics using the Tarsia puzzles.

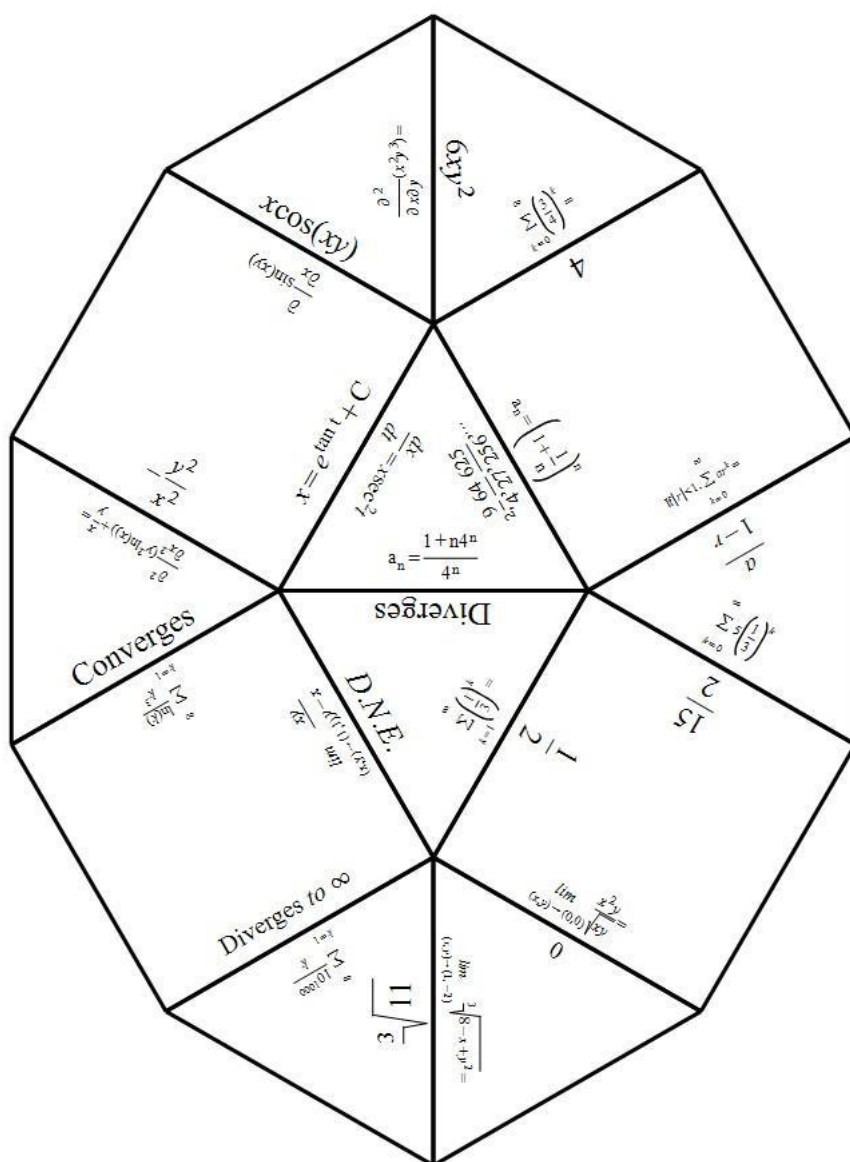


Figure 5: Mixed topic puzzle

5. Reflection, Future Research and Conclusion

This case study was not empirical research. Our intention was to try out the Tarsia puzzles and to get informal feedback from the students, as well as to ascertain the number of students who would participate in this activity. We consider this intervention as a pilot that will be followed up with a full-scale research study with the Research Ethics Board approval.

A number of observations were made during these tutorial sessions, and we grouped them into three themes: enhanced understanding, collaboration, and increased motivation and engagement.

Enhanced Understanding: Tarsia puzzles enable students to engage in discussions about mathematical objects, expressions, equations, terms and terminologies and to deduce the logical connections between them. By actively engaging in discussions about the logical links between mathematical objects, expressions, equations, terms and terminologies and seeking correct matches between pieces of the Tarsia puzzle, students reinforce and consolidate their understanding of the mathematical content they have learned. This active engagement promotes deeper comprehension compared to passive listening or rote memorization (Prince, 2004). When these tutorial sessions were scheduled, the initial thought was that perhaps 1 or 2 students will show up on Friday afternoon from 4 pm to 6 pm. Surprisingly, there was a good attendance every week. It is tempting to suggest that the students who attended were those who were highly motivated. However, after 8 weeks, the first author came to know almost all students who attended. We believe that students came to the tutorial sessions because the kind of activities they were doing, including the Tarsia puzzles, were different from what they would normally do in labs. Lab questions are usually routine textbook exercises, and for these exercises students have to write their own solutions. However, with the Board rounds and the Tarsia puzzles, students worked together and more importantly engaged in discussions and communications about mathematical ideas.

Collaboration: While the Tarsia puzzles can be solved individually, we observed that when solved in pairs or small groups, collaboration can be fostered among students through their discussion and conferring and explaining why logical links exist between pieces. As they discuss potential matches and strategies, students articulate their reasoning and learn from each other. This collaborative environment enhances communication skills and promotes a deeper learning and understanding (Johnson et al., 1998).

Increased Motivation and Engagement: The game-like nature of Tarsia puzzles makes learning more enjoyable. The challenge of completing the puzzle provides a sense of achievement and can motivate students to persist in solving complex problems. This increased engagement can lead to a more positive attitude towards mathematics and greater overall participation in the classroom (Freeman et al., 2014).

The Tarsia puzzles used in the tutorial sessions were physical samples. However, we envision the development of a mobile or web-based app that could simulate the puzzle experience. Such an app would allow these activities to be used in a large lecture setting or on the move, addressing some of the limitations associated with traditional TACTivities. Implementing active learning in the classroom could be challenging, particularly in a large lecture. In labs, where attendance is expected and class sizes are typically below 50, it is more feasible to incorporate active learning strategies. Future experimental research could examine the effectiveness of Tarsia puzzles in promoting engagement and conceptual understanding in mathematics education.

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