CASE STUDY

Using Annotation Sheets to Teach Undergraduates to Learn Proofs and Derivations

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Abstract

On arrival at university, undergraduate mathematics students often struggle to take effective notes in lectures, particularly when they are learning content which requires a high level of understanding, such as proofs and derivations. Annotation sheets, which contain the algebraic steps of proofs or derivations with spaces left for students to add explanations, can provide a structure for taking notes which allows students to focus on the reasons behind the steps. These sheets have been introduced into two applied maths modules and initial observations suggest that they have a positive impact on students' note-taking and mathematical learning.

Keywords: Note-taking, notes, proof, understanding, transition.

1. Introduction

When mathematics undergraduates arrive at university, their approach to note-taking is often very simplistic. As described by Freitag (2020), many students attempt to record verbatim anything that their lecturers write down or display in a written format. This means that students have a complete record of the content they have been taught, which they can then refer back to later. It also encourages them to engage, to a certain extent, in the lecture as they must be continually reading and copying. However, students are often so busy copying what their lecturer is writing, that they do not listen to what the lecturer is saying. These spoken comments can be very important for understanding the written content (Lew et al., 2016) but students who are too focused on copying down written information often overlook them (Fukawa-Connelly et al., 2017).

This is particularly problematic when students are learning ideas which they need to understand rather than just memorise, such as proofs or, in more applied modules, derivations (which I define as procedures using logical steps to produce a given equation or formula from a set of assumptions or known statements). In the case of either proofs or derivations, students often write down the algebraic lines of working, which they might consider to be the "final answer" to a question, without making any note of the explanations for why each step might be expected to follow from the previous one or lead to the desired final result (Lew et al., 2016). Such notes are ineffective for students trying to understand the content later, as they do not contain the key information which aids understanding.

This inability of incoming undergraduates to take effective notes on proofs and derivations is perhaps unsurprising, as school-level mathematics in the UK is far more focussed on mathematical procedures which can be practised and memorised without being fully understood (Darlington, 2014). Although A-Level (pre-university) mathematics examinations do assess students' ability to "construct extended arguments and proofs" (Joint Council for Qualifications, 2021), these skills constitute a small part of the course and are rarely mastered by students achieving modest (B-C) grades. Students therefore often transition to university with little experience of learning proofs and derivations and subsequently find these aspects of their course challenging (Laugwitz et al., 2025; Iannone & Miller, 2019).

In an attempt to address these problems and help students to learn proofs and derivations effectively, some lecturers use partial lecture notes (otherwise known as gappy, guided or skeletal notes). These

notes contain a significant part of the content with strategically placed gaps to be filled in by the student. This gives students time to listen to the lecturer, as they do not always have to be copying down information, whilst also requiring them to pay attention so as to fill in the gaps. There is evidence that partial lecture notes can improve students' understanding during lectures (Feudel & Panse, 2022) and their subsequent academic performance (Cardetti et al., 2010), although this may not always be the case (Machida et al. 2018). Regardless of this, however, the vast majority of students still end up with identical notes either provided by or copied directly from the lecturer (Feudel & Panse, 2022; Iannone & Miller, 2019). This is considered to be a disadvantage by some students who prefer to organise their notes in their own way (Feudel & Panse, 2022) but it also poses the problem that students are not developing note-taking skills whilst at university.

The ability to take notes is a valuable skill for graduates in any profession (BPM Team, 2025; Indeed Editorial Team, 2025), as they will often be required to identify the most important points from a large quantity of written or verbal information and summarise these points succinctly in a format which will be useful for them later on. This applies to numerate careers as much as any others (ACCA, 2025; Ban, 2024) and so it is highly beneficial for mathematics undergraduates to improve their note-taking whilst at university. This is not generally achieved by using partial lecture notes, as students take very few notes and these are usually stipulated by the lecturer, nor is it achieved if students simply copy verbatim from the board.

Overall, it would be ideal to provide a structure whereby students actively learn to take notes which include the reasons behind the steps of proofs and derivations, as well as the algebraic steps and formal statements. These notes would be personal to the student, containing whatever information they needed to understand the content and revise it later. In this article, I introduce a method of note-taking for proofs and derivations with the aim of achieving these goals.

2. Implementation

To support undergraduate mathematics students with notetaking, so-called annotation sheets were introduced into a first-year Classical Mechanics module and a second-year Vibrations & Waves module. Annotation sheets are hand-out-style documents which contain the key algebraic lines of a derivation or proof with appropriately sized gaps for students to write their own notes between the lines. These notes are individual to the student, unlike in many types of partial lecture notes where there is a correct answer which goes in each space (Feudel & Panse, 2022). The students were provided with annotation sheets during lectures and were encouraged to write down whatever they thought would help them to understand or revise the derivation when they looked at it again later.

Some examples of annotation sheets are shown in figure 1. Figure 1A is an annotation sheet used in first-year Classical Mechanics for the derivation of the work-energy principle. Figure 1B shows an example annotation sheet which could be used in a foundational analysis module for the proof of the limit of a sum.

Annotation sheets can be used flexibly, depending on the context in which they are used. For example, in the small-cohort module, the sheets were printed out and provided to the students at the start of each lecture, whereas in the large-cohort module, digital versions of the sheets were provided to the students in advance of each lecture. Students could then choose either to print the sheets for themselves or to annotate them digitally.

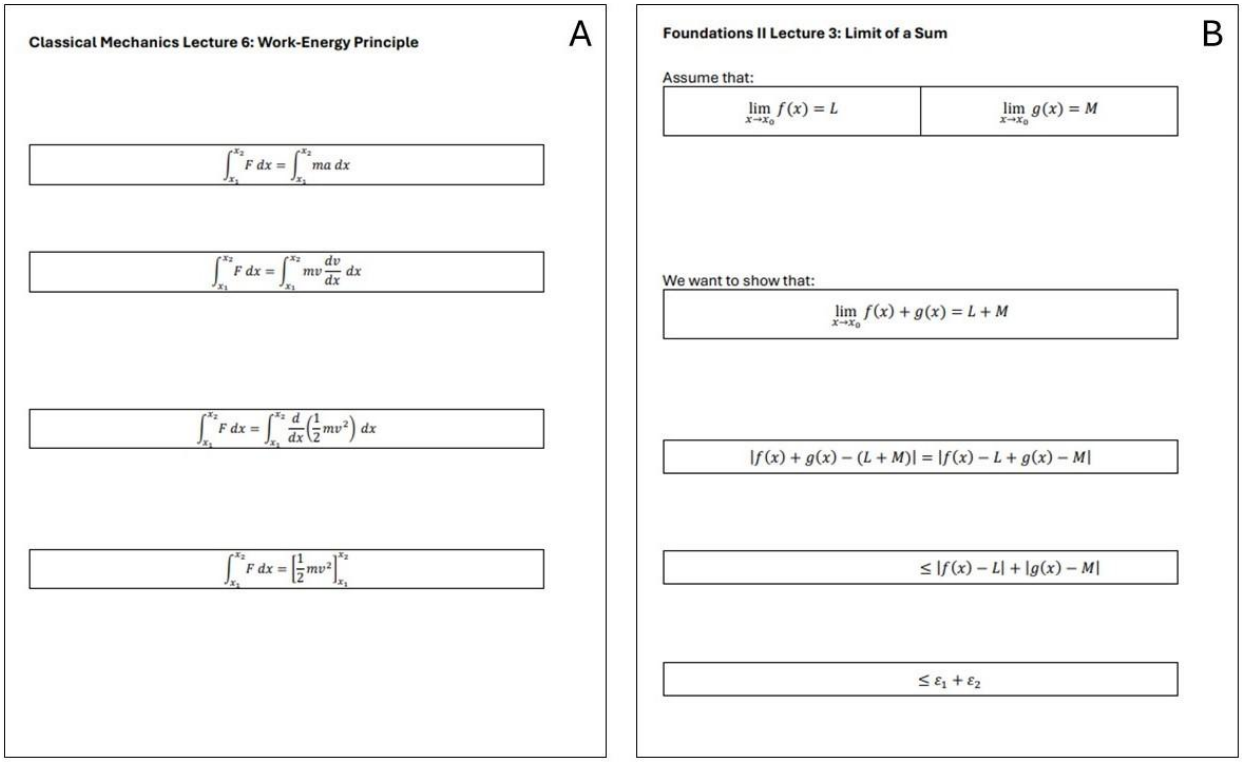


Figure 1. Examples of blank annotation sheets.

Annotation sheets are also very flexible in terms of their content. The lecturer can leave gaps for the students to write down any notes, explanations or lines of working which they think the students will benefit from copying out or writing in their own words. For example, in the sheet for the proof of the limit of a sum (figure 1B), spaces have been left for students to write out the definition of a limit, as well as to make more personalised notes about how the steps follow from one another. This ensures that the students will write out this definition plenty of times, helping them to memorise it.

The annotation sheets were also used to teach the students how to take notes by reducing the scaffolding provided over the course of the semester. At the start of each module, the lecturer showed the students an example of how to fill in the sheet by adding notes to their own sheet under a visualiser. The lecturer thereby provided the students with examples of how they could annotate their sheets, whilst also encouraging them to personalise their notes and only include the content which they found helpful. Over the course of the module, the lecturer provided less guidance on what notes should be included. By the end of the semester, the lecturer explained the derivations verbally whilst making less-structured notes on the board and then gave the students time to formulate their own notes and add them to their sheets. The length of time required by the students varied depending on the length and nature of the derivation but, as students often took some notes whilst the lecturer was talking, it was not generally longer than a couple of minutes.

Figure 2 shows some examples of how the annotation sheet for the work-energy principle could be completed. The notes in figure 2A are relatively sparse. The student has used the abbreviation “N2” for Newton’s second law to justify the first line of working. They have then stated a key formula which is needed to get to the second line and included the method used to show that the second and third lines are equivalent. Finally, they have interpreted the meaning of the terms in the final equation. The notes in figure 2B, on the other hand, are much more detailed. Among other things, they contain a reference back to a previous derivation from lecture 2, a more detailed explanation of the use of the chain rule and an explicit statement that the last line describes the work-energy principle. Both

of these sets of notes are equally valid, as long as they contain the information that these individual students need to understand the derivation.

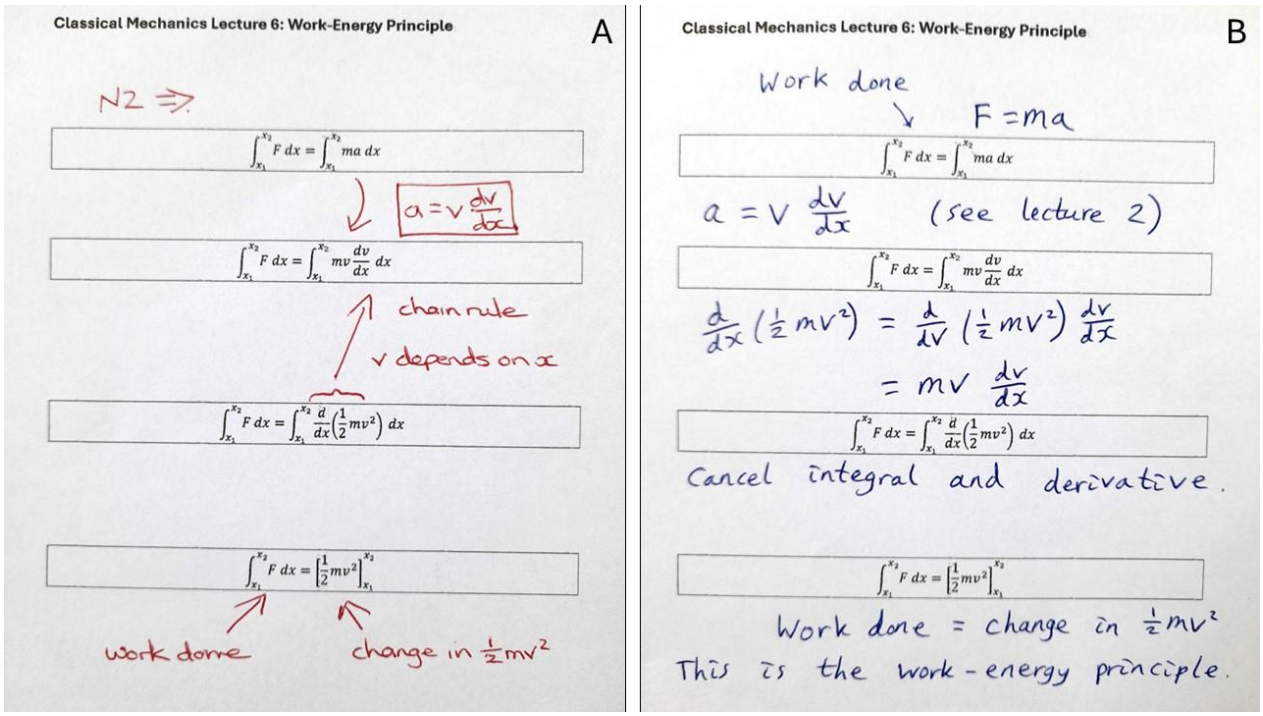


Figure 2. Examples of completed annotation sheets.

To ensure the accessibility of the annotation sheets, a number of different formats were made available to students. All sheets were provided digitally, well in advance of the lectures, in both PDF and Microsoft Word formats. The printed sheets were provided in different background colours for students with visual stress and large-print copies were available on request. Moreover, students were not required to use the sheets if they preferred to take notes in a different way. The lectures were also recorded, allowing any students who could not attend to add their annotations at a later stage. This was important as the module lecture notes contained only the steps from the annotation sheets, without any explanation, to prevent students from copying all of their annotations from the lecture notes.

3. Feedback & Observations

Student feedback on the annotation sheets was extremely positive. When asked, “What has worked well on your module?” in the end-of-module feedback survey, many students mentioned the annotation sheets. In particular, they commented on how the sheets provided structure to the derivations, thereby making them easier to follow in the lectures. The students also said that the sheets provided a more manageable way to learn derivations, as they were broken down into smaller steps. A couple of students on the smaller-cohort module mentioned the benefits of having the sheets printed.

One student also described how they used the sheets for revision. They reported that they practised filling in the gaps on the sheets, then removed the sheets and derived the algebraic lines of working using their understanding of the steps. Another student explained that the sheets had helped them to learn how to take notes in lectures. They reported that they had previously copied down everything

their lecturers wrote on the board, but the sheets gave them the confidence to make more personalised notes for other modules, for instance by annotating their lecture notes.

The students' notetaking abilities in the modules which used the annotation sheets also appeared to improve over time, both when using the sheets and making notes on other aspects of the module. As the semester progressed and the lecturer provided less instruction on how to complete the sheets, the students were noticeably more attentive to what the lecturer was saying and began to take more individualised notes. Some students who had initially copied down everything that was written on the board clearly became more considered in their notetaking, as they began to be selective about what they copied, included additional information in their notes and rephrased content in ways that made sense to them.

The annotation sheets may also have helped to improve students' academic outcomes. Most students scored well on the derivations-based questions in their assessments following the introduction of the annotation sheets and, in revision sessions, students were generally confident in explaining the steps of the derivations, something which previous cohorts of students had struggled with. The annotation sheets also have benefits for the lecturer. They are quick and easy to make, particularly if the derivations or proofs already exist in a digital format (for instance, in the lecture notes). The algebraic steps can simply be copied from the original document into a blank annotation sheet, and appropriately sized spaces added between the lines.

The structure of the sheets also provided an alternative type of exam question. Students were given the algebraic steps of a derivation and were asked to explain how particular lines followed from the preceding lines. This had several benefits over asking students to reproduce the full derivation. Firstly, these questions were much quicker for students to answer, allowing time for more content from the module to be assessed. Secondly, these questions forced students to demonstrate their understanding and so prevented them from obtaining marks simply by memorising algebra with no understanding whatsoever. Finally, the structure of the questions supported students who were struggling; even if they could not remember the beginning of the derivation, they could still show their understanding of later parts. The steps which students were asked to explain in these questions had to be chosen carefully to ensure fair and transparent marking. In particular, steps which rely on one key idea, assumption or formula work well, as any correct explanation of this idea can then be awarded the marks.

The annotation sheets also saved time in lectures, compared with having to allow students the time to copy everything from the board. As well as this, they allowed more flexibility than having to stick rigidly to the structure of partial lecture notes. Of course, time still needed to be set aside during lectures to teach the students how to use the sheets and to allow them to construct their notes. This was potentially more time-consuming than traditional approaches at the start of the module, when the sheets were unfamiliar to the students and their note-taking skills were limited. However, once the students got used to using the sheets, their notetaking became quicker.

Another potential problem is the financial cost and environmental impact of printing the sheets. This can, of course, be mitigated against by providing the sheets digitally. This worked well in the large-cohort module, especially as each sheet was labelled with the lecture in which it would be used, allowing students to prepare their sheets beforehand. Many of the students had tablets which allowed them to annotate PDF versions of the sheets. Others typed their notes into Word document versions, or else printed the sheets themselves and annotated them by hand. However, this does potentially cause a problem for students who lack the necessary technology to annotate sheets digitally or the funds to print their own hard copies.

4. Discussion

By providing a structure for students to take their own notes, annotation sheets can improve students' general note-taking skills and support them to understand the steps of proofs and derivations. They provide a middle ground between traditional chalk-and-talk lectures and lectures based on partial lecture notes, allowing lecturers to provide some material in a more flexible and spontaneous way and yet removing the need for students to copy down large quantities of algebra whilst the lecturer gives valuable oral explanations. Using these sheets, the lecturer can actively encourage students to develop their note-taking skills, rather than removing the need for them to take notes or expecting them to learn this skill on their own.

Annotation sheets are a good way for students to learn and revise proofs and derivations, as they encourage students to focus on the reasons behind the steps, rather than just the algebra and technical wording which constitutes the final derivation or proof. This is highly beneficial for the students' deeper understanding of the content and, by learning the reasons rather than rote-learning the algebra, it provides a much quicker and easier method for learning proofs and derivations in preparation for an exam, which is strongly recommended by lecturers (Houston, 2009; Alcock, 2012). The sheets also enable the lecturer to focus on the explanations more heavily themselves, as they are not spending all of their time writing algebra on the board, which in turn emphasises the importance of understanding the steps to the students.

The purposes of annotation sheets may be best met when they are used with first-year or foundation-year students as they transition to university. At this stage, many students have little experience of note-taking other than the expectation for them to copy verbatim from the board at school (Squelch, 2015). They are also unfamiliar with learning proofs and derivations and may only ever have learned mathematics by practising procedures or memorising methods. By using annotation sheets with new students, they may be prevented from developing unhelpful habits when learning proofs and derivations, such as trying to memorise the algebraic steps without understanding them. Hopefully, they will then be able to develop sufficient note-taking skills as to be able to take notes independently in the latter stages of their course.

Annotation sheets, or some variation on them, may also be useful in disciplines other than mathematics. Undergraduates in chemistry and economics are reported to struggle with note-taking (Bucat & Williams, 1989; Artz et al., 2022) and even students studying English, history and psychology need to develop their note-taking skills at university (Baker & Lombardi, 1985; Locke, 1977). An annotation sheet essentially contains the formal steps of an argument, with spaces for explanations of these steps to be added, and this principle could be applied to a wide variety of subject areas.

In the mathematical sciences, annotation sheets could also be used in Maths Support settings, as well as in formal lectures. Proofs and derivations are common areas of difficulty for undergraduates (Laugwitz et al., 2025) and so are frequently the cause of enquiries in Maths Support. To help with this, a set of blank and completed annotation sheets could be created for those proofs and derivations which are most often poorly understood. These would not necessarily help with note-taking but could still fulfil their purpose of structuring proofs and derivations and encouraging students to focus on the reasons behind the steps.

Annotation sheets could also be beneficial for learning mathematics beyond just proofs and derivations. For example, undergraduate mathematicians are often taught procedures which they are expected to understand as well as be able to implement. Students often focus on memorising how to carry out the procedure (MacBean, 2011) but giving them these steps on an annotation sheet

and encouraging them to add the reasons behind the steps could highlight the importance of deeper understanding.

Going forward, it would be beneficial to trial the annotation sheets in a wider range of modules. Thus far, they have only been used to teach derivations in the applied maths modules which the researcher was delivering at the time, and so it is important to find out whether or not they work equally well in teaching proofs in pure maths modules. Data should also be collected on the effectiveness of annotation sheets for students' note-taking skills and understanding of mathematical proofs and derivations. A more formal survey of students' perspectives might also clarify how students use the sheets and in what ways they find them helpful.

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