CASE STUDY

E-learning/e-assessment systems based on webMathematica for university mathematics education

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Abstract

After-class learning is quite important to understand and master college mathematics. To promote students' after-class learning, we developed the web site "MATH ON WEB", which currently consists of two systems; Web-based Mathematics Learning System (WMLS) and Web-based Assessment System of Mathematics (WASM). Both systems have been developed with web*Mathematica*. WMLS is a system for assisting students' self-learning and WASM is a system for an online assessment. We implemented blended learning environments with these systems and more than 600 students at our university learn with the systems every year. In this article, we summarize the feature of the systems and report the result of our teaching practice for first-year engineering students with these systems.

Keywords: university mathematics education, e-assessment, e-learning, CAS.

1. Background and motivation

1.1. Role of e-learning/e-assessment systems for university mathematics education

After-class learning is quite important to understand and master college mathematics. Traditionally, after-class learning at university level has been promoted by giving students paper-based exercises. Such traditional after-class learning has some problems. If paper-based exercises are given as homework, teachers have to mark students' results. Otherwise, students want to know answers for the exercises. Each mathematics class at university in Japan includes 60-80 students. So marking students' homework every week is hard work for teachers. Should teachers give answers to students? Giving answers to students is also problematic. In college mathematics, there are many mathematics problems whose correct answers are not uniquely determined. In linear algebra, many problems have infinitely many correct answers. For such problems, students cannot check their answer even if teachers give answers to them. Another problem is that students cannot have any support during their work at home. Many students do their homework in the evening, night, or weekend. Therefore, when students have questions about how to solve problems at home, they can neither ask their teachers nor obtain real-time feedback.

E-learning is a useful tool to overcome the above issues and to build an effective after-class learning environment. We can use e-learning system to give and mark homework. Using e-learning system, we can develop a "homework system" without a heavy workload for teachers. Teachers cannot help students overnight, but e-learning systems can help students all day, even at midnight. And e-learning systems enable students to determine whether their answers are correct or not, even when there are infinitely many correct answers for the mathematical problem.

There is another merit of e-learning. To understand and master college mathematics, it is important to understand the meaning of mathematical concepts deeply. And to do so, it is important for students to explore various examples. E-learning is a good solution to develop effective interactive learning materials which promote students' understanding of mathematical concepts.

1.2. Why is CAS needed?

In college mathematics, there are many mathematical problems whose correct answer is not unique. For an example, let us consider the following problem: “Find a parametric representation of the plane in space given by the equation $x+y+z=1$”. Let $(a_1,a_2,a_3),(b_1,b_2,b_3),(c_1,c_2,c_3)$ be any three points, which satisfy the equation but are not on the same line. Then, we always get the correct answer for the problem as follows:

$$(x,y,z)=(a_1,a_2,a_3)+s(b_1-a_1,b_2-a_2,b_3-a_3)+t(c_1-a_1,c_2-a_2,c_3-a_3)$$

where s and t are any real numbers. Conversely, let us consider to determine whether the following student’s answer is correct or not.

$$(x,y,z)=(p_1,p_2,p_3)+s(u_1,u_2,u_3)+t(v_1,v_2,v_3)$$

How can we do that? Since the correct answer is not unique, pattern-matching methods are not applicable. To check the above student’s answer, it is needed to verify the following conditions:

- $(x,y,z)=(p_1,p_2,p_3)$ satisfies the equation $x+y+z=1$;
- $(x,y,z)=(u_1,u_2,u_3)$ and $(x,y,z)=(v_1,v_2,v_3)$ satisfy the equation $x+y+z=0$;
- The two vectors (u_1,u_2,u_3) and (v_1,v_2,v_3) are non-zero and non-parallel.

A computer algebra system (CAS) allows us to verify them easily. Finding a basis for the column space of the given matrix A is another example. For the second example, we need a more complicated algorithm to check students’ answers.

Another reason is that CAS enables us to develop a simulation type of e-learning content. Using rich functions of CAS, we can develop effective interactive contents with which students can learn the meaning of mathematical concepts. We give an example of such content in section 2.2.

2. MATH ON WEB: webMathematica-based e-learning/e-assessment systems for university mathematics education

We developed e-learning/e-assessment systems for university mathematics education and offer them through the website “MATH ON WEB”. Currently, the website consists of the two systems; Web-based Mathematics Learning System (WMLS) and Web-based Assessment System of Mathematics (WASM). WMLS is a system for assisting students’ self-learning; It consists of the drill section and the simulation section (Kawazoe, Takahashi and Yoshitomi, 2013). WASM is a system for online assessment; It consists of online assessment materials associated to learning units (Kawazoe and Yoshitomi, 2016). Both systems have been developed with web*Mathematica* and cover all topics in the standard courses of calculus and linear algebra for the first-year university students.

2.1. WMLS: Drill section

The drill section of WMLS offers an online mathematics exercise environment. It has a large number of mathematics problems in calculus and linear algebra courses for the first-year university students. Actually, it has more than 200 learning units and each unit consists of five problems. Figure 1 shows an example of contents. For each unit, the five problems are presented in a fixed order and students have to solve them correctly in the order. Each ordered list of problems has been implemented carefully from the educational viewpoints. Only when a student solves a problem correctly, he/she can go to the next problem. After when all the five problems are correctly solved by a student, a mark indicating the “completion” of the learning unit appears in his/her learning summary page. There are three types of mark indicating the status of the task: *completed*, *suspended*, and *given-up*. The system shows various data to teachers. Teachers can see the learning status of students in their mathematics class.

Figure 1. An example from the drill section of linear algebra (Kawazoe et al., 2013)

Translation: "For the matrix A, let V be the subspace of R^3 consisting of all solutions of $Ax = 0$. Find the dimension of and a basis for V."

The problem data consists of a problem template (Text), a problem seed (a list of mathematical expressions in *Mathematica* form; sometimes given as a *Mathematica* program which generates mathematical expressions), an answer column (HTML form), an answer analyser (a *Mathematica* program which analyses students' answers and identifies types of errors), a list of feedback messages (CSV), and a problem example with its model answer and a guide for how-to-solve (PDF). Each element in a list of feedback messages corresponds to the error type, and the system shows an appropriate feedback message to a student depending on the identified error type. Figures 2 and 3 show how the system works. Table 1 shows examples of feedback messages for the problem in figure 1. *Mathematica* programs analysing students' answers are written as "Which statement" of Mathematica, and error analysis is done by first-match method. There is no time limit for solving problems. The correct answer is never shown. Students can retry the same problems repeatedly until they get correct answers, reading textbooks, notebooks, model answers and guides for how-to-solve, or asking friends and the teacher.

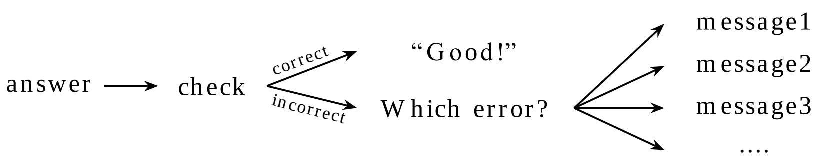

Figure 2. The flow of analyzing students' answer (Kawazoe et al., 2013)

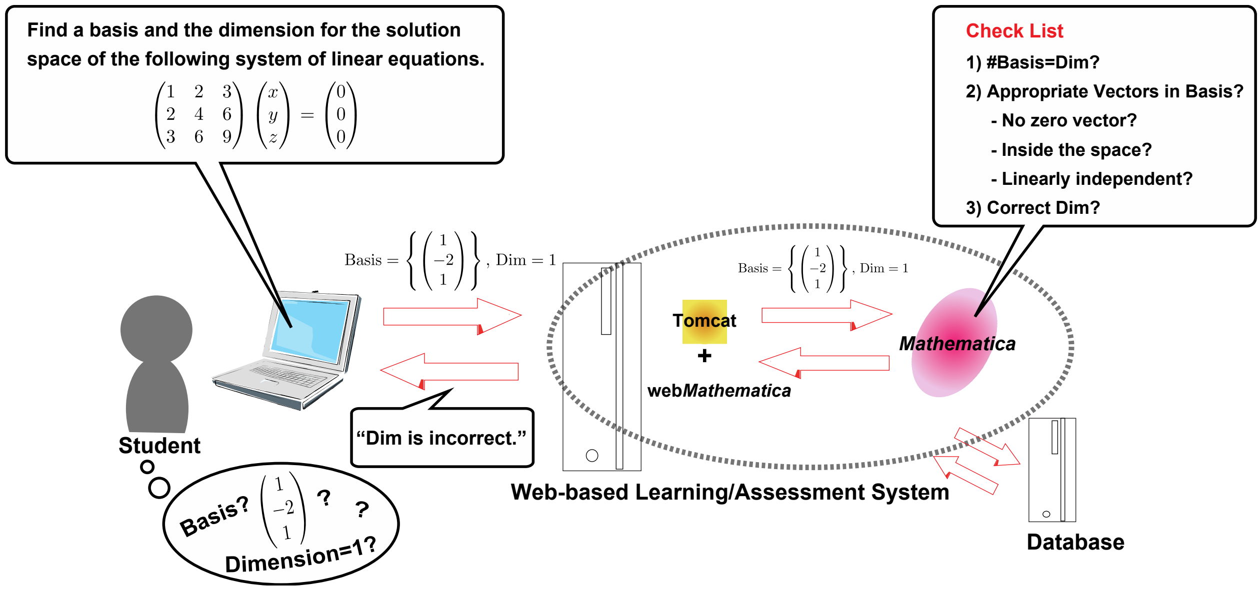

Figure 3. The basic structure of the drill section of WMLS

Table 1. Examples of feedback for the problem in figure 1

Student' answer	Feedback message
Basis=$\left\{\begin{pmatrix}1\\-2\\1\end{pmatrix}\right\}$, Dim=2	The number of vectors in Basis is not equal to Dim. See the definition of basis and dimension.
Basis=$\left\{\begin{pmatrix}1\\-2\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix}\right\}$, Dim=2	Basis contains the zero vector. See the definition of basis.
Basis=$\left\{\begin{pmatrix}1\\-2\end{pmatrix},\begin{pmatrix}2\\-4\end{pmatrix}\right\}$,	Vectors in Basis are linear dependent. See the definition of basis.
Basis=$\left\{\begin{pmatrix}1\\-2\end{pmatrix},\begin{pmatrix}2\\-1\end{pmatrix}\right\}$,	Good!
Basis=$\left\{\begin{pmatrix}3\\0\end{pmatrix},\begin{pmatrix}0\\3\end{pmatrix}\right\}$,	Good!

2.2. WMLS: Simulation section

The simulation section of WMLS offers a simulation type of e-learning contents. A simulation type of content consists of interactive learning material to learn a mathematical concept and assessment material to assess students' understanding. Students learn mathematical concepts with a simulation type of content by varying parameter values and observing the results (figure 4). Assessment materials are the same as in the drill section.

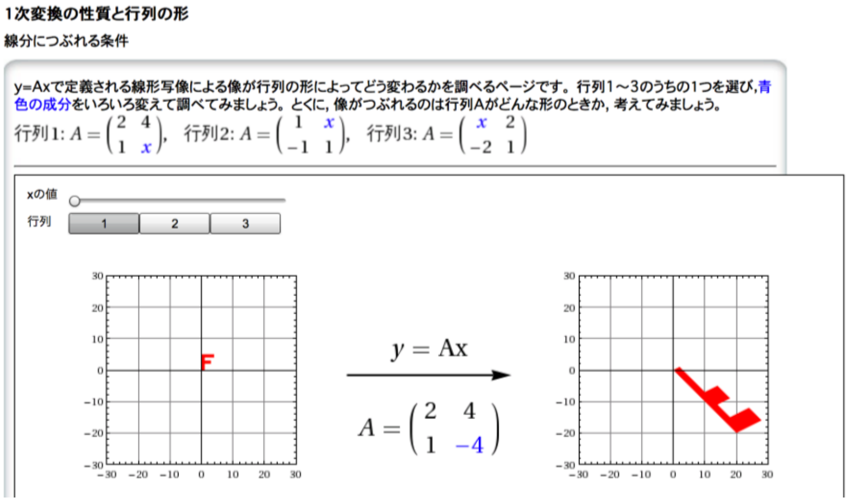

Figure 4. An example from the simulation section of linear algebra (Kawazoe et al., 2013)

Translation: “When does the linear transformation $y = Ax$ map the figure to a line segment?”

2.3. WASM: Assessment materials

WASM has two different modes: Assessment mode and Exercise mode. The exercise mode is almost the same as the drill section of WMLS. The main difference is that problems are presented to students in completely random order or randomly generated by the *Mathematica* program. Using the exercise mode, students can practise solving problems included in an assessment test before they try the assessment. Online assessment materials are associated to learning units and three levels of achievement: *basic*, *standard*, and *advanced.* An assessment test associated to a learning unit consists of multiple problem modules that are presented in random order. WASM has currently 140 problem modules. In the assessment mode, students need to solve each problem within the time limit. We show an example of assessment materials in figure 5.

Figure 5. An example of assessment materials in calculus

Translation: “Find an antiderivative of the following function.”

In WASM, JavaScript-based popup keyboards based on jQuery Keypad (Wood, 2014) assist students to input their answers. In most CAS based e-learning/e-assessment systems, it is inconvenient that students have to input their answers in the input format of CAS. Actually, many of them have difficulty in inputting their answers into the answer fields. Since required symbols and functions differ with problems, the keyboard is provided in different layout depending on the problem (figure 6).

Figure 6. A popup keyboard for the problem in figure 5

In WASM, a tool for content developers has been improved in comparison with WMLS. A problem text, a problem seed, an answer column in HTML-form, a *Mathematica* program for analysing students' answers, a list of feedback messages and so on can all be edited online, while these data have to be uploaded as files separately by content developers in WMLS. In WMLS, handling and managing of variables in input forms of an answer column had been inconvenient to content developers. In WASM, all variables in input forms can easily be managed and handled online in a developer's environment.

2.4. Some remarks on the history of the systems

The project started from 2001 and the first version of MATH ON WEB started with simulation-type materials in 2002 (Kawazoe et al., 2003) and then drill-type materials were added in 2004. All materials are written in Japanese, but all of them are open to the public from the beginning of the project. In the academic year 2005, MATH ON WEB was officially released to all students at our university. Funded by the Japanese government, these materials have been developed as WMLS in 2009. Funded by the government again, WASM has been developed and added to MATH ON WEB in 2013.

3. Effectiveness of the blended learning class with our systems

3.1. A new blended learning class with WLMS and WASM

In the fall/winter semester of the academic year 2014/2015, we implemented a blended learning class with WMLS and WASM for the linear algebra course of the first-year engineering students. We had already implemented blended learning classes with WMLS (Kawazoe, Takahashi and Yoshitomi, 2013), but a blended learning class with both WMLS and WASM was a new approach. The class was designed as follows. Every week, students were recommended to use WMLS for after-class learning and their achievement was assessed with a mini-exam (paper-based test) in the next lesson. For a student who failed the mini-exam, a supplementary exam with WASM was imposed to him/her after class. We illustrate the design in figure 7.

3.2. Result of the questionnaire survey

At the end of the semester, we conducted a questionnaire survey about WASM in the class (63 students in total) and 56 students answered it. The questionnaire consisted of multiple choice questions; especially for measuring the usability and the usefulness of the system, we used a Likert scale. 94.6% of the 56 students answered that they used WASM during the semester. In the questionnaire, we asked the additional questions to those students who used WASM during the semester. When asked for the reason that they used WASM, 84.9% of them answered that they used it because supplementary exams were imposed, while four students answered that they used it because they wanted to check their understanding. On the modes they used, 49.1% of them answered that they only used the assessment mode, while 47.2% of them answered that they used both the assessment mode and the exercise mode. Only two students answered that they used only the exercise mode. On the usability of the system, they answered "very easy" (11.3%), "somewhat easy" (50.9%), "neutral" (18.9%), "somewhat difficult" (11.3%) and "very difficult" (7.5%). On the usefulness for developing mathematical skills and self-evaluating of understanding, they answered "very useful" (18.9%), "somewhat useful" (64.2%), "neutral" (11.3%), "somewhat unuseful" (5.7%) and "very unuseful" (0%). We also asked them which supplementary work they prefer: a paper-based homework or a WASM exam. For that question, 58.5% of them answered that they prefer a WASM exam, while 26.4% of them answered that they prefer a paper-based homework. The remaining 15.1% of them answered that they prefer to use both works. The results indicate that this new blended learning environments using both WMLS and WASM is preferable to students.

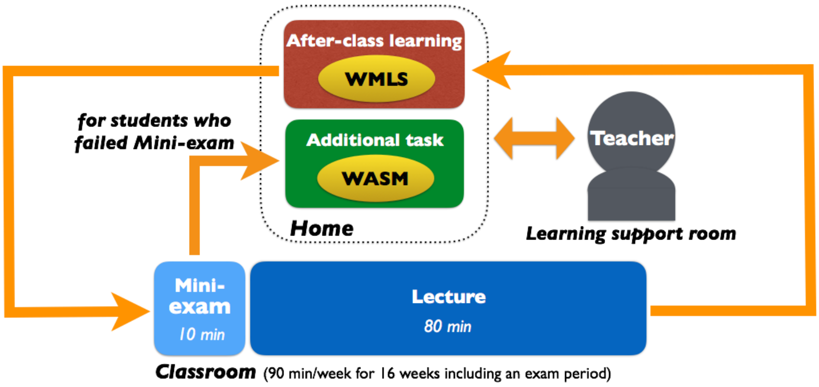

Figure 7. A new blended learning class with WMLS and WASM

4. Future works

4.1. Development of Moodle plugin of WMLS and WASM

WMLS and WASM are standalone all-in-one systems and isolated from any existing learning management system (e.g., Moodle). Most universities have learning management systems (LMS) and use them to enrich students' learning environments and improve the educational qualities. If WMLS and WASM are seamlessly connected to LMS, then the mathematics teaching and learning environments with these systems become much more confortable to students and teachers. To develop a connection to LMS is one of the next goals of our research project. We plan to develop Moodle plugin of WMLS and WASM. The plugin is planned to be able to handle the actual WMLS/WASM problem data. The prototype version of the plugin was developed in 2016 (Nakahara, Yoshitomi and Kawazoe, 2016). We are planning to release the first version of the plugin in 2017.

4.2. Development of a contents sharing framework among heterogeneous systems

E-learning is undoubtedly a powerful tool for university mathematics education. Many systems have been developed and used at many universities in many countries. However, the development of learning content is a heavy workload for content developers. At least in Japan, content developers are usually mathematics teachers. Thus it is very hard for them to develop sufficient quantity of learning content. This situation seems to prevent e-learning becoming widespread. There are some attempts to share content, but they are only for users of the specific system: e.g., Maple T.A. Cloud by Maplesoft (2016), mathbank.jp (Taniguchi et al. 2016) for STACK. To overcome the above issue, we proposed to develop a contents sharing framework among the different mathematical e-learning/e-assessment systems (Kawazoe and Yoshitomi, 2013). As for the structure of content data in the e-learning/e-assessment systems for university mathematics education, the essential part of it is almost the same, we believe. Actually, at least between STACK and our systems, we can manually translate the question/problem data from each other (Yoshitomi, 2013). This fact implies the essential design of question/problem data is mutually compatible in heterogeneous systems. We started the research project called MeLQS (the Mathematics e-Learning Question Specification) to this aim. It is a collaborative research with other researchers who use Maple T.A. or STACK. The goal of the project is to develop a framework to share the MeLQS formatted data via a cloud service, which could be exported to as many as possible heterogeneous systems. In the near future, we hope, will come an era when teachers need only select the question design and use them in their own systems.

5. References

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