CASE STUDY

Experience sharing: Mathematical Contest in Modelling (MCM)

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Abstract

In January 2016, Coventry University's **sigma** Mathematics Support Centre (MSC) funded three students for MCM, a multi-day mathematics competition held annually in the USA. This is organised by the Consortium for Mathematics and Its Applications (COMAP) and sponsored by the Mathematical Association of America (MAA), the Society of Industrial and Applied Mathematics (SIAM) and the Institute for Operations Research and the Management Sciences (INFORMS). In this article the team leader and advisor reflect on their experience.

Keywords: mathematical contest in modelling, real-world problems, interdisciplinary research.

1. Introduction

The MCM is a four-day international mathematics competition for high school students and undergraduates, organised by the Consortium for Mathematics and Its Applications (COMAP), a US-based charity with the aim to improve mathematics education for students of all ages. This involves real-world mathematical modelling where research, analytics and applied intelligence reign along with less-quantifiable factors like timing and luck. It challenges teams of students to clarify, analyse, and propose solutions to open-ended problems. The contest attracts a diverse range of students and faculty advisers from over 900 institutions around the world (see MCM homepage). The MCM is designed to encourage effective discussion supporting informed modelling decisions, improved student problem solving, and to promote technical writing. Students participate as team members rather than as individuals, creating an environment for peer learning and skills development. The MCM has been increasingly popular with rapidly rising numbers of teams participating (Table 1).

Table 1. Numbers of teams participated for the last six years

Year	2011	2012	2013	2014	2015	2016
Number of teams participated	2,775	3,697	5,636	6,755	7,636	12,446

In January 2016, we entered the first Coventry University team, whose student members included Ji Wang (2nd year Mathematics and Statistics), Kaiyuan Lin (3rd Finance exchange student), ChingYi Ng (1st year Mathematics student), along with Dr Aiping Xu, the Manager of Coventry University's sigma Mathematics Support Centre (MSC), as an advisor. In this short article, we reflect upon our experience.

2. The Contest

2.1. Before the contest was open

As a team we went through many past questions and analysed our strengths and weaknesses. The problems tend to be open-ended, and are drawn from all fields of science, business and public policy. Preparation for the contest was in excess of five months (approximately five hours per week per team-mate) and entailed extensive literature review on Mathematical Modelling, Machine Learning, Simulation and Programming, anling with additional topics in mathematics and statistics. This intensive research enabled the team to greatly expand their knowledge of these topics. Coventry University's MSC was of great assistance during this period, having tutors covering a wide range of areas of expertise.

2.2. Our Project

For the contest we were offered a list of six problems (http://www.comap.com/undergraduate/contests/mcm/contests/2016/problems/), consisting of three on mathematical modelling and three on interdisciplinary modelling. After careful consideration, we chose an interdisciplinary modelling problem: 'Are we heading towards a thirsty planet?' (2016 ICM Problem E), which was of interest to the whole team. Moreover, it fell on the areas of data analysis and optimisation, which we have confidence in.

We were asked to address the following six tasks:

1. To develop a model to evaluate a country's ability to produce clean water, which should take into account the dynamic nature of factors that affect both supply and demand.
2. To choose a country or region from the UN water scarcity map (Figure 1) and analyse its water condition using the model built in Task 1.

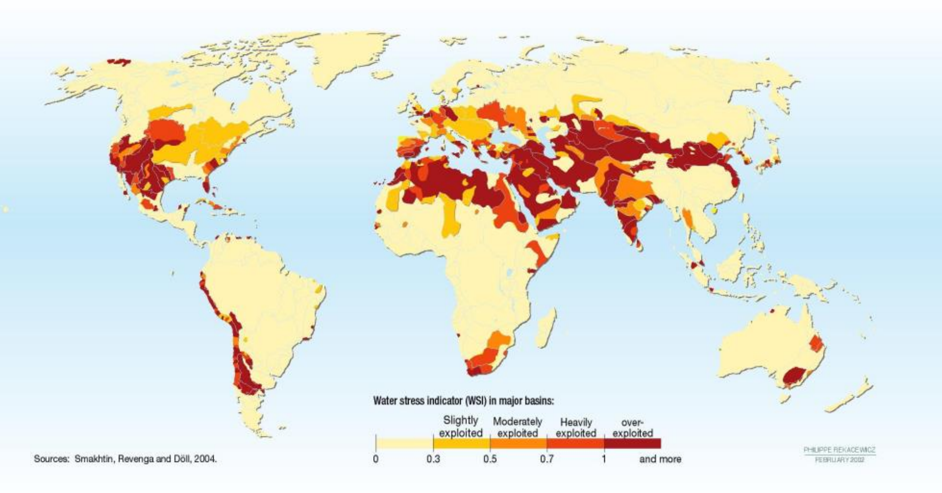


Figure 1. UN water scarcity map

3. To predict what the water situation will be like in 15 years.
4. To design an intervention plan to improve the water situation in the chosen area based on our clean water production ability model built in Task 1.
5. To evaluate the water condition after imposing our intervention plan on the chosen area.
6. To discuss the advantages and disadvantages of our models.

For other users to practise, we recommend starting from the Analytic Hierarchy Process, which provides good theory for the evaluation of problems like these (Saaty 2008).

2.3. Data collection

Clean water production ability can be affected by society, economics and environmental conditions. To evaluate their effects, we carried out some basic analysis, for instance, consideration of the Gross Domestic Product (GDP) as a factor for the economics condition. All economic data was sourced from the World Bank (the World Bank homepage). Regarding water condition data, we checked all the websites of national water resource departments of potential target countries.

2.4. Solution

We started with a Comprehensive Evaluation Model (Figure 2), which enabled us to assess Clean Water Production Ability (CWPA) in light of Water Availability Indicators (WAI). All of the acronyms in Figure 2 are defined in Table 2.

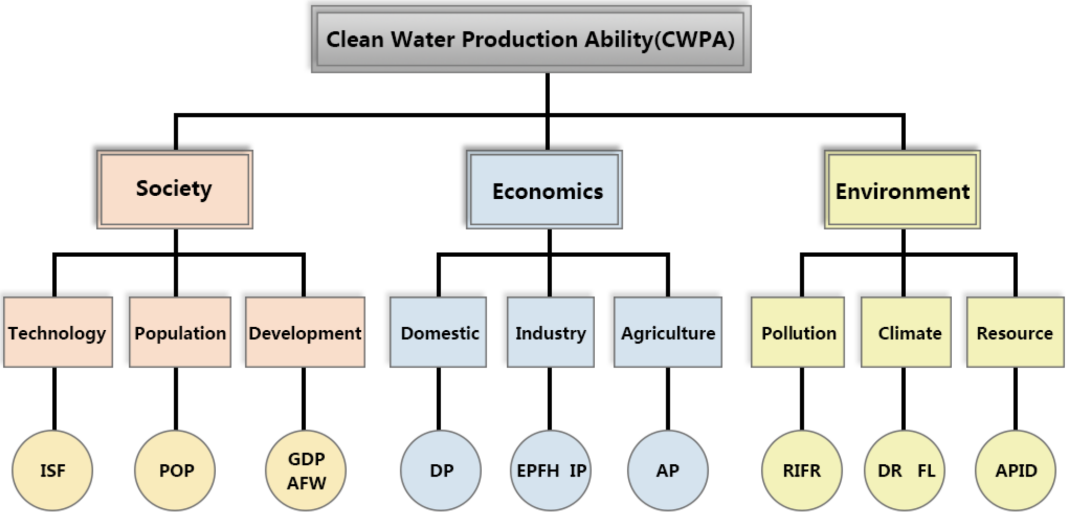


Figure 2. Comprehensive Evaluation Model

Table 2. Definition of acronyms

Abbreviation	Meaning
CWPA	Clean Water Production Ability
DP	Annual Freshwater Withdrawals, domestic (% of total freshwater withdrawal)
AP	Annual Freshwater Withdrawals, Agriculture (% of total freshwater withdrawal)
ISF	Improved Sanitation Facilities (% of population with access)
FL	Flood Occurrence (index)
DR	Drought Severity (index)
EPFH	Electricity Production from Hydroelectric Sources (% of total)
POP	Population in the area
GDP/POP	GDP per capita (dollar)
LA/POP	Land Area per capita (square kilo meter)
AFW/POP	Annual Freshwater Withdrawals per capita (10^3 cube meter)
RIFR/POP	Renewable Internal Freshwater Resources per capita (billion cubic meters)
APID/POP	Average Precipitation in Depth per capita (mm per year)
IP	Annual Freshwater Withdrawals, Industry (% of total freshwater withdrawal)

The model is based on Multiple Linear Regression, which has been proven to be accurate and robust through sensitivity testing. Germany was then chosen as an object for observation because its water resources are moderately exploited in the UN water scarcity map (Figure 1). We have analysed the change in indicators over time, which is essential for determining the future trend for other models. The current situation is interpreted with the model representation and empirical evidence.

Our forecast models included Logistic Regression and AutoRegressive Integrated Moving Average (ARIMA) models for the estimation of indicators. With the Comprehensive Evaluation Model, we could simulate CWPA changes in 15 years without any intervention. A comparison between present and future situations is then made to evaluate the impact of changes including environmental effects on citizens.

We then set up our Best Development Plan and an alternative Minimum Requirement Plan for Germany using Non-Linear Programming. Our Best Development Plan is to figure out the optimal combinations of inductors that lead to the optimal CWPA. The alternative way is to work out the minimum changes in inductors to achieve a given level of water availability. The feasibility of all indicators is explained in this part.

We finished with an Influence Model, which was adopted to estimate the effect of the intervention plan on surrounding areas. Based on Graph Theory, Geopolitics and Game Theory, our Influence Model could measure the systematic influence between countries quantitatively (Figure 3). The weights in the graphical model in Figuare 3 are measured by the ratio of the GDPs between the adjoined countries, which describes the influence of power. The arrows demonstrate the influence direction between countries. For example, Germany is the country that influences all its adjoining countries. The Czech Republic is most influenced by Germany. All the influences shown in the graphical model are negative because we have assumed that all the countries play a zero-sum game.

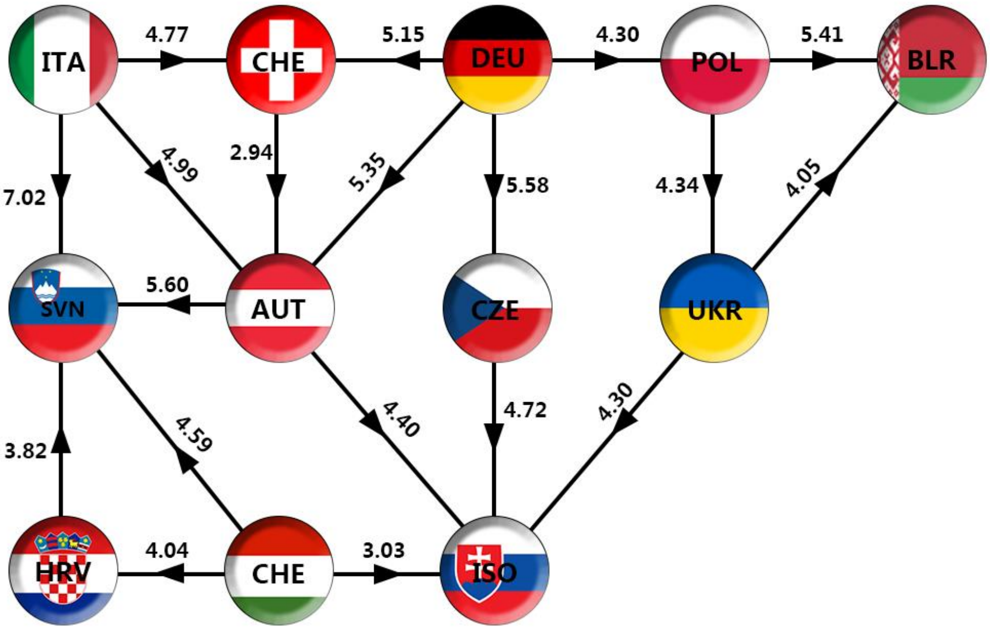


Figure 3. Directed graph with weights in adjusted log-form

2.5. *Strength and Weakness of our models*

Strengths:

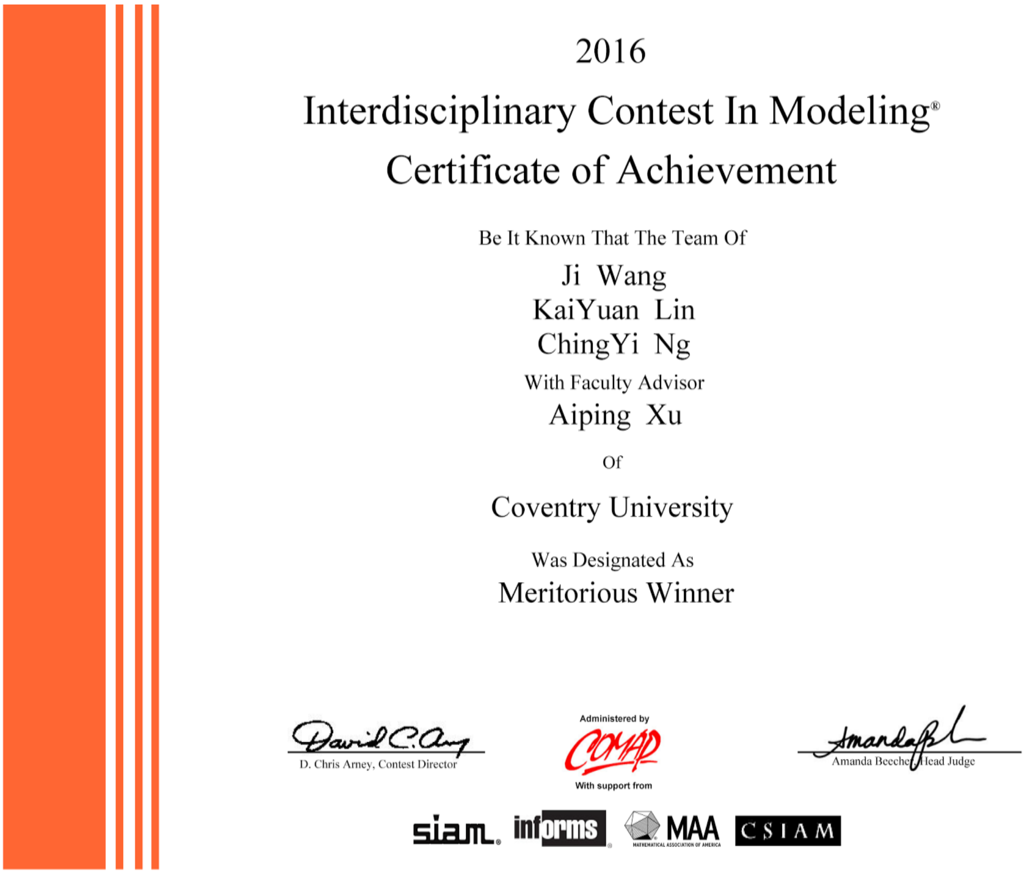
- Different levels of a minimum development plan can be worked out based on different targets in the future, which can be taken into consideration for macro-policy making.
- Most of the proposals are proved to be doable in both the Best Development Plan and the Minimum Development Plan based on analysis.
- The Influence Model measures the systematic influence between countries. The basics are derived from geopolitical concepts.

Weaknesses:

- Restrictions are difficult to set accurately. The range is sometimes vague, depending on historic data.
- The assumptions on the Influence Model are too theoretical.
- Lack of data makes some estimation less accurate in the forecast models.

3. Our achievement

There were 12,446 teams from 900 institutions around the world participating in 2016. The awards and percentages were: Outstanding Winners (0.2%), Finalist Winners (0.3%), Meritorious Winners (12.3%), Honourable Mentions (39.3%), and Successful Participants (46.9%). We are delighted to be a Meritorious Winner (Figure 4).


2016

Interdisciplinary Contest In Modeling®

Certificate of Achievement

Be It Known That The Team Of

Ji Wang

KaiYuan Lin

ChingYi Ng

With Faculty Advisor

Aiping Xu

Of

Coventry University

Was Designated As

Meritorious Winner

D. Chris Arney, Contest Director

Administered by COMAP

With support from

siam. informs MAA CSIAM

Amanda Beecher, Head Judge

Figure 4. Winner Certificate

4. Reflection

On reflection, working out how to apply mathematics and statistics to real-world problems was the contest's most challenging aspect. In order to complete the tasks within the given time frame, we had to comprehend the problems and quickly develop suitable approaches, as well as to read through relevant literature so as to better understand the background and methodologies of related fields. Finding a complete and accurate water condition dataset for the model was more difficult than anticipated. However, we have thoroughly enjoyed the contest. It greatly improved our understanding of mathematics and statistics, especially with respect to solving real-world problems. It required knowledge outside of Mathematics and Statistics (e.g. Ecology and Hydrology for our problem), the development of which was challenging but satisfying. Additionally, our ability to communicate and cooperate effectively within a group whilst under pressure was hugely improved by taking part in the contest.

In hindsight, we would change the following aspects:

- To choose better teammates. It would be beneficial if all the teammates contribute evenly so careful selection of the team is really important.
- To determine which question to focus on at an earlier stage, instead of considering all of them, which turned out to be very time-consuming. We had only 96 hours to research and submit our solutions in the form of a research paper thus time was a big issue.
- To read more widely before the contest. As our problem is interdisciplinary, better models could be produced if we had gained more knowledge of other subjects.

This international contest provided us with opportunities and challenges. We strongly recommend wider participation, where many different skills can be learnt and consolidated.

5. Acknowledgements

All team members are extremely grateful for the support (including financial) that they obtained from the **sigma** Mathematics Support Centre.

6. References

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