SHORT UPDATE

maths e.g. as a learning resource

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Abstract

This update describes the use of the "*maths e.g.*" question database in enhancing any sort of learning material by easily including specific 'Try one yourself' links to any of the over 5000 individual (randomised) questions or allowing student selection from the numerous topics/sub-topics that span the school/university interface and selected service mathematics content. A new question type is also presented to facilitate the (partial) testing of more theoretical material.

Keywords: learning resource, service mathematics, e-assessment, feedback.

1. Introduction

Since 2000 we have been developing the *maths e.g.* e-assessment system at: https://www.mathcentre.ac.uk:8081/mathseg/ for casual use (no sign up required) and a teachers' interface at: https://www.mathcentre.ac.uk:8081/mathsegteacher/teacher.jsp where (after you sign up) tests may be composed by teachers from the 5000 or so question spaces, in a manner similar to shopping on Amazon (but entirely free).

In common with most other e-assessment systems such as Stack, Numbas, Dewis and Webworks, *maths e.g.* uses **question spaces** that encode the algebraic and pedagogic structure of each question which is then realised at runtime by choosing randomised parameters (numbers, words, scenarios). Thus, each question space generates thousands or millions of questions seen by students, thereby allowing virtually unlimited practice. If a student goes wrong, feedback is given with the question's choice of parameters carried through into all features of the feedback (wording, equations using MathML and diagrams using SVG), see Greenhow (2015). This represents a rich learning environment and, being a standard web page, works accessibly on all browsers, PC, Mac or smart phone, using browser-native zoom and translate capabilities and a "Fonts and colours" link to allow a student's display choices to be implemented.

Whilst I make no claims for the efficacy of *maths e.g*. as an e-assessment package, still less present evidence or comparisons with other systems, the experience of remote teaching last year during Covid restrictions suggests *maths e.g*. could be a useful addition to the students' learning, as follows.

2. Embedding resources in curriculum delivery

Last year I sat weekly in-term *maths e.g.* tests, primarily to keep remotely-learning, and possibly assessment-driven, students on task; to assess students I did not need so many marks. Moreover, continual testing did run the risk of downplaying their engagement in the underlying theory, which with no marks associated with it, "didn't count" in the eyes of those students who would benefit most from mastering it. With a return to 'normality', a sensible balance might be to embed e-assessment formatively within such theory, thus breathing life into what they can perceive as otherwise dry material, followed by just two or three in-term e-assessments and a traditional exam. I have no hard evidence that this will work, but anecdotally students do attempt questions, then very actively engage with the feedback, learn how to do a particular question and then follow it with generalising what they have learned in conjunction with the theory they now see the need for. On running a new realisation of a question, I have observed that students initially treat it as completely new, starting from scratch, and then say “Hang on, I have done this question before.” This is where we want students to be, i.e., able to do all questions of that class. The flipped approach of example-to-theory may help, especially as students can self-test their newfound understanding by running another question, or several, before moving on. Without such examples, students may move on anyway and fail to learn much. *maths e.g.* now provides a trivially-easy way to access questions and feedback outside of formal tests and doesn’t require student login, so can be also used for pre-sessional revision or schools outreach material without falling foul of privacy issues.

The interface is based around a tree structure currently comprising 29 main topics and numerous subtopics spanning GCSE, A level and year 1 undergraduate mathematics and some mathematically-oriented topics from Economics, Biosciences, Chemistry and Health. Whilst familiarity with this tree structure is educational in itself, one should not simply point students at the above links and hope they will engage. Even for keen students, it will not always be clear where to find questions: for example, an integration involving partial fractions could be in algebra or integration. This difficulty will be compounded for students tackling new subject areas and, especially, expending effort in answering questions they may, or may not, need. Thus, teachers will need to direct them: I think this is best done by embedding links to individual questions or whole sub-topics into any of your (existing) learning resources that supports web links (word, power point, pdfs, other web pages etc). For an individual question, just run any question and add the link to the url at the top of the question window to your resource. For a topic or subtopic, note its pid number (displayed top right when a topic is selected in the student interface) and edit the following link: https://www.mathcentre.ac.uk:8081/mathseg/topic.jsp?pid=114 (the 114 at the end links to Differentiation\Chain rule\Natural logarithms as shown in Figure 1). It’s as easy as that.

Screenshot of the *maths e.g.* student interface and selected sub-topic.

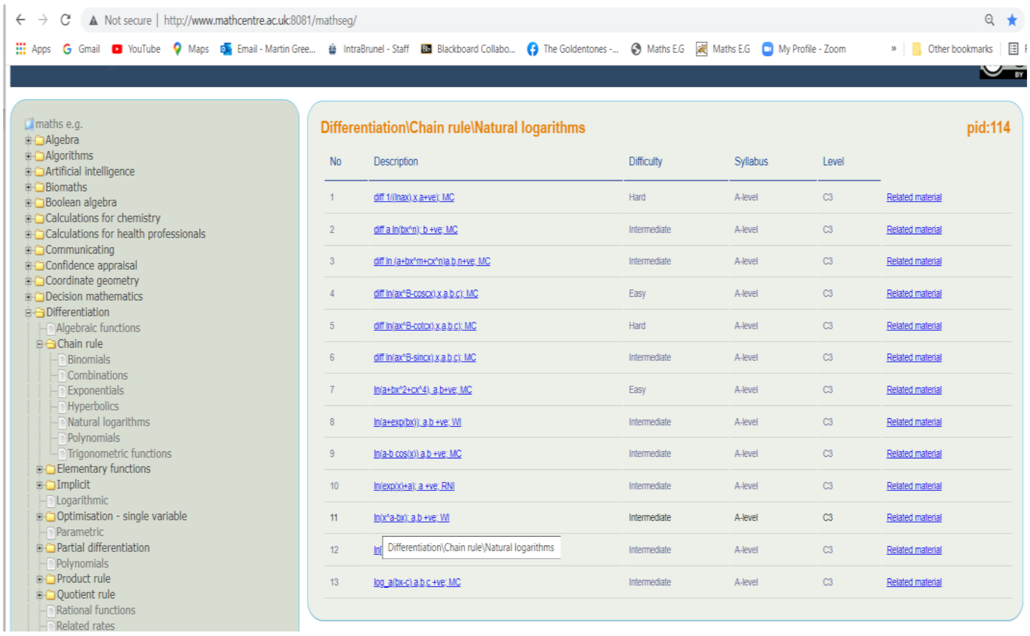

Figure 1. Part of the topic tree expanded on the left and questions in the selected topic on the right, showing pid 114.

It will not have escaped you that *maths e.g.* is a potentially-useful source of questions, generally 'reverse engineered' so that the answers come out nicely, that can be used in traditional assessments and exams. Just take what you want. If you want to re-author any question in any other e-assessment package, or any learning package such as a scripted web page, take a look at the Javascript code using View Source and take what you want. Some questions use global functions which I can provide on request. If you do use *maths e.g.* in this way, please make whatever you create publicly available for all.

3. A new question type

It is obvious that e-assessment can only form part of the student's journey and is limited in addressing our overarching aim of getting students to be able to 'do mathematics'. This undoubtedly involves moving away from standard questions to at least being able to mathematically model, make approximations and find approximate or limiting-case solutions, implement suitable computer packages or numerical methods, make conjectures (and hopefully prove them), make generalisations and extensions, interpret results and finally effectively communicate to others at an appropriate level. Such lofty aims will fail unless students have done the groundwork. e-assessment is an excellent way to provide the necessary fluency they will need to be able to use an array of techniques and tools in unstructured tasks.

There are many different question types available to help students master algorithms and procedures needed in basic algebra and calculus for example, but I have found these difficult to implement effectively in definition-based or analysis-type questions. For the latter, true/false questions are useful and versatile, but for the former I have needed to develop list-based questions, as in Figure 2. Both will be included in the next *maths e.g.* update, but on paper it is not obvious how the question in Figure 2 was randomised. In fact, all correct and any number of incorrect (but feasible) statements are held in arrays from which the code randomly chooses a number (usually between 5 and 8) of statements to display (again in a randomised order). Given that the functionality of the question is independent of these arrays or question topic, the author only really needs to focus on creating incorrect statements. Feedback for such definition-based questions is currently limited to just stating the correct answers, but could be extended to give counter-examples showing why chosen incorrect statements are wrong. Another extension could be to replace the word **must** in the question wording with **must not** or **may** but this has not been attempted yet.

Clearly this question type needs evaluating by collecting student views and success rates. Indeed, a reviewer states: *It seems that more natural input (from student's perspective) would be achieved via checkboxes* and this is a good point. However, for partial credit, the marking scheme becomes complicated: should one reward students for identifying correct choices AND not selecting incorrect choices, and penalise them for not selecting correct choices AND selecting incorrect choices? This could result in negative marks which students generally do not understand and think is unfair. At least for now, the question is marked dichotomously as a mastery question where the response must be fully correct and in the correct syntax (although spurious white spaces are removed before marking). Insisting on syntax may seem picky, but such discipline may help students when later writing their own computer code.

Screenshot of a list question.

In this example Declan is considering using the Intermediate Value Theorem (IVT) on a function f(x) to infer that for some value, c, then:

if u is any number between f(a) and f(b), then there is some c in the interval (a,b) such that f(c) = u.

Which of the following properties **must** the function have in order to apply the IVT theorem?

Given such properties, and any others necessary for the IVT to be applied, which, if any, properties below hold for *c*?

You may assume a < b

1. The function is negative in the closed interval [a,b]
2. c is in the closed interval [a,b]
3. The function f(x) is piecewise continuous on the open interval (a,b).
4. The value(s) of c cannot be determined from IVT theorem.
5. The domain of the function f(x) contains the closed interval [a,b].
6. f(a) > f(c)
7. The function f(x) is differentiable on the closed interval [a,b].
8. The function f(x) is continuous on the open interval (a,b) but not necessarily on the closed interval [a,b].

A =

Important: You must select all properties that apply to the question and nothing else.

Input your answer string in the form of a sequence of increasing numbers e.g. 1,3,9 without spaces, separating each property number with a comma and with no full stop at the end.

If you think none of the above properties apply, input *none*.

Figure 2. A question to test student understanding of the necessary conditions for the IVT.

4. Reference

Greenhow, M., 2015. Effective computer-aided assessment of mathematics; principles, practice and results. *Teaching Mathematics and its Applications*, 34(3), pp.117-137. https://doi.org/10.1093/teamat/hrv012