CASE STUDY

Designing a blended delivery foundation mathematics course: Targeting self-efficacy, algebraic skill development and social connectedness

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Abstract

Foundation mathematics courses play a crucial role in allowing students who have not achieved the pre-requisite mathematics credits for tertiary studies to re-engage with STEM studies. This paper describes how targeting self-efficacy, skill development and social connectedness have influenced the design of a foundation mathematics course at a New Zealand university. These design goals are grounded in education literature, and the paper outlines how focusing on these goals has resulted in the design of a course that incorporates novel approaches to learning and assessment, including online and face-to-face components. These approaches include the use of large, automatically graded weekly quizzes; adaptive learning quizzes for essential skills such as fraction arithmetic; and an emphasis on face-to-face learning and connection through tutorials and collaborative problem-solving workshops. The paper concludes with reflections on the success of the course so far and raising questions for future investigation.

Keywords: Foundation mathematics, self-efficacy, blended, computer-aided assessment, STACK

1. Introduction

Foundation mathematics courses play a crucial role in allowing students to re-engage with science, technology, engineering and mathematics (STEM) studies, when they have not achieved the pre-requisite mathematics for tertiary studies. Such courses provide another avenue to address equity of opportunity in education since many of the students in these courses have been previously disadvantaged in their access to secondary education (Martin et al., 2021). However, retention and providing adequate preparation for students in STEM studies is a challenging problem (Patterson & Sallee, 1986) because of the wide range of mathematical experiences students have when starting these courses (Perkin & Bamforth, 2011). This challenge is also exacerbated by the fact that many students choose not to participate in the support programs on offer (Hillock & Khan, 2019). It is therefore important that tertiary institutions provide foundation courses that enable students to succeed and progress to further studies in their chosen field.

This paper describes how the design goals of self-efficacy, skill development and social connectedness have been incorporated into a foundation mathematics course at a New Zealand university. These design goals are grounded in education literature, and their description for the purpose of this study is shared with the reader, before outlining the course design and the intended effect of various activities within the course. The foundation mathematics course described here acts as a prerequisite for entry into first year calculus.
courses for students who did not study this content in high school. The course usually has about 700 students spread between two semesters, including students with a wide range of mathematics experience and attainment. For example, some students only narrowly failed the prerequisite requirements at school, whereas others haven’t studied any algebra in the final three years of high school. This is a 12-week, 150-hour course with no assumed prerequisite, and it has previously had low levels of student engagement and pass rates as low as 40%.

We begin the paper with an overview of some common issues that arise in the delivery of foundation courses. This reveals several key design goals that the course design needs address: self-efficacy, algebraic skill development, and social connectedness. Identifying these issues gives design goals for the course as well as a framework for evaluating the final product. The second part of the paper outlines the assessment and learning activities for the course and how they work towards these design goals. Activities include a mix of computer-aided assessment alongside an emphasis on face-to-face interactions. We then reflect on initial implementations of the course, including a discussion of how students responded to the various activities and raising further research questions.

2. Literature review

Foundation mathematics courses and remediation have been the subject of much study as both a difficult and important issue for tertiary institutions. Before describing the specifics of the course design, an outline of some of the key issues facing foundation students sets up a framework for the design by providing design goals for the course. The first of these key issues is student self-efficacy, understood for the purpose of this study as low mathematics confidence. The second is the development of essential algebraic skills which we emphasise alongside the more advanced learning outcomes. The final issue we focus on in the course design is whanaungatanga, a te reo Māori term that can be understood here as social connectedness. In this section we also discuss computer-aided assessment as one of the tools we use to achieve these design goals.

Each of the issues mentioned here is a crucial aspect of designing this foundations course and must be addressed in the design, noting that “one of the ways that teaching can take place is through shaping the landscape across which students walk. It involves the setting in place of epistemic, material and social structures that guide, but do not determine, what students do.” (Goodyear, 2015, p. 34). We argue below that each of the goals mentioned in the previous paragraph is an essential part of this landscape. The activities and assessments we incorporate into the course encourage students towards productive activity in line with the constructive alignment approach of Biggs and Tang (2007).
2.1. Self-efficacy

Self-efficacy is “beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments” (Bandura, 1997, p. 3), that is, the belief that one can succeed at a specific task. Figure 1 summarises literature showing that improved student self-efficacy drives persistence, motivation and help-seeking behaviours (see a more detailed version in (Skaalvik et al., 2015)). For instance, Skaalvik et. al. (2015) investigated the effect of each of these connections, and Williams and Williams (2010) found that the reciprocal relationship between performance and self-efficacy was relevant in most of the 33 nations they studied, so it appears that this is a connection that can be fairly broadly applied across cultures. The converse also seems intuitive: if a student believes they can’t succeed at a task then they are more likely to have little reason to persist or seek help if it becomes difficult. Bengmark et. al. (2017) found that not only are self-efficacy and motivation predictors of success in tertiary mathematics, but there is opportunity to improve these during first-year tertiary studies. By targeting student self-efficacy – examining our communications and attitudes as instructors and providing students with opportunities to succeed – we can impact student outcomes in mathematics.

As indicated in Figure 1, experiencing success is known to be one of the main factors that improves self-efficacy as it reinforces the belief that one can succeed at mathematics. Jansen et al. (2013) found that providing opportunities for students to succeed at small tasks, and setting questions at an appropriate level of difficulty, encouraged students to attempt many questions and improved their self-efficacy. According to Lynch (2010), acknowledging success at a larger task through a ‘mastery’ grade above 70 – 80% provides meaningful feedback and encouragement for students (Zientek et al., 2019), who have many experiences of obtaining 50% or less on such tasks. This gives us insight into the factors that should be in place for tasks we set students in the design of the course.

Zientek et al. (2019) argue that teacher support and language choices are also important factors in determining self-efficacy, they also refer to (Hattie & Timperley, 2007) who emphasise the role of feedback in self-efficacy and learning. For example, suppose a teacher states dismissively that a particular exercise is very easy. A first-year student who listens to this but finds the new content difficult could interpret this as meaning that they do
not belong in that classroom. Carefully examining teacher attitudes and language can prevent this scenario, and instead language can be used to encourage students in their persistence and motivation and acknowledge the quality of effort expended (Barton, 2018). Though it can be difficult to manage at large scale, support should be readily available even if it is a combination of lecturers, teaching assistants, and online support.

2.2. Algebraic skill development

Foundation mathematics students often have difficulty with essential algebraic skills, which can prevent a thorough understanding of the material (Hillock et al., 2013). For example, this can manifest as persistent errors or misuse of notation such as negatives (Cangelosi et al., 2013) or the equals sign (Vincent et al., 2015). By identifying and addressing the underlying algebraic misconceptions, rather than just correcting the individual error, we can equip students to develop these skills and transition to an algebraic way of thinking.

The course design needs to identify, and provide support for, these underlying misconceptions. These identified skills require a mastery level understanding, even if mastery thresholds aren’t appropriate for or applied to the rest of the course. Students therefore need consistent feedback on minor errors as well as more conceptual errors. Co-requisite models for these foundational skills have been shown to benefit students (Hillock et al., 2013; Logue et al., 2019) and emphasise that these are skills to be improved and mastered over time rather than attempted in an isolated unit. Repeated, targeted practice is also a useful tool against misconceptions (Hattie & Timperley, 2007).

2.3. Whanaungatanga/ Social connection

Another challenge to address in the course design is to facilitate whanaungatanga; community connections and a sense of belonging among students. The te reo Māori phrase is used here because this is an aspect to learning and engagement that is of particular cultural importance to Māori and Pasifica students, although its importance for the general cohort is also increasingly acknowledged in the literature (Kinnear et al., 2022; Mullen et al., 2022). In order to effectively learn and engage, students must have a sense of belonging not only in the content (self-efficacy), but also in the community and the institution (Kalantzis & Cope, 2010). Explaining and collaborating with peers is one of the best ways to engage with content and facilitate effective learning, according to Lynch (2010) and Sofroniou and Poutos (2016) who found that students who participated in group learning had improved grades. Furthermore, interactions with teaching staff not only provide the most effective means of feedback, explanation and identifying misconceptions, but also play a large role in determining student self-efficacy as discussed in Section 2.1 (see Figure 1).

Encouraging whanaungatanga can be challenging because students often give little value to peer connections and can be hesitant to approach teaching staff. In studying student attitudes towards tutorials, Herrmann (2014) found that students with a “surface approach” to learning (aiming to minimally satisfy requirements rather than taking a “deep approach” and aiming to develop an understanding of concepts) often viewed their peers as “in a neutral sense, fellows in ignorance”. Input from peers was seen as a distractor while they waited for the tutor to give them the correct answer, rather than viewing peer interactions and discussions as part of the learning process. Students with low self-efficacy may also be
hesitant to seek help and can attribute less value to help-seeking behaviours than teaching staff do; they may not view this as central to the learning process (Lynch, 2007). The challenge in designing a course then is to incorporate whanaungatanga into the teaching and learning activities in a way that encourages students to see its value.

2.4. Computer-aided assessment

Computer-aided assessment (CAA) has been used in mathematics education for many years and has included numerous programs such as MathLab, MapleTA and, more recently, STACK (Dorko, 2020b; Hannah et al., 2014; Higgins et al., 2019; Sangwin & Kinnear, 2021). The use of CAA gives scope to include more formative assessment in a large course (Kinnear et al., 2022), which reinforces the value that teachers place on regular practice (Lynch, 2007). However, this must be implemented with care so to encourage effective study techniques and to mitigate risks from relying on automated feedback (Rønning, 2017).

Implementations of CAA often allow students to reattempt a question or assignment after receiving feedback. It has been observed that this leads to cyclic problem-solving activity among students, and that students are likely to finish any problem that they start (Dorko, 2020a; Hirsch & Weibel, 2003). Students also prefer online homework, even in studies where it leads to similar outcomes compared to paper-based homework (Halcrow & Dunnigan, 2012). However, CAA is not without disadvantages to consider, for example the use of CAA can lead to an overemphasis on the final answer to an exercise, at the expense of understanding the process and concepts involved (Rønning, 2017). Students have also been observed guessing answers once they receive initial feedback on an exercise. A reliance on CAA can also exacerbate equity and accessibility issues if students have less access to computers through disability or disadvantage.

Over recent years, there is a growing community of instructors using the STACK Moodle plugin for CAA (Sangwin & Kinnear, 2021). STACK is underpinned by a computer algebra system so has powerful and versatile computing capabilities for both generating random variations of questions as well as grading student answers algebraically (https://stack-assessment.org). This means that students can input algebraic expressions and equations, as well as other mathematical objects such as lists and sets. The open-ended nature of student input means that a STACK-based quiz can closely mimic paper-based homework questions.

CAA will be a necessary component of the course to provide students with sufficient practice and feedback at scale. By implementing CAA using STACK the questions can mostly replicate what would otherwise be on a traditional homework sheet but with the added benefit of immediate feedback.

3. Design overview for course

Here we describe the design of the course and give a brief description of the various learning and teaching activities, including how they will contribute towards the design goals introduced in Section 2. Figure 2 shows how the main activities of the course align with the goals of self-efficacy, algebraic skill development and whanaungatanga (also see Table 1 for an overview of the assessment weighting). Each main course component contributes towards the design goals in some way, while this is highlighted in the diagram below we
elaborate on each aspect throughout this section. It is important that each activity contribute towards the goals, for example we want to avoid further exacerbating any low self-efficacy. While some of the activities described below are individual study, opportunities for whanaungatanga are built into a few aspects of the course design to provide some balance against the individualised components.

![Diagram of course design goals](image)

**Figure 2: Course design goals (arrows show main goals of each activity)**

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Assessment Weighting</th>
<th>Hours per week (total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lectures</td>
<td>In person, livestreamed, recorded</td>
<td>-</td>
<td>4 lectures/week (48 hours)</td>
</tr>
<tr>
<td>Lab Quiz</td>
<td>Weekly STACK quiz, 50 questions, automated feedback</td>
<td>24%</td>
<td>2 – 4 hours self-study per week (36 hours)</td>
</tr>
<tr>
<td>Computer Lab Attendance</td>
<td>Tutor support, working on LQS</td>
<td>6%</td>
<td>2-hour weekly lab (24 hours)</td>
</tr>
<tr>
<td>Core Skills Module</td>
<td>Online videos and CAP quiz (STACK questions)</td>
<td>5%</td>
<td>(4 hours)</td>
</tr>
<tr>
<td>Collaborative Workshops</td>
<td></td>
<td>5%</td>
<td>1 hour workshop, 10 weeks (10 hours)</td>
</tr>
<tr>
<td>Mid-course test</td>
<td>Closed-book, invigilated, written test</td>
<td>15%</td>
<td>1.5 hour test plus revision time (9 hours)</td>
</tr>
<tr>
<td>Final Exam (≥40% on exam required for passing grade)</td>
<td>Closed-book, invigilated, written exam</td>
<td>45%</td>
<td>3 hour exam, plus revision time (20 hours)</td>
</tr>
</tbody>
</table>
3.1. Lab Quiz

The weekly Lab Quiz is the main activity that students interact with each week, grades achieved on these quizzes contributing a total of 24% of the course assessment. Each quiz consists of 50 STACK questions with a ‘check’ button beside each question (Figure 3). Students can check their answers to each question immediately and, if the answer is incorrect, they can resubmit with a penalty of only 10% of the question mark. Up to half of the questions each week cover the content from that week while the remaining questions serve as consolidation for earlier content.

Good feedback is feedback that makes students think (Barton, 2018). Students are only provided with minimal feedback: correct or incorrect. The goal in providing such minimal feedback is for students to check their written working for errors or seek help from a tutor. Students get unlimited attempts at each question with only a small penalty applied to each incorrect attempt. Worked examples and lecture notes are provided, so students could refer to these as a starting point. The online questions also go alongside computer lab classes with a high tutor-student ratio. Tutors encourage students to persist with problems and help them work out the material. The weekly lab quiz provides students with the opportunity to succeed at small tasks (individual questions), thereby contributing to improved self-efficacy. This encourages students to persist at an individual question until they obtain the correct answer, but also values this weekly work by contributing marks towards their final grade.

Figure 3: Instant feedback in weekly Lab Quiz
3.2. Core Skills Module

The Core Skills Module is a small component within the course consisting of online, self-paced content covering fraction arithmetic, order of operations and factorisation. This content requires a mastery threshold so is best suited to self-paced learning, as students will require differing amounts of time to consolidate this learning. These three areas were identified as being a common source of errors and hindering the development of further skills. For example, fractions and ratios are required to understand and apply trigonometric functions, as well as being a useful step in developing a more robust understanding of number. Factorisation is an essential skill that is relied on in most topics throughout the course and has been identified as a topic students often struggle with (O’Connor, 2022).

This module is implemented using a Computer Adaptive Practice Quiz (CAP Quiz) alongside a library of short videos and worked examples. CAP Quiz is a Moodle plugin that uses an ELO rating system (a method of ranking students against the question bank) in order to facilitate a mastery threshold. The CAP Quiz presents students with one question at a time, selecting each question so that students have an estimated 75% chance of getting the question correct. By working through questions until their score reaches a desired threshold, students achieve mastery of the required mathematical skills while being exposed to questions of an appropriate difficulty. Further details about the implementation of CAP Quiz can be found in (Cameron, 2022).

3.3. Whole-year course

Some students find the course content quite challenging and time consuming as they have previously missed background mathematics knowledge and skills. The whole-year course therefore offers students the opportunity to take the same course spread over two semesters. This allows extra time to consolidate their learning without placing a prerequisite requirement on the course. Enrolment is optional and students can change into the whole-year course until about two thirds of the way through the first semester.

3.4. Collaborative workshops

The collaborative workshops are the missing piece to the design so far, and they allow students to meet with each other and meaningfully collaborate on mathematical tasks. Group work and problem-solving are useful for student learning and self-efficacy (Evans et al., 2020; Sofroniou & Poutos, 2016) and the workshops also place an emphasis on communicating mathematical ideas. The tasks cover a variety of mathematical topics and applications and are intended to be accessible to all students while still having potential for extension to higher-level concepts. Workshop topics expose students to a variety of mathematical fields including cryptography, number theory and probability. For example, one workshop explores graph theory through map colouring and encourages students to explore open problems and pose their own mathematical questions and hypotheses by creating variations on the original problem.
4. Reflection and Discussion

Here I (the author) reflect on initial evidence for how the course implementation met the design goals of student self-efficacy, skill development and whanaungatanga. The diagram in Figure 1 shows that self-efficacy drives persistence, motivation, and help-seeking behaviours, so observing these student behaviours would give indirect evidence of self-efficacy. The author obtained ethics approval to publish the following data (Ref: HEC 2021/116).

Initial gradebook data from the course implementation reveals evidence of student persistence, as seen through the continued engagement and high completion rate of the weekly lab quizzes. For example, in Semester 1 of 2022, 64% of students obtained at least 75% in at least 10 of the labs, and a further 11% of students obtained at least 60% in at least 10 of the labs. This data points towards a high level of persistence on a week-to-week basis. In the lab classes, I also observed improved student persistence and help-seeking behaviours as they approached questions. At the start of semester, I observed that students who got a wrong answer would often just move to the next question or seem stuck, neither asking for help nor knowing how to proceed. Tutors were encouraged to intervene at this point because they could observe that the answer was wrong. However, as the semester progressed, I also observed many students start by checking their working to identify errors, and then either look up similar examples or ask a tutor for guidance. This problem-solving behaviour demonstrated an increased persistence to continue working on each problem until it was correct, seeking out the appropriate resources. Similar behaviour has also been observed in the literature in response to online homework (Dorko, 2020a). Further research on the effect of the lab quizzes and feedback structure would be very useful especially as
many institutions increasingly rely on STACK questions and the potential they have for detailed feedback and solutions.

1. **How does this feedback model effect student confidence and motivation?**

Students had different motivations for taking the whole-year course: time-management and wanting to spend less time per week on the course; self-awareness that they needed extra time to understand concepts in order to pass the course; wanting to consolidate skills and aiming for a high grade. In the small cohort, several students worked hard consistently throughout the year and achieved their goal grade whereas others seemed to fall behind and disengage or withdraw from the course. This alternative pathway is still a work in progress and leads to the question:

2. **How can we best support and engage students who need a longer timeframe to consolidate content from this foundations course?**

Many students enjoyed the collaboration and exploring new mathematical problems in the workshops, as evidenced by responses to a reflective question in the test. Multiple students responded that they appreciated the collaborative aspect, and/or that they discovered that mathematics is a fascinating and varied subject, not just restricted to algebra and arithmetic. However these responses were not universal and some students resisted spending time on activities that did not directly affect their exam performance.

3. **Investigate student attitudes and how the workshops influenced students’ views of mathematics and collaborative activities.**

The new course design appears to be successful so far, with improved student engagement and pass rates. The questions outlined above point towards further research to investigate the identified goals of self-efficacy, skill development and whanaungatanga.

5. **Acknowledgements**

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6. **References**


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