Contents

For information about how to submit an article, notifications of new issues and further information relating to *MSOR Connections*, please visit https://journals.gre.ac.uk/index.php/msor.

This journal is published with the support of the **sigma** network and the Greenwich Maths Centre.

Editorial

Tony Mann, School of Computing and Mathematical Sciences, University of Greenwich, UK
Email: a.mann@gre.ac.uk

Welcome to the first issue of *MSOR Connections* for the academic year 2023/24. We are happy to present articles about mathematics and statistics support in the UK, Czech Republic, New Zealand and Norway. Bailey, Claridge, and Partner present a research report on students' views of calculus; Cameron describes a blended delivery foundation course; Joy, Curran, Webb and Capinski provide a case study about helping a blind mathematics student; Pawley and Neale present a report on a workshop exploring how mathematics and statistics support and teaching are evolving post-pandemic; and Rebenda, Pátíková, Chvátal, Rogovchenko, Bakke, and Croft report on the concluding meeting of a major international project.

The short update by Glaister provides charts presenting data on A-level results in England between 2019 and 2023. The author's position means that he cannot comment on the data provided, but we feel that the data is sufficiently important for our readers to merit publication in *MSOR Connections*. If any readers wish to provide analysis of this data, we would be happy to consider their submissions for publication in a future issue.

As always, we are grateful to the authors for their contributions which we are sure readers will find useful, informative, and sometimes provocative.

MSOR Connections can only function if the community it serves continues to provide content, so we strongly encourage you to consider writing research articles or case studies about your practice, accounts of your research into teaching, learning, assessment and support, and your opinions on issues you face in your work.

Another important way readers can help with the functioning of the journal is by volunteering as a peer reviewer. When you register with the journal website, there is an option to tick to register as a reviewer. It is very helpful if you write something in the 'reviewing interests' box, so that when we are selecting reviewers for a paper we can know what sorts of articles you feel comfortable reviewing. To submit an article or register as a reviewer, just go to http://journals.gre.ac.uk/ and look for *MSOR Connections*.

THIS PAGE INTENTIONALLY LEFT BLANK

tf

RESEARCH ARTICLE

Investigating students' perception of the importance of calculus: a cross-discipline comparison to inform module development

Joseph D. Bailey, Department of Mathematical Sciences, University of Essex, Colchester, Essex, UK. Email: jbailef@essex.ac.uk (contact author)
Jessica Claridge, Department of Mathematical Sciences, University of Essex, Colchester, Essex, UK. Email: jessica.claridge@essex.ac.uk
Alexander Partner, Department of Mathematical Sciences, University of Essex, Colchester, Essex, UK. Email: akpart@essex.ac.uk

Abstract

This study compares perceptions of calculus across disciplines in university education. As highlighted by Rasmussen et al. (2014) more evidence is needed to understand the "relationship between calculus and the client disciplines of engineering, physics, biology, and chemistry", with calculus courses often designed from the perspective of mathematicians. This work aims to provide insight as to when it is appropriate to group different disciplines together for taught calculus modules in Higher Education (HE).

This short study assesses how students perceive the importance of calculus across disciplines including Mathematics, Electronic Engineering, Economics and Business.

Specifically, we consider the following:

1) Are there differences in how students from different disciplines perceive the importance of studying calculus?

2) Do students view the field of calculus as: something to be learned to pass their course; something to be fully understood; or a tool for future study/career?

Whilst this small study cannot provide definitive answers to these open yet important questions, the work presented here does reveal that students in Mathematics and Economics had mostly similar and positive perceptions about calculus. In contrast, however, Business students often viewed calculus differently when compared to the other disciplines. Therefore, these results suggest some potential groupings for teaching cross-discipline- calculus. Importantly, they also suggest how further research should develop regarding how such groupings could affect attainment, pass rates and other pedagogical considerations noted in HE calculus modules.

Keywords: calculus, undergraduate, mathematics, cross-disciplinary, student perception.

1. Introduction

Calculus education underpins many undergraduate programmes including Mathematics, Engineering, Computer Science, Economics and Business Studies (Hagman et al, 2017). As such, it can be used to serve multiple purposes; from being a foundation for preparing students for later modules, to being a benchmark indicating the level of mathematical ability required for the discipline being studied (Hagman, 2019). This, along with the volume of topics covered under the banner of *calculus*, means that such courses will often contain a large amount of content which can exacerbate or create negative feelings within students towards the STEM field as a whole (Hagman et al, 2017). This can lead to calculus modules being linked with various challenges within Higher Education (HE) such as STEM degree drop-out rates (Rasmussen & Ellis, 2013; Leu, 2017), failure or underachievement in degree courses (Eng et al., 2010) and under-preparation for future professional careers (Hensel & Hamrick, 2012). As such, it is necessary to carefully consider the teaching environment, including how to group students from different disciplines in order to increasing students' perceptions of the importance of calculus., This, in turn, could help to reduce the above-mentioned challenges which are specific to calculus and allow for their underlying causes to be better explored.

Due to calculus' ubiquity across many STEM disciplines, it is often the subject which is used to trial new or alternative teaching styles and learning environments. These include flipped-learning classes (Sahin et al., 2015), the use of virtual environments (Bognar et al., 2010) and other technological setups and inclusions (Tall et al., 2008; Kay & Kletskin, 2012; Sevimli, 2016) as well as educational initiatives such as peer teaching (Weulander et al., 2016) and self-regulation (Johns, 2020). However, there is little existing research which has demonstrated how students engage with calculus itself, especially given that students tend to be taught in broadly the same manner with similar content regardless of their field of study (Czocher et al., 2013). Hence, there is a need to explore and understand how calculus relates to the separate disciplines in order to identify specific content that is the most beneficial for students and, therefore, develop individual curricula which reflect the needs and interests of different programs. This should increase students' interest and engagement with the content itself, with the added benefit of exploring different teaching approaches (Rasmussen et al., 2014; Hitt & González-Martín, 2016).

Understanding how students view the usefulness of learned material is necessary since perceived difficulty and importance can be the most significant factor of a student's behaviour (Ting & Lee, 2012) and academic success (Harackiewicz et al., 2016). Students' perceptions of the importance of a topic can vary depending on many factors including: the learning environment (Ahmed et al., 2018; Yoo & Kim, 2019); their preconceived notions of the topic (Ferreira & Santoso, 2008); the module structure (Tudor et al., 2010); and the use of relevant assignments (Fedesco et al. 2017). It is known that to increase perceived importance of a subject one needs to develop problem solving skills and increase attainment, as well as incorporate applicable examples of mathematical concepts which are relevant to one's field of study (Marrongelle, 2001; Osman et al., 2013; Willmot & Simms, 2018). Therefore, understanding the motivation and key areas of importance for different disciplines allows educators to tailor module content to their students. However, calculus cohorts are often treated uniformly with little analysis on the subtleties in perception and motivation across different disciplines which clearly identifies a research gap that needs addressing (Hitt & González-Martín, 2016; Rasmussen et al., 2014).

When designing modules, disciplines may be grouped together without careful consideration for the differences in learning styles and motivations. For instance, different disciplines may have different overall aims for learning a module, varying from fully understanding the underlying theory, to simply providing a mechanism to cope with later modules. It is common for first year university students who intend to study a range of STEM and quantitative degrees to study the same calculus module, a practice that is common across the world in HE such as Scotland (Kinnear, 2018), America (Rasmussen & Ellis, 2013), Brazil (Maderia et al., 2019) and Malaysia (Tang et al., 2013). This is often the case as it is assumed that since the underlying content needs to be taught in various degree schemes, students can be taught together to minimise teaching strain within departments. However, differences in learning styles across disciplines studying calculus have been noted (Alamolhodaei, 1996; Dündar, 2015; Johns, 2020), implying some combinations may be more appropriate than others. Indeed, groupings can be inconsistent even within the same university institution. For example, the University of Essex will teach calculus content to Economics and Business students together in a foundation year (Year 0), whereas Economics and Mathematics students will be taught calculus content together in Year 1. Which leads to the question; which, if any, such groupings are appropriate when teaching across disciplines?

Here, by using a simple survey study, we look at how students perceive the importance and relevance of calculus depending on their degree scheme. The aim is to provide evidence of similarities or dissimilarities that can lead to more appropriate module groupings, minimising repeated taught courses across HE institutions whilst ensuring that students are taught in the manner most beneficial for their discipline's requirements.

2. Research Questions

We consider the following open and important questions:

1) Are there differences in how students from different disciplines perceive the importance of studying calculus?

2) Do students view the field of calculus as: something to be learned to pass their course; something to be fully understood; or a necessary and useful tool for their future study and/or career? Does this vary by discipline, by gender, by progression through HE?

We aim to give an initial indication of disciplines which may be appropriate to group together for teaching according to students' differing interests and perceptions. We hypothesised that Mathematics students would see the direct relevance of calculus to their degree scheme and therefore understand the need to study the underlying principles. By contrast, we hypothesised that students of Economics, Business Studies and Engineering will perceive calculus as a tool or a set of rules to be used to answer applied problems i.e., less interest in understanding why the techniques work and little concern about how all the methods from across calculus are linked together.

3. Materials and Methods

This study was approved by the ethics committee of the University of Essex (ETH1920-1755). As an incentive, participants were automatically entered into a prize draw to win a £20 Amazon voucher upon successful completion of the survey.

3.1 Participants

Participants were recruited based on whether they had studied a first-year module that incorporates calculus content, in their respective departments of either; Mathematical Sciences (M), Electronic Engineering (EE), Business (B) or Economics (Ec) at the University of Essex. The modules studied were: MA101 - Calculus; CE142 - Mathematics for Engineers; BE300 - Quantitative Methods & Finance; and EC115 - Methods of Economic Analysis.

A total of 145 responses were used in the analysis; the demographic breakdown is given in Table 1. Of these, the largest set of respondents had studied the Business module (n=57) with a similar number taking the Mathematics (n=34) and Economics module (n=33) and fewer taking the Electronic Engineering module (n=21). These reflect the comparative sizes of the departments. The responses were a majority male (n=86) with first year students having the highest number of responses (n=68), compared to second year/year abroad students (n=45) and final year students (n=33). In general, the small sample size can raise methodological concerns, however, our numbers are comparable to other similar studies (Newton & McCunn, 2015; Darmaji et al., 2019; Ballantine & Larres, 2004).

Table 1: Demographic characteristics of the study population.

Discipline	Total responses	Gender				Year of Study (current)			
		Male	Female	Other	Prefer not to say	1st	2nd	Final	Placement/ Year Abroad
M	34	*19*	*15*	*0*	*0*	*13*	*12*	*9*	*0*
B	57	*35*	*22*	*0*	*0*	*29*	*13*	*11*	*4*
Ec	33	*15*	*17*	*0*	*1*	*13*	*9*	*10*	*1*
EE	21	*17*	*2*	*1*	*1*	*12*	*6*	*3*	*0*
Total	**145**	***86***	***56***	***1***	***2***	***67***	***40***	***33***	***5***

Disciplines specified as Mathematics (M), Business (B), Economics (Ec), Electronic Engineering (EE).

3.2 Materials

Participants were asked to complete some initial demographic questions followed by 16 items that assessed their perception of calculus using statements and a 5-point Likert scale (*1=strongly disagree, 5=strongly agree*). Participants were also asked to complete a multiple-choice question asking them to select the sentence which most described their opinion of studying calculus. The questions used in this study are given in Table S1 in Appendix A. Questions were designed to give insight into the perceived importance of calculus and its role within the larger context of the degree and work prospects. To ensure clarity and consistency, students were instructed to use the definition of *calculus* as:

> *Calculus only includes topics such as differentiation, integration, algebraic manipulation, sequences and series, trigonometric functions, vectors, complex numbers. This does not include any probability, statistics nor financial content.*

3.3 Procedure and Experiment

The survey was compiled in Qualtrics (https://www.qualtrics.com). All qualifying students from the chosen departments were emailed with details about the study that included a link to the survey which they were invited to complete in July 2020. The survey was available for one week after participants had completed their end of year examinations after which time the survey was closed and the link was disabled. Students were advised that responses would be kept anonymous but may be linked to group-level outcomes and used in publication or further research.

3.4 Data Analysis

Descriptive statistics for each question were found by analysing the mean composite score for each Likert scale question (Boone & Boone, 2012; Kale, et al. 2015). Significant differences between responses were calculated using Mann-Whitney-Wilcoxon tests (Sullivan & Artino, 2013). All analysis was performed using R (R Core Team, 2020).

4. Results

Table 2: Means and standard deviations of the responses to the items in the calculus questionnaire described in Table S1.

Item	Mean				S.D.			
	M	B	Ec	EE	M	B	Ec	EE
1^	2.03†	2.88*	2.15	2.57	0.953	1.019	0.906*	1.248†
2^	1.97	2.70*	2.09	1.71†	0.999	1.149	1.182†	0.956*
3	3.71	3.82†	3.67*	3.76	1.00	1.037	0.990*	1.221†
4^	1.79†	2.79*	2.00	2.62	1.122*	1.292	1.270	1.465†
5	4.44†	3.53*	4.31	3.81	0.959	1.136	0.78*	1.250†
6	4.29†	3.18*	3.94	3.71	0.938	1.269	0.914*	1.419†
7	3.03	2.98	3.09†	2.81*	1.058	1.009*	1.088	1.209†
8	4.44†	3.47*	3.84	3.50	0.894*	1.197	1.036	1.395†
9	4.26†	3.70*	4.03	3.70	0.864*	1.195	1.016	1.380†
10	3.94†	3.49*	3.84	3.50	1.043*	1.151	1.068	1.504†
11	4.27†	3.37*	4.10	3.45	0.674*	1.080	0.978	1.468†
12^	2.58*	3.21†	2.68	2.80	0.867*	0.940	1.166	1.322†
13	4.03†	3.26	3.55	2.75*	0.951*	1.188	1.060	1.209†
14	3.84†	3.16	3.52	2.85*	0.939*	1.099†	1.061	1.089
15	3.64†	3.49	3.52	3.25*	1.270*	1.297	1.313	1.517†
16	3.27†	3.11	2.94*	3.15	0.977	0.859*	1.031	1.268†

*Mean and standard deviation values that are denoted with † and * symbols indicate the highest and smallest values, respectively, for each item. Items that are denoted with a ^ symbol indicate items which were negatively coded and so the symbols for the highest and lowest values are also reversed for ease of interpretation.*

Mathematics students gave the strongest opinions with the highest mean score for 12 questions, along with the lowest standard deviation for 10 questions, indicating that their responses were the most aligned at the intradisciplinary level (Table 2). Whereas, Electronic Engineering students had the widest range of answers, demonstrated by having the highest standard deviation in 14 questions.

When making comparisons between disciplines, Mathematics and Business were seen to differ the most with significant differences in responses in nine items (Q1, Q2, Q6, Q8, Q9, Q11, Q12, Q13, Q14; Table 3 & Table S4). Business and Economics had significant differences in two items (Q1, Q6), whereas, Mathematics and Economics had no significant differences. Four items were significantly different between Mathematics and Electronic Engineering (Q8, Q11, Q13, Q14), one item was significantly different between Business and Electronic Engineering (Q2), and no significant differences were found between Economics and Electronic Engineering (Table 3 & Table S4).

Only five items (Q3, Q7, Q10, Q15, Q16) were answered similarly by all disciplines (Table 3). Q3 and Q7 explored whether students believed they needed to repeat methods and techniques to succeed in calculus rather than concentrate on the underlying principles; with which the majority of students agreed. Q15 asked about the novelty of the calculus content in the course and returned the most varied responses for all disciplines across all items (demonstrated by having the highest standard deviation for each discipline [Table 2]). Q1 asked if students thought of calculus as many disconnected topics, this was answered broadly by Electronic Engineering and Business students, however, both Mathematics and Economic students strongly disagreed (Table 2 & Table S3). Although Q16, which asked if the contents of the taught module covered many seemingly separate topics compared to other taught modules, was generally agreed with by all disciplines demonstrated by all means being above 2.5 (Table 1), all disciplines included many responses who disagreed with this statement (Tables S2 & S3). Interestingly apart from Q3, these items (Q7, Q10, Q15, Q16) all had relatively flat distributions, with responses not highly skewed towards any answer indicating that there was a wide range of opinions across the students.

Table 3: The pairings of disciplines where there was a significant difference in the responses (indicated by Mann-Whitney test p<0.05) for each item in the calculus questionnaire described in Table S1. The statistical values for each test can be found in Table S5.

Item	Pairings of disciplines with a significant difference in responses			
1	M-B	B-Ec		
2	M-B		B-EE	
6	M-B	B-Ec		
8	M-B			M-EE
9	M-B			
11	M-B			M-EE
12	M-B			
13	M-B			M-EE
14	M-B			M-EE

Disciplines specified as Mathematics (M), Business (B), Economics (Ec), Electronic Engineering (EE). Where an item is not listed then no pairings of disciplines had significant differences for that item.

When asked about their agreement with studying calculus as being necessary for their future career (Q10), the responses displayed qualitatively different results between the disciplines despite their similar mean values (Table S3). Whilst, on average, all disciplines indicated that they perceived calculus as necessary for their career, it was noticeable that a substantial proportion of Business (21%) and Electronic Engineering (30%) students disagreed (Table S2 & S3). In comparison, disciplines agreed studying calculus was beneficial to performing well in their degree (Q9; Table 2) though still a notable proportion of students from Business and Electronic Engineering disagreeing with the statement (24.6% and 20.0% respectively; Table S3).The statement *there is only one correct way to solve a calculus problem* (Q2), exhibited a high proportion of Business students with strong agreement, resulting in a mean of 2.70 (Table 1). This was the only item where one discipline gave an average response different to the other three disciplines; the other disciplines generally disagreed with the statement. Interestingly, however, 14% of Mathematics students still showed agreement with the statement.

Disciplines differed over the connection of calculus and their degree, with students from Business and Electronic Engineering generally not seeing the connection at the beginning of studying their modules (Q4; $mean_B = 2.80$, $mean_{EE} = 2.60$; Table 2), whereas Mathematics and Economics did see the connection at the outset ($mean_M = 1.80$, $mean_{Ec} = 2.00$; Table 2). Upon completion of their respective modules all disciplines tended to agree there was a connection (Q5), however, there was still a high proportion of Business (21%) and EE (19%) students who did not consider there to be a connection (Table S3). All disciplines claimed to have enjoyed studying calculus as part of their degree (Q17) with over 60% selecting either sentence 1 or 3 (Table 4). The most selected answer stated that students both 'enjoyed' and found the topic 'useful'. However, this was most pronounced for Mathematics (87%) and Economics (67.7%) with under half of respondents selecting this sentence for Business (45.6%) and Electronic Engineering (40%). Over a third of Business and EE students intimated that studying calculus was not useful for their degree (Q8) which was similar to the results asking if calculus is useful to study in their subject (Q5). Although, of those Business students who did not consider calculus useful, a larger proportion (21%) enjoyed it, compared to those who did not (12%) (Q17; Table 4).

Table 4: Number of participants which picked the given sentence which best described their opinion of studying calculus.

Descriptive sentence	M	B	Ec	EE	Total
I enjoyed studying calculus and the module was useful for my degree.	27 (82%)	26 (46%)	21 (68%)	8 (40%)	82 (58%)
Although I don't enjoy calculus the module was useful for my degree.	3 (9%)	12 (21%)	9 (29%)	4 (20%)	28 (20%)
I enjoyed studying calculus but the module was not very useful for my degree.	2 (6%)	12 (21%)	1 (3%)	4 (20%)	19 (13%)
I don't enjoy calculus and the module was not very useful for my degree.	1 (3%)	7 (12%)	0 (0%)	4 (20%)	12 (9%)

Disciplines specified as Mathematics (M), Business (B), Economics (Ec), Electronic Engineering (EE). The values in brackets show the proportion of each discipline which selected the given sentence.

Business students had the highest proportion of students expecting to go onto a specific career post university (B-65%; M-37%; Ec-52%; EE-52%; Table 5) compared to going into further study (B-18%; M-26%; Ec-33%; EE-33%; Table 5). Mathematics students were the least sure of their postgraduate development with almost a third of students stating that they were undecided, which was double the proportion of both Business and Economics students (Table 5). Although it should be highlighted that Business had a slim majority of respondents who were in their first ear (51%; Table S4).

Table 5: Future plans of participants after finishing their undergraduate studies for each discipline. This question was asked to all participants regardless of their current year of study.

Next Plans	M	B	Ec	EE	Total
Employment	13 (37.1%)	37 (64.9%)	17 (51.5%)	11 (52.4%)	78 (53.4%)
Further study	9 (25.7%)	10 (17.5%)	11 (33.3%)	7 (33.3%)	37 (25.3%)
Undecided	11 (31.4%)	9 (15.8%)	5 (15.2%)	2 (9.5%)	27 (18.5%)
Other	2 (5.7%)	1 (1.8%)	0 (0%)	1 (4.8%)	4 (2.7%)

Disciplines specified as Mathematics (M), Business (B), Economics (Ec), Electronic Engineering (EE). The values in brackets show the proportion of each discipline who selected the given option.

5. Discussion

Calculus is at the core of many aspects of mathematical education. Students from distinct disciplines have differing perceptions of the usefulness and importance of the subject depending on how they identify links to their specific field. Therefore, teaching certain cohorts in an identical manner might be detrimental for how a student engages with the content. However, identifying similarities may allow HE institutions to optimise the effectiveness of their teaching by efficiently grouping students according to disciplines which share similar perceptions of the subject. In this simple study we considered how students studying a variety of quantitative degrees perceived calculus and, importantly, how they felt it affected their studies and future careers. This is a vital yet largely unexplored question in the literature (Hitt & González-Martín, 2016; Rasmussen et al., 2014).

Our results indicate that students from different disciplines do have marked differences in their perception of calculus and its importance for their studies, however there are also notable similarities observed (Table 1; Table S5). Economics and Mathematics have similar perceptions about calculus, including: the usefulness of studying it for their degree and future career, the need to understand the underlying principles, and how calculus relates to other aspects of their studies. In contrast, Business students were seen to have more varied responses, with many questions answered differently compared with other disciplines. Interestingly, Electronic Engineering responses placed them closer to Economics rather than Mathematics; however, this could in part be due to the smaller number of responses. Electronic Engineering were also most divided in their responses (shown by having the highest standard deviation in 14 questions), indicating that these students have a wide range of perceptions of calculus.

Business students were the only discipline to largely agree that there is only one way to solve calculus questions (Q2). This is supported by Q1 and Q3 where Business students felt they only needed to memorize methods to successfully answer questions and that calculus was formed of many disconnected topics. These findings potentially indicate that Business students view calculus as a collection of methods for solving unconnected problems rather than one coherent topic. Business students were also the respondents with the highest proportion expecting to go onto a specific career post university, potentially explaining why they were more focused on applications than underlying theory. These results support the idea of including exercises and examples based around the core concepts of calculus with clear applications in settings with which Business students will be familiar (Marrongelle, 2001; Osman et al., 2013; Willmot & Simms, 2018).

For both Mathematics and Economics students, the majority indicated that they enjoyed the study of calculus and found it useful. They also shared an agreement that calculus was connected to their degree and did not see calculus as many disconnected topics. These results would suggest that these students share a similar perception of mathematics: namely that they view the underlying theory as important as well as valuing a comprehensive understanding in the concepts. Given these similarities, Mathematics and Economics may be a suitable grouping for joint teaching of calculus, thus allowing for cross-disciplinary study which has been shown to be beneficial across age ranges (Kokotovich, 2008; Vahey et al. 2012).

There are many ways that this work could be expanded upon in future research. For example, whilst these views reflect the opinions of the students, they do not demonstrate the thoughts of those teaching it. It is reasonable to conject that the student views of a subject are affected by the way in which a teacher presents the ideas, which in turn may be influenced by the teacher's own perceptions. Interestingly, this has been shown not to be the case in environmental biology in secondary school students (Kiarie, 2016), where there was found to be no statistically significant relation between teachers and students' perception and attainment in the subject. However, enthusiasm in teaching is known to be positively correlated to an increase attainment in mathematics and STEM subjects in secondary education (Kunter et al., 2008; Lazarides et al., 2018; Jungert et al., 2020) as well as in subjects in HE (Entwistle et al., 2002; Devlin & O'Shea, 2012).

Understanding calculus education is a complex issue which goes beyond simply identifying differences and similarities when stratified by discipline. It is always hard to make precise conclusions about students' perceptions and outcomes due to the complex array of attributing factors, however, these results do indicate some tentative results which could be used in cross-discipline module design. Previous work has highlighted the effect various attributing factors can have on the performance of students studying calculus including gender (Ellis et al, 2016), ethnicity (Riegle-Crumb & Grodsky, 2010; Minor, 2016), and socio-economic background (Byun et al, 2015). A larger study looking to combine these factors together with the teaching environment, including groupings and student/staff perceptions of the importance of calculus, would allow for a more in-depth analysis of attainment, pass rate and value added.

Our results did not indicate any major differences when considering the additional factors of gender, year of study or future plans (see Appendix C for further information). However, a natural extension of this work would be to consider these factors in a mixed effect model, to determine if subgroups within disciplines have differing perceptions. This could also be used to identify what, if any, of these factors are the driving force behind the similarities and dissimilarities found to exist between groups. Such work would require a larger set of respondents to ensure a more even distribution amongst all combinations of factors (such as discipline, gender, year of study and future plans).

In conclusion, in this small study we find some tentative evidence to support cross-teaching in calculus, with certain groupings more effective than others, such as Mathematics and Economics compared with Mathematics and Business. These results can be used to inform module design to optimise teaching environments, thus allowing for the factors causing the observed challenges in calculus such as relatively low attainment, STEM degree drop-out rates and under preparation for future careers to be better understood, analysed and addressed.

6. Copyright

The anonymised data used in this study is available upon request from the authors.

7. Appendices

Appendix A - Questionnaire

Table S1: Tests and questionnaires items.

Demographic questions

1. Which degree course are/were you studying towards? (e.g Mathematics, Data Science and Analytics, Maths with Finance)

2. Which year of study have you just completed? (Oct 2019 - Jun 2020; excluding any Foundation or Repeated years)
   (1st; 2nd; Placement/Year Abroad; Final)

3. Which of the following modules have you taken most recently?
   (MA101 – Calculus; BE300 – Quantitative Methods & Finance; CE142 – Mathematics for Engineers; EC115 – Methods of Economic Analysis; None of the above)

4. What age are you?
   (18-22; 23-25; 26+; Prefer not to answer)

5. To which gender do you most identify?
   (Female; Male; Other; Prefer not to answer)

6. Did you study post-16 education in England, Wales or Northern Ireland?
   (Yes; No)

7. Did you study a mathematics or statistics qualification between the ages of 16-18?
   (Yes; No)

8. Which of the following post-16 qualifications have you studied? Select all that apply.
   (AS-/A-Level Mathematics; AS-/A-Level Further Mathematics; AS-/A-Level Statistics; IB Mathematics SL/HL; IB Further Mathematics HL; Level 3 Core Maths; None of the above)

9. Did you study a foundation year at XXX?
   (Yes; No)

10. Which of the following module(s) did you study in your foundation year?
    (IA112 – Essential Mathematics; IA115 – Mathematical Methods and Statistics; IA124 – Mathematics and Statistics; None of the above)

11. What are your plans immediately following your degree?
    (Further study; Employment; Undecided; Other - please specify below)

Items in the calculus questionnaire

For the remainder of the questionnaire the term ‘calculus’ refers to specific content of [selected module]. This only includes topics such as differentiation, integration, algebraic manipulation, sequences and series, trigonometric functions, vectors, complex numbers. This does not include any probability, statistics nor financial content.

You will now be shown a series of statements that will require you to reflect on your perceptions of the calculus content of [selected module]. For each statement, please select the option that indicates to which you agree with each statement. *(1 = strongly disagree, 2 = somewhat disagree, 3 = neither agree nor disagree, 4 = somewhat agree, 5 = strongly agree)*

Please note that these questions refer only to the *content* of the module. They are <u>not</u> designed to comment on the lecturer, lecturing style or module material.

1. I think of calculus as many disconnected topics.
2. There is only one correct way to solve a calculus problem.
3. In order to solve problems in calculus, you need to memorize many different methods and techniques.
4. At the time of studying [selected module] I did not consider there to be a connection between calculus and my degree scheme.
5. Having completed [selected module] I consider there to be a connection between calculus and my degree scheme.
6. Outside of [selected module], I often came across questions in my studies which required knowledge of calculus.
7. In order to succeed in my degree, I mostly needed to repeat problems and techniques used in calculus, I do/did not need to learn the underlying principles.
8. It is necessary to study calculus to: - study my subject.
9. It is necessary to study calculus to: - get a high mark in my subject.
10. It is necessary to study calculus to: - do well in my anticipated field of work.
11. I think that the calculus component of [selected module] was important for my overall understanding of the contents of my degree.
12. The subject of calculus has little relation to what I will experience in the outside world.
13. The calculus content of [selected module] increased my interest in: - mathematics.
14. The calculus content of [selected module] increased my interest in: - my subject.
15. The majority of the calculus contents of [selected module] was either new to me or involved a deeper learning of topics I had previously studied.
16. The calculus contents of [selected module] included a large number of separate topics compared to my other modules.

Final question regarding perception of calculus

Select the sentence which is most applicable for you:

1. I enjoyed studying calculus and the module was useful for my degree.
2. Although I don't enjoy calculus the module was useful for my degree.
3. I enjoyed studying calculus but the module was not very useful for my degree.
4. I don't enjoy calculus and the module was not very useful for my degree.

Appendix B – Additional Tables

Table S2: Mean, standard deviation and the number of participants for the responses to each item in the calculus questionnaire in Table S1.

Item	Mean	S.D.	Disagree or strongly disagree	Neither agree nor disagree	Agree or strongly agree
1	2.47	1.07	79 (54.1%)	40 (27.4%)	27 (18.5%)
2	2.24	1.15	101 (69.2%)	13 (8.9%)	32 (21.9%)
3	3.75	1.03	21 (14.4%)	19 (13%)	106 (72.6%)
4	2.35	1.33	92 (63.4%)	15 (10.3%)	38 (26.2%)
5	3.96	1.1	19 (13.1%)	16 (11%)	110 (75.9%)
6	3.69	1.22	29 (20%)	20 (13.8%)	96 (66.2%)
7	2.99	1.06	54 (37.2%)	37 (25.5%)	54 (37.2%)
8	3.79	1.18	23 (16.1%)	18 (12.6%)	102 (71.3%)
9	3.91	1.12	17 (11.9%)	25 (17.5%)	101 (70.6%)
10	3.68	1.16	25 (17.5%)	31 (21.7%)	87 (60.8%)
11	3.75	1.1	26 (18.3%)	16 (11.3%)	100 (70.4%)
12	2.89	1.06	52 (36.6%)	44 (31%)	46 (32.4%)
13	3.44	1.17	32 (22.5%)	30 (21.1%)	80 (56.3%)
14	3.36	1.09	30 (21.1%)	42 (29.6%)	70 (49.3%)
15	3.5	1.31	42 (29.6%)	19 (13.4%)	81 (57%)
16	3.12	0.98	39 (27.5%)	48 (33.8%)	55 (38.7%)

The percentages in brackets show the proportion of the entire study population who gave the given response.

Table S3: Proportion of each discipline for each item's responses in the calculus questionnaire in Table S1.

Item	Disagree or strongly disagree				Neither agree nor disagree				Agree or strongly agree			
	M	**B**	**Ec**	**EE**	**M**	**B**	**Ec**	**EE**	**M**	**B**	**Ec**	**EE**
1^	71.4%	38.6%	63.6%	52.4%	20.0%	33.3%	30.3%	19.0%	8.6%	28.1%	6.1%	28.6%
2^	82.9%	52.6%	72.7%	85.7%	2.9%	14.0%	9.1%	4.8%	14.3%	33.3%	18.2%	9.5%
3	14.3%	12.3%	15.2%	19.0%	14.3%	10.5%	15.2%	14.3%	71.4%	77.2%	69.7%	66.7%
4^	77.1%	52.6%	78.1%	47.6%	8.6%	10.5%	3.1%	23.8%	14.3%	36.8%	18.8%	28.6%
5	5.7%	21.1%	3.1%	19.0%	5.7%	14.0%	9.4%	14.3%	88.6%	64.9%	87.5%	66.7%
6	5.7%	35.1%	9.4%	19.0%	5.7%	17.5%	15.6%	14.3%	88.6%	47.4%	75.0%	66.7%
7	37.1%	33.3%	34.4%	52.4%	25.7%	31.6%	21.9%	14.3%	37.1%	35.1%	43.8%	33.3%
8	5.7%	24.6%	9.7%	20.0%	0.0%	15.8%	12.9%	25.0%	94.3%	59.6%	77.4%	55.0%
9	2.9%	17.5%	3.2%	25.0%	8.6%	21.1%	29.0%	5.0%	88.6%	61.4%	67.7%	70.0%
10	8.6%	22.8%	9.7%	30.0%	20.0%	24.6%	25.8%	10.0%	71.4%	52.6%	64.5%	60.0%
11	2.9%	28.1%	9.7%	30.0%	5.9%	14.0%	12.9%	10.0%	91.2%	57.9%	77.4%	60.0%
12^	47.1%	21.1%	45.2%	50.0%	35.3%	35.1%	25.8%	20.0%	17.6%	43.9%	29.0%	30.0%
13	5.9%	26.3%	19.4%	45.0%	14.7%	22.8%	29.0%	15.0%	79.4%	50.9%	51.6%	40.0%
14	2.9%	26.3%	19.4%	40.0%	32.4%	31.6%	22.6%	30.0%	64.7%	42.1%	58.1%	30.0%
15	26.5%	28.1%	29.0%	40.0%	11.8%	15.8%	12.9%	10.0%	61.8%	56.1%	58.1%	50.0%
16	20.6%	26.3%	32.3%	35.0%	35.3%	36.8%	35.5%	20.0%	44.1%	36.8%	32.3%	45.0%

Disciplines specified as Mathematics (M), Business (B), Economics (Ec), Electronic Engineering (EE). Items that are denoted with a ^ symbol indicate items which were negatively coded.

Table S4: Number of participants by year of study and discipline.					
Year of Study	**M**	**B**	**Ec**	**EE**	**Total**
1st	14 (40%)	29 (50.9%)	13 (39.4%)	12 (57.1%)	68 (46.6%)
2nd	12 (34.3%)	13 (22.8%)	9 (27.3%)	6 (28.6%)	40 (27.4%)
Final	9 (25.7%)	11 (19.3%)	10 (30.3%)	3 (14.3%)	33 (22.6%)
Placement/Year Abroad	0 (0%)	4 (7%)	1 (3%)	0 (0%)	5 (3.4%)
Disciplines specified as Mathematics (M), Business (B), Economics (Ec), Electronic Engineering (EE). The percentages in brackets show the proportion of each discipline who gave the given response.					

Table S5: Results of the Mann-Whitney-Wilcoxon test for similarities between responses for each item in the calculus questionnaire shown in Table S1 when comparing between pairs of disciplines.

Item	M-B		M-Ec		M-EE		B-Ec		B-EE		Ec-EE	
	W-stat	***p-value***	**W-stat**	***p-value***	**W-stat**	***p-value***	**W-stat**	***p-value***	**W-stat**	***p-value***	**W-stat**	***p-value***
1	642.5*	*0.002**	540.5	*0.586*	282.5	*0.093*	1237*	*0.007**	656	*0.493*	281.5	*0.193*
2	706.5*	*0.006**	523.5	*0.364*	379.5	*0.760*	1134	*0.064*	798*	*0.009**	391.5	*0.276*
3	942.5	*0.563*	587.5	*0.884*	387.5	*0.684*	1008.5	*0.459*	663	*0.346*	359.5	*0.787*
4	736.5	*0.015*	559	*0.993*	262.5	*0.035*	1137	*0.025*	603	*0.960*	245.5	*0.050*
5	1237.5	*0.012*	564	*0.938*	449	*0.048*	692	*0.016*	586	*0.872*	410.5	*0.057*
6	1418.5	*<0.001*	634	*0.167*	449	*0.048*	625*	*0.006**	473	*0.120*	369.5	*0.448*
7	986.5	*0.929*	527	*0.663*	411.5	*0.429*	865.5	*0.676*	679.5	*0.334*	393	*0.268*
8	1334*	*0.001**	630	*0.061*	482.5*	*0.001**	712	*0.075*	579.5	*0.906*	380	*0.101*
9	1277.5*	*0.004**	652.5	*0.048*	422	*0.062*	788.5	*0.333*	546	*0.749*	325	*0.732*
10	1209.5	*0.052*	578.5	*0.578*	408	*0.225*	747.5	*0.184*	557	*0.872*	345	*0.439*
11	1303.5*	*0.001**	600.5	*0.126*	451*	*0.005**	691	*0.047*	566	*0.963*	373	*0.132*
12	627*	*0.003**	487	*0.576*	326	*0.793*	1106.5	*0.039*	725	*0.056*	319	*0.859*
13	1270*	*0.004**	679.5	*0.017*	493.5*	*0.001**	848.5	*0.742*	668	*0.219*	377.5	*0.163*
14	1261.5*	*0.009**	591.5	*0.330*	499*	*0.002**	744	*0.189*	666	*0.238*	407	*0.045*
15	1015	*0.672*	546.5	*0.775*	387	*0.345*	875	*0.937*	625	*0.480*	342	*0.496*
16	1056	*0.448*	611	*0.243*	365	*0.638*	946.5	*0.562*	567	*0.975*	289	*0.674*

Disciplines specified as Mathematics (M), Business (B), Economics (Ec), Electronic Engineering (EE).

Appendix C – Additional Analyses

Here we include the results of responses when grouped by gender (Male, Female), year of study (1st, 2nd or Final year) and future plans (employment, Further Education, undecided). These analyses were ancillary to the main aim of the work in which differences by discipline studied was the major concern.

In each case, visual inspection does not reveal any appreciable differences (see Figs S1-S3), certainly when compared with the results isolated by discipline (Fig S4). However, to fully examine the combined effect of these factors, an inter-/intra- analysis would be required. Here we note that to get robust results a larger data set with more evenly dispersed participants from all combinations of subcategories would be required (e.g., here Electronic Engineering had only two Female respondents compared to 17 Male; Table 1).

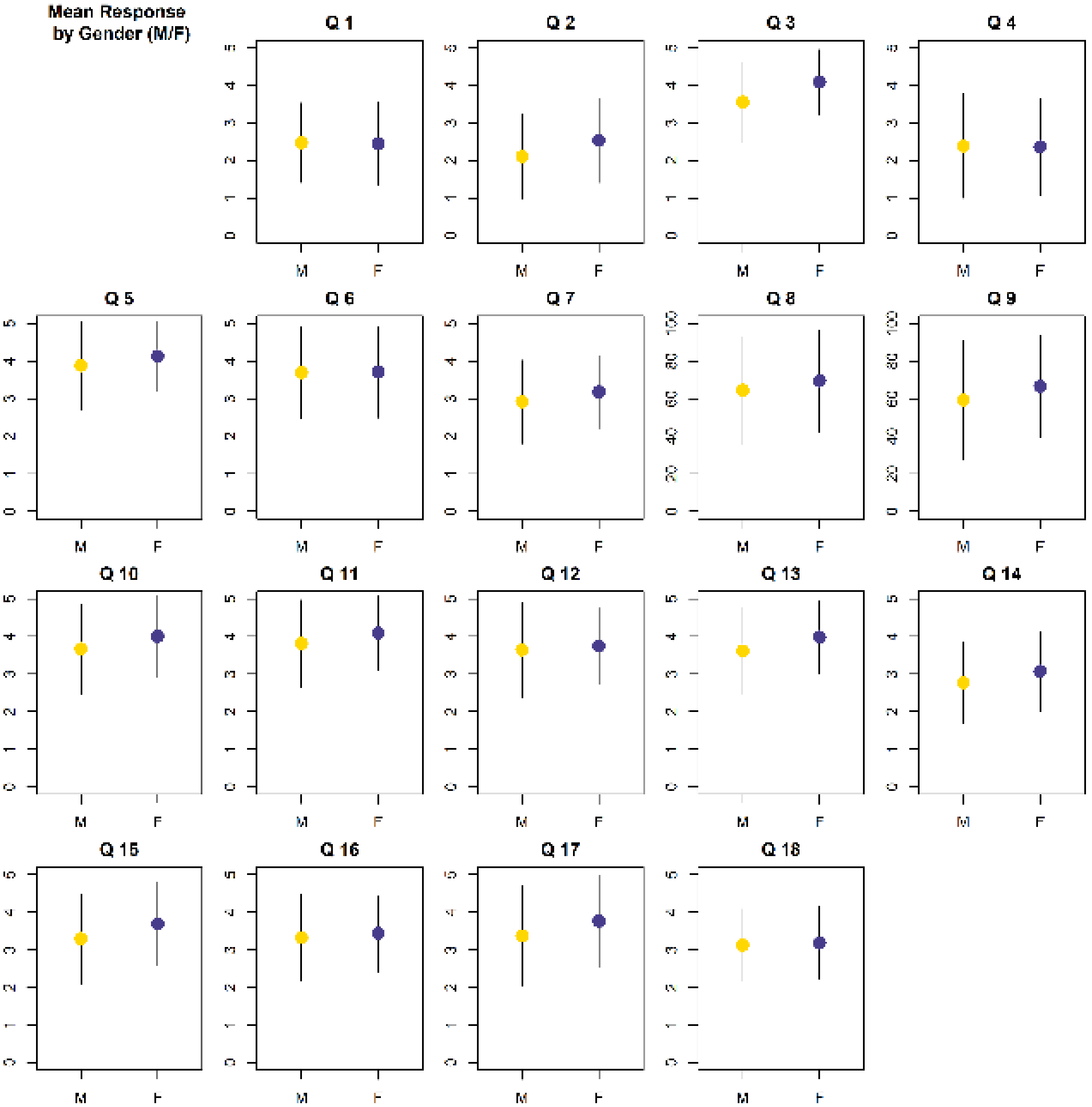


Figure S 1. Panel showing the mean response for each question by gender (Male – yellow circle; Female – purple circle). Bars depict s.d.

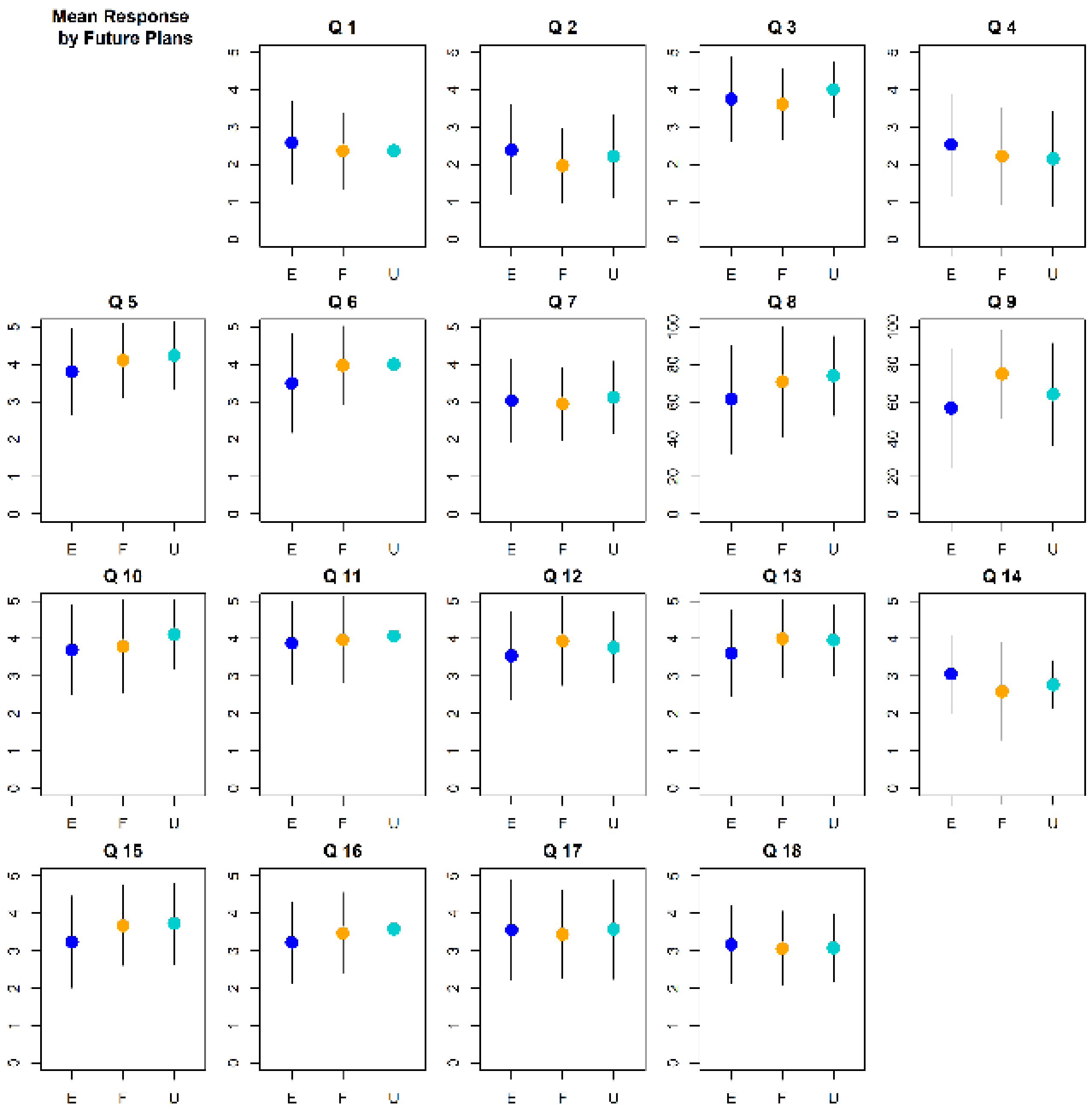


Figure S 2. Panel showing the mean response for each question by future plans ('E' – Employment, dark blue circle; 'F' – Further Education, orange circle; 'U' – undecided, cyan circle.). Bars depict s.d.

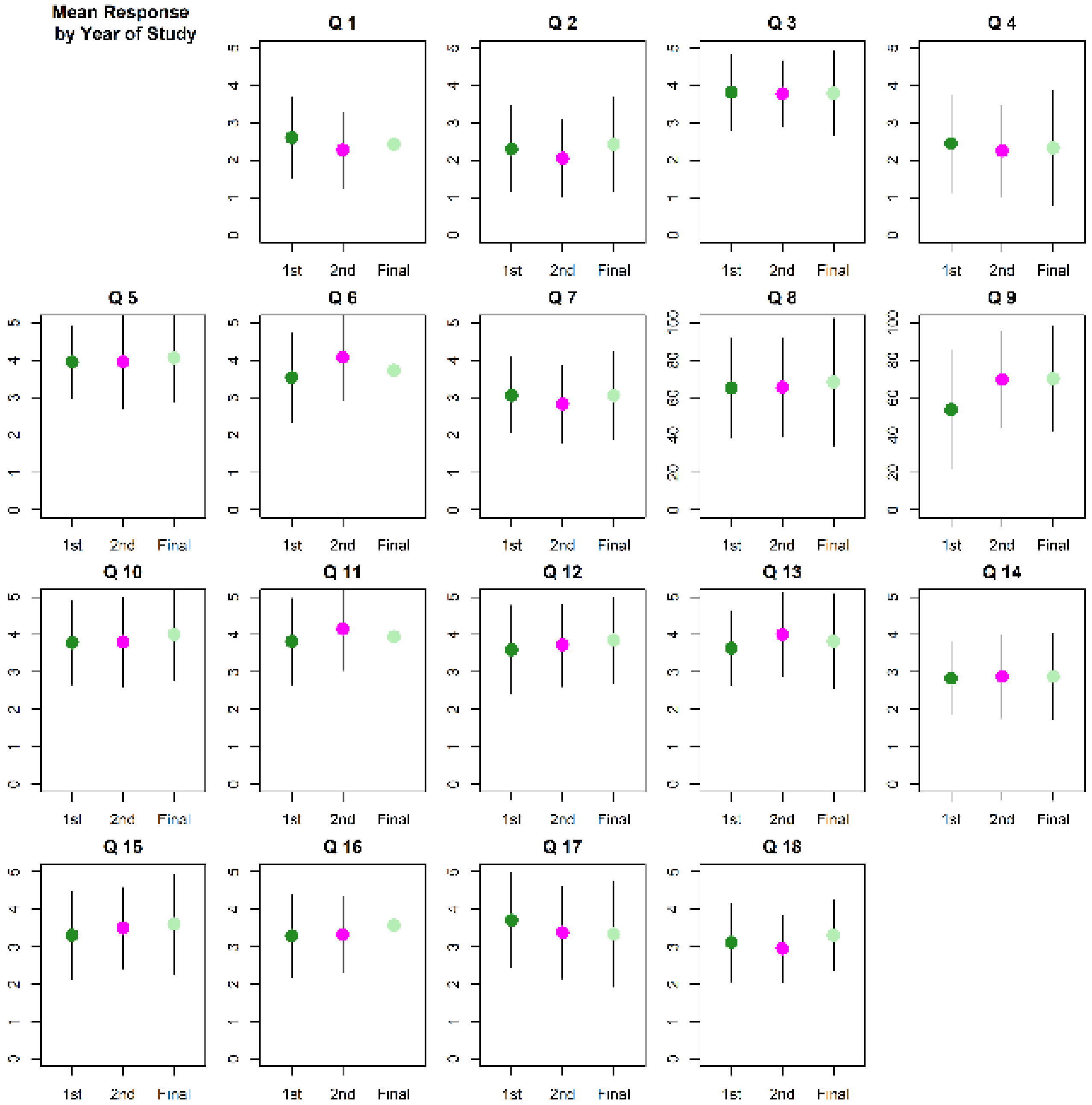


Figure S 3 Panel showing the mean response for each question by year of study (1st year, dark green circle; 2nd year, magenta circle; final year, light green circle). Bars depict s.d.

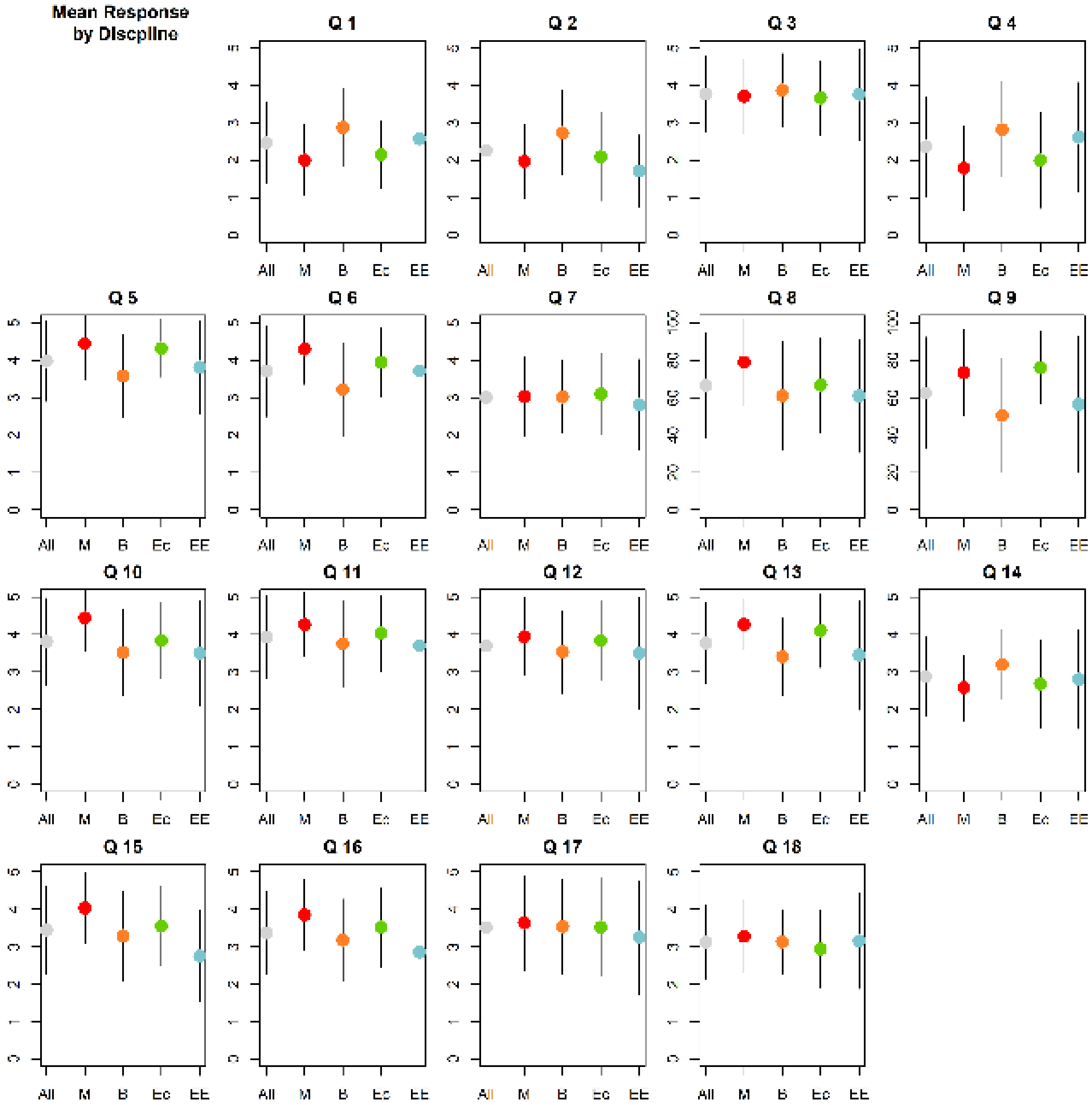


Figure S 4 Panel showing the mean response for each question by discipline (overall, grey circle; M - Maths, red circle; B - Business, orange circle; Ec - Economics, green circle; EE – Electronic Engineering, blue circle). Bars depict s.d.

8. References

Ahmed Y., Taha M. H., Al-Neel S., & Gaffar A. M. (2018). Students' perception of the learning environment and its relation to their study year and performance in Sudan. *Int J Med Educ*, 9, 145-150. doi:10.5116/ijme.5af0.1fee

Alamolhodaei, H. (1996). A study in higher education calculus and students' learning styles. PhD thesis, University of Glasgow. https://eleanor.lib.gla.ac.uk/record=b1594527

Ballantine, J. A. & Larres, P. M. (2004). A critical analysis of students' perceptions of the usefulness of the case study method in an advanced management accounting module: the impact of relevant work experience. *Accounting Education*, 13(2), 171-189. doi:10.1080/09639280410001676885

Bognar, L., Fáncsikné, É. H., Horvath, P., Joos, A., Nagy, B. & Strauber, G. (2018). Improved learning environment for calculus courses. *Journal of Applied Technical and Educational Sciences*, 8(4), 35-43. doi:10.24368/jates.v8i4.59

Boone, H. N., & Boone, D. A. (2012). Analyzing Likert Data. *The Journal of Extension*, 50(2), Article 48. doi:10.34068/joe.50.02.48

Byun, S. Y., Irvin, M. J., & Bell, B. A. (2015). Advanced math course taking: Effects on math achievement and college enrolment. *The Journal of Experimental Education*, 83(4), 439-468. doi:10.1080/00220973.2014.919570

Czocher, J. A., Tague, J., & Baker, G. (2013). Where does the calculus go? An investigation of how calculus ideas are used in later coursework. International *Journal of Mathematical Education in Science and Technology*, 44(5), 673-684. doi:10.1080/0020739X.2013.780215

Darmaji, D., Kurniawan, D. A., Astalini, A., Kurniawan, W., Anwar, K., & Lumbantoruan, A. (2019). Students' Perceptions of Electronic's Module in Physics Practicum. *Journal of Education and Learning*, 13(2), 288-294. https://eric.ed.gov/?id=EJ1325540

Devlin, M., & O'Shea, H. (2012). Effective university teaching: Views of Australian university students from low socio-economic status backgrounds. *Teaching in Higher Education*, 17(4), 385-397. doi:10.1080/13562517.2011.641006

Dündar, S. (2015). Mathematics teacher-candidates' performance in solving problems with different representation styles: The trigonometry example. EURASIA Journal of Mathematics, *Science and Technology Education*, 11(6), 1379-1397. doi:10.12973/eurasia.2015.1396a

Eng, T. H., Li, V. L., & Julaihi, N. H. (2010). The relationships between students' underachievement in mathematics courses and influencing factors. *Procedia-Social and Behavioral Sciences*, 8, 134-141. doi:10.1016/j.sbspro.2010.12.019

Entwistle, N., McCune, V., & Hounsell, J. (2002). Approaches to studying and perceptions of university teaching-learning environments: Concepts, measures and preliminary findings. Enhancing Teaching and Learning Environments in Undergraduate Courses Occasional Report, 1, 1-19. http://www.ed.ac.uk/etl/docs/ETLreport1.pdf

Ellis, J., Fosdick, B. K., & Rasmussen, C. (2016). Women 1.5 times more likely to leave STEM pipeline after calculus compared to men: Lack of mathematical confidence a potential culprit. *PloS one*, 11(7), e0157447. doi:10.1371/journal.pone.0157447

Fedesco, H. N., Kentner, A., & Natt, J. (2017). The effect of relevance strategies on student perceptions of introductory courses. *Communication Education*, 66(2), 196-209. doi:10.1080/03634523.2016.1268697

Ferreira, A., & Santoso, A. (2008). Do students' perceptions matter? A study of the effect of students' perceptions on academic performance. *Accounting & Finance*, 48(2), 209-231. doi:10.1111/j.1467-629X.2007.00239.x

Hagman, J. (2019). Towards a Forward-Thinking College Calculus Program. In Theorizing STEM Education in the 21st Century. *IntechOpen*. doi:10.5772/intechopen.77870

Hagman, J. E., Johnson, E., & Fosdick, B. K. (2017). Factors contributing to students and instructors experiencing a lack of time in college calculus. *International Journal of STEM Education*, 4(1), 1-15. doi:10.1186/s40594-017-0070-7

Harackiewicz, J. M., Smith, J. L., & Priniski, S. J. (2016). Interest matters: The importance of promoting interest in education. *Policy Insights from the Behavioral and Brain Sciences*, 3(2), 220-227. doi:10.1177/2372732216655542

Hensel, R. A., & Hamrick, T. R. (2012). Comparison of paths to calculus success. In American Society for Engineering Education. *American Society for Engineering Education*. doi: 10.18260/1-2--21092

Hitt, F., & González-Martín, A. S. (2016). Generalization, covariation, functions, and Calculus. In A. Gutiérrez, G. L. Leder & P. Boero (Eds.), Second Handbook of Research on the Psychology of Mathematics Education. The Journey Continues, 3-38. Rotterdam: Sense Publishers. doi:10.1007/978-94-6300-561-6_1

Johns, C. (2020). Self-Regulation in First-Semester Calculus. *International Journal of Research in Undergraduate Mathematics Education*, 6, 404-420. doi:10.1007/s40753-020-00114-9

Joshi, A., Kale, S., Chandel, S., & Park, D. K. (2015). Likert scale: explored and explained. *British Journal of Applied Science & Technology*, 7(4), 396-403. doi:10.9734/BJAST/2015/14975

Jungert, T., Levine, S., & Koestner, R. (2020). Examining how parent and teacher enthusiasm influences motivation and achievement in STEM. *The Journal of Educational Research*, 113(4), 275-282. doi:10.1080/00220671.2020.1806015

Kiarie, S. M. (2016). Effects of Teachers' Perceptions on Students' Perceptions and Achievement in Environmental Education in Secondary School Biology in Gilgil Sub-County Nakuru County, Kenya. International *Journal of Environmental and Science Education*, 11(12), 5736-5761. https://eric.ed.gov/?id=EJ1115678

Kay, R. and Kletskin, I., 2012. Evaluating the use of problem-based video podcasts to teach mathematics in higher education. *Computers & Education*, 59(2), 619-627. doi:10.1016/j.compedu.2012.03.007

Kinnear, G. (2018). Delivering an online course using STACK. In Proceedings of the STACK Conference.

Kokotovich, V. (2008, September). Cooperative, Cross-Discipline Teaching and Learning. In International Conference on Cooperative Design, Visualization and Engineering, 216-224. Springer, Berlin, Heidelberg. doi:10.1007/978-3-540-88011-0_30

Kunter, M., Tsai, Y.-M., Klusmann, U., Brunner, M., Krauss, S. & Baumert, J. (2008). Students' and mathematics teachers' perceptions of teacher enthusiasm and instruction. *Learning and Instruction*, 18(5), 468-482. doi:10.1016/j.learninstruc.2008.06.008

Lazarides, R., Buchholz, J., & Rubach, C. (2018). Teacher enthusiasm and self-efficacy, student-perceived mastery goal orientation, and student motivation in mathematics classrooms. *Teaching and Teacher Education*, 69, 1-10. doi:10.1016/j.tate.2017.08.017

Leu, K. (2017). Beginning College Students Who Change Their Majors within 3 Years of Enrollment. Data Point. NCES 2018-434. National Center for Education Statistics. https://eric.ed.gov/?id=ED578434

Li, Y., & Singh, C. (2021). Effect of gender, self-efficacy, and interest on perception of the learning environment and outcomes in calculus-based introductory physics courses. *Physical Review Physics Education Research*, 17(1), 010143. doi:10.1103/PhysRevPhysEducRes.17.010143

Madeira, V. R., de Souza, A. L. L., Peixoto, A., & da Gama Afonso, H. C. A. (2019). A Calculus Project to Support Students that Enter Engineering Courses. In 2019 IEEE Global Engineering Education Conference (EDUCON), 1224-1227. IEEE. doi:10.1109/EDUCON.2019.8725142

Marrongelle, K. A. (2001). Physics experiences and calculus: How students use physics to construct meaningful conceptualizations of calculus concepts in an interdisciplinary calculus /physics course. Unpublished doctoral dissertation, University of New Hampshire, Durham.

Minor, E. C. (2016). Racial differences in mathematics test scores for advanced mathematics students. *The High School Journal*, 193-210. https://www.jstor.org/stable/44075323

Newton, G., & McCunn, P. (2015). Student perception of topic difficulty: Lecture capture in higher education. *Australasian Journal of Educational Technology*, 31(3). doi:10.14742/ajet.1681

Osman, K., Hiong, L.C. & Vebrianto, R. (2013). 21st century biology: an interdisciplinary approach of biology, technology, engineering and mathematics education. *Procedia-Social and Behavioral Sciences*, 102, 188-194. doi:10.1016/j.sbspro.2013.10.732

R Core Team (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/.

Rasmussen, C., & Ellis, J., (2013). Who is switching out of calculus and why. In Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education (Vol. 4, pp. 73-80). PME.

Rasmussen, C., Marrongelle, K., & Borba, M.C. (2014) Research on calculus: what do we know and where do we need to go? *ZDM Mathematics Education*, 46, 507–515. doi:10.1007/s11858-014-0615-x

Riegle-Crumb C & Grodsky E. (2010). Racial-Ethnic Differences at the Intersection of Math Course-taking and Achievement. *Sociology of Education*. 83(3), 248-270. doi:10.1177/0038040710375689

Sahin, A., Cavlazoglu, B., & Zeytuncu, Y.E., (2015). Flipping a college calculus course: A case study. *Journal of Educational Technology & Society*, 18(3), 142-152. https://www.jstor.org/stable/10.2307/jeductechsoci.18.3.142

Sevimli, E. (2016). Do calculus students demand technology integration into learning environment? case of instructional differences. *Int J Educ Technol High Educ*, 13, 37.doi:10.1186/s41239-016-0038-6

Sullivan, G. M., & Artino, A. R., (2013). Analyzing and interpreting data from Likert-type scales. Journal of Graduate Medical Education. 5(4), 541-2. doi:10.4300/JGME-5-4-18

Tall, D., Smith, D., & Piez, C. (2008). Technology and Calculus. In Heid, M. K., & Blume, G. W (Eds), Research on Technology and the Teaching and Learning of Mathematics, Volume I: Research Syntheses, 207-258.

Tang, H. E., Julaihi, N. H., & Voon, L. L. (2013). Attitudes and perceptions of university students towards calculus. *Social and Management Research Journal*, 10(1), 1-39. https://ir.uitm.edu.my/id/eprint/13111

Ting, D. H., & Lee, C. K. C. (2012). Understanding students' choice of electives and its implications. *Studies in Higher Education*, 37(3), 309-325. doi:10.1080/03075079.2010.512383

Tudor, J., Penlington, R., & McDowell, L. (2010). Perceptions and their influences on approaches to learning. *Engineering Education*, 5(2), 69-79 doi:10.11120/ened.2010.05020069

Vahey, P., Rafanan, K., Patton, C., Swan, K., van't Hooft, M., Kratcoski, A., & Stanford, T. (2012). A cross-disciplinary approach to teaching data literacy and proportionality. *Educational Studies in Mathematics*, 81(2), 179-205. doi:10.1007/s10649-012-9392-z

Weurlander, M., Cronhjort, M. and Filipsson, L. (2017). Engineering students' experiences of interactive teaching in calculus. *Higher Education Research & Development*, 36(4), 852-865. doi:10.1080/07294360.2016.1238880

Willmot, P. & Simms, R. (2018). Mathematics Education for 21st Century Engineering; extended abstract. Proceedings EERN spring Colloquium (pp. 69-70)

Yoo, D. M., & Kim, D. H. (2019) The relationship between students' perception of the educational environment and their subjective happiness. *BMC Med Educ*, 19, 409 (2019). doi:10.1186/s12909-019

THIS PAGE INTENTIONALLY LEFT BLANK

CASE STUDY

Designing a blended delivery foundation mathematics course: Targeting self-efficacy, algebraic skill development and social connectedness

Rosie A. Cameron, School of Mathematics and Statistics, The University of Canterbury, Christchurch, New Zealand. Email: rosalind.cameron@canterbury.ac.nz

Abstract

Foundation mathematics courses play a crucial role in allowing students who have not achieved the pre-requisite mathematics credits for tertiary studies to re-engage with STEM studies. This paper describes how targeting self-efficacy, skill development and social connectedness have influenced the design of a foundation mathematics course at a New Zealand university. These design goals are grounded in education literature, and the paper outlines how focusing on these goals has resulted in the design of a course that incorporates novel approaches to learning and assessment, including online and face-to-face components. These approaches include the use of large, automatically graded weekly quizzes; adaptive learning quizzes for essential skills such as fraction arithmetic; and an emphasis on face-to-face learning and connection through tutorials and collaborative problem-solving workshops. The paper concludes with reflections on the success of the course so far and raising questions for future investigation.

Keywords: Foundation mathematics, self-efficacy, blended, computer-aided assessment, STACK

1. Introduction

Foundation mathematics courses play a crucial role in allowing students to re-engage with science, technology, engineering and mathematics (STEM) studies, when they have not achieved the pre-requisite mathematics for tertiary studies. Such courses provide another avenue to address equity of opportunity in education since many of the students in these courses have been previously disadvantaged in their access to secondary education (Martin et al., 2021). However, retention and providing adequate preparation for students in STEM studies is a challenging problem (Patterson & Sallee, 1986) because of the wide range of mathematical experiences students have when starting these courses (Perkin & Bamforth, 2011). This challenge is also exacerbated by the fact that many students choose not to participate in the support programs on offer (Hillock & Khan, 2019). It is therefore important that tertiary institutions provide foundation courses that enable students to succeed and progress to further studies in their chosen field.

This paper describes how the design goals of self-efficacy, skill development and social connectedness have been incorporated into a foundation mathematics course at a New Zealand university. These design goals are grounded in education literature, and their description for the purpose of this study is shared with the reader, before outlining the course design and the intended effect of various activities within the course. The foundation mathematics course described here acts as a prerequisite for entry into first year calculus courses for students who did not study this content in high school. The course usually has about 700 students spread between two semesters, including students with a wide range of mathematics experience and attainment. For example, some students only narrowly failed the prerequisite requirements at school, whereas others haven't studied any algebra in the final three years of high school. This is a 12-week, 150-hour course with no assumed prerequisite, and it has previously had low levels of student engagement and pass rates as low as 40%.

We begin the paper with an overview of some common issues that arise in the delivery of foundation courses. This reveals several key design goals that the course design needs address: self-efficacy, algebraic skill development, and social connectedness. Identifying these issues gives design goals for the course as well as a framework for evaluating the final product. The second part of the paper outlines the assessment and learning activities for the course and how they work towards these design goals. Activities include a mix of computer-aided assessment alongside an emphasis on face-to-face interactions. We then reflect on initial implementations of the course, including a discussion of how students responded to the various activities and raising further research questions.

2. Literature review

Foundation mathematics courses and remediation have been the subject of much study as both a difficult and important issue for tertiary institutions. Before describing the specifics of the course design, an outline of some of the key issues facing foundation students sets up a framework for the design by providing design goals for the course. The first of these key issues is student self-efficacy, understood for the purpose of this study as low mathematics confidence. The second is the development of essential algebraic skills which we emphasise alongside the more advanced learning outcomes. The final issue we focus on in the course design is *whanaungatanga*, a te reo Māori term that can be understood here as social connectedness. In this section we also discuss computer-aided assessment as one of the tools we use to achieve these design goals.

Each of the issues mentioned here is a crucial aspect of designing this foundations course and must be addressed in the design, noting that "one of the ways that teaching can take place is through shaping the landscape across which students walk. It involves the setting in place of epistemic, material and social structures that guide, but do not determine, what students do." (Goodyear, 2015, p. 34). We argue below that each of the goals mentioned in the previous paragraph is an essential part of this landscape. The activities and assessments we incorporate into the course encourage students towards productive activity in line with the constructive alignment approach of Biggs and Tang (2007).

2.1. Self-efficacy

Self-efficacy is “beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments” (Bandura, 1997, p. 3), that is, the belief that one can succeed at a specific task. Figure 1 summarises literature showing that improved student self-efficacy drives persistence, motivation and help-seeking behaviours (see a more detailed version in (Skaalvik et al., 2015)). For instance, Skaalvik et. al. (2015) investigated the effect of each of these connections, and Williams and Williams (2010) found that the reciprocal relationship between performance and self-efficacy was relevant in most of the 33 nations they studied, so it appears that this is a connection that can be fairly broadly applied across cultures. The converse also seems intuitive: if a student believes they *can’t* succeed at a task then they are more likely to have little reason to persist or seek help if it becomes difficult. Bengmark et. al. (2017) found that not only are self-efficacy and motivation predictors of success in tertiary mathematics, but there is opportunity to improve these during first-year tertiary studies. By targeting student self-efficacy – examining our communications and attitudes as instructors and providing students with opportunities to succeed – we can impact student outcomes in mathematics.

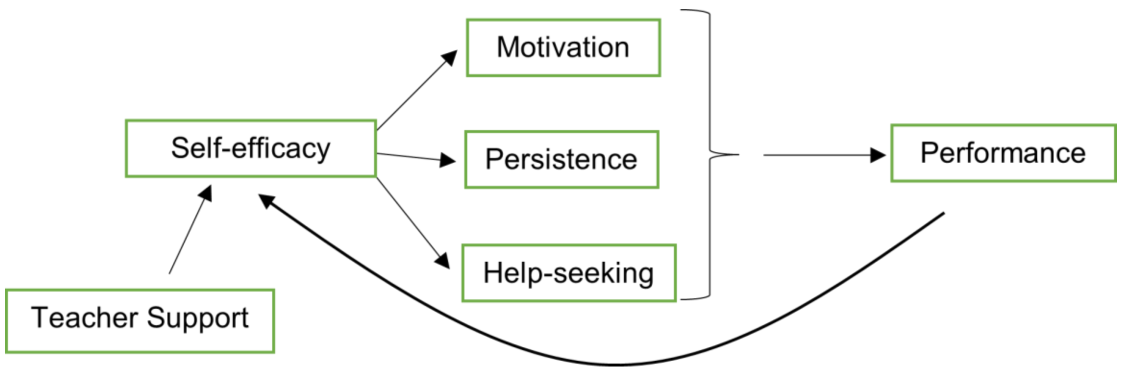


Figure 1: Summary of self-efficacy interactions

As indicated in Figure 1, experiencing success is known to be one of the main factors that improves self-efficacy as it reinforces the belief that one *can* succeed at mathematics. Jansen et al. (2013) found that providing opportunities for students to succeed at small tasks, and setting questions at an appropriate level of difficulty, encouraged students to attempt many questions and improved their self-efficacy. According to Lynch (2010), providing ‘easy wins’ for students can make the overall task more manageable. Finally, acknowledging success at a larger task through a ‘mastery’ grade above 70 – 80% provides meaningful feedback and encouragement for students (Zientek et al., 2019), who have many experiences of obtaining 50% or less on such tasks. This gives us insight into the factors that should be in place for tasks we set students in the design of the course.

Zientek et al. (2019) argue that teacher support and language choices are also important factors in determining self-efficacy, they also refer to (Hattie & Timperley, 2007) who emphasise the role of feedback in self-efficacy and learning. For example, suppose a teacher states dismissively that a particular exercise is very easy. A first-year student who listens to this but finds the new content difficult could interpret this as meaning that they do not belong in that classroom. Carefully examining teacher attitudes and language can prevent this scenario, and instead language can be used to encourage students in their persistence and motivation and acknowledge the quality of effort expended (Barton, 2018). Though it can be difficult to manage at large scale, support should be readily available even if it is a combination of lecturers, teaching assistants, and online support.

2.2. Algebraic skill development

Foundation mathematics students often have difficulty with essential algebraic skills, which can prevent a thorough understanding of the material (Hillock et al., 2013). For example, this can manifest as persistent errors or misuse of notation such as negatives (Cangelosi et al., 2013) or the equals sign (Vincent et al., 2015). By identifying and addressing the underlying algebraic misconceptions, rather than just correcting the individual error, we can equip students to develop these skills and transition to an algebraic way of thinking.

The course design needs to identify, and provide support for, these underlying misconceptions. These identified skills require a mastery level understanding, even if mastery thresholds aren't appropriate for or applied to the rest of the course. Students therefore need consistent feedback on minor errors as well as more conceptual errors. Co-requisite models for these foundational skills have been shown to benefit students (Hillock et al., 2013; Logue et al., 2019) and emphasise that these are skills to be improved and mastered over time rather than attempted in an isolated unit. Repeated, targeted practice is also a useful tool against misconceptions (Hattie & Timperley, 2007).

2.3. Whanaungatanga/ Social connection

Another challenge to address in the course design is to facilitate *whanaungatanga*; community connections and a sense of belonging among students. The te reo Māori phrase is used here because this is an aspect to learning and engagement that is of particular cultural importance to Māori and Pasifica students, although its importance for the general cohort is also increasingly acknowledged in the literature (Kinnear et al., 2022; Mullen et al., 2022). In order to effectively learn and engage, students must have a sense of belonging not only in the content (self-efficacy), but also in the community and the institution (Kalantzis & Cope, 2010). Explaining and collaborating with peers is one of the best ways to engage with content and facilitate effective learning, according to Lynch (2010) and Sofroniou and Poutos (2016) who found that students who participated in group learning had improved grades. Furthermore, interactions with teaching staff not only provide the most effective means of feedback, explanation and identifying misconceptions, but also play a large role in determining student self-efficacy as discussed in Section 2.1 (see Figure 1).

Encouraging whanaungatanga can be challenging because students often give little value to peer connections and can be hesitant to approach teaching staff. In studying student attitudes towards tutorials, Herrmann (2014) found that students with a "surface approach" to learning (aiming to minimally satisfy requirements rather than taking a "deep approach" and aiming to develop an understanding of concepts) often viewed their peers as "in a neutral sense, *fellows in ignorance*". Input from peers was seen as a distractor while they waited for the tutor to give them the correct answer, rather than viewing peer interactions and discussions as part of the learning process. Students with low self-efficacy may also be hesitant to seek help and can attribute less value to help-seeking behaviours than teaching staff do; they may not view this as central to the learning process (Lynch, 2007). The challenge in designing a course then is to incorporate whanaungatanga into the teaching and learning activities in a way that encourages students to see its value.

2.4. Computer-aided assessment

Computer-aided assessment (CAA) has been used in mathematics education for many years and has included numerous programs such as MathLab, MapleTA and, more recently, STACK (Dorko, 2020b; Hannah et al., 2014; Higgins et al., 2019; Sangwin & Kinnear, 2021). The use of CAA gives scope to include more formative assessment in a large course (Kinnear et al., 2022), which reinforces the value that teachers place on regular practice (Lynch, 2007). However, this must be implemented with care so to encourage effective study techniques and to mitigate risks from relying on automated feedback (Rønning, 2017).

Implementations of CAA often allow students to reattempt a question or assignment after receiving feedback. It has been observed that this leads to cyclic problem-solving activity among students, and that students are likely to finish any problem that they start (Dorko, 2020a; Hirsch & Weibel, 2003). Students also prefer online homework, even in studies where it leads to similar outcomes compared to paper-based homework (Halcrow & Dunnigan, 2012). However, CAA is not without disadvantages to consider, for example the use of CAA can lead to an overemphasis on the final answer to an exercise, at the expense of understanding the process and concepts involved (Rønning, 2017). Students have also been observed guessing answers once they receive initial feedback on an exercise. A reliance on CAA can also exacerbate equity and accessibility issues if students have less access to computers through disability or disadvantage.

Over recent years, there is a growing community of instructors using the STACK Moodle plugin for CAA (Sangwin & Kinnear, 2021). STACK is underpinned by a computer algebra system so has powerful and versatile computing capabilities for both generating random variations of questions as well as grading student answers algebraically (https://stack-assessment.org). This means that students can input algebraic expressions and equations, as well as other mathematical objects such as lists and sets. The open-ended nature of student input means that a STACK-based quiz can closely mimic paper-based homework questions.

CAA will be a necessary component of the course to provide students with sufficient practice and feedback at scale. By implementing CAA using STACK the questions can mostly replicate what would otherwise be on a traditional homework sheet but with the added benefit of immediate feedback.

3. Design overview for course

Here we describe the design of the course and give a brief description of the various learning and teaching activities, including how they will contribute towards the design goals introduced in Section 2. Figure 2 shows how the main activities of the course align with the goals of self-efficacy, algebraic skill development and whanaungatanga (also see Table 1 for an overview of the assessment weighting). Each main course component contributes towards the design goals in some way, while this is highlighted in the diagram below we elaborate on each aspect throughout this section. It is important that each activity contribute towards the goals, for example we want to avoid further exacerbating any low self-efficacy. While some of the activities described below are individual study, opportunities for whanaungatanga are built into a few aspects of the course design to provide some balance against the individualised components.

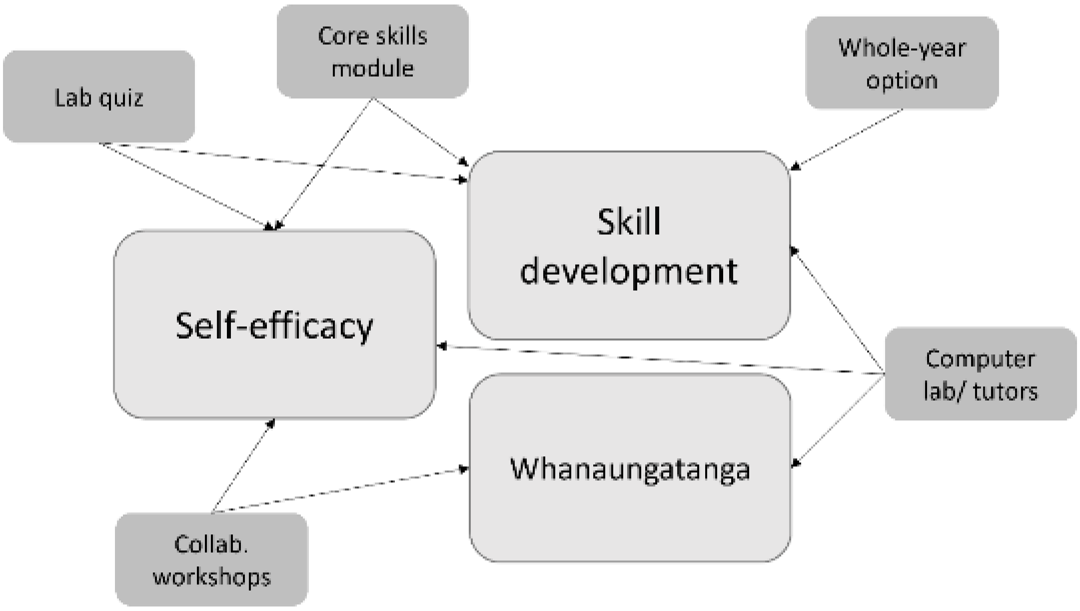


Figure 2: Course design goals (arrows show main goals of each activity)

Table 1: Course assessment and activity structure

Item	Description	Assessment Weighting	Hours per week (total)
Lectures	In person, livestreamed, recorded	-	4 lectures/week (48 hours)
Lab Quiz	Weekly STACK quiz, 50 questions, automated feedback	24%	2 – 4 hours self-study per week (36 hours)
Computer Lab Attendance	Tutor support, working on LQS	6%	2-hour weekly lab (24 hours)
Core Skills Module	Online videos and CAP quiz (STACK questions)	5%	(4 hours)
Collaborative Workshops		5%	1 hour workshop, 10 weeks (10 hours)
Mid-course test	Closed-book, invigilated, written test	15%	1.5 hour test plus revision time (9 hours)
Final Exam (≥40% on exam required for passing grade)	Closed-book, invigilated, written exam	45%	3 hour exam, plus revision time (20 hours)

3.1. Lab Quiz

The weekly Lab Quiz is the main activity that students interact with each week, grades achieved on these quizzes contributing a total of 24% of the course assessment. Each quiz consists of 50 STACK questions with a 'check' button beside each question (Figure 3). Students can check their answers to each question immediately and, if the answer is incorrect, they can resubmit with a penalty of only 10% of the question mark. Up to half of the questions each week cover the content from that week while the remaining questions serve as consolidation for earlier content.

Good feedback is feedback that makes students think (Barton, 2018). Students are only provided with minimal feedback: correct or incorrect. The goal in providing such minimal feedback is for students to check their written working for errors or seek help from a tutor. Students get unlimited attempts at each question with only a small penalty applied to each incorrect attempt. Worked examples and lecture notes are provided, so students could refer to these as a starting point. The online questions also go alongside computer lab classes with a high tutor-student ratio. Tutors encourage students to persist with problems and help them work out the material. The weekly lab quiz provides students with the opportunity to succeed at small tasks (individual questions), thereby contributing to improved self-efficacy. This encourages students to persist at an individual question until they obtain the correct answer, but also values this weekly work by contributing marks towards their final grade.

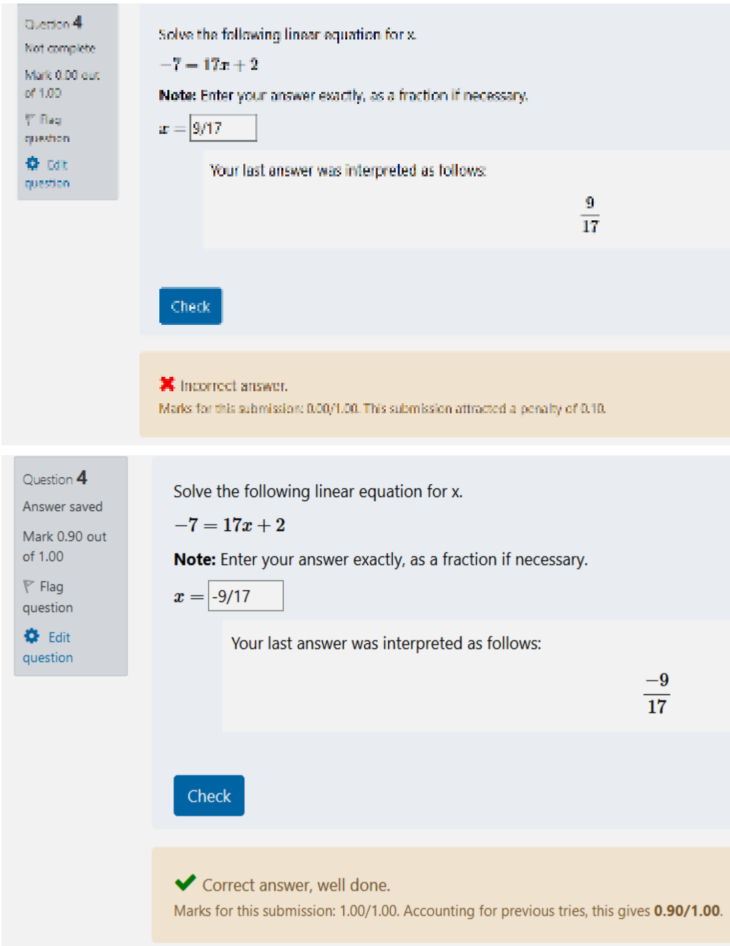


Figure 3: Instant feedback in weekly Lab Quiz

3.2. Core Skills Module

The Core Skills Module is a small component within the course consisting of online, self-paced content covering fraction arithmetic, order of operations and factorisation. This content requires a mastery threshold so is best suited to self-paced learning, as students will require differing amounts of time to consolidate this learning. These three areas were identified as being a common source of errors and hindering the development of further skills. For example, fractions and ratios are required to understand and apply trigonometric functions, as well as being a useful step in developing a more robust understanding of number. Factorisation is an essential skill that is relied on in most topics throughout the course and has been identified as a topic students often struggle with (O'Connor, 2022).

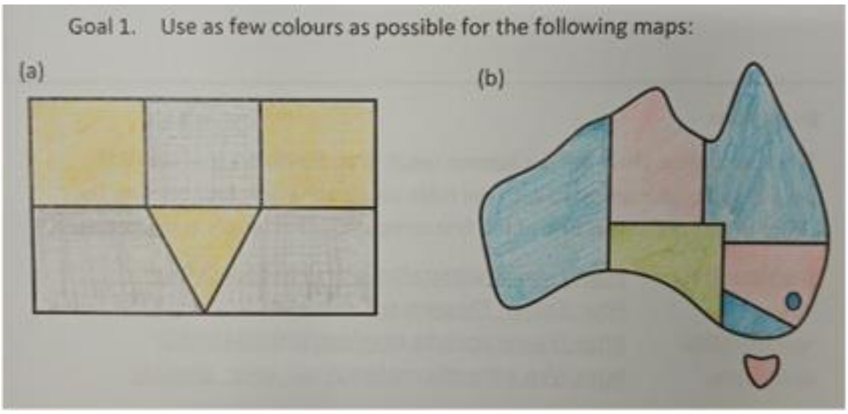
This module is implemented using a Computer Adaptive Practice Quiz (CAP Quiz) alongside a library of short videos and worked examples. CAP Quiz is a Moodle plugin that uses an ELO rating system (a method of ranking students against the question bank) in order to facilitate a mastery threshold . The CAP Quiz presents students with one question at a time, selecting each question so that students have an estimated 75% chance of getting the question correct. By working through questions until their score reaches a desired threshold, students achieve mastery of the required mathematical skills while being exposed to questions of an appropriate difficulty. Further details about the implementation of CAP Quiz can be found in (Cameron, 2022).

3.3. Whole-year course

Some students find the course content quite challenging and time consuming as they have previously missed background mathematics knowledge and skills. The whole-year course therefore offers students the opportunity to take the same course spread over two semesters. This allows extra time to consolidate their learning without placing a prerequisite requirement on the course. Enrolment is optional and students can change into the whole-year course until about two thirds of the way through the first semester.

3.4. Collaborative workshops

The collaborative workshops are the missing piece to the design so far, and they allow students to meet with each other and meaningfully collaborate on mathematical tasks. Group work and problem-solving are useful for student learning and self-efficacy (Evans et al., 2020; Sofroniou & Poutos, 2016) and the workshops also place an emphasis on communicating mathematical ideas. The tasks cover a variety of mathematical topics and applications and are intended to be accessible to all students while still having potential for extension to higher-level concepts. Workshop topics expose students to a variety of mathematical fields including cryptography, number theory and probability. For example, one workshop explores graph theory through map colouring and encourages students to explore open problems and pose their own mathematical questions and hypotheses by creating variations on the original problem.



Goal 2. Create a map that needs at least four different colours. Make sure you check that it is impossible to colour it with only three colours.

Goal 3. Can you create a map that needs five different colours?

Goal 4. Change one of the rules of the game or add a new rule to make it more interesting/challenging. Explain how your new rule changes the game and why it is more interesting/challenging.
Ask your tutor for rule suggestions.

Figure 4: Snapshot from map colouring workshop

4. Reflection and Discussion

Here I (the author) reflect on initial evidence for how the course implementation met the design goals of student self-efficacy, skill development and whanaungatanga. The diagram in Figure 1 shows that self-efficacy drives persistence, motivation, and help-seeking behaviours, so observing these student behaviours would give indirect evidence of self-efficacy. The author obtained ethics approval to publish the following data (Ref: HEC 2021/116).

Initial gradebook data from the course implementation reveals evidence of student persistence, as seen through the continued engagement and high completion rate of the weekly lab quizzes. For example, in Semester 1 of 2022, 64% of students obtained at least 75% in at least 10 of the labs, and a further 11% of students obtained at least 60% in at least 10 of the labs. This data points towards a high level of persistence on a week-to-week basis. In the lab classes, I also observed improved student persistence and help-seeking behaviours as they approached questions. At the start of semester, I observed that students who got a wrong answer would often just move to the next question or seem stuck, neither asking for help nor knowing how to proceed. Tutors were encouraged to intervene at this point because they could observe that the answer was wrong. However, as the semester progressed, I also observed many students start by checking their working to identify errors, and then either look up similar examples or ask a tutor for guidance. This problem-solving behaviour demonstrated an increased persistence to continue working on each problem until it was correct, seeking out the appropriate resources. Similar behaviour has also been observed in the literature in response to online homework (Dorko, 2020a). Further research on the effect of the lab quizzes and feedback structure would be very useful especially as many institutions increasingly rely on STACK questions and the potential they have for detailed feedback and solutions.

1. *How does this feedback model effect student confidence and motivation?*

Students had different motivations for taking the whole-year course: time-management and wanting to spend less time per week on the course; self-awareness that they needed extra time to understand concepts in order to pass the course; wanting to consolidate skills and aiming for a high grade. In the small cohort, several students worked hard consistently throughout the year and achieved their goal grade whereas others seemed to fall behind and disengage or withdraw from the course. This alternative pathway is still a work in progress and leads to the question:

2. *How can we best support and engage students who need a longer timeframe to consolidate content from this foundations course?*

Many students enjoyed the collaboration and exploring new mathematical problems in the workshops, as evidenced by responses to a reflective question in the test. Multiple students responded that they appreciated the collaborative aspect, and/or that they discovered that mathematics is a fascinating and varied subject, not just restricted to algebra and arithmetic. However these responses were not universal and some students resisted spending time on activities that did not directly affect their exam performance.

3. *Investigate student attitudes and how the workshops influenced students' views of mathematics and collaborative activities.*

The new course design appears to be successful so far, with improved student engagement and pass rates. The questions outlined above point towards further research to investigate the identified goals of self-efficacy, skill development and whanaungatanga.

5. Acknowledgements

The author was supported by the Distributed Leadership in Teaching Programme at The University of Canterbury. Many thanks to colleagues at Te Kura Pāngarau who were involved in developing STACK questions.

6. References

Bandura, A. (1997). *Self-efficacy: The exercise of control.* (pp. ix, 604). W H Freeman/Times Books/ Henry Holt & Co.

Barton, C. (2018). *How I Wish I'd Taught Maths: Lessons Learned from Research, Conversations with Experts, and 12 Years of Mistakes*. John Catt Educational Limited.

Bengmark, S., Thunberg, H., & Winberg, T. M. (2017). Success-factors in transition to university mathematics. *International Journal of Mathematical Education in Science and Technology*, *48*(7), 988–1001.

Cameron, R. A. (2022). Computer Adaptive Practice for a foundation mathematics course. *International Journal of Emerging Technologies in Learning*, *17*(23).

Cangelosi, R., Madrid, S., Cooper, S., Olson, J., & Hartter, B. (2013). The negative sign and exponential expressions: Unveiling students' persistent errors and misconceptions. *The Journal of Mathematical Behavior*, *32*(1), 69–82.

Dorko, A. (2020a). Red X's and Green Checks: A Model of How Students Engage with Online Homework. *International Journal of Research in Undergraduate Mathematics Education*, *6*(3), 446–474.

Dorko, A. (2020b). What Do We Know about Student Learning from Online Mathematics Homework? In *Teaching and Learning Mathematics Online*. Chapman and Hall/CRC.

Evans, T., Thomas, M. O. J., & Klymchuk, S. (2020). Non-routine problem solving through the lens of self-efficacy. *Higher Education Research & Development*, 1–18.

Halcrow, C., & Dunnigan, G. (2012). Online Homework in Calculus I: Friend or Foe? *PRIMUS*, *22*(8), 664–682.

Hannah, J., James, A., & Williams, P. (2014). Does computer-aided formative assessment improve learning outcomes? *International Journal of Mathematical Education in Science and Technology*, *45*(2), 269–281.

Hattie, J., & Timperley, H. (2007). The Power of Feedback. *Review of Educational Research*, *77*(1), 81–112.

Herrmann, K. J. (2014). Learning from tutorials: A qualitative study of approaches to learning and perceptions of tutorial interaction. *Higher Education*, *68*(4), 591–606. JSTOR.

Higgins, K., Huscroft-D'Angelo, J., & Crawford, L. (2019). Effects of Technology in Mathematics on Achievement, Motivation, and Attitude: A Meta-Analysis. *Journal of Educational Computing Research*, *57*(2), 283–319.

Hillock, P. W., Jennings, M., Roberts, A., & Scharaschkin, V. (2013). A mathematics support programme for first-year engineering students. *International Journal of Mathematical Education in Science and Technology*, *44*(7), 1030–1044.

Hillock, P. W., & Khan, R. N. (2019). A support learning programme for first-year mathematics. *International Journal of Mathematical Education in Science and Technology*, *50*(7), 1073–1086.

Hirsch, L., & Weibel, C. (2003). Statistical evidence that web-based homework helps. *Focus*, *23*(2), 14.

Jansen, B. R. J., Louwerse, J., Straatemeier, M., Van der Ven, S. H. G., Klinkenberg, S., & Van der Maas, H. L. J. (2013). The influence of experiencing success in math on math anxiety, perceived math competence, and math performance. *Learning and Individual Differences*, *24*, 190–197.

Kalantzis, M., & Cope, B. (2010). The Teacher as Designer: Pedagogy in the New Media Age. *E-Learning and Digital Media*, *7*(3), 200–222.

Kinnear, G., Wood, A. K., & Gratwick, R. (2022). Designing and evaluating an online course to support transition to university mathematics. *International Journal of Mathematical Education in Science and Technology*, *53*(1), 11–34.

Logue, A. W., Douglas, D., & Watanabe-Rose, M. (2019). Corequisite Mathematics Remediation: Results Over Time and in Different Contexts. *Educational Evaluation and Policy Analysis*, *41*(3), 294–315.

Lynch, D. J. (2007). 'I've studied so hard for this course, but just don't get it!' Differences between student and faculty perceptions. *College Student Journal*, *41*(1), 22–24. SPORTDiscus with Full Text.

Lynch, D. J. (2010). Motivational Beliefs and Learning Strategies as Predictors of Academic Performance in College Physics. *College Student Journal*, *44*(4), 920–927. SPORTDiscus with Full Text.

Martin, G., Anthony, G., Brown, J., Dalrymple, M., Ell, F., Greenwood, S., Higgins, J., Murray, R., Roskruge, M., Trinick, T., & Yoon, C. (2021). *Pāngarau Mathematics and Tauanga Statistics in Aotearoa New Zealand*. Royal Society Te Apārangi.

Mullen, C., Pettigrew, J., Cronin, A., Rylands, L., & Shearman, D. (2022). The rapid move to online mathematics support: Changes in pedagogy and social interaction. *International Journal of Mathematical Education in Science and Technology*, *53*(1), 64–91.

O'Connor, B. R. (2022). Improving student proficiency in solving quadratic equations. *Australian Mathematics Education Journal (AMEJ)*, *4*(2), 29–38.

Patterson, D., & Sallee, T. (1986). Successful remedial mathematics programs: Why they work. *The American Mathematical Monthly*, *93*(9), 724–727.

Perkin, G., & Bamforth, S. (2011). A variety of approaches to the provision of mathematics help for first-year engineering undergraduates. *International Journal of Electrical Engineering Education*, *48*(1), 79–91.

Rønning, F. (2017). Influence of computer-aided assessment on ways of working with mathematics. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, *36*(2), 94–107.

Sangwin, C. J., & Kinnear, G. (2021). Coherently Organized Digital Exercises and Expositions. *PRIMUS*, *32*(8), 927–938.

Skaalvik, E. M., Federici, R. A., & Klassen, R. M. (2015). Mathematics achievement and self-efficacy: Relations with motivation for mathematics. *International Journal of Educational Research*, *72*, 129–136.

Sofroniou, A., & Poutos, K. (2016). Investigating the Effectiveness of Group Work in Mathematics. *Education Sciences*, *6*(3), Article 3.

Vincent, J., Bardini, C., Pierce, R., & Pearn, C. (2015). Misuse of the Equals Sign: An Entrenched Practice from Early Primary Years to Tertiary Mathematics. *Australian Senior Mathematics Journal*, *29*(2), 31–39.

Williams, T., & Williams, K. (2010). Self-efficacy and performance in mathematics: Reciprocal determinism in 33 nations. *Journal of Educational Psychology*, *102*(2), 453–466. APA PsycArticles.

Zientek, L. R., Fong, C. J., & Phelps, J. M. (2019). Sources of self-efficacy of community college students enrolled in developmental mathematics. *Journal of Further and Higher Education*, *43*(2), 183–200.

THIS PAGE INTENTIONALLY LEFT BLANK

CASE STUDY

An accessible maths journey

Lilian Joy, Digital Education Team, University of York, York, England. Email: lilian.joy@york.ac.uk
Natalie Curran, Maths Department, University of York, York, England. Email: nsec500@york.ac.uk
Cordelia Webb, Maths Department, University of York, York, England. Email: cordelia.webb@york.ac.uk
Maciej Capinski, Maths Department, University of York, York, England. Email: mc545@york.ac.uk

Abstract

The four narratives that follow bring together the stories from the lived experiences of a post-graduate blind maths student, her tutor, her transcriber and a learning technologist over the course of five years. It provides an insight into what is needed to help one student with their own way of learning maths. It also demonstrates how pulling at one thread can help to unravel and reveal the many lenses through which accessible maths needs to be approached.

Keywords: Accessible, Disabled, Blind, Technology, Narrative.

1. Introduction

Stories can help people to come to a shared understanding and to connect with one another (Connelly and Clandinin, 1990). The four narratives that follow bring together the stories from the lived experiences of a blind maths student, her tutor, her transcriber and a learning technologist. We started a 'story conga', a way to capture the lived experience of each person connected to a particular student and letting each other's stories sink in and change our own perspectives of our connection in the story. Each person wrote their own experience and after reading each other's narratives, they edited their story to reflect further insights.

2. Our stories

2.1. A blind maths student's story - Natalie

I lost my sight when I was 8/9 years old. The school I was at did their best to support me, but when it reached the time to start choosing my Standard Grade (GCSE) subjects, we all agreed that it would be in my best academic interests to switch to a school with a specialised `Visual Impairment Unit`.

One of the main reasons we thought this was a good idea was that `they'll know how to do the maths properly and how to do different languages in braille and stuff.` Well, they didn't. It's true that they definitely had more time and resources than my previous school, but their approach to braille in the sciences and languages was no more advanced than our own. In other words, write it in braille and then have someone sighted translate the gibberish. The only difference was that they used an electronic braille device for everything rather than swapping out for the mechanical Perkins Brailler as I did for Maths.

Perhaps this is a good place to stop and give a quick explanation as to what I am talking about.

- Perkins Brailler: a large, clunky, extremely heavy mechanical braille device greatly resembling an old-fashioned type-writer; paper is physically rolled into it and rows of tiny needles are pushed through the paper in the formation the user types, thereby producing lines of braille.

- Braille Note Apex: an electronic device, much smaller, lighter and hence more portable than the aforementioned Perkins. It has much the same capacity as a PC from about 15/20 years ago, an in-built voice function and a strip of 32 (or 18) braille cells which form an electronic braille display in place of a screen.

Apart from the pros and cons already mentioned, the key advantage of the Apex over the Perkins when in text-based classes was the ability to listen to what I'd previously written which is much faster than reading the physical braille. On the other hand, the primary benefit of the Perkins compared to the Apex when in science-based classes was the ability to quickly and easily scan up and down previous lines of working. The bonus of the voice provided by the Apex was almost entirely lost by the fact that the Apex wasn't smart enough - and braille rules were too complicated - for it to be able to tell the difference between English and mathematical constructions. For example, the Apex would read (as in speak and print) the equation: `2+3 =5`, as: "2 ;to 3 ;(5" [ignoring the various quotation marks]. I was informed by my new school that I would be expected to type my maths in such a form - i.e. one which only made sense when physically reading the braille - and then pass it to a specialist assistant teacher for translation, before it could then be handed in to the class teacher.

I thought this was a rubbish idea.

I decided there must be a better way. I'd already started lightly exploring some of the deeper, more technical aspects of my Apex, particularly as the thought of maybe wanting to study something mathematical at university had started to flicker across the edges of my imagination. So, I went home and experimented with every kind of file type the Apex had to offer. I read various sections in the user guide and eventually settled on the `computer braille table` file type. I discovered that if I used a different set of braille commands, I could produce braille that felt right to me, sounded right when the Apex read it back and looked right to my class teacher without it having to go through any sort of transcription process! This was computer braille, computer maths and Unicode.

The next day, I went into class and told the assistant teacher about this. I think it's fair to say she was impressed but cautious. We agreed to give my new discovery a go, with the understanding that I'd revert to their original method if we found it wasn't working. That was eleven years ago now, and I am still using the same method.

I would be the first to admit that my system is far from perfect. My Apex does not have access to the entire set of unicode tables, hence there are some symbols which it cannot interpret and we have to work round. There are many symbols which sound and look correct but are displayed in braille as a random combination of dots, or perhaps even worse, written out in words. For example: "left-brace" appears on my display whenever the "{" character is used; far from ideal when dealing with set notation. I also have to spend a bit of time undoing all the quirks which appear when reading a file sent to me because, for some reason, the Apex does some strange things such as swapping all Greek characters for normal letters of the alphabet but with different accents added to them. e.g. “α” becomes "á". And then there's the obvious fact that most people do not use computer maths for extended amounts of mathematical text. It is regarded as somewhat crude compared to the more advanced approaches such as LaTeX.

Nevertheless, these disadvantages are far outweighed by the benefits:

1. I can produce maths myself which is then immediately legible and understandable to sighted readers;
2. I can use the speech feature on my Apex fairly effectively;
3. I can assign shortcuts and commands to the symbols I use most frequently;

4. My materials can be made accessible to me by anyone at a similar level of mathematical understanding or higher, without any need for specialist training or knowledge of braille;
5. I can email my lecturers questions quickly and directly, and read their replies, without the need for any transcription;
6. If my Apex were to suddenly break, I would be able to continue my studies, though not as easily, with a laptop and braille display.

It has even proved useful in other subjects, such as Philosophy - when learning about logical notation; and French/Spanish - because it gave me access to all the different accents used in these languages.

In short, whilst unicode, computer maths and computer braille create nothing like an ideal system, I am yet to find anything better.

Since working with Lilian (learning technologist), I have been made aware that it may be possible to read mathematical material on websites or online journal articles that are not written in computer maths. If we can find a way of doing this easily and effectively, then it could give me access to a whole host of content which has hitherto been closed off to me, thereby bringing me that tiny bit closer to an equal playing field with my peers.

We managed to get JAWS (on my pc), combined with the Mathjax[1] menu, and Voiceover (on my iPhone) to successfully read equations aloud. However, no one in their right minds would learn maths simply by listening to it, and the braille output each was able to present had its own problems.

The other most obvious issue with this solution is that it is purely web-based. It does not address the problems of how I save this information; edit it in order to make my own notes or comments; or create and output my own maths, which is clearly legible to both myself and my tutors. Even if we allow for my own system neutralising the final negative here, the first two elements are still significant barriers in their own right.

2.2. A tutor's story – Maciej

I first came across Natalie when I first started marking her exams. This was a bit challenging as the format was quite different: her solutions were submitted in plain text so were less readable than the LaTex or Lyx used by other students, and the text was also more scattered making it harder to navigate. Otherwise, it was a standard student submission. There were issues when the exercise was to produce a graph in Excel but instead, Natalie made tables with the data. Geometric exercises were also much harder for Natalie as in some areas, much can be read from a picture which posed problems for her. It is also really important that Natalie has the transcription as it would be impossible to do anything without them.

Knowing it would be a challenge to teach her, I talked to my father (also an instructor on the course) who would make an effort to reply quickly when Natalie would post questions in text format. Mathematics is a peculiar subject as it can take weeks to understand a text but the same information can be conveyed within an hour's seminar. There is some magic involved in talking over real-time mathematics. With COVID, tools have been developed to teach online but it always seems to work if you start with a blank file and create the maths in real-time. This approach also allows the students

[1] Mathjax is "a JavaScript display engine for mathematics that works in all browsers". When it is implemented, people can right-click (or select) the maths (and open the Applications menu) to see extra accessibility functions and they can set these for the whole page.

to ask questions when they feel something is wrong: often the maths is correct and they don't understand something but this allows for discussion and further explanation. It was therefore important to find a format that allowed me to explain things to Natalie in real time that was not just limited to text. In higher mathematics, you need to discuss arguments and this is much harder if you cannot "see" the equations.

After some trial and error, we found Framapad[1] which is a web page where I can type text and Natalie can read it on her braille reader in real time. If you copy and paste mathematical text symbols such as ∫ or √, these appear as required. This still requires simplification as very long formulas are less readable but this compactification is common in mathematics: we understand F to be F(x, t) which saves writing the latter every time. I was astonished and surprised by the technology limitations that Natalie must cope with: her screen reader has under 20 characters that can be read in a single line. I had imagined that they were like A4 paper but instead she can only read one line at a time and that line cannot be very long. Frequently, Natalie would have to jump between lines to see the text that I was typing. This system also works for the coding aspects of the course compared to other sites that required Natalie to jump between windows.

Lecturer's notes for students.

Lemma. (Ito) If $X(t)$ is an Ito process:

$$dX(t) = a(t)dt + b(t)dW(t)$$

and $F(t, x)$ is a $C^{1,2}$ function. Then $F(t, X(t))$ is an Ito process; moreover

$$dF(t, X(t)) = \frac{\partial F}{\partial t}(t, X(t))dt + \frac{\partial F}{\partial x}(t, X(t))a(t)dt + \frac{\partial F}{\partial x}(t, X(t))b(t)dW(t) + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}(t, X(t))b^2(t)dt.$$

Figure 4 The original set of lecture notes used for sighted students.

Lecturer's notes for Natalie.

Ito lemma.
If X(t) is an Ito process:
dX(t)=a(t)dt+b(t)dW(t)
and F(t,x) is a C^(1,2) function. Then F(t,X(t)) is an Ito process, moreover
dF(t,X(t)) = F_t(t,X(t))dt + F_x(t,X(t)) a(t) dt + F_x(t,X(t)) b(t) dW(t) + 0.5*F_xx(t,X(t)) b^2(t) dt.

Figure 5 The compactified version produced with the online text editor for Natalie.

For Excel documents, we use Google Spreadsheets as we can work on the same spreadsheet, and I could see where her cursor is so I can direct her to the next cell. I could add text which she could read in real-time. This works well but is much much slower than when meeting with other students.

[1] Framapad (https://framapad.org/abc/en/) is an installation of Etherpad (https://etherpad.org/), a simple open source collaborative online text editor.

To be honest, I'm amazed at how she is able to work with big and complicated spreadsheets with only 20 characters at a time. It's mind-blowing!

Key tips for working with Natalie

Having shorter meetings on a regular basis: We have a system where we have an hour-long meeting with each student concentrating on 3 different modules. This is an efficient and effective system for most students but, for Natalie, everything takes much more time. It is better if she sends the questions upfront so the answers can be prepared to suit her. For most students, I can just share my screen, highlight the relevant parts of the lecture notes for example and maybe write some more explanation if needed. For Natalie, there's no point in sharing my screen and I need to think about the best way to write the particular part in a compact form, find an explanation which avoids technical derivations or construct a simple example to sell the main point. Doing three modules in one meeting is therefore difficult so it is better to meet more regularly.

Understand her limitations: I was surprised by the limitations of the technology that Natalie has: there is a real need for technological advancement in the tools that blind students have access to and there should be readers which cover more. Mathematical equations are often one after the other and you need a grasp of, say 3 lines simultaneously. Some proofs are very technical and a sequence of equations in a PDF can take two pages. In this case, having an overview is different to being able to scroll line by line. This is an engineering problem and providing help to maths students is not quite enough of an incentive but there should be a "braille ipad" which would help a wider audience.

2.3. A transcriber's story – Cordelia

My first step to becoming a transcriber was replying to a general email sent around the maths department to find someone to help transcribe diagrams for Natalie. This was in March 2020 and, one lockdown later, I was employed by the Learning Support Centre (LSC) who coordinated the support work for students with disabilities at the University of York.

I've since helped students in subjects including Maths, Physics and Electronic Engineering. These are very different subjects both in the type of technical content they have and how it is taught: this means note taking is varied and needs to be tailored to suit both the subject and the individual student.

Notetaking: how the lecturers help (or don't!)

From a notetaking perspective, the easiest lectures for me to transcribe are those where the lecturer handwrites (clearly!) on a board. The handwriting ensures the pace of the lecture is reasonable and helps clarify what information is most important.

With COVID and the subsequent shift to more online content, I have notetaken almost exclusively online, remotely and asynchronously. This has enabled me to help more students but also made me reliant on Panopto, the lecture capture system used in York.

It is much easier to read and make accurate notes when handwritten lectures are done digitally or through a visualiser. The only downside to this is that content is lost from view at the "end of the page" rather than on chalkboards where, when used systematically, content can remain visible for a much longer period. Generally chalkboard lectures are hard to read with lecture capture as cameras are typically at the back of the room and don't always successfully follow the lecturer across the boards.

For most of these subjects, slides are not extensively used. When they are, the pace is often faster and the content often more quantitative: Maths almost never uses slides while the less calculation-based Physics content has many more slide-based course notes.

Lecturers mostly engage positively with recording lectures although some have the opinion that students should be attending in person. While it is important that students fully engage with their courses, it is also important to remember that not everyone watching the lecture capture is having another hour in bed: there are a range of reasons students watch the lecture capture and this may include students who attended the original lecture in person or, like me, support workers. There were times where I was timetabled to be supporting 4 students simultaneously. It is only possible for these 4 students to have the support work they require if I can work from the lecture recordings.

Notetaking: what I do

I've supported students with a range of different educational support needs and have made notes mostly in Word as this was what the original training suggested and what most students requested as accessible and useful for them. However, there is now more debate about this training for science and maths subjects as well as more discussion about encouraging students to embrace other software and technologies.

Recently, I have been using MathType[1] to input more mathematical content. This did slow me down at first as it took time to find a way to use this efficiently and smoothly among non-technical content. However, now I am more used to the technology, it is faster and makes it easier to output alternative formats.

Transcription: what I did and what I do now

For Natalie, the bulk of the transcription work normally occurs over the summer before term begins. I receive the material as PDF documents in a Google Drive which I transcribe into Rich Text Format (RTF) documents.

Previously, I copied and pasted the PDF content to a Word document. However:

- line breaks occurred after each line from the PDF;
- within the text, certain letter combinations (e.g. ff, fi) were interpreted as symbols rather than letters so needed to be corrected;
- mathematical content lost many symbols (e.g. σ, ∑) and formatting (1/2 pastes as 1 2 with a line break);
- punctuation was incorrectly copied (, as ; for example)

All of these had to be corrected manually which was time consuming and error prone. Find and Replace was the most important shortcut to help with this and it could also be used to help meet Natalie's particular needs and preferences. For example, Natalie requires no space after an = sign which can occur hundreds of times within a document.

---

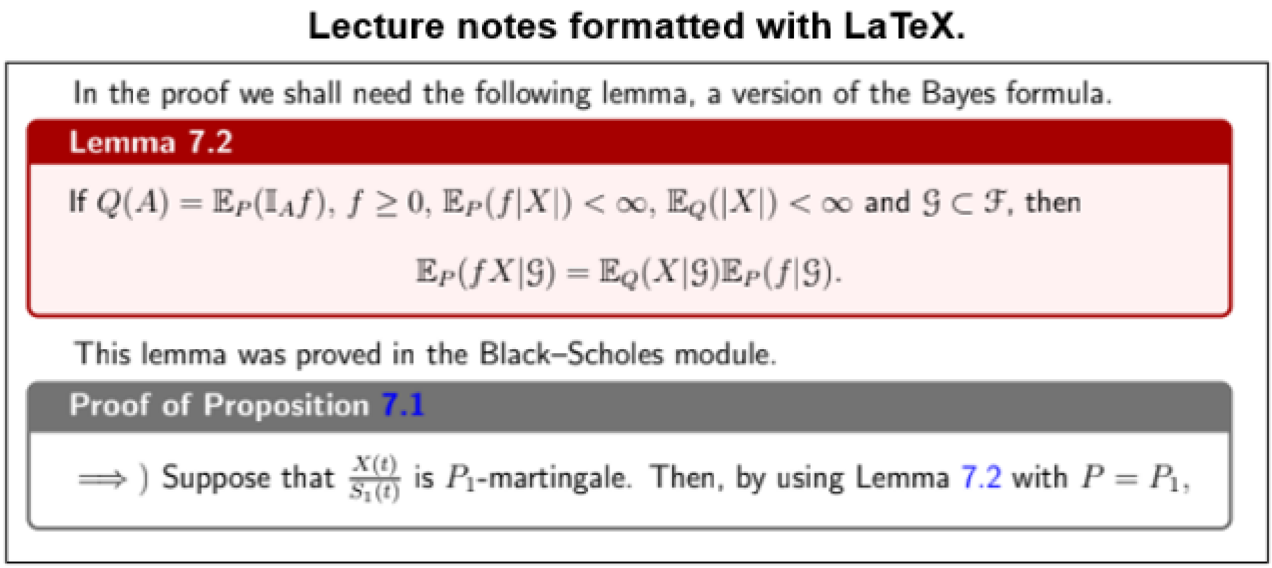
[1] MathType (https://www.wiris.com/en/mathtype/) is a formula editor and equation writer by the company, Design Science. Also known as Wiris MathType, it is the Maths editor found in the Blackboard VLE. It can also be bought as an add-on for Microsoft Office.

After meeting Lilian (learning technologist), I found out about Equatio[1] and used that to screenshot and copy mathematical content into LaTeX. I later found out that I could get the LaTeX file from the lecturers. This made my life easier as nearly all the issues from above were fixed. However, there were downsides:

- additional LaTeX formatting and comments had to be removed;
- mathematical content had to be translated from LaTeX.

Again, Find and Replace went a long way to addressing these problems: I could easily and unambiguously replace \sigma with σ but LaTeX has a lot of formatting syntax so more needed to be stripped out for Natalie. For example, \frac{σ^2}{2} can be simplified to σ^2/2. Lecturers also need to supply the LaTeX file which is not always comfortable, especially if the file is not "tidy" and contains lots of comments.

In terms of technologies that can help, MathPix[2] is the fastest alternative we have found to date. I can take the PDF supplied for the whole course and output a Word document where the text appears as required; there are no additional line breaks and mathematical content is displayed using the Word equation editor. With Mathtype installed, I can quickly translate all the maths to plain text with a keyboard shortcut which is much closer to what Natalie needs. There are also no additional commands and text is translated accurately and with no additional line breaks. Although I still have to go through the document for particular quirks, it has more than halved the transcribing time and reduced the errors that can slip through.



Lecture notes formatted with LaTeX.

In the proof we shall need the following lemma, a version of the Bayes formula.

Lemma 7.2

If $Q(A) = \mathbb{E}_P(\mathbb{I}_A f)$, $f \geq 0$, $\mathbb{E}_P(f|X|) < \infty$, $\mathbb{E}_Q(|X|) < \infty$ and $\mathcal{G} \subset \mathcal{F}$, then

$$\mathbb{E}_P(fX|\mathcal{G}) = \mathbb{E}_Q(X|\mathcal{G})\mathbb{E}_P(f|\mathcal{G}).$$

This lemma was proved in the Black–Scholes module.

Proof of Proposition 7.1

$\Longrightarrow$) Suppose that $\frac{X(t)}{S_1(t)}$ is P_1-martingale. Then, by using Lemma 7.2 with $P = P_1$,

Figure 6 A section of the PDF lecture notes shows an equation in a call out box.

---

[1] Equatio (https://www.texthelp.com/en-gb/products/equatio/) is a tool by Texthelp that allows users to create equations, formulas, mathematical and scientific notation, through voice, screenshot, handwriting or typing at a keyboard.

[2] Mathpix (http://mathpix.com) is a conversion tool that can convert handwritten maths, images and PDFs to a range of outputs including Markdown, LaTeX (including straight to Overleaf), DOCX, Excel or even ChemDraw. It also has a markdown editor.

Lecture notes for Natalie with formatting removed.

In the proof we shall need the following lemma, a version of the Bayes formula.

Lemma 7.2

If Q(A) =ℰ_P(𝒥_A *f), f ≥ 0, ℰ_P(f * |X|) < ∞, ℰ_Q(|X|) < ∞ and 𝒢 ⊂ ℱ, then ℰ_P(f *X | 𝒢) =ℰ_Q(X | 𝒢) *ℰ_P(f | 𝒢).

This lemma was proved in the Black -Scholes module.

Proof of Proposition 7.1

⟹ Suppose that X(t)/S_1(t) is P_1 -martingale. Then, by using Lemma 7.2 with P =P_1, Q =P_2, f =M_1^2(t), Y =X(t)/S_2(t), and 𝒢 =ℱ_s, we get

Figure 7 The transcribed version of Fig 3 for Natalie has formatting removed and the equations are linear.

General advice to lecturers

1. **Be aware of the limitations of the technology you are using:** lecture capture does not always accurately follow you and PDF documents are not always the best output for students
2. **Embrace the technology**: use lecture capture as it benefits many students in numerous ways and using a visualiser or tablet to write content is easier to read than a blackboard but still retains that "chalk and talk" feel.
3. **Help students help themselves**: releasing material in alternative formats such as HTML lets students be more independent and gives them a range of ways to engage with content. Moreover, students may not be aware how they can best use these different formats so consider pointing them towards different software and assistive technologies.
4. **Lead by example**: by making your content more accessible, you can also encourage students to make their content accessible which benefits them but may also make it easier for lecturers to mark their work too! Lecturers and markers have accessibility needs too!

2.4.A learning technologist's story - Lilian

I work as an education adviser and learning technologist with a specialist area of digital accessibility at the University. I first started on my accessible maths journey in 2019 after spending a day with Natalie, who was then an undergraduate maths student. I wanted to know how materials were being converted for her and how she was learning higher level maths. She did her best to explain the challenges and why her own maths method worked for her, but I didn't yet have the baseline knowledge about braille, maths and the various formats and tools to do her justice. It led me to start my research journey into accessible maths.

My colleague and I organised a discussion about accessible STEM which made us realise that most lecturers produce their notes using LaTeX output as Beamer[1] slides or PDF, followed by Microsoft Equation Editor or Mathtype in Word or PowerPoint and, to a lesser extent, R. There was also a lot

[1] Beamer is a LaTeX editor for creating slides featuring overlays and animation. It is an alternative to PowerPoint and provides support for more complex mathematical notation.

of handwritten content. As I delved more into the world of accessible maths, I realised that I had opened a can of worms - there was a lot of material on the virtual learning environment (VLE) that had mathematical or STEM content. These could not easily be converted into other formats by the accessibility tool, Ally[1], that we had acquired for our VLE, unless the maths was accessible in the first place. I set about looking at each format and how we could make it more accessible. The university funded several projects with student interns so that we could work out what needed to be done and the best workflows for us. Through the research on these projects, we found some tools that were useful to help with the creation of alternative formats when necessary.

We acquired licences for Mathtype to encourage people away from using Microsoft equation editor in Word and PowerPoint. Microsoft's equation editor has its own ecosystem for users but doesn't allow us to output alternative formats easily. MathType however can be output to a range of formats including HTML with MathJax. MathType is also the maths editor within Blackboard, which is our VLE at the University.

We acquired Texthelp Equatio as this enabled us to capture handwritten maths and convert it into digital formats. Instructors could also input maths easily by handwriting, typing or using LaTeX, thereby encouraging a digital-first workflow. Equatio could be used with Microsoft Office documents, Google Documents as well as on the VLE. It was also a valuable tool that enabled students to help themselves. We developed a worksheet to help students to learn the tool and we would run workshops with departments to help students with it.

LaTeX took me a while to grasp but four things really helped:

1. Jim Tyson from University College London (UCL) had kindly shared his notes on creating accessible files from LaTeX. I sent this to lecturers who reported back that they were able to follow Jim's methods and produce nice web pages or Word documents from their LaTeX files. However, It didn't always produce the kind of referenced equations that maths lecturers like.
2. Having my own digital maths intern who could talk me through the basics of LaTeX and experiment with me to work out a template that could be useful to lecturers, providing them with accessibility advice and a format they could use.
3. Having access to Overleaf[2] meant I could dabble with LaTeX without too much effort.
4. Getting Pandoc[3] made available through our software centre meant lecturers could easily add this to their managed computers. Pandoc can be used to convert documents from one format to another.

We produced a LaTeX template with accessibility guidance built-in. Although it uses an accessibility package that is no longer supported (since 2022), it provides us with a way to output PDFs that pass

---

[1] Ally (https://www.anthology.com/products/teaching-and-learning/learning-effectiveness/anthology-ally) is an add-on to the Blackboard, Moodle or Canvas VLE (Ally for web is a sister-product for other types of websites). Ally provides guidance and feedback to instructors/page owners on the accessibility of their online information, and it allows the students/readers to download documents in a variety of accessible formats.

[2] Overleaf (https://www.overleaf.com/) is an online collaborative LaTeX editor.

[3] Pandoc (https://pandoc.org/index.html) is a tool run on a PC that can convert documents between a wide range of file types.

the accessibility checkers while we're waiting for LaTeX to become more accessible by design. We are currently working on advice around using Mathpix for this process too.

Web-based materials are seen as the gold-standard for digital accessibility, but students are still keen to get their learning materials as PDFs (Hughes, 2019). If the lecturers can easily generate different formats, students can help themselves to the format that works best for them. So far, this is the main challenge for some lecturers, especially those who use LaTeX to produce more complicated content. Some lecturers have successfully pivoted to using alternative systems like R Markdown to produce their notes (Dias, 2022). However, students also need to be aware of the benefits of using web-based versions and they need access to the right tools to help them use PDFs more flexibly for notetaking and revision.

Both Mathpix and EquatIO are tools that we feel can really help staff and students to create accessible documents for themselves. They can convert handwritten materials to digital, depending on how clear the original handwritten materials were in the first place! Editing the digital conversion is quite straightforward in both software.

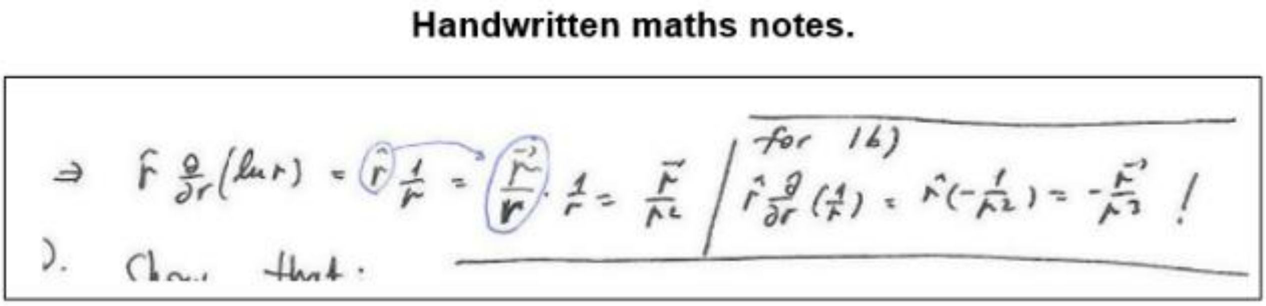


Figure 8 Some handwritten annotations that Mathpix struggled to understand and therefore imported as an image

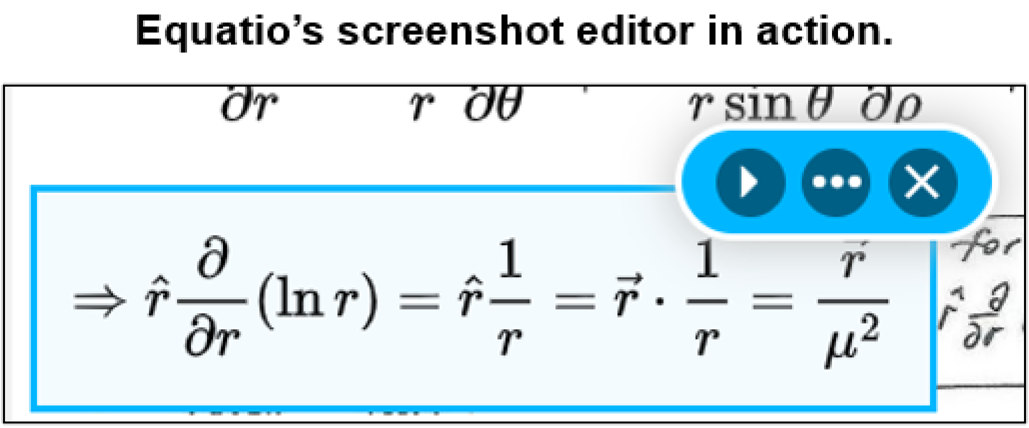


Figure 9 EquatIO provides a first stab at converting the handwritten maths to digital maths.

Students already have access to Equatio but we're considering whether to provide Mathpix too as it would allow them to convert PDFs into more editable formats so they can make notes and learn some LaTeX too.

Our journey towards digital-first accessible maths is still on-going, but we now have several students, instructors and departments working together as part of a maths accessibility working group. We output everything we learn to our Accessible Equations web site[1]. We help to chair the Jisc Accessible Maths working group[2] where several members are working together to pool our knowledge and skills to progress accessible maths across the sector.

My journey was started by one conversation with Natalie, one attempt to understand things from her perspective that launched me into a whole chain of events, creating further opportunities to work with her and to bring others on board. Natalie has driven my passion for user research and the ambition to make mathematical materials more accessible at the University. This 'story conga' has revealed to me how much more there is to be aware of as I marvel at Maciej's inventive approach to teaching Natalie online, and Cordelia's persistence with different technologies to work out the most efficient approach to transcription. By working together, we have built an infrastructure and mapped a way towards accessible maths (Webb, 2022) that benefits all staff and students at the University.

Advice for learning technologists:

- If you're a learning technologist interested in progressing accessible maths at your organisation, you'll have to collaborate with many people on several projects to make it happen. See Accessible Maths blog post[3].
- There isn't a one size fits all and learning technologists and organisations need to engage with a diverse range of tools and workflows to enable all staff and all students to produce accessible maths.
- Start by speaking to
  - one student about their workflow and their needs and go from there.
  - one lecturer who has had to teach a visually impaired student.
  - transcribers who work with and support the students.
  - your disability team to find out what they recommend and who they work with.

3. References

Connelly, F. M. and Clandinin, D. J. (1990) 'Stories of Experience and Narrative Inquiry', *Educational Researcher*, 19(5), pp.2–14. Available at: https://doi.org/10.2307/1176100.

Dias, A. (2022) 'Developing digital accessibility using R Markdown', *Forum Magazine*, 49, p.19. Available at https://bit.ly/adiasMarkdown

Hughes, C. (2019) 'Making lecture notes more accessible', unpublished.

Webb, C. (2022) 'Mapping the way to accessible mathematics', *Forum Magazine*, 50, p.25. Available at https://bit.ly/webbcMapping

[1] Accessible Equations web site for the University of York, http://bit.ly/eaccess-equations2.

[2] Jisc Accessible Maths working group have a web site at https://github.com/A11yMaths.

[3] Accessible maths blog post (https://elearningyork.wpcomstaging.com/2021/09/27/accessible-maths/)

THIS PAGE INTENTIONALLY LEFT BLANK

SHORT UPDATE

A level Mathematics (England) grade distributions and grade boundaries 2019-2023

Paul Glaister, Department of Mathematics and Statistics, University of Reading, UK.
Email: p.glaister@reading.ac.uk

Abstract

This article sets in context the A level Mathematics qualifications (in England) awarded in Summer 2023 with those awarded in 2019-2022, along with year-on-year overall comparisons of grade profiles, and year-on-year comparisons of grade boundaries and grade profiles for each of the four A level Specifications: Pearson/Edexcel, AQA, OCR A and OCR B (MEI).

Keywords: AS/A level qualifications; mathematics, grade profiles, grade boundaries, 2019-2023.

1. Introduction

A levels in Mathematics have been awarded since 1951, and underwent a major reform in England in 2017 (see Department for Education, 2016) which were first examined in 2018, although the 2019 examinations were the first ones sat by a cohort after two years' of A level study.

In 2020 & 2021 A levels in England were awarded on a different basis from previous years following the cancellation of national examinations because of the disruption to schools and colleges due to Covid.

2. 2019-2023 policy

The background policies and implementation to the awards In England in 2019-2023 can be summarised as follows:

- ***Summer 2019***: National examinations were held. Grades were awarded as in previous years - primarily on a statistical basis to ensure comparable outcomes with candidates in previous years.
- ***Summer 2020***: Centre Assessed Grades (CAGs) – students did not sit national examinations (unlike in 2019 and previously). They were awarded Centre Assessed Grades – schools/colleges were asked to submit a Centre Assessed Grade for their students which were expected to be a realistic judgement of the grade each student would have been most likely to get if they had taken their examinations in each subject.
- ***Summer 2021***: Teacher Assessed Grades (TAGs) – students did not sit national examinations (unlike in 2019 and previously). Instead of using the same approach as in 2020 using CAGs, students were instead awarded Teacher Assessed Grades – schools/colleges were asked to submit a Teacher Assessed Grade for their students which were expected to represent the student's performance in the subject content they had been taught, representing a subtle but important difference from CAGs.
- ***Summer 2022***: National Examinations were held. The Office of Qualifications and Examinations Regulator for qualifications, examinations and assessments in England (Ofqual) set out their intentions in September 2021 for Summer 2022 awards (Ofqual, 2021):

"*Our aim is to return to a pre-pandemic grade profile.*"

"*In 2022 we will aim, therefore, to reflect a midway point between 2021 and 2019.* ***In 2023 we aim to return to results that are in line with those in pre-pandemic years*****.**"

and Dr Jo Saxton, Chief Regulator at Ofqual, confirmed on the publication of A level results on 18 August 2022 (Ofqual, 2022):

"*Today's results are higher than those of 2019, and – as we have always said – lower than in 2021, when there was a different method of assessment. It makes sense to compare this year's results with those of 2019 when exams were last sat. I felt strongly that it would not have been right to go straight back to pre-pandemic grading in one go but accept that we do need to continue to take steps back to normality. These results overall, coming as they do broadly midway between 2021 and 2019, represent a staging post on that journey*."

- ***Summer 2023***: National Examinations were held. Dr Jo Saxton, Chief Regulator at Ofqual, confirmed on the publication of A level results on 17 August 2023 (Ofqual, 2023):

"Two years ago we set out a clear plan to return to pre-pandemic grading – a system that schools, colleges, universities and employers are all familiar with. As we said then, we expected overall A level results would be similar to 2019, and lower than in 2022. However, recognising the disruption that students have experienced, we put in place important grading protection to make sure that a student who would have secured a particular grade in 2019, would be just as likely to achieve that same grade this year. It is therefore more meaningful to compare this year's results with those of 2019, the last summer exam series before the pandemic.

"There has been a return to pre-pandemic grading this summer in England with protection in place for students. It is most meaningful to compare results to 2019, the last summer exam series before the pandemic.

"Overall A level results in England are similar to 2019. Outcomes at grade A and above are 26.5% compared with 25.2% in 2019, and outcomes at grade B and above are 52.7% compared with 51.1% in 2019."

3. Entry numbers

Figure 1 shows the entry numbers for each of the A level Specifications: Pearson/Edexcel, AQA, OCR A and OCR B (MEI), (Pearson, 2023a), (AQA, 2023a), (OCR, 2023a).

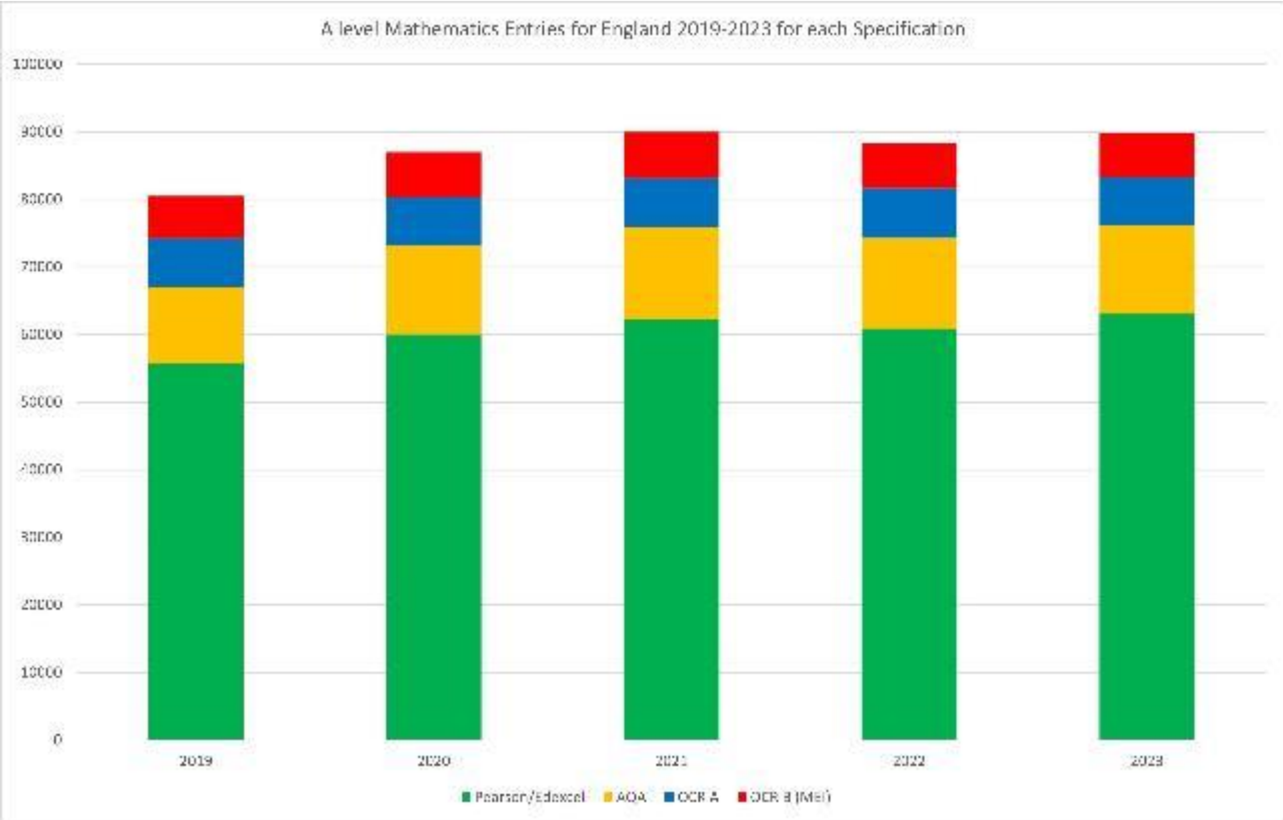


Figure 1. A level Mathematics Entries for England: 2019, 2022, 2023 (examinations), 2020 (CAGs), 2021 (TAGs) for each Specification.

4. Grade distributions

4.1. Overall national data

Figure 2 shows the grade distribution for National Examination results for A level Mathematics in England in Summer 2023 compared with the National Examination results in 2019 and 2022 and those from TAGs in 2021 and CAGs in 2020 (JCQ, 2023).

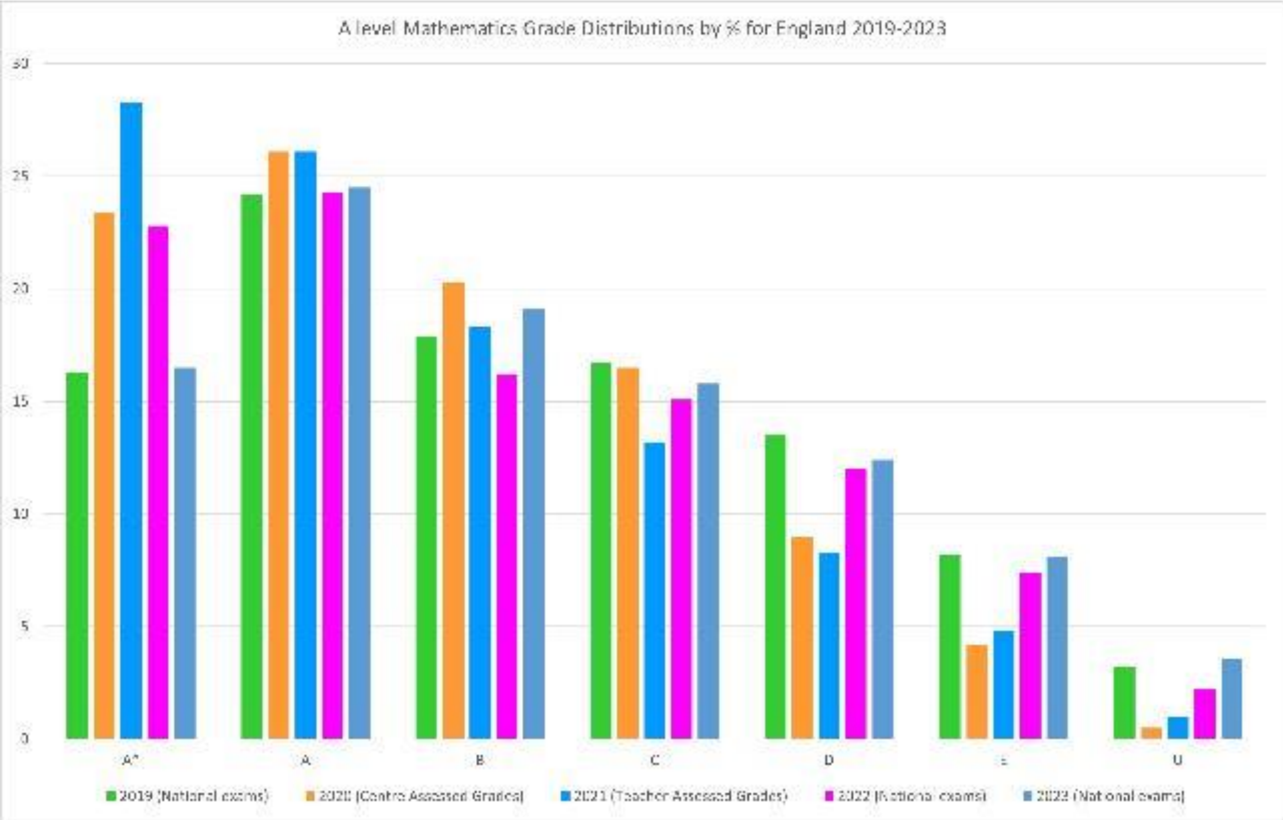


Figure 2. A level Mathematics Grade Distributions by % for England: 2019 (examinations), 2020 (CAGs), 2021 (TAGs), 2022, 2023 (examinations).

4.2. Data by year

Figures 3a, 3b and 3c show the grade distribution for the reformed Specifications: Pearson/Edexcel, AQA, OCR A and OCR B (MEI) as stacked column charts for each of the National Examination results for A level Mathematics in Summer 2019, 2022 and 2023, respectively, (Pearson, 2023a), (AQA, 2023a), (OCR, 2023a).

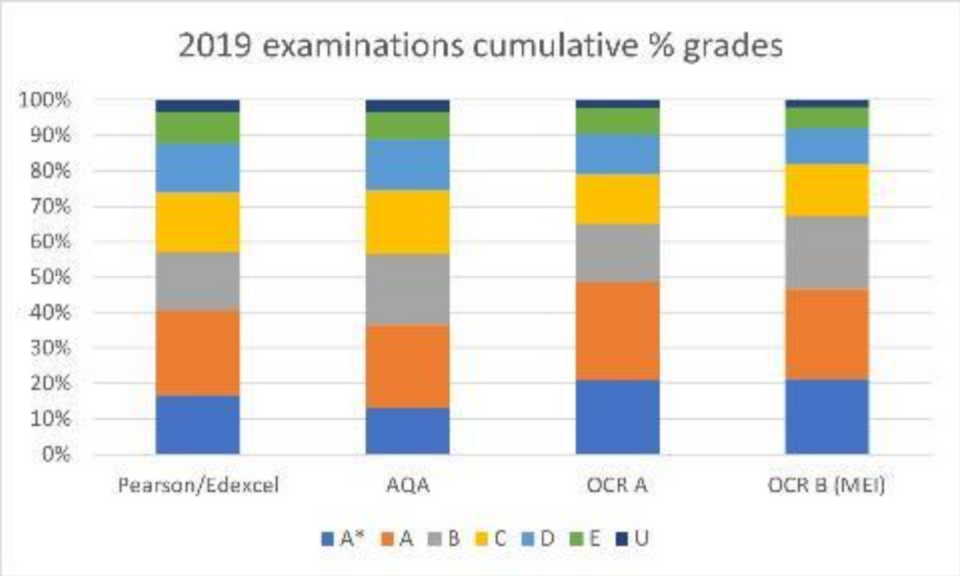


Figure 3a. A level Mathematics Grade Distributions by % for England 2019 for each Specification.

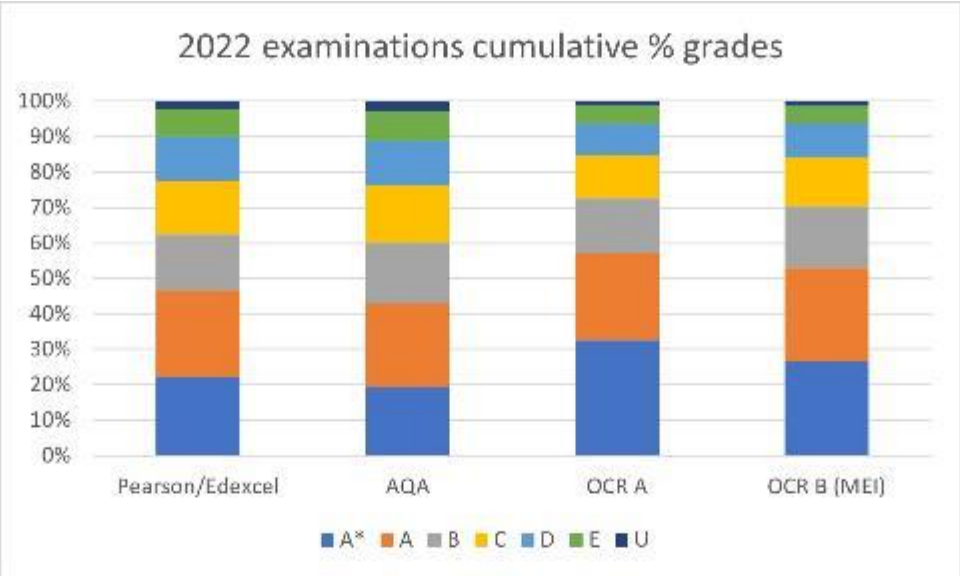


Figure 3b. A level Mathematics Grade Distributions by % for England 2022 for each Specification.

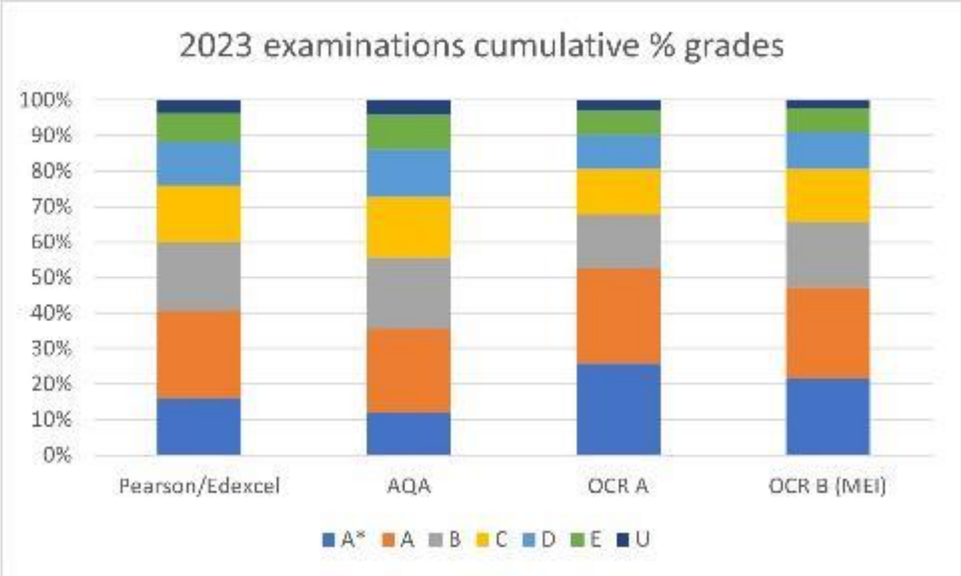


Figure 3c. A level Mathematics Grade Distributions by % for England 2023 for each Specification.

4.3. Data by specification

Figures 4a, 4b, 4c and 4d show the grade distribution for each the reformed Specifications: Pearson/Edexcel, AQA, OCR A and OCR B (MEI), respectively, as stacked column charts for each of the National Examination results for A level Mathematics in Summer 2019, 2022 and 2023, (Pearson, 2023a), (AQA, 2023a), (OCR, 2023a).

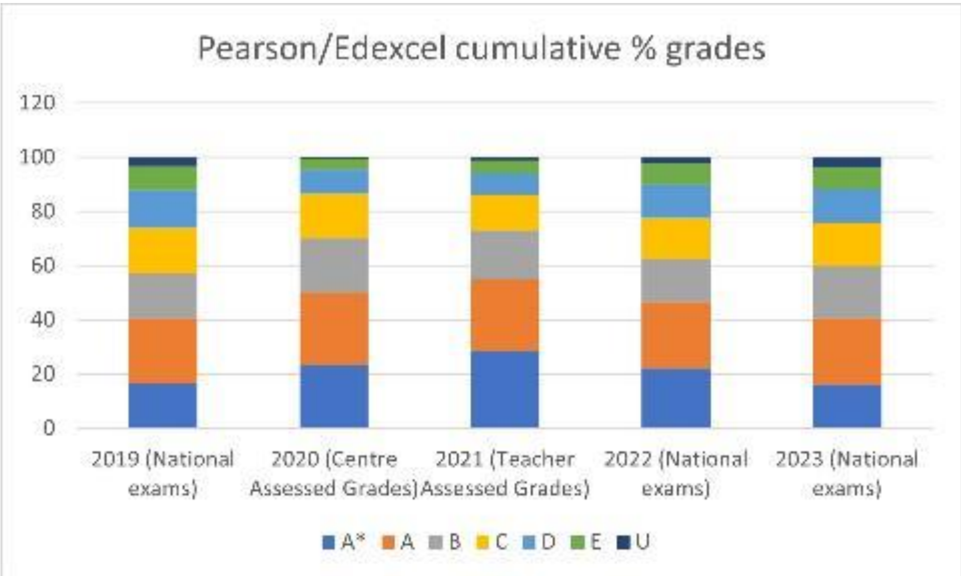


Figure 4a. A level Mathematics Grade Distributions by % for England 2019-2023 for Pearson/Edexcel.

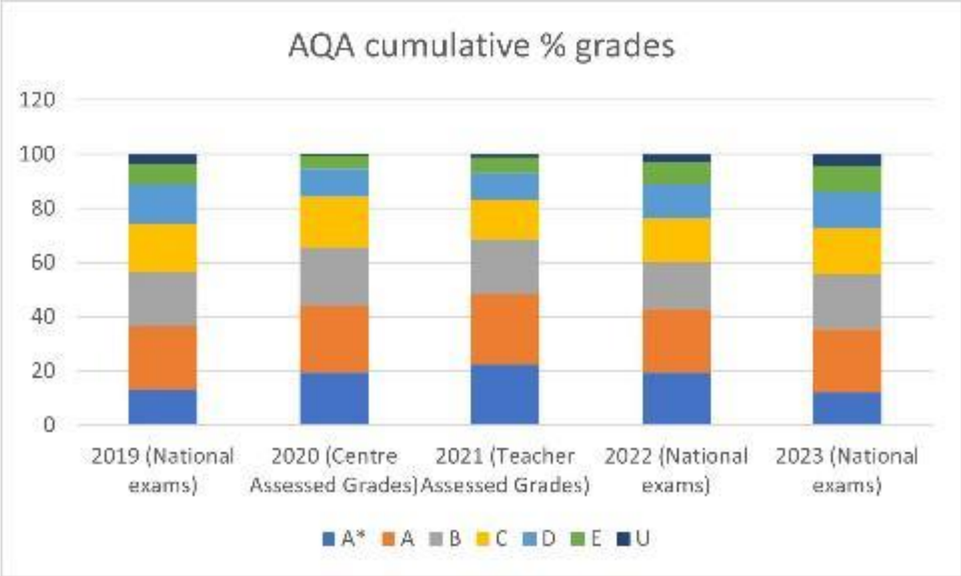


Figure 4b. A level Mathematics Grade Distributions by % for England 2019-2023 for AQA.

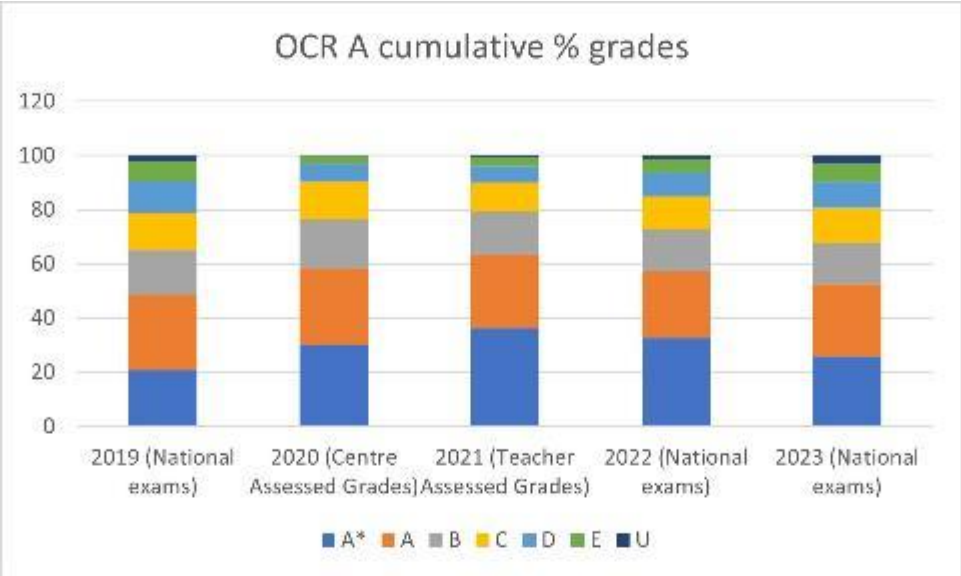


Figure 4c. A level Mathematics Grade Distributions by % for England 2019-2023 for OCR A.

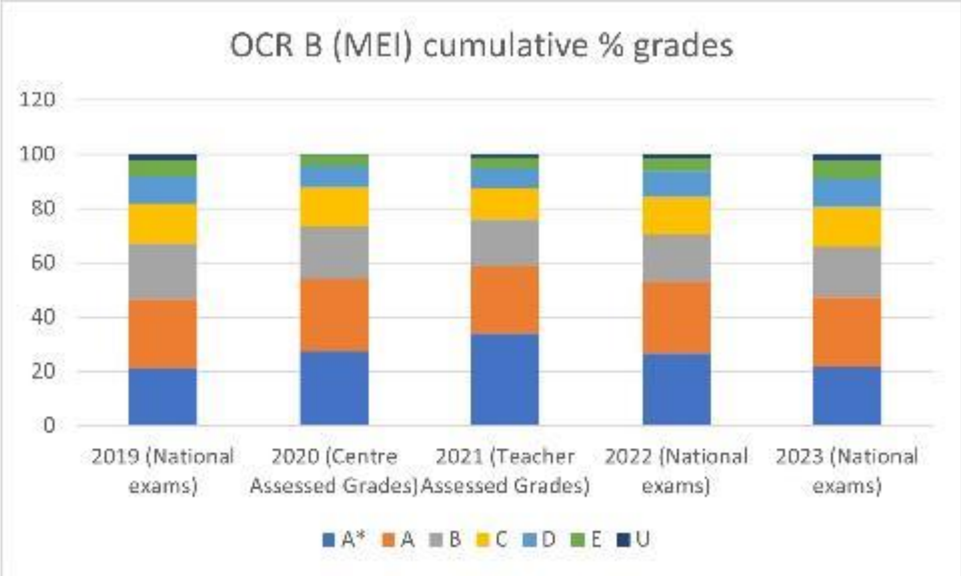


Figure 4d. A level Mathematics Grade Distributions by % for England 2019-2023 for OCR B (MEI).

(See Figures 5 and 6 in Glaister (2023) for all grade distributions shown on the same graph - ordered by Specification and by Year, respectively - for the National Examination results for A level Mathematics in Summer 2023, compared with those from TAGs in 2021, CAGs in 2020, and the National Examination results in 2019 and 2022, for the reformed Specifications: Pearson/Edexcel, AQA, OCR A and OCR B (MEI).)

4.4. Overall national data by Specification and by Year for grades A, A and B*

Grades A*, A and B are typical grade requirements for A level Mathematics for students applying for entry to degree programmes in mathematical sciences.

Figures 5a and 5b show the detailed profiles for grade A* only - ordered by Year and by Specification, respectively; the corresponding detailed profiles for grade A only are shown in Figures 6a and 6b, while Figures 7a and 7b show the corresponding detailed profiles for grade B only, (Pearson, 2023a), (AQA, 2023a), (OCR, 2023a).

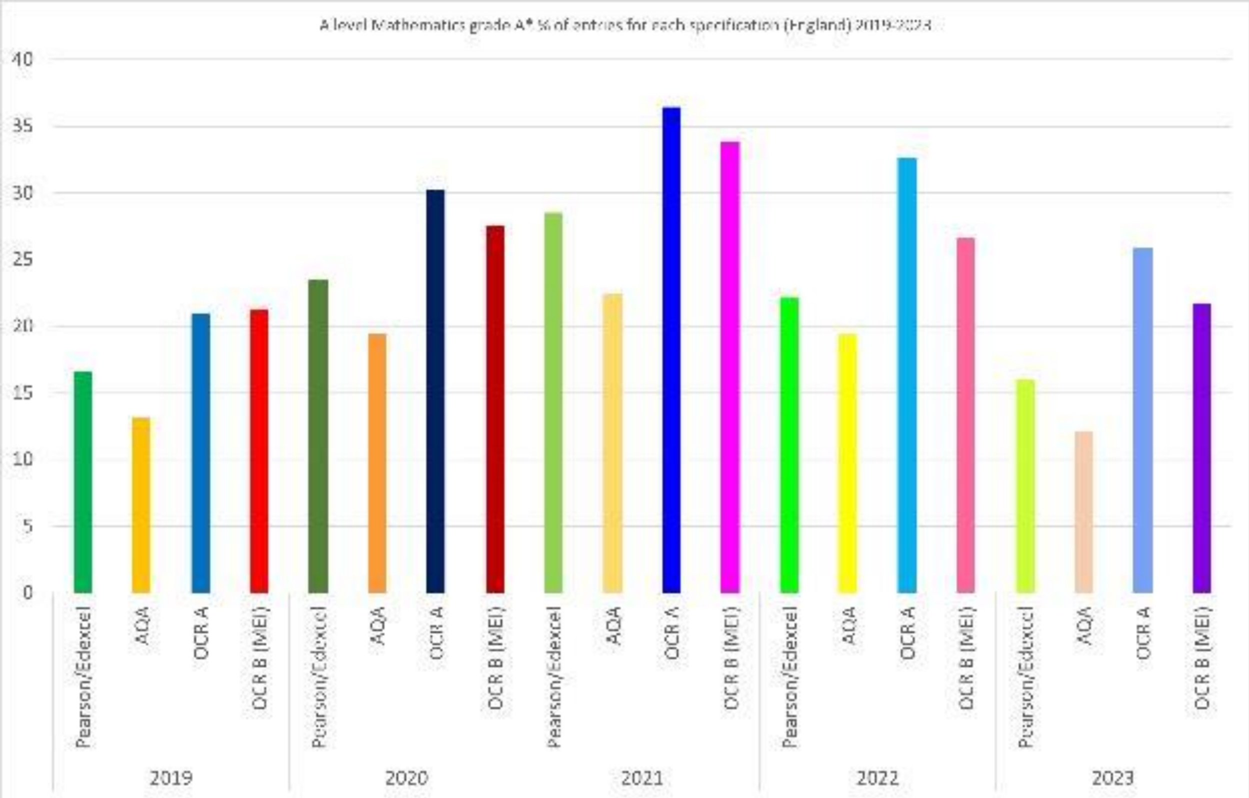


Figure 5a. A level Mathematics Grade A* Profiles by % for England 2019-2023 for each Specification.

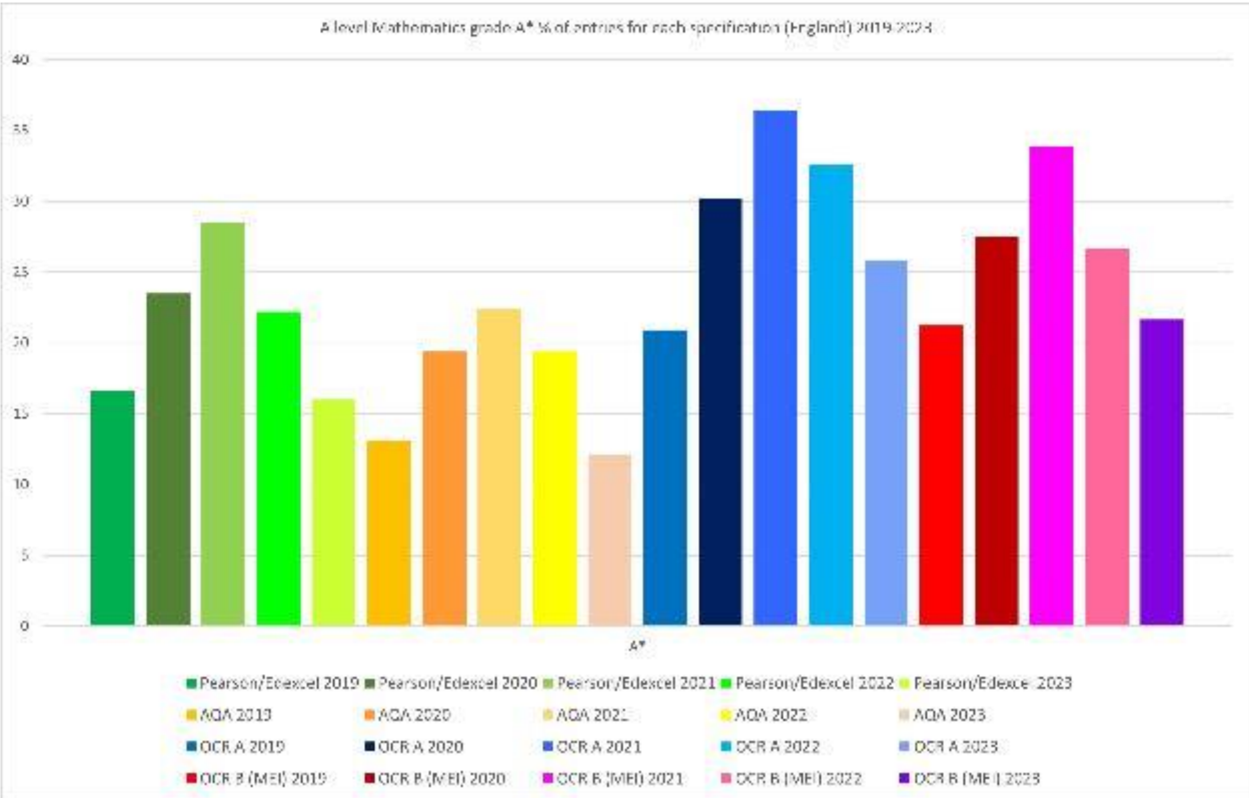


Figure 5b. A level Mathematics Grade A* Profiles by % for each Specification for England 2019-2023.

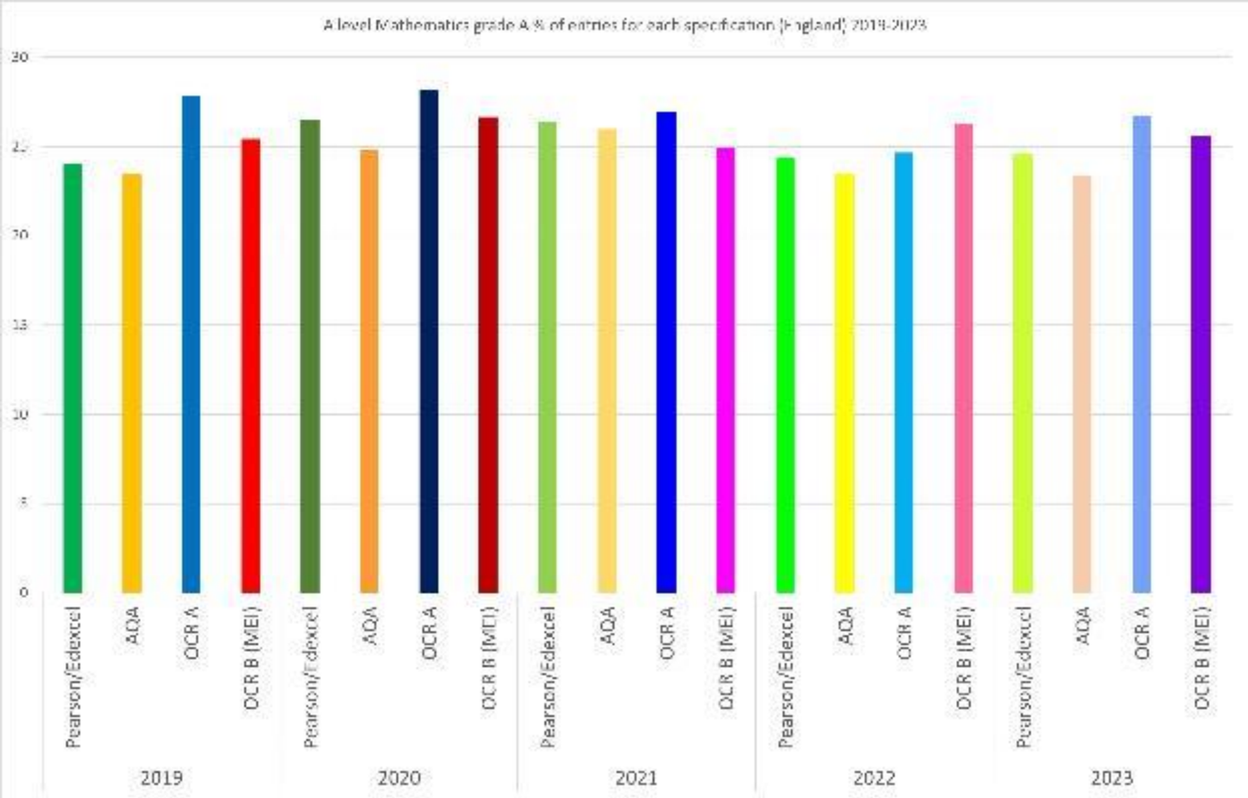


Figure 6a. A level Mathematics Grade A Profiles by % for England 2019-2023 for each Specification.

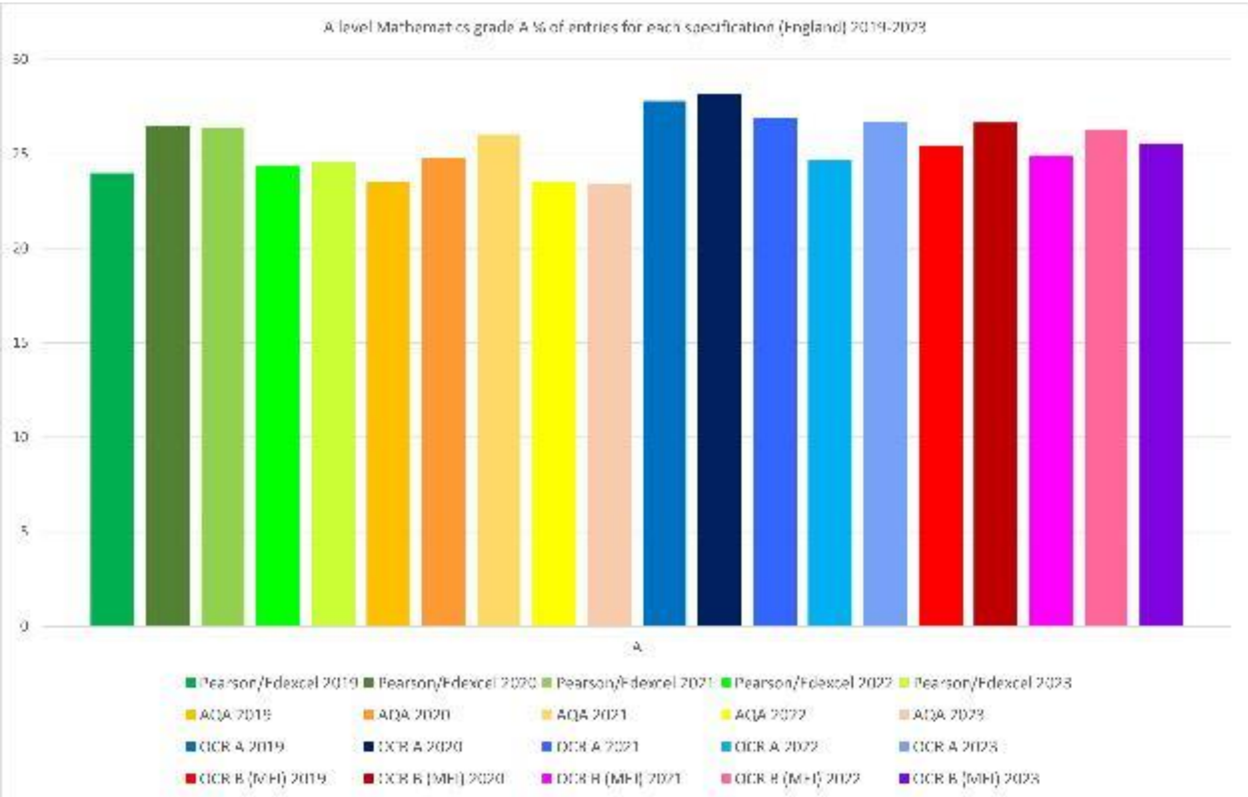


Figure 6b. A level Mathematics Grade A Profiles by % for each Specification for England 2019-2023.

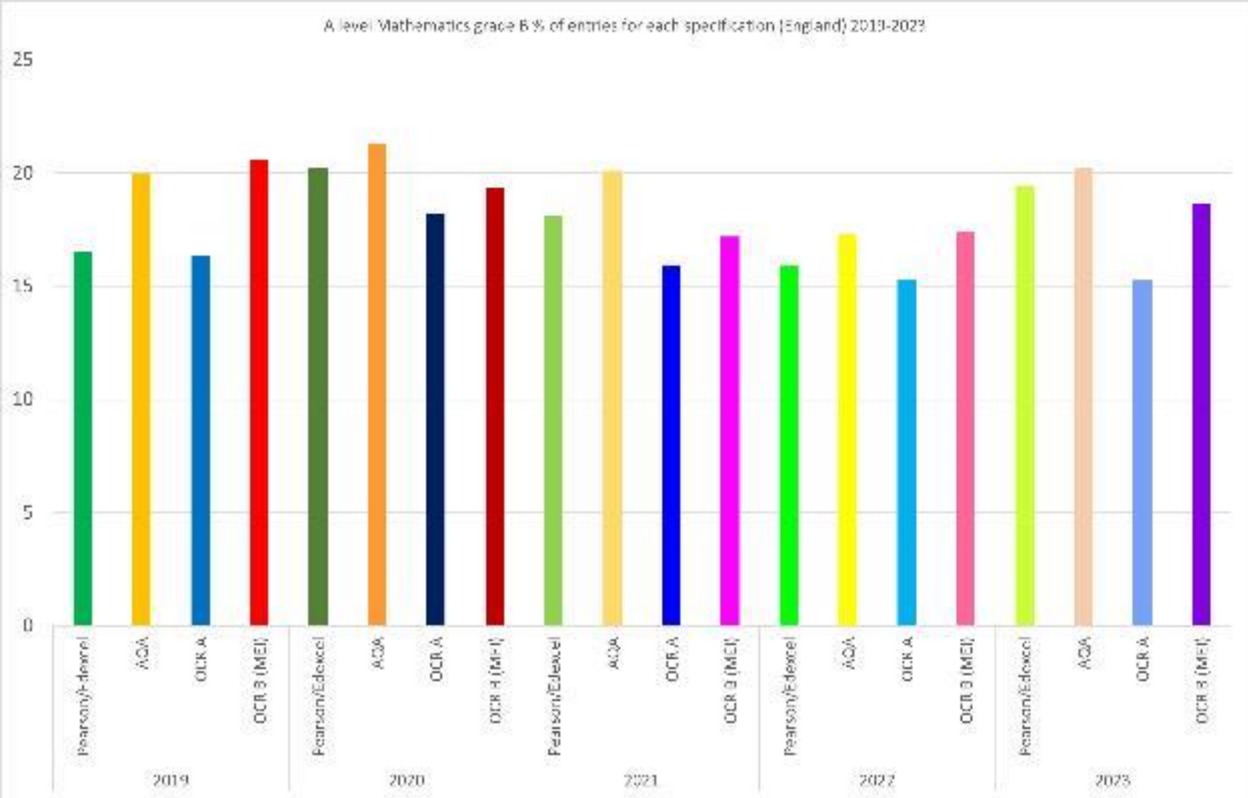


Figure 7a. A level Mathematics Grade B Profiles by % for England 2019-2023 for each Specification.

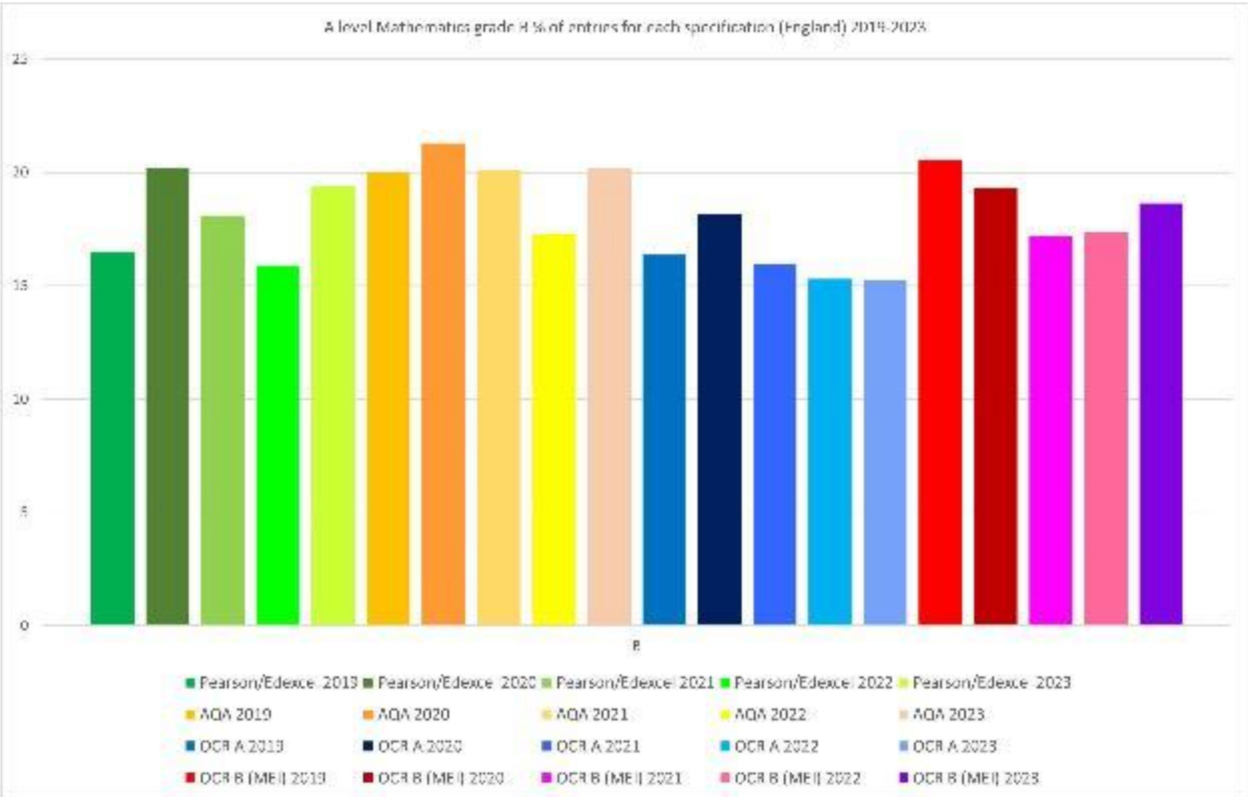


Figure 7b. A level Mathematics Grade B Profiles by % for each Specification for England 2019-2023.

5. Grade boundaries

Figures 8a, 8b and 8c show the A level Mathematics grade boundaries as % for each of 2019, 2022 and 2023 National Examinations, respectively, for each Specification, (Pearson, 2023b), (AQA, 2023b), (OCRb, 2023).

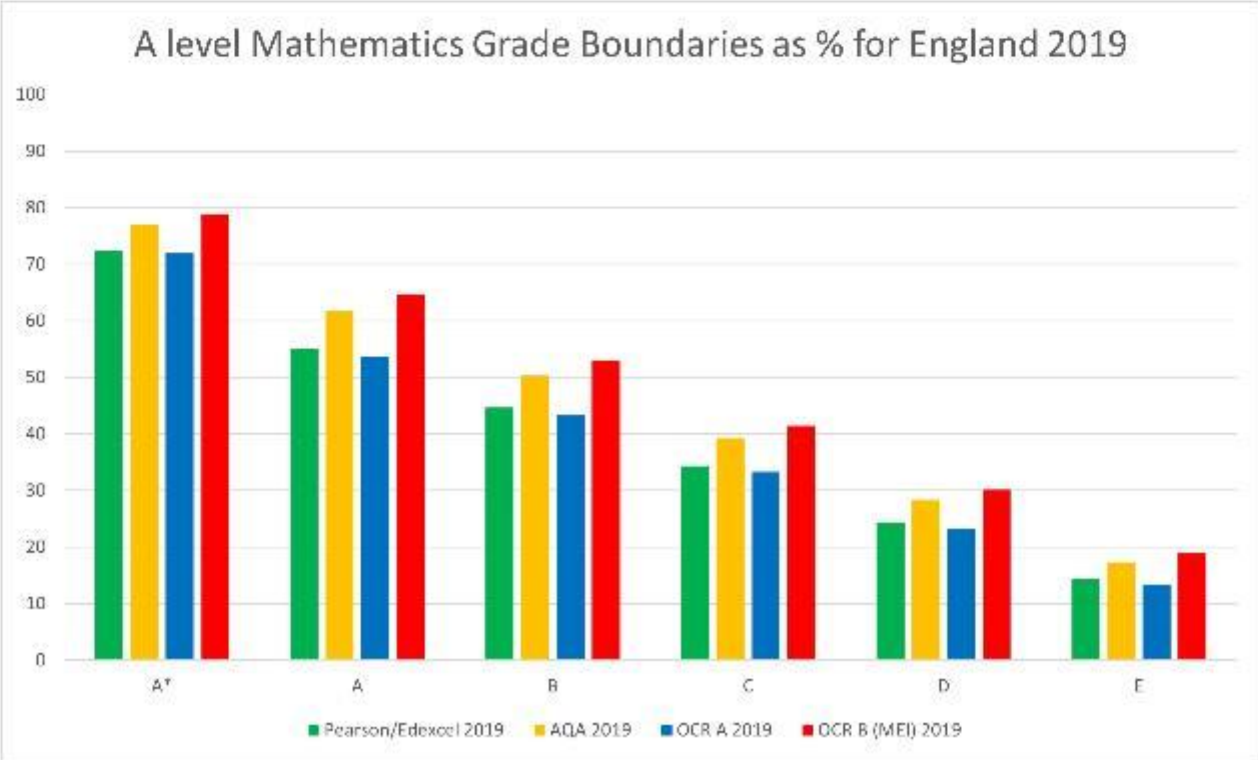


Figure 8a. A level Mathematics grade boundaries as % for 2019 National Examinations for each Specification.

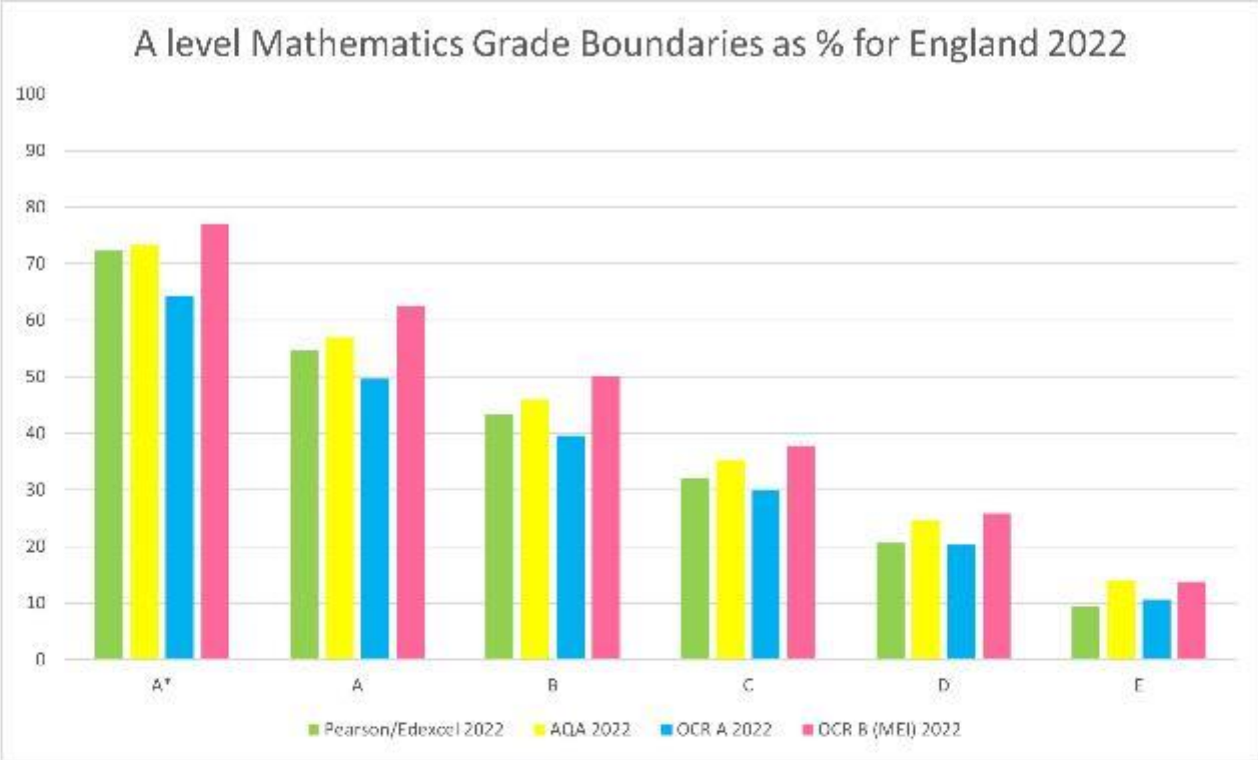


Figure 8b. A level Mathematics grade boundaries as % for 2022 National Examinations for each Specification.

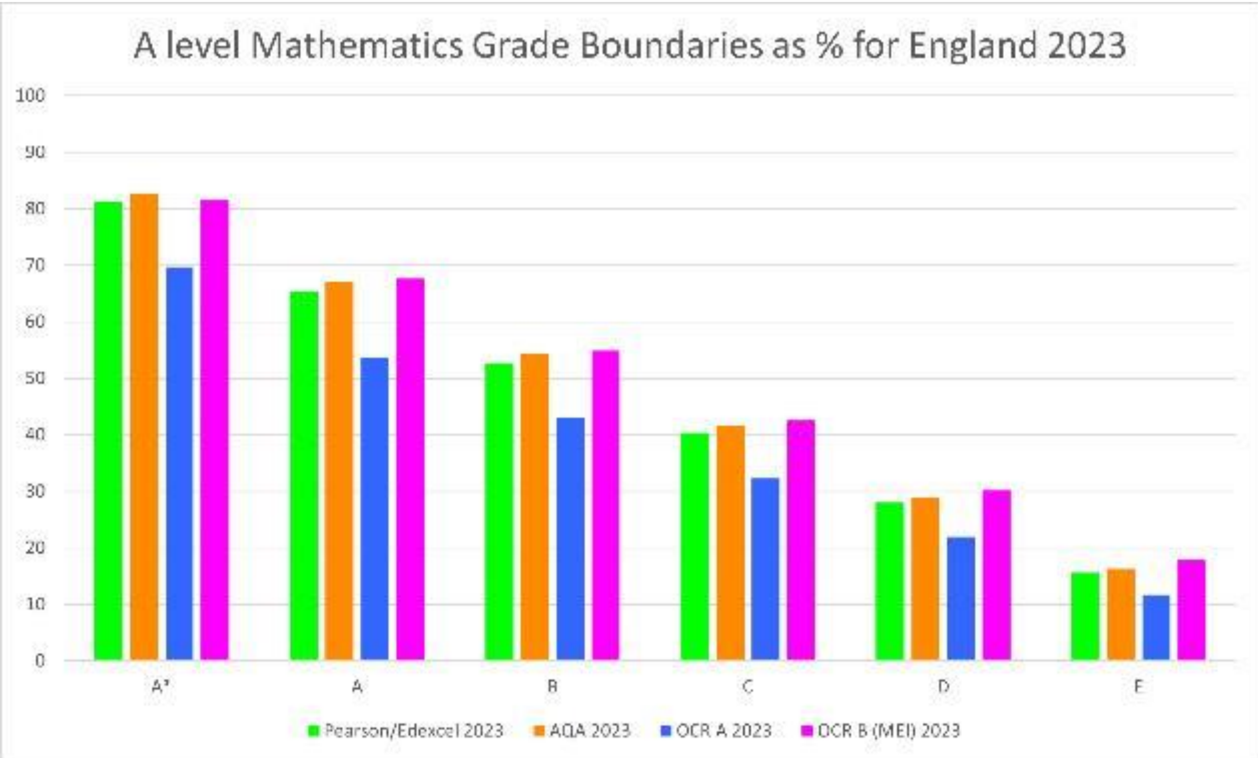


Figure 8c. A level Mathematics grade boundaries as % for 2023 National Examinations for each Specification.

(See Figures 14 and 15 in Glaister (2023) for all grade boundaries shown on the same graph as % for 2019, 2022 and 2023 National Examinations - ordered by Specification and by Year, respectively.)

6. References

AQA, 2023a. AQA | Exams admin | Results days | Results statistics. Available at: https://www.aqa.org.uk/exams-administration/results-days/results-statistics [Accessed 17 August 2023].

AQA, 2023b. AQA | Exams admin | Results days | Grade boundaries. Available at: https://www.aqa.org.uk/exams-administration/results-days/grade-boundaries [Accessed 17 August 2023].

Department for Education, 2016. GCE AS and A level subject content for mathematics. Available at: https://www.gov.uk/government/publications/gce-as-and-a-level-mathematics [Accessed 2 October 2023].

JCQ, 2023. Examination results - JCQ Joint Council for Qualifications. Available at: https://www.jcq.org.uk/examination-results/ [Accessed 17 August 2023].

OCR, 2023a. Results statistics (ocr.org.uk). Available at: https://www.ocr.org.uk/administration/results-statistics/ [Accessed 17 August 2023].

OCR, 2023b. Grade boundaries (ocr.org.uk). Available at: https://www.ocr.org.uk/administration/results-statistics/ [Accessed 17 August 2023].

Glaister, P., 2023. A level Mathematics (England) grade distributions and grade boundaries 2019-2023. Available at: https://www.personal.reading.ac.uk/~smsglais/A_level_Mathematics_2017_reformed_specifications-grade_distributions_and_grade_boundaries_2019-2023.pdf [Accessed 17 August 2023].

Ofqual, 2021. Ofqual's approach to grading exams and assessments in summer 2022 and autumn 2021. Available at: https://www.gov.uk/government/speeches/ofquals-approach-to-grading-exams-and-assessments-in-summer-2022-and-autumn-2021 [Accessed 17 August 2023].

Ofqual, 2022. Guide to AS and A level results in England, summer 2022. Available at: https://www.gov.uk/government/news/guide-to-as-and-a-level-results-in-england-summer-2022 [Accessed 17 August 2023].

Ofqual, 2023. Guide to AS and A level results for England, summer 2023. Available at: https://www.gov.uk/government/news/guide-to-as-and-a-level-results-for-england-summer-2023 [Accessed 17 August 2023].

Pearson, 2023a. Grade statistics | Pearson qualifications. Available at: https://qualifications.pearson.com/en/support/support-topics/results-certification/grade-statistics.html?Qualification-Family=A-Level [Accessed 17 August 2023].

Pearson, 2023b. Grade boundaries | Pearson qualifications. Available at: https://qualifications.pearson.com/en/support/support-topics/results-certification/grade-boundaries.html?Qualification-Family=AS-and-A-level [Accessed 17 August 2023].

WORKSHOP REPORT

The new normal: What does maths and stats support and teaching look like post pandemic?

Susan Pawley. School of Mathematics and Statistics, The Open University, Milton Keynes, UK. Email:susan.pawley@open.ac.uk
Andrew Neate. School of Mathematics and Statistics, The Open University, Milton Keynes, UK. Email:andrew.neate@open.ac.uk

Abstract

The workshop "The new normal: What does maths and stats support and teaching look like post pandemic?" took place on the 1st of December 2022 and discussed the changes to mathematics and statistics support since the lifting of restrictions after the pandemic. The event consisted of five short talks where presenters explored the changes that had occurred to mathematics and statistics support at their institutions and concluded with a discussion on how to effectively combine online and face-to-face support and how to increase engagement in all forms of support. This report summarises the talks and discussion, concluding with some thoughts on the changes still required and how we can support each other.

Keywords: maths and stats support, online student support, **sigma** Network, post COVID-19 pandemic.

1. Overview

A workshop to consider the new normal in mathematics and statistics support (MSS) was held in December 2022. The aim of the workshop was to consider the changes in support from pre-pandemic methods and how well students were engaging with these new approaches. The workshop was a **sigma** Network (**sigma** Network, 2023) event hosted by The Open University. It took place online and was attended by 37 participants from 26 different higher education establishments in four different countries.

Prior to the COVID-19 pandemic, national surveys of MSS indicated that whilst there was widespread provision in the UK (Grove et al, 2019), and Ireland (Cronin et al., 2016), most of this support was face-to-face, with only limited on-line support being provided (Mac an Bhaird et al., 2021). The introduction of pandemic lockdown measures in March 2020, led to a rapid change in the provision of support (Hodds, 2020; Johns, 2021) with universities restricting campus access for students and staff so that MSS, along with most other teaching, had to move almost completely online. Consequently, as Hodds (2020) reports, most institutions saw a drastic reduction in student numbers participating in MSS when compared to pre-pandemic levels. As the restrictions were relaxed, Gilbert *et al.* (2021) reported that, of 12 practitioners from around the world interviewed in January 2021, most indicated that when in-person support was permitted, they would continue with some online support but as a supplementary offering. However, as reported in O'Sullivan & Pfeiffer (2023), there remain challenges in getting students to engage with MSS since the pandemic.

The workshop started with Holly Gilbert discussing the findings she had gathered over the period May 2020 to June 2021 on MSS in the "new normal". It then followed with practitioners from four different institutions discussing how they were now offering MSS and finished with a discussion on how to effectively combine online and in-person support and how to increase engagement in all forms of support. The workshop recordings, and other resources, can be found on the **sigma** Network website (**sigma** Network, 2023b).

2. MSS in the “new normal”: The practitioner and student perspective

The workshop was opened by Holly Gilbert who is a third-year PhD student at Coventry University researching MSS in the “new normal” under the supervision of Professor Duncan Lawson and Dr Mark Hodds. She presented on the three areas of her research: the practitioner perspective, the student perspective, and observations of pedagogy. She then concluded by giving some recommendations on good practice for MSS provision in our “new normal”.

The practitioner perspective examined the types of MSS institutions offered prior to and during the pandemic and their current opinion of online MSS and how that has changed through the pandemic. This took the form of a longitudinal study, where she explored the results from:

- a questionnaire conducted by Hodds (2020), in particular the final open-ended question, “should we be changing our approach or replicating what we did face-to-face?”;
- data from 12 interviews undertaken in early 2021 (Gilbert et al 2021);
- and the results from a comparative survey between practitioners in Germany, Great Britain and Ireland which was completed in June 2021 (Gilbert et al 2023).

The main theme that arose from the 2020 study is that practitioners were unsure whether they should be replicating online what they did face-to-face or adopting new approaches. By the interviews in early 2021, all interviewees were seeing benefits from the flexibility of online MSS such as the ability to be more accessible to students that are away from campus but were noticing reduced interaction and the inability to see unspoken cues. In the June 2021 survey, most practitioners were still finding it difficult to determine how well students were engaging, however there were some that were finding it easier. The overall message from practitioners across the three studies was that returning to face-to-face MSS was positively anticipated but online methods should be used as a compliment.

The student perspective focused on the question “what are the current barriers to students’ engagement and how can we reduce them?”. She asked both users and non-users of MSS and found the two main reasons for non-use were that students were unaware MSS existed and that they were embarrassed to use MSS.

Gilbert then outlined her current research on observations in pedagogy, where she is comparing the behaviour of MSS tutors in online and face-to-face sessions with reference to a protocol she has designed and investigating whether her observations match the impressions of the participants involved.

Finally, Gilbert concluded by offering several recommendations to MSS providers, with the key advice being providers needed to advertise extensively, explaining not only what is on offer and where, but also the advantages of MSS, what students should expect and regular reassurance that MSS is for everyone. They also need to increase their advertising reach by utilising lecturers and making adverts more personal, based on for example subject groups, to highlight that MSS is not just for mathematics students.

3. Current MSS provision in four UK Universities

The workshop continued with four presenters from different institutions discussing the MSS provisions at their university, what their experiences are post pandemic, the challenges they are facing and how they are addressing them.

Lesley Davis, from the University of Leicester, spoke about how their support had changed over the last few years. Before the pandemic, the MSS team had access to a suitable permanent room, with tables and chairs, where they were able to offer a dedicated drop-in support clinic every Thursday and a mix of drop-in support and booked appointments for the rest of the week. Post pandemic, they have been moved to a different room, which whilst more visible, is only suitable for booked appointments. The demand for booked sessions increased significantly after the end of the pandemic, but staff were aware they have a much larger capacity if they run drop-in sessions, as the time taken for the support is generally less than the booked session. To address this, they attempted to reintroduce their drop-in service on a Monday and Thursday. The sessions were promoted widely and directed at the same bodies of students that made good use of the drop-ins pre-pandemic. However, attendance at these has been very low, which could be due several factors including the sessions being in different rooms on a Monday and Thursday, and a perception amongst students that the drop-ins were only to be used if a booked appointment was not available. Therefore, from 2023, the centre will only be offering booked sessions, and they will be looking at how improvements can be made by changing the booking methods and offering a variety of appointment lengths, with the idea that they will be able to offer short appointments quickly, but a longer appointment will need to be booked in advance.

Inna Namestnikova discussed the current MSS provision at Brunel University where support had remained completely online during the last two academic years, only returning to blended provision in October 2022. Prior to the pandemic, a blended structure for MSS had been introduced in 2016/17, however the online sessions had very low attendance, with most students preferring to see support staff in person. During the pandemic, when only online support was available, the number of students attending these sessions increased to levels similar to pre-pandemic uptake of face-to face support. However, when the option of face-to-face support was reintroduced in October 2022 students continued to prefer to access online support. Moreover, the way in which students want to be supported appears to have also changed. There is a reduction in students attending central workshops, such as “Back to Maths”, which aims to improve their overall ability in mathematics, with students instead preferring individual tailored support. Looking towards provision for the future, MSS will need to include further support for their increasing post graduate population, trying to establish the best methods to support mathematics software packages and exploring the suitability of different conferencing facilities to support particular teaching activities. Inna’s final comment on the change to MSS, is that for Brunel students online support appears to work best.

Rob Wilson, from Cardiff University, focussed his presentation on the ways they were revitalising mathematics learning communities. This was framed by a discussion of the evidence around the potential impact of learning communities and how these support retention, motivation, well-being, engagement and other aspects of the broader student experience. Prior to the pandemic, by way of a student partnership project, Cardiff University had already begun looking at learning communities to help increase engagement. However, the pandemic had given them the opportunity to go back and consider what they value, what they are trying to encourage and how they go about it. The two key findings that arose from the student partnership project were that students can feel isolated from each other and from their department and faculty, and that communications could give a sense of hierarchy to students which can be seen as a barrier to belonging. From this, several changes, both formal and informal, had been made, and whilst they were trying to develop community, they were aware that there are lots of different sub-communities within the student body. Staff and students have been given responsibility and time to focus on community and there is now a student engagement officer and a student voice coordinator. They have completely changed the emphasis of their induction sessions encouraging students to speak to each other and find out who their peers are.

Alongside this they had also recently moved to a new building which provides both lecture theatres and shared social spaces where students can work and meet in an informal setting, creating an environment that has a “buzz” around it. MSS has an informal space in the new building but it does not have a dedicated room. Consequently, since the pandemic, MSS has changed its physical location, the type of study space it uses and the times it operates. They are now trying to re-establish their presence and have seen an upward trend in drop-in numbers. Other changes since the pandemic include the introduction of a flipped classroom approach for a large core first year module which helps staff to see how students are coping, allowing them to react in real time to any problems. Informal activities such as an away day, a weekly board games hour, a coding club, pizza and movie nights have also been established, giving choice to students about what they get involved in and trying to integrate the Maths Society into the school further. Rob's initial reflections on these changes were:

- not to ignore small potentially positive changes that could be easily made. For instance, changing the way we address students via email to be more inclusive and less hierarchical (e.g. changing, "Dear students" to "Dear fellow mathematicians"),
- to keep in mind diversity, choice and ownership in changes made. Different students will engage at different times with different things for different reasons, so it is important to consider different options and choices within those options. It is also important to recognise that staff and students are part of the community and so it is important to get students involved in the planning and organising of activities.
- Staff engagement with the community requires just as much emphasis as student engagement.

Alun Owen discussed the past, present and future of the **sigma** Maths and Stats Support Centre at Coventry University. This is a large MSS centre, which offers support both face-to-face and online. Pre-pandemic, centre visits were in their tens of thousands per year, but during the pandemic period these numbers reduced significantly when all support was moved online. They observed that the abrupt changes brought about by the pandemic had led to increased levels of general anxiety which contributed to students having additional support needs, struggling with understanding content, and also potentially affecting their ability to access support. Students have also been observed to work in different ways, creating more online informal groups and working together online. More recently they have seen a significant return to in-person visits to the centre as students are seeing the benefit of face-to-face support. In 2021/22 all lectures at Coventry were hybrid, but students were only offered the option of online if required. Consequently, the number of visits to the centre increased and it continues to offer mostly face-to-face and some limited online drop-in sessions, which can be challenging for staffing as they cannot be run in parallel.

Since the pandemic and following on from Gilbert's research (Gilbert et al, 2021; 2023), Coventry University have tried to make their MSS centre more inclusive by, for example, placing inspirational quotes in the centre and changing advertising to make it clear the centre is not just for mathematicians. They have also tried to enhance the centre by making the lighting softer and are introducing sound reduction to help neurodiverse students. They seek regular, simple, immediate feedback from students to ensure they are happy with their visit. They have written a code of practice for centre staff, which is displayed to students so they know what to expect. This includes providing a friendly welcome, adapting their approaches where required, being honest about what centre staff can and cannot do. They are also trying to tackle plagiarism, by indicating that whilst they cannot directly help a student to answer coursework questions, they can help with course content that is relevant to coursework assignments. As well as continuing to run diagnostic tests for new students on a number of courses, they have also started several additional proactive measures to support students including running: refresher workshops (similar to Brunel's back to maths), consolidation workshops, maths anxiety workshops, and staff networks to connect with staff who may be working in isolation to support students with maths or statistics. There are also reactive measures that have been put in place, such as adapting timetables to run both online and face-to-face support sessions, offering appointments of differing lengths in both modes of delivery, and making resources more accessible. To increase inclusivity, they also have academic staff from different subject teaching teams, such as nursing or engineering, contributing some hours in the centre so that their students can see it is not only for mathematics students.

Owen then discussed the changing situation of drop-in statistics consultations. Prior to the pandemic, the demand for statistics consultations was predictable with a clear peak every March corresponding to undergraduate project deadlines. However, in 2020/21 a second peak in demand occurred in June, due to students deferring their projects (as was allowed over pandemic). In 2021/22, demand had recovered to pre-pandemic levels with mainly face-to-face support but now with a far larger project peak in March and a second deferred project peak in the summer. This increased peak demand has been covered through an increase in staff numbers. So far, in 2022/23, statistics support demand has been even higher!

Looking to the future, the centre's students are coming from an increasingly wider range of subject areas, with many more courses, such as the biosciences for example, becoming much more data driven. They currently have students with increased general anxiety as well as mathematics/statistics anxiety and are unsure whether this will reduce once the effects of the pandemic lessen. There is a greater focus on MSS to contribute to access and participation plans, to facilitate support for those with additional learning needs and neurodiversity differences, with more accountability for the services provided and resources needing to be accessible. The support required is also changing with mathematics becoming more reliant on programming languages. Skills in data handling and coding are becoming more important with an increase in demand for new data science courses, computational social sciences and combined studies of data science and healthcare.

4. Discussion on effectively combining online and face-to-face support and how to increase engagement in MSS

After a short break we continued with a discussion on how to effectively combine online and face-to-face support and how to increase engagement in all forms of support. A couple of questions about the online diagnostic tests used at Coventry highlighted that whilst mathematics support was now mainly face-to-face there, which was seen as most effective, statistics support could sometimes be more suitably delivered online, where screens could be shared and online statistical packages supported more efficiently. This demonstrated the need for both online and face-to-face support to continue.

Further discussions on the development of a learning community at Cardiff displayed how many institutions were returning to concentrate on the face-to-face aspects however it has been noted that many students are not returning to face-to-face lectures and are instead relying on using uploaded materials and recordings to study courses. This in turn is affecting the number of students that are seeking face-to-face support and so support centres will need to adapt their provision to facilitate the ways students are learning and appreciate that students may be looking for alternative resources to supplement their learning more than they have done in the past.

The discussion then moved on to talk about the differences between formal and informal environments for MSS and what the benefits and disadvantages are of these two types of locations. With a formal physical space for MSS it is easy for staff to see students outside that may be reluctant to come in and who can then be gently encouraged. In contrast, when locating MSS within informal social areas, it is difficult for staff to tell if students need help or are just using the space. In a small formal space it maybe more obvious that a student is getting help, which could lead to anxiety or create a stigma. However, in a larger or informal social space, it can be less obvious that students are receiving help from staff as students are studying there independently which can make access easier. Having a large flexible space available for MSS on a regular basis may encourage engagement from students but with space at a premium in most universities this is not always possible.

Looking at ways to increase engagement in support, a post pandemic initiative at the University of Bedfordshire was discussed where support staff coordinated with lecturers and visited as many lectures as possible, particularly those where there are known support requirements, to introduce themselves and remind students about the support available. This had resulted in increased attendance at the MSS centre. This has the added advantage of students seeing the faces of the support staff so they become more familiar, as well as showing that the support they offer is endorsed by the lecturer.

5. Conclusions

The use of MSS services changed dramatically during the pandemic and there remain questions about what best practice will look like in future. The way students engage with support has changed and the opportunities and lessons learnt have offered some improvements that now need to be incorporated into our more traditional methods. For some institutions there has been a change in student behaviour, and they are now more likely to book an online support session than turn up at a drop-in face-to-face session. At other universities, the demand for face-to-face support has returned alongside a provision of online support. How we adapt to the new style of support is still emerging and the discussions held in workshops like these are invaluable to help support each other and share practice.

6. Acknowledgements

The authors are grateful to the **sigma** Network Steering Group for organising and running the workshop for which this report has been generated.

7. References

Cronin, A., Cole, J., Clancy, M., Breen, C. and Ó Sé, D. (2016). An Audit of Mathematics Support Provision on the Island of Ireland in 2015*. Dublin: National Forum for the Enhancement of Teaching and Learning in Higher Education.* Available via http://www.sigma-network.ac.uk/wp-content/uploads/2019/02/Audit-of-MLS-provision-Ireland.pdf (last accessed 24 April 2023).

Gilbert, H., Hodds, M. and Lawson, D. (2021). "Everyone seems to be agreeing at the minute that face-to-face is the way forward": practitioners' perspectives on post-pandemic mathematics and statistics support. *Teaching Mathematics and its Applications*, 40(4), pp. 296–316.

Gilbert, H, Schürmann, M, Liebendörfer, M, Lawson, D and Hodds, M (2023). Post-pandemic online mathematics and statistics support: Practitioners' opinions in Germany and Great Britain & Ireland. *International Journal of Mathematical Education in Science and Technology.* https://doi.org/10.1080/0020739X.2023.2184282

Grove, M., Croft, T. and Lawson, D. (2020). The extent and uptake of mathematics support in higher education: results from the 2018 survey. *Teaching Mathematics and its Applications*, 39(2), pp. 86-04.

Hodds, M. (2020). *A report into the changes in Mathematics and Statistics support practices due to Covid-19.* Available via https://www.sigma-network.ac.uk/wp-content/uploads/2020/07/Report-into-the-changes-in-Maths-and-Stats-Support-practice-during-Covid-19.pdf (last accessed 24 April 2023).

Johns, C. and Mills, M. (2021). Online Mathematics Tutoring During the COVID-19 Pandemic: Recommendations for Best Practices. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 31(1), pp. 99–117.

Mac an Bhaird, C., Mulligan, P. and O'Malley, J. (2021). Mathematics support centres' online presence: provision in Ireland and the UK in 2018. *Teaching Mathematics and Its Applications*, 40(3), pp.190-209.

O'Sullivan, C. and Pfeiffer, K. (2023). Mathematics Learning Support - Linking Practice to Research in the New Normal - A report of the IMLSN Workshop 2022. *MSOR Connections*, 21(3), pp. 39-43. https://doi.org/10.21100/msor.v21i3.1364

sigma Network (2023). ***sigma*** *Network for excellence in mathematics and statistics support*. Available via http://www.sigma-network.ac.uk/ (last accessed 24 April 2023).

sigma Network (2023, b) *The new normal: What does maths and stats support and teaching look like post pandemic?* Available via https://www.sigma-network.ac.uk/calendar/the-new-normal/ (last accessed 24 April 2023).

THIS PAGE INTENTIONALLY LEFT BLANK

WORKSHOP REPORT

Capacity Building in Mathematics and Statistics Learning Support in Norway and the Czech Republic (MSLS Net)

Josef Rebenda, CEITEC BUT, Brno University of Technology, Purkyňova 656/123, 612 00 Brno, Czech Republic.
Email: rebenda@vutbr.cz

Zuzana Pátíková, Faculty of Applied Informatics, Tomas Bata University in Zlín, Nad Stráněmi 4511, Zlín 76005, Czech Republic.
Email: patikova@utb.cz

Martin Chvátal, Faculty of Economics and Administration, Masaryk University, Lipová 41a, Brno
Email: chvatal.m@mail.muni.cz

Svitlana Rogovchenko, University of Agder, Jon Lilletuns vei 9, 4879 Grimstad, Norge.
Email: svitlana.rogovchenko@uia.no

Tørris Koløen Bakke, Faculty of Engineering Science and Technology, UiT The Arctic University of Norway, Alta, Norway
Email: torris.k.bakke@uit.no

Tony Croft, Mathematics Education Department, Loughborough University, Loughborough, UK.
Email: a.c.croft@lboro.ac.uk

Abstract

This report describes the final meeting of the project "Capacity Building in Mathematics and Statistics Learning Support in Norway and the Czech Republic (MSLS Net)" held at the Tomas Bata University in Zlín, Czech Republic (June 12-14, 2023). Provision of mathematics and statistics learning support (MSLS) is developing rapidly in many parts of the world and activity in Norway and the Czech Republic has been accelerated significantly through this EEA Grants funded project. Representatives of each of the five partner institutions worked on creating a summary of good practices in tutor training, designing learning resources, and in delivering, monitoring and evaluation of mathematics and statistics support. Provision varied considerably across the institutions and the centres represented demonstrated diverse and innovative ways in which mathematics support is evolving. Outputs from the project include a Handbook on good practice and a booklet concerned with mathematics support centre tutor training, including pedagogic training and learning resources for the development of the tutors as described below. Finally, consideration turned to the value of establishing a professional network to continue this important work. The report will be relevant to other international groups interested in working in university level mathematics and statistics support.

Keywords: Mathematics support, statistics support, European collaboration, mathematics support network.

1. Background and aims of the project

In many parts of the world there is ongoing concern about the quantitative skills of undergraduates, and in some cases, postgraduate university students. In mitigation, universities have introduced a variety of forms of support including drop-in centres and online provision. A key element is that such support is provided *in addition* to students' regular and credit-bearing programmes of lecturing, seminars etc. (see for example Lawson, Croft & Halpin, 2003). Surveys in the UK, Australia, Ireland, and the United States (see, for example, Grove, Croft, & Lawson, 2020, MacGillivray, 2009, Cronin, Cole, Clancy et al., 2016 and Mills, Rickard & Guest, 2020) demonstrate that such provision is now widespread in these countries. Whilst the main objective of the project reported here is to improve cooperation between higher education institutions in Norway and the Czech Republic in the learning and teaching of mathematics itself, a further objective is to share experiences in the provision of mathematics and statistics support, to promote the idea of such support in higher education and to support the development of national networks bringing together those who work in this developing field. The project addresses four key goals for European higher education: tackling future skills mismatches and promoting excellence in skills development, building inclusive and connected higher education systems, ensuring higher education institutions contribute to innovation, and supporting effective and efficient higher education systems. The project ran from August 2021 until July 2023. The outputs, or products, of the project include a Handbook on good practice and a booklet concerned with the training of tutors who work in mathematics support, both published in English. These include: summaries of the project partners' good practice in setting up, operating, promoting and evaluating MSLS, five case studies describing MSLS practice of the partner universities, an overview of resources for tutor development and learning resources in MSLS, and tips for use of those resources. In addition, there is a collection of materials - training resources, video tutorials, feedback questionnaires, promotional leaflets and other resources related to MSLS provision - developed throughout the project and available in three languages: English, Czech and Norwegian. One of the important outcomes of the project was initiating discussion about establishing a network of practitioners connecting Czech and Slovak MSLS centres tentatively called π (Pi) Network, aiming to foster collaboration and knowledge sharing among the participating institutions. The Pi Network is envisioned as a virtual space where individuals from various universities and educational backgrounds can come together to contribute, seek inspiration, and find support in their MSLS endeavours. This network should serve as a platform to foster collaboration and growth in mathematics education, ensuring that everyone interested in MSLS, regardless of experience or access to a support centre, can actively participate.

2. Universities involved in the project and past meetings

The consortium includes five higher education institutions: Brno University of Technology (BUT) in the Czech Republic (Project Promoter coordinating the efforts of four other participants), two further Czech universities, Masaryk University (MU) also in Brno and Tomas Bata University in Zlín (TBU), and two Norwegian Universities, The Arctic University of Norway (UiT) and University of Agder (UiA). The consortium features a balanced mixture of experienced organisations and those new to mathematics support. UiA, MU and TBU had established mathematics and statistics support centres in autumn 2015, spring 2016 and autumn 2016 respectively. BUT later set up a centre in autumn 2021, and UiT has since developed an innovative form of "remote support" offered to students who are not on campus using the platform Discord (https://discord.com). To date, three project workshops have taken place in Brno, Kristiansand and Alta. The first workshop in Brno (CZ) was arranged as a hybrid event because some colleagues including the guest speaker David Bowers from the **sigma** Network could not participate physically due to COVID-19 travel restrictions in place at the time. Presentations of partners' MSLS activities and **sigma** Network experience laid the groundwork for plenary discussions and group work where the focus and main topics of the project outputs were outlined. The second workshop in Kristiansand (NO), where a list of chapters of the two outputs was created, was fully face-to-face and benefited from the experience of Professor Duncan Lawson, former chair of the **sigma** Network, as well as other experts in the field invited by The Centre for Research, Innovation and Coordination of Mathematics Teaching (MatRIC) to the MatRIC Annual

Conference 2022 (https://www.uia.no/en/centres-and-networks/matric). Duncan Lawson also participated in the third workshop in Alta (NO) and arranged a virtual excursion to the Coventry University mathematics and statistics support service. The structure of the two outputs was discussed and content for several chapters was developed during the third workshop. In addition to the three workshops, online meetings and working sessions have taken place approximately once a month. Details of the project were presented at two conferences in Norway and one in the Czech Republic, and a dedicated promotional event took place in Brno (CZ) in January 2023 which was reported in the **sigma** Newsletter, Issue 26: Spring 2023.

3. The purpose and realisation of the final workshop

The purpose of the final workshop was to collect together all the information and learning that has taken place throughout the two years of the project. Of particular focus was creating a summary of good practices in tutor training, creating learning resources, and in delivering, monitoring and evaluation of mathematics and statistics support provision. The workshop took place over three days at the Tomas Bata University in Zlín, within the Faculty of Applied Informatics (June 12-14, 2023).
On each day the programme had a specific focus.

On the first day work focussed on the development of training materials for new support centre tutors and learning resources for students. The workshop started with a brief introduction by each participant and a project update from each partner university. Then, there followed a presentation of training and learning resources of the **sigma** Network by the guest speaker Professor Tony Croft. The workshop continued with two collaborative work sessions the aim of which was to put together material for the booklet for training MSLS tutors. The collaboration took place in a shared document on Google Drive displayed on the wall, so that everybody could see what was being written and what was the direction of the plenary discussion.

On Day 2, focus was upon the concept and history of mathematics support in the partner institutions, with partners explaining the rationale for support in their own institution and the way support has evolved there. Then, the different aspects of provision were considered including how to set up and run a centre, collecting feedback and evaluation of the service, promotion of the service at the university, and dissemination beyond. An introduction was again given by Tony Croft and was followed by six collaborative work sessions aimed at collecting material for the intended Handbook on good practice.

Day 3 started with an excursion to the Tomas Bata Memorial, a memorial of the founder of the world-famous Bata shoe company. Following this most interesting visit, delegates returned to focus upon the value of networking and establishing a form of professional association which would enable this important collaborative work to continue. Discussion was informed by the development of the **sigma** network. A special hybrid session (Figure 1) was arranged with focus on network building. Remote participants from three universities joined us in listening to Tony Croft's experience of establishing the **sigma** Network and in the follow-up discussion about possible means and forms of collaboration within the emerging Pi Network.

Figure 1: The hybrid session with a focus on network building.

Finally, one more special session should be mentioned which took place in the evening of Day 1. After the grill party in the bar on the roof of the student dormitory, the event manager arranged a pantomime game with challenging, mostly abstract words that had been used during the workshop on Day 1. We saw plenty of creative performances (in the end all words had been guessed), and we thank Dominika for this unique experience. Figure 2 shows some of the workshop participants.


Figure 2: Some of the workshop participants.

For further information about the project contact Josef Rebenda, Brno University of Technology, (rebenda@vutbr.cz) or visit the project website https://msls-net.ceitec.cz/en where the project outputs can also be found.

4. Acknowledgment

This report has been written with financial support from the project "Capacity Building in Mathematics and Statistics Learning Support in Norway and the Czech Republic (MSLS Net)", registration number EHP-CZ-ICP-3-009, financed by the EEA Grants 2014-2021, programme Education. Iceland, Liechtenstein, and Norway contribute through EEA Grants to a more equal Europe, both socially and economically, and strengthen the relations between Iceland, Liechtenstein and Norway, and the 15 Beneficiary States in Europe.

Iceland Liechtenstein Norway grants

5. References

Cronin, A., Cole, J., Clancy, M., Breen, C., and O'Se, D. (2016). An audit of mathematics support provision on the island of Ireland in 2015. Dublin: National Forum for the Enhancement of Teaching and Learning in Higher Education. Retrieved from http://www.sigma-network.ac.uk/wp-content/uploads/2019/02/Audit-of-MLS-provision-Ireland.pdf

Grove, M., Croft, T., and Lawson, D. (2020). The extent and uptake of mathematics support in higher education: results from the 2018 survey. *Teaching Mathematics and Its Applications*, 39(2), 86-104. https://doi.org/10.1093/teamat/hrz009

Lawson, D. A., Croft, A. C., & Halpin, M. (2003). Good practice in the provision of mathematics support centres (2nd ed.). LTSN Maths, Stats & OR Network.

MacGillivray, H. (2009). Learning support in mathematics and statistics in Australian universities – A guide for the university sector. New South Wales: The Australian Learning and Teaching Council. Retrieved from https://www.mathcentre.ac.uk/resources/uploaded/guide--altc-learning-support-in-maths-and-stats.pdf

Mills, M., Rickard, B., and Guest, B. (2020). Survey of mathematics tutoring centres in the USA. *International Journal of Mathematical Education in Science and Technology.* Advance online publication. https://doi.org/10.1080/0020739X.2020.1798525