MSOR connections

Articles, case studies and opinion pieces relating to innovative learning, teaching, assessment and support in Mathematics, Statistics and Operational Research in HE.

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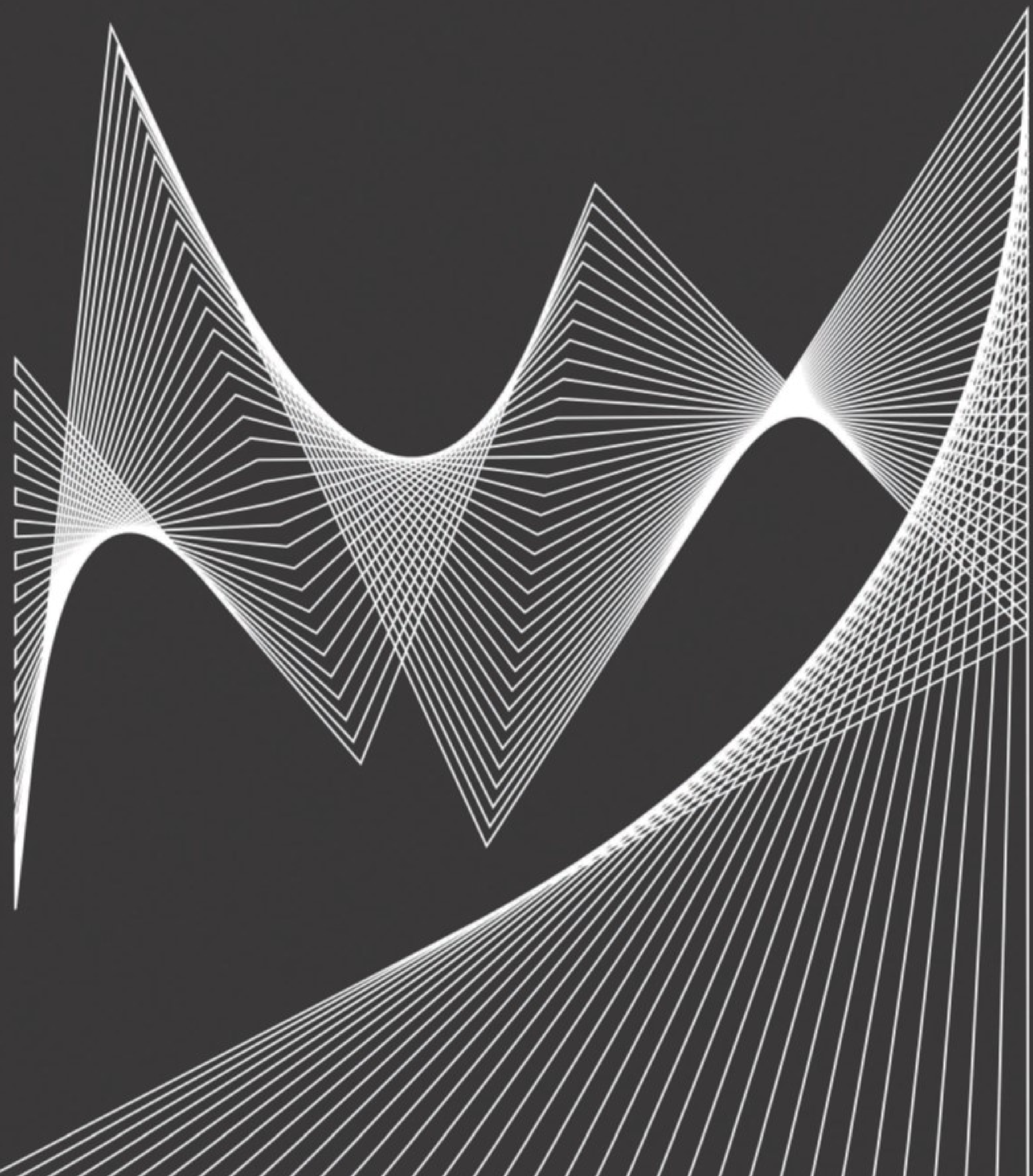

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EDITORIAL

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I'm delighted to welcome you to this latest edition of MSOR Connections. This edition includes Case Studies related to classroom practice in a range of MSOR subjects, a Research Report that brings together LaTeX and R to create quizzes in Moodle, and a Resource Overview of Pingo.

This particular edition kicks off with three excellent articles with classroom practice very much the focus, which includes some very helpful ideas and examples of good practice. The first, by Corner and Cornock, presents a case study using problems based on applications or physical objects in a first year pure mathematics module. This is followed by another case study by Lennerstad which considers design of an undergraduate calculus course, with the aim of facilitating higher student activity, teacher-student exchange, continuous teacher learning about students' mathematics knowledge and immediate feedback to improve the course as it progresses. This first group of three articles is then completed by Evans who presents a resource review of Pingo, a free, web-based system that provides an excellent way of introducing interaction in large group teaching.

These are followed by two engaging articles related to e-assessment. Firstly we have a research article by Jach, which describes how several free, open-source and popular tools can be combined to produce data-driven, up-to-date quizzes for Moodle, using examples from economics. These free tools include LaTeX and a new associated package called `moodle`, together with R the statistical computing software. This is followed by a case study by Erskune and Metsel, which describes the implementation of e-assessment in *STACK M820 Calculus of Variations and Advanced Calculus*, which is part of the Open University's Masters Programme in Mathematics.

A significant element of the motivation for these first five articles relates to improving student retention. It is fitting therefore, that we round of this edition with a case study by Khan, which addresses the very interesting related question of whether various factors affect attendance rates at university lectures in mathematics, and whether there is any relationship between attendance and exam performance?

I thoroughly enjoyed reading all of the articles in this edition and I hope you do to. If you would like to submit an article for publication in a future edition of MSOR Connections, we always welcome contributions and details of how to submit can be found at https://journals.gre.ac.uk/index.php/msor/about/submissions. Submissions could include case studies, opinion pieces, research articles, student-authored or co-authored articles, resource reviews (technology, books, etc.), short updates (project, policy, etc.) or workshop reports. Please consider contributing and encourage others you know to contribute also. We are always looking for willing volunteers to review articles for MSOR Connections, so please get in touch with us if you are interested in this role.

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CASE STUDY

Applications and props: the impact on engagement and understanding

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Abstract

Problems based on applications or objects were added into a first year pure module in gaps where real-life problems were missing. Physical props were incorporated within the teaching sessions where it was possible. The additions to the module were the utilities problem whilst studying planar graphs, data storage when looking at number bases, RSA encryption after modular arithmetic and the Euclidean algorithm, as well as molecules and the mattress problem when looking at group theory. The physical objects used were tori, molecule models and mini mattresses. Evaluation was carried out through a questionnaire to gain the students' opinions of these additions and their general views of applications. Particular attention was paid to the effect on engagement and understanding.

Keywords: Use of applications; physical props; manipulatives; engagement and understanding

1. Introduction and background

The module 'Number and Structure' is situated within a BSc Mathematics programme which focusses on real-world applications and mathematical modelling. The module is one of the purer modules on the course. The content currently consists of sets, graph theory, digraphs, matrices, Boolean algebra, number bases, modular arithmetic, proof, the Euclidean algorithm, irrational numbers, rings, groups and equivalence relations. Some topics were already linked with physical and practical examples. These included Rubik's cubes (discussed by Cornock (2015) for another module) and playing cards when looking at permutations, circuits whilst studying Boolean algebra, the instant insanity problem for graph theory, and symmetries of physical 2D-shapes in group theory.

In a review of the module in 2017, some topics were removed if they could not immediately be linked to any real-life objects / applications and did not feature in subsequent modules. 'Applied mathematical problems' were introduced. Such a problem is where *"the situation and the questions defining it belong to some segment of the real world and allow some mathematical concepts, methods and results to become involved"* (Blum and Niss, 1991). The problems are *"well suited to assist students in acquiring, learning and keeping mathematical concepts, notions, methods and results, by providing motivation for and relevance of mathematical studies"*.

Manipulatives were used where possible. These are physical, pictorial, or virtual representations of abstract concepts, widely used in pre-university mathematics education (Furner and Worrell, 2007), as well as other subjects. Those discussed in this article are often referred to as physical props, though other, virtual and pictorial, manipulatives are used in other parts of the module. Much has been written (Cope, 2015; McNeil and Jarvin, 2007) about the use and effectiveness of manipulatives in teaching mathematics to pre-university students, especially in primary education, but little is written about their use for teaching mathematics in higher education. There has been some discussion as to the use of manipulatives in other areas of undergraduate education such as Chemistry (Saitta, Gittings, and Geiger, 2011), Engineering (Pan, 2013) and particularly in areas related to Biology (Jungck, Gaff, and Weisstein 2010; Krontiris-Litowitz, 2003; Guzman and Bartlett, 2012). The molecular models discussed in this article were themselves borrowed from the university's Biochemistry team.

McNeil and Jarvin (2007) weigh up both sides of the debate on the effective use of mathematical manipulatives, though the counterevidence they provide focuses mainly on the *"cognitive resources"* of children and so is harder to reconcile with the abilities of adults in higher education. We believe that other recommendations, such as principles (c) and (d) described by Laski, Jor'dan, Daoust, and Murray (2015), are in line with the use of manipulatives within the scope of this module, with their other principles again focussing on the cognitive capabilities of children rather than adults. If used in the right way, physical objects can be used to represent abstract ideas in physical form to *"help students deeply understand the math they are learning and needing to apply to our everyday life"* (Furner and Worrell, 2007).

The following problems, topics and activities were brought into the module:

Utilities problem: The utilities problem is a well-known recreational mathematics puzzle (Kullman, 1979). The problem can be described using Figure 1 below in which each house (A, B, C) has to be connected to each of the utilities (Gas, Water, Electricity) in such a way that none of the connections cross one another.

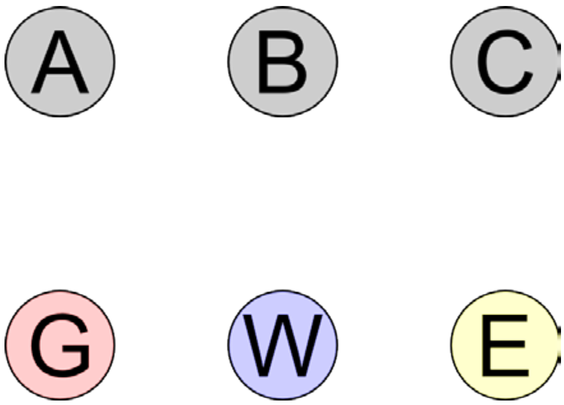

Figure 1. The utilities problem

The students were asked to try and find a solution to this problem, which is not possible, before trying to solve the problem on a labelled polystyrene torus (donut) which was used as a physical prop (see Figure 2). String was attached to represent the connections.

Figure 2. Utility problem on a torus

Data storage: A section on data storage was added after the work on different number bases. This included general information about how data is stored and specific information on the storage of pictures, numerical data, text, and logical data.

RSA encryption: Following a section on the Euclidean algorithm, RSA encryption was used to encrypt and decrypt messages (Katz, Menezes, Van Oorschot, and Vanstone, 1996). An assignment was introduced on RSA encryption which required the students to decrypt an individualised message.

Chemistry: The students were required to look at physical 3D-models of molecules of ammonia (NH_3) (Figure 3) and tetrabromogold ($AuBr_4^-$) (Figure 4) and produce the group tables for their symmetries (Walton, 1998). Such manipulatives are widely used in teaching chemistry, models having been used to demonstrate molecular structures since the middle of the 19th century (Perkins, 2005).

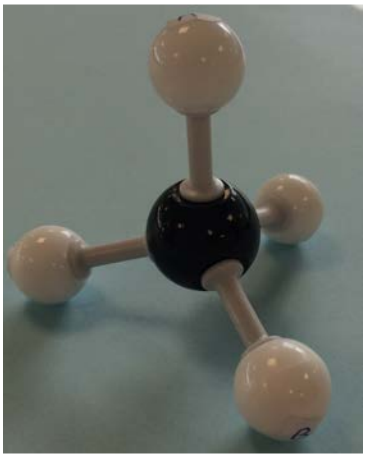

Figure 3. Ammonia molecule model

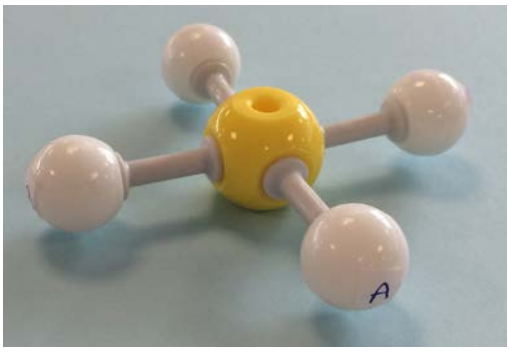

Figure 4. Tetrabromogold molecule model

Mattress problem: The mattress-flipping problem is described by Hayes (2008) as the task of finding *"some set of geometric manoeuvres that you could perform in the same way every time in order to cycle through all the configurations of the mattress"*. The students were given a mini mattress, as shown in Figure 5, and were them to describe the possible movements which would change its orientation. They were then asked whether any combination of such movements would cycle through every possible orientation without as much setup as the tasks with the other physical props.

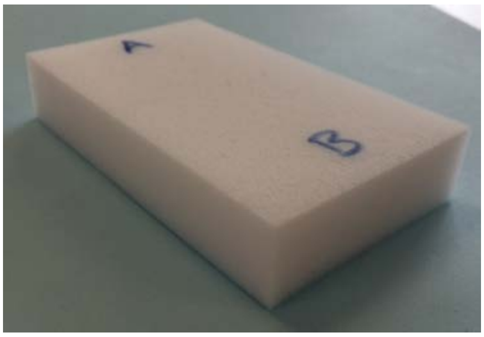

Figure 5. Mini mattress model

Following the introduction of the new topics and practical tasks, evaluation was carried out to gain the students' opinions on the changes, and the impact on their engagement and understanding.

2. Methodology

Evaluation was undertaken through an in-class online questionnaire towards the end of the 2017/18 academic year. Out of the 68 students who attended the classes that week (70% of students taking the module), 51 students filled in the questionnaire.

The questionnaire started with some general questions about applications. The first question was about whether seeing the uses of a topic changes their approach to work. Then the students were asked about their preferences in terms of theory and applications.

The general questions were followed by some statements about specific applications used in the module and whether they helped with understanding topics as well and whether the connections between the applications and the topics were clear. The students were asked to give an indication of how much they agreed or disagreed with the statements by using a 5 point Likert scale. These were followed by statements about physical props used within class, again with a 5 point scale.

Given that more applications were considered when studying group theory than ring theory, a question was included on whether there was a difference learning the topics. This was followed by questions on whether their views of particular topics had changed once the application had been considered.

The students were asked a general question about whether having physical examples in class clarified the concepts that were introduced. The questionnaire concluded with a statement about the module being beneficial to the real-world, which was part of a 5 point scale question and was followed by a comments box for reasons why.

There was a good response rate throughout the questionnaire. A total of 32 students (63% of students who filled in the questionnaire) answered the question about comparing group theory with ring theory, which was the lowest response to a question. All other questions had at least 42 (82%) students answering them. All subsequent percentages will be of the number of students who answered the question, unless otherwise stated.

3. Results

3.1 Use of applications

When asked whether seeing the uses of a topic change how they approach work, 60% of the students gave a positive response, whereas 40% gave a negative response. The most common reason was that it makes them more motivated or interested (36% of 60%). One student said that *"if [they are] able to see the uses of a topic [they are] more likely to want to understand why the procedures work"* and that *"without seeing [an] application [they] tend to just remember the procedure and not ask question[s] about why certain things work".* Other reasons provided by the students included that it helps them, they use different methods for different topics, they may alter the layout, it gives a second angle, it allows the work to be seen differently, it encourages focus, they want to understand it more, they prioritise work that is more useful and their approaches are more analytical.

When asked what they prefer when working on a problem, the majority of students said that they prefer a mixture of theoretical problems and problems based on an application or did not have a preference (82%), some prefer theoretical problems (10%) and some prefer problems being based on an application (8%).

The reasons provided by the students who like a mixture of theoretical and applied questions included that a mixture helps with understanding (27%), it is more appropriate within jobs to do both (4%), it gives an appreciation of the theory and the application (11%), different types are better for different topics (7%), sometimes it is easier out of context first (4%), it enables students to answer a range of questions (7%), and makes it more interesting and enjoyable (9%). One student said that *"the application lets you see the uses of the technique while the theoretical part helps you [to see] how to understand how to perform it"*. Another said that they *"like to learn the theory of something then see its application"* as *"it tends to make [them] think deeper about why processes work and forces [them] to ask knowledge-furthering questions"*. Also, one student said that *"the more theoretical approach…allows [them] to get a base understanding of the concept but [they] also like to be able to apply it to a real world scenario and get some practice with it."*

The students who expressed a preference for problems being based around an application said this was because it would otherwise *"seem pointless",* it is easier to understand, and it makes *"a topic seem more interesting to know how it is used"*. The ones who prefer theoretical problems said they find them interesting, that they can understand the problems better, and they prefer looking at the *"raw maths"*.

3.1.1 The Utilities Problem

The students were asked whether the utilities problem helped them with their understanding of planar graphs. The results in Figure 6 show there was a small number who did not think that the problem helped them. There was a substantial number who agreed with statement (65%) or provided a neutral response (29%).

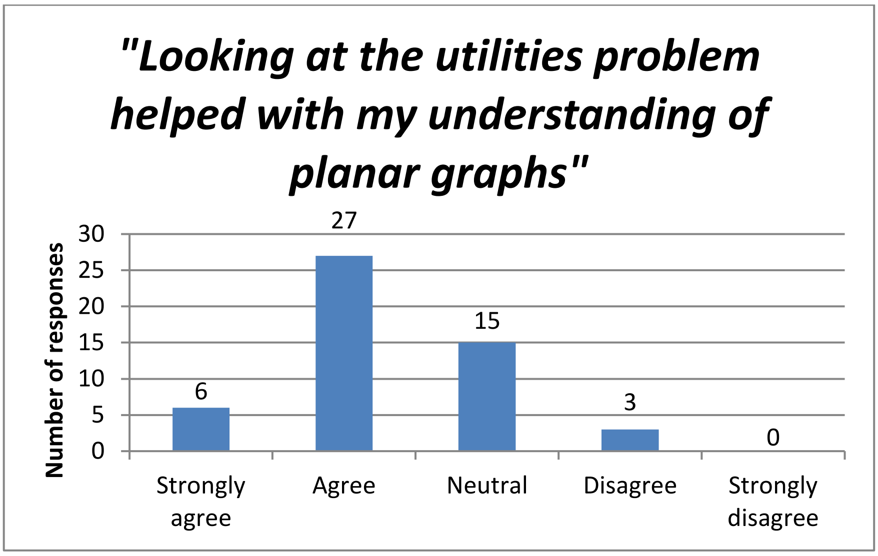

Figure 6

3.1.2 Group Theory

When asked whether learning about ring theory differed from learning about group theory, a large proportion of the students (53%) said it was not different and some said it was (18%). One student in particular was unable to remember ring theory. Another said that it was different *"because [they] used physical objects to help with group theory which helped [them] understand it more"*.

The students were asked whether they agreed with statements about whether the work on molecules and the mattress question helped with their understanding of group theory. A large proportion of the responses were positive (61% for molecules and 73% for the mattress problem) or neutral (29% and 20%) as seen in Figures 7 and 8.

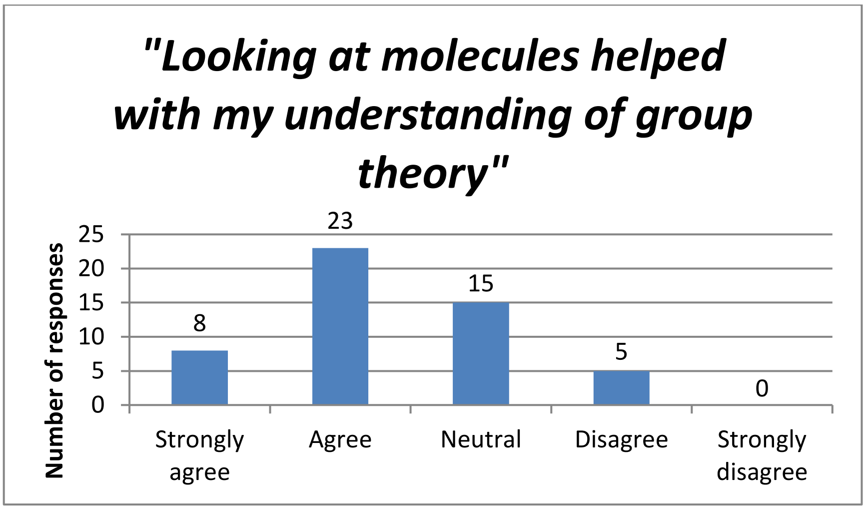

Figure 7

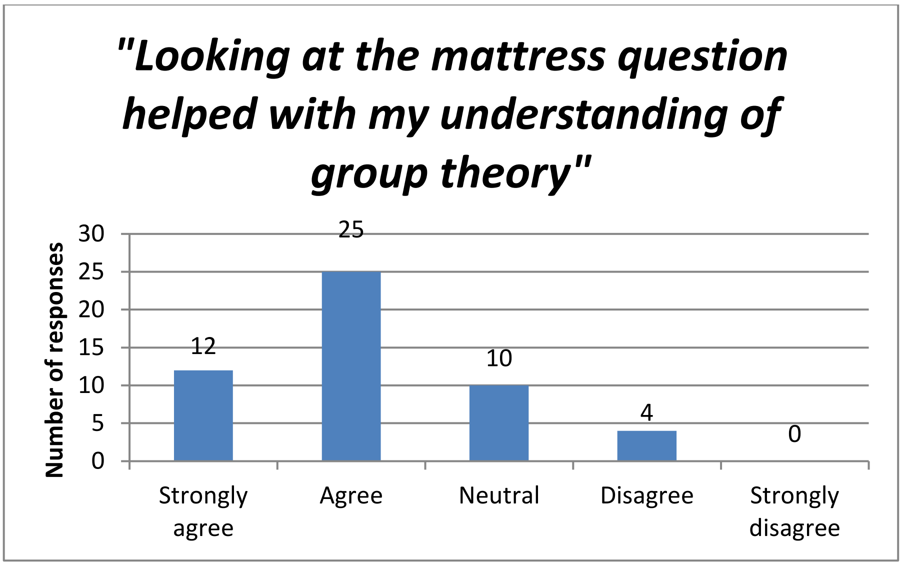

Figure 8

3.1.3 Binary and Hex Numbers

When asked whether their view of hex and binary numbers had changed once they had studied data storage, 40% said it had and 60% said it had not. Comments about why it had not changed included that they had studied it before (19% of 60%), that they had not studied the topic before (12%), data storage was confusing (12%), it did not help (8%), and they did not understand it (12%). Amongst the positive responses, comments about why their view changed included that it was useful to know why it was being studied or how it could be used (53% of 40%), it helped with understanding (35%), and the real-life application helped (12%). One student said that it gave them an *"interesting context"*.

When asked whether they agreed with the statement that looking at data storage helped with the understanding of binary and hex numbers, there was much more of a mixed response as presented in Figure 9. Just over half the students (52%) who answered this question said that it helped, a third provided a neutral response and 15% thought it did not help.

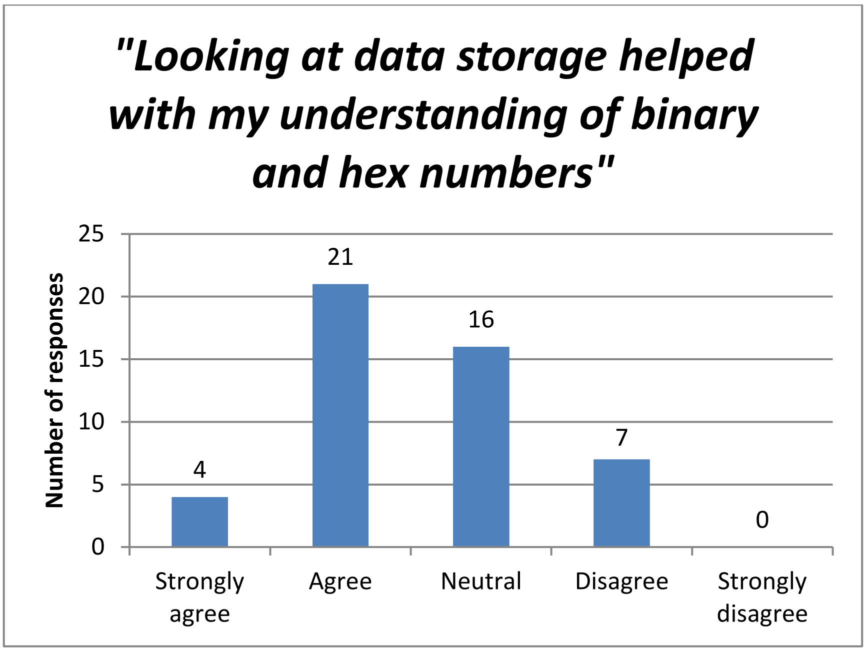

Figure 9

3.1.4 The Euclidean Algorithm and Number Theory

When asked whether their view of number theory and/or the Euclidean algorithm changed once RSA encryption had been studied the response was much more positive than for data storage. A larger proportion of students (69%) found that it had changed their view. The reasons provided included that they could see how it was useful and how it could be applied (52%), it gave them a clearer understanding (23%), it became easier to do (7%) and made it interesting (7%). One student said that *"it was good to see a useful situation where it would be used and [it] motivated [them] to work harder and understand more"*. Other comments included that it allowed them to see a different side of the topic, it *"sparked different ideas"*, they could visualise the ideas and the RSA encryption gave the topics *"purpose"*.

There was one student who was not sure. The students who said it had not changed their view (29%) said that it seemed quite similar (8%), they already had an interest in number theory or had seen the application (23%) and that it made it more complicated (15%).

When asked whether they agreed with a statement that RSA encryption helped them to understand number theory and the Euclidean algorithm, the response was very positive with 82% of the students answering the question either agreeing or strongly agreeing with the statement that was provided (Figure 10).

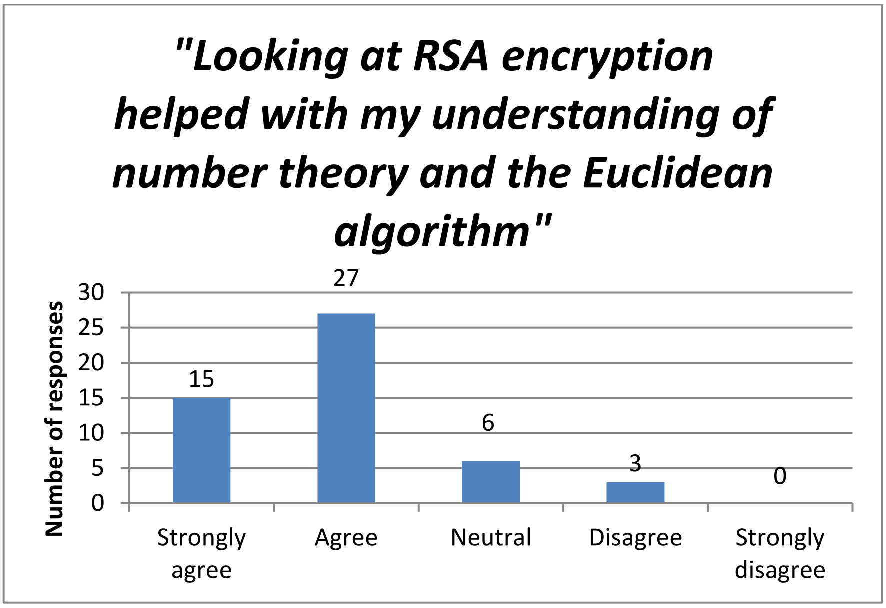

Figure 10

More students were positive about RSA encryption than the other additions. A potential reason is that the students had an assignment on the topic, whereas they did not have any assignment questions on the other additions.

3.2 Manipulatives in Class

Open responses were sought as to whether having physical examples in class clarifies the concepts that are introduced, and why. The majority of these responses were positive (70%), giving varied reasons such as more easily understanding concepts *"viscerally"* or *"physically rather than just reading content about them"*, enabling them to better visualise or *"see the problem"*, and keeping them focused and engaged. Of these responses being able to visualise problems was the most common reason given for the usefulness of the physical props (36% of 70%). Some of the positive comments use the words *"real example"*, *"real life scenario"*, and *"real and understandable context"*, though it is unclear whether the word *real* is being used as a synonym for *physical*, especially as some of the physical props used do not represent an actual real-world problem (for example, the utilities problem on the torus).

About a quarter (26%) of responses to the question were a mixture of positive and negative, generally saying that *"sometimes"* the props were useful in clarifying concepts. These students addressed things like the amount of time spent using the physical props in class (for example, a student said that *"physical examples take up a little too much time in class but may help others"*), that *"sometimes they can help but sometimes they can just be a hassle and seem pointless"*, and *"sometimes as it can help visualise things, other times it makes things more confusing"*.

Of the remaining comments the reasons given for the physical props not helping to clarify concepts were that *"physical examples never really helped [them] in general"* and *"sometimes it just created more confusion"*.

3.2.1 The Utilities Problem on a Torus

When asked whether the torus and string helped with their understanding of planar graphs, 72% agreed that it did (Figure 11). Of the remaining responses 20% were neutral, while 8% disagreed.

Responses to whether they could clearly see the connection between the activity and the topic of planar graphs the results were similar though slightly less strongly positive. About 75% of responses agreed but only 6 of these strongly agreed in contrast to 13 of the responses of the previous question. Of the remaining responses 16% were neutral while 10% disagreed.

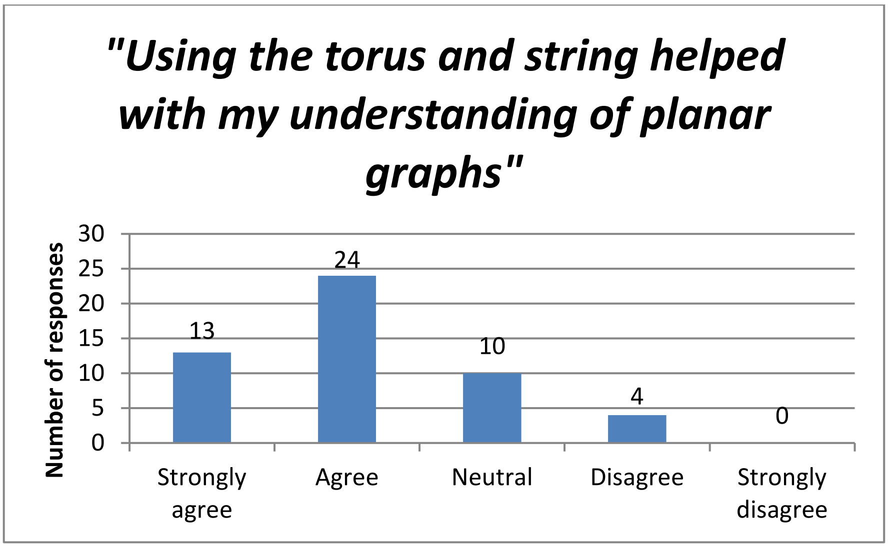

Figure 11

3.2.2 Molecules

The responses in Figure 12 show that 71% of students agreed that looking at physical molecule models helped with their understanding of group theory, while 21% gave neutral responses, and 8% disagreed. Many of the students (79%) agreed that they could clearly see the connection between the molecule models and group theory, 19% gave a neutral response, and 2% disagreed.

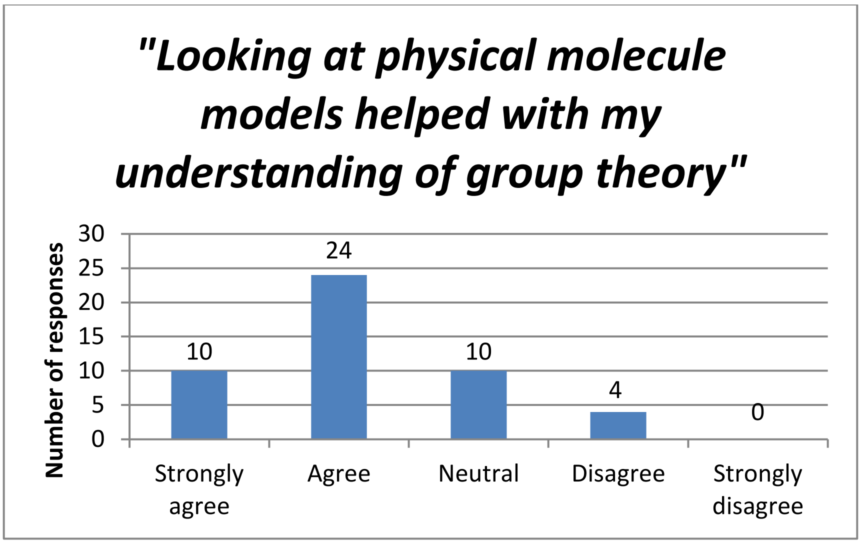

Figure 12

3.2.3 Mattress Task

When asked whether using the mattress models helped their understanding of group theory an 81% majority agreed that it did, 15% were neutral, and 4% disagreed (Figure 13). The distribution of these responses was the same when asked whether they could clearly see the connection between the mattress task and group theory. The responses here were, however, slightly more positive than the other tasks involving physical props.

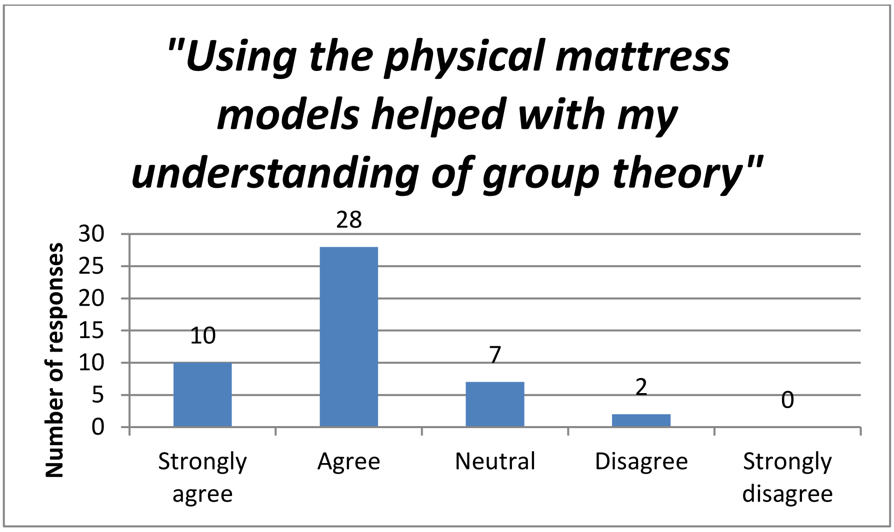

Figure 13

3.3 Benefit of the Content to the Real-World

When asked whether they could see how the content of Number and Structure is beneficial in the real-world, 91% of the students either agreed or strongly agreed that they could as shown in Figure 14.

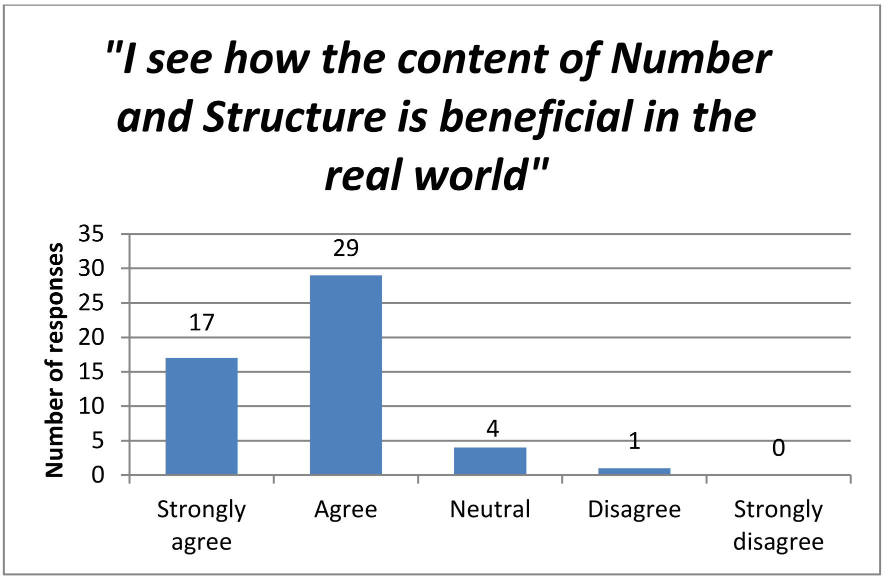

Figure 14

The student who disagreed said that they *"don't see how it could be useful"*. Out of the four students who provided a neutral response one didn't know, one did not always *"understand why [they were] doing it"*, one did not understand the real-world applications, and the other said that they did not know how all of the topics are used in the real-world.

The reasons provided by the students who either agreed or strongly agreed with the statement included comments about how they can see how the ideas are used in everyday life and the real-world (35% of 91%), the content was linked to applications (26%), and there are many applications of the topics (15%). One student mentioned that *"almost every single topic covered this year has had valuable real-world applications"*. Specific mentions of topics were encryption (15%), binary numbers (4%), and graph theory (4%). One student commented about developing problem-solving and logical thinking skills. Other comments included that it *"helped to see how all these ideas are used in everyday lives"*, "*it allows [them] to appreciate and understand more as to how it impacts the real world",* and they *"can see where it is useful"*. In some topics it was easier to see the link than others. One student commented that *"there are times when [they] struggle to see the application of certain theories but in general it has been clearly conveyed in which way some theories are used in the real world"*, whereas another said that *"it was sometimes surprising how some real life situations can be looked at mathematically"*. One student summarised that *"the module uses real world applications to help visualise and solve theoretical questions"*.

4. Conclusions

Following the addition of applied problems and objects, a substantial amount of evidence was collected to suggest that they had an impact on the students. The majority of them can see how the content of the module is beneficial in the real-world as they can see the uses and the links. A fair number of students (60%) said they approach work differently after seeing the uses of a topic, with increased motivation or interest being mentioned as the top reason. The students tend to like a mixture of theoretical and applied problems, which they said increases their understanding and interest / enjoyment. The application of RSA encryption helped the most with understanding (82%), followed by the mattress problem (73%), the utilities problem (65%), molecules (61%) and then data storage (52%). The use of manipulatives was also received well, with 70% of the students being positive about the physical examples. Reasons for this included understanding and engagement. The use of the mattresses helped the most with understanding (81%), followed by the tori (72%) and the molecules (71%).

5. References

Blum, W. and Niss M.,1991. Applied Mathematical Problem Solving, Modelling, Applications, and Links to other Subjects - State, Trends and Issues in Mathematics Instruction. *Educational Studies in Mathematics*, 22(1), pp.37-68. https://doi.org/10.1007/BF00302716

Cope, L., 2015. Math manipulatives: Making the abstract tangible. *Delta Journal of Education*, 5(1), pp.10-19. Available at: http://www.deltastate.edu/PDFFiles/DJE/spring-2015/dje_spring_2015_cope-final.pdf [Accessed 3 July 2018].

Cornock, C., 2015. Teaching group theory using Rubik's cubes. *International Journal of Mathematical Education in Science and Technology*, 46(7), pp.957-967. Available at: https://www.tandfonline.com/doi/pdf/10.1080/0020739X.2015.1070442 [Accessed 5 July 2018].

Furner, J. M., and Worrell, N. L., 2017. The Importance of Using Manipulatives in Teaching Math Today. Transformations, 3(1), p.2. Available at: https://nsuworks.nova.edu/transformations/vol3/iss1/2 [Accessed 3 July 2018].

Guzman, K. and Bartlett, J., 2012. Using simple manipulatives to improve student comprehension of a complex biological process: Protein synthesis. *Biochemistry and Molecular Biology Education*, 40(5), pp.320-327. https://doi.org/10.1002/bmb.20638

Hayes, B., 2005). Group theory in the bedroom. *American Scientist*, 93, pp.395-399. Available at: https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.208.8157&rep=rep1&type=pdf [Accessed 3 July 2018].

Jungck, J.R., Gaff, H. and Weisstein, A.E., 2010. Mathematical manipulative models: In defense of "Beanbag Biology". *CBE—Life Sciences Education*, 9(3), pp.201-211. https://dx.doi.org/10.1187%2Fcbe.10-03-0040

Katz, J., Menezes, A.J., Van Oorschot, P.C. and Vanstone, S.A., 1996. *Handbook of applied cryptography*. Boca Raton, FL: CRC press. Available at: https://cacr.uwaterloo.ca/hac/ [Accessed 3 July 2018].

Krontiris-Litowitz, J., 2003. Using manipulatives to improve learning in the undergraduate neurophysiology curriculum. *Advances in physiology education*, 27(3), pp.109-119. https://doi.org/10.1152/advan.00042.2002

Kullman, D.E., 1979. The utilities problem. *Mathematics Magazine*, 52(5), pp.299-302. https://doi.org/10.2307/2689782

Laski, E.V., Jor'dan, J.R., Daoust, C. and Murray, A.K., 2015. What makes mathematics manipulatives effective? Lessons from cognitive science and Montessori education. *SAGE Open*, 5(2), p.2158244015589588. https://doi.org/10.1177/2158244015589588

McNeil, N. and Jarvin, L., 2007. When theories don't add up: disentangling he manipulatives debate. *Theory into Practice*, 46(4), pp.309-316. https://doi.org/10.1080/00405840701593899

Pan, E.A., 2013. *The use of physical and virtual manipulatives in an undergraduate mechanical engineering (Dynamics) course*. University of Virginia. Available at: https://search.proquest.com/docview/1435605243 [Accessed 3 July 2018].

Perkins, J.A., 2005. A history of molecular representation. Part one: 1800 to the 1960s. *J Biocommun*, 31(1), p.1. Available at: https://jbiocommunication.org/issues/31-1/features3.html [Accessed 3 July 2018].

Saitta, E.K., Gittings, M.J. and Geiger, C., 2011. Learning dimensional analysis through collaboratively working with manipulatives. *Journal of Chemical Education*, 88(7), pp.910-915. https://doi.org/10.1021/ed100019f

Walton, P.H., 1998. *Beginning Group Theory for Chemistry*. London: Oxford University Press.

CASE STUDY

A calculus course in knowledge feedback format

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Abstract

This paper presents a design of an undergraduate calculus course that aims at achieving four connected goals: 1. higher student activity, 2. teacher-student exchange, 3. continuous teacher learning about the present students' mathematics knowledge as the course progresses, and 4. immediate feedback from this learning to improve the ongoing course. In this process, students can perceive the course as one where one's own activity contributes to other students' learning and even improves the course in the long term. The main content of this paper is the *course design* to fulfil or come close to these goals.

By the design it is uncomfortable for a student to be passive (breaking the course pattern), and comfortable to be active (following the course pattern). However, all course features except examination are truly voluntary. Student passivity is accepted.

Keywords: Learning, dialogue, quiz, diagnostic, course design.

1. Introduction

This course is designed for teachers who enjoy interaction with students, and are interested in listening to students' mathematical questions and to provide feedback on their level, or who wish to develop such competencies. It is based on the idea that a teacher, besides the subject knowledge itself, needs to develop a complementary knowledge: a detailed understanding of how students understand and misunderstand the subject in conjunction with practical ways to overcome such difficulties.

This complementary knowledge can be gradually and systematically developed during the service, resulting in courses developing year by year. If this learning also affects the ongoing course, students may feel included and invited to participate, which may by itself lead to an extra effort in their learning.

This course contains several orchestrated activities to provide opportunities for teachers to develop such competence, which is not possible without active students and thus must be dependent on the student group. How to design such a course is not a trivial task.

Part of the means to activate students is mini exams during some course weeks. This is popular among many students for this very reason – increased activity. But questioning, investigating and deeper learning requires a more open error-tolerant attitude than "examination-attitude". To this end, activities between the small exams are designed to have a non-examination character: investigative, open, inclusive, (slightly) mathematically provocative, and viewing all errors as constructive and as occasions of learning.

It must be stressed that resources for a systematic evaluation of the course and production of evidence that underpin the intentions that have guided course design choices have not been available (see however Section 5).

In Wittman (1995), mathematics education is described as a design science, as is engineering, architecture, business and medicine. Here the most essential question is how to design artefacts that are successful, which may be an engine, a building, an enterprise, a medical treatment method, or a course. All these areas are strongly dependent of knowledge of different sciences, of course, but the goal of research is not only to understand. The main point is that creative design has a decisive importance in design sciences. The design must result is an artefact, that can be scrutinized. In the case of education: how a course is designed is essential for the result of it. The questions “What?” and “Why?” may be enough in basic research, but must in a design science lead to a particular answer to “How?”.

A further basic inspiration for this course’s design has been Donald Shön’s reflective practice (Shön, 1983), in terms of professional development. Here professionals learn from their own professional experiences, and engage in a process of continuous learning.

The idea of reflective practice can be taken further when the profession is teaching. In this case there are always learners to engage in dialogue with, in the aim of reflective practice and continuous professional development. Then professional development becomes entangled with the learners’ learning, and thus to the immediate goal of the professional activity.

The willingness of students to interact and share the teacher’s knowledge and skills is discussed in Kindberg (2013) in a perspective that is both pedagogical and rhetorical. The teacher’s ability to construct a safe and open learning atmosphere, which is also a rhetorical issue because of its focus on verbal communication, is essential for the students’ learning outcome.

In the special case of mathematics education, verbal reflection is scarce by tradition, despite the subject’s fundamental dialogical nature (Lennerstad, 2008, Ernest 1994). One can argue that mathematics problem solving, and mathematics in general, has a characteristic and important vagueness, making mathematics in need of verbalisation. This is expressed by P. Ernest in the preface of Rowland (2000):

“*Precision is the hallmark of mathematics and a central element in the ‘ideology of mathematics’. Tim Rowland, however, comes to the startling conclusion that vagueness plays an essential role in mathematics talk. He shows that vagueness is not a disabling feature that detracts from precision in spoken mathematics, but is a subtle and versatile device which speakers deploy to make mathematical assertions with as much precision, accuracy and confidence as they judge the content and context warrant.*”

Thus, dialogue and its natural vagueness is at the core of the nature of mathematics.

2. The calculus course: the practicalities

The course ‘Analys 2’ is a basic calculus course at the Blekinge Institute of Technology in Sweden. Its eight weeks contains three weeks about integrals, three weeks about ordinary differential equations, and one week about Taylor series. The final week is devoted to an overview of the course’s results and methods, and to old exams. The immediately preceding course ‘Analys 1’ covers numbers, the function concept, different function classes, limits, derivatives, and applications. Both courses are mandatory in five civil engineering programs at Blekinge Institute of Technology, and approximately 100 students from two or three of these programs participate at each course occasion. For each student and week, the course entails eight hours of lectures, and two hours of scheduled exercise solving.

The assessment of the course is in two parts: an individual written exam of 4.5 credit points and a project of 1.5 credit points. A student must pass both. In the written exam, 50% of the points are needed to pass, and no points can be gained to this exam from course activities. Each teacher has some liberty in how to design the project part – which I have used for the design of this course.

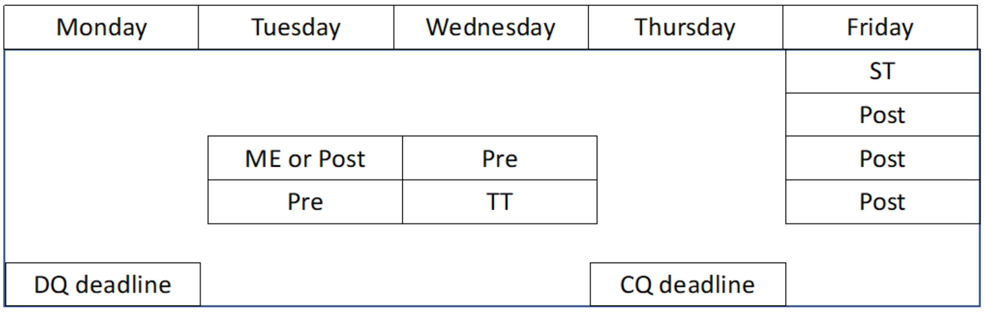
The six tasks in the written exam are conventional except for the last, which is conceptual. This task requires an account of the relations between concepts in the course, and possibly proofs of theorems. An example of such a task is the following:

"*Q6. How is differentiation and integration related? Are there elementary functions that do not have an elementary primitive function, although they are integrable? What is the difference between a definite and an indefinite integral? Which are the main methods to calculate a primitive function? Is there any differential equation that can be solved by integration? How can one prove that a continuous function on a bounded interval is integrable?*"

Grading the conceptual task has not turned out to be more difficult than other tasks, however slightly more time consuming. Also, students recognize that only solving problems, and learning proofs word by word, is not enough to pass this task.

3. Design of the course

To clarify the course design within the frames defined by Section 2; there are 2 hours lectures on Tuesday and on Wednesday, and 4 hours on Friday. The two hours of scheduled exercises occur during each week in addition to the plan described in this section.



Weekly design DQ = diagnostic quiz, CQ = Conceptual quiz, ME = Mini exam (course week 3, 5, 7), Pre = Pre-lecture, Post = Post-lecture, TT = Teacher's tasks, ST = Students' tasks

Figure 1. Weekly plan of calculus course

3.1 The guide, the pre-lectures and the post-lectures

The content to learn during one week is presented for students by a one page document, called a 'guide' – one for each week except the last. The guide contains:

1. the new topics of the particular week;
2. what students are supposed to learn;
3. recommended exercises;
4. six 'Teacher's tasks';
5. six 'Student's tasks'.

Typically, new mathematics is 'pre-lectured' during the second half of Tuesday's and during Wednesday's lectures. 'Pre-lecturing' means giving typical examples, main ideas and connections with motivations, usually no proofs, and solving typical and not very complicated problems.

The last hour of Wednesday is devoted for the teacher to solve the Teacher's problems.

The first hour of Friday is the Students' session: students solve the Student's problems in the lecture on the whiteboard, commented on in an affirmative way by the teacher. This is not an examination, but voluntary. If no student wishes to solve a problem, the teacher does it.

The remaining hours of Friday and the first hour of Tuesday is used for 'post-lectures'. Post-lecturing means proving statements not proved in the pre-lecture, generalizing and deepening the week's mathematical content, and solving more complex problems. This can be assumed to me more fruitful after the students have calculated and attempted problems themselves.

3.2 The quizzes

To complement the problem solving activity in Student's tasks, there are two quizzes about the theory: a Diagnostic quiz and a Concept quiz. Both occur every week on the course site except during the last week. Each contains about 12 mathematical claims concerning the concepts of the week. For each quiz statement, a student must choose: True or False. After the choice, the correct answer appears immediately together with an explanation of the answer; why it is false, if it is, and under which cases it might be true, and how it connects to other concepts of the course.

These explanations provide a lot of mathematical understanding, at least about how the concepts connect with each other, and the limits of them. Here is an example of a part of a quiz.

True or false?

1. The differential equation $y' = 1/y$ is linear.

   Answer: False. Comment: Not even after multiplication into $y'y = 1$ does it have the form $f(x)y' + yg(x) = h(x)$, as it should for linear equations of the first order (for some functions $f(x)$, $g(x)$ and $h(x)$);

2. Linear differential equations can only be solved if the coefficients are constant.

   Answer: False. Comment: With the integrating factor method, many first order linear differential equations can be solved that do not have constant coefficients;

3. No differential equation is both separable and linear.

   Answer: False. Comment: The differential equation $y' = y$, for example, has both properties. In the form $y' – y = 0$ it is linear, and in the form $y'/y = 1$ the variables are separated. So it can be solved by both methods.

Three examples with "False" as right answer were here chosen, since such statements may be more difficult for us teachers to find and formulate. The Diagnostic quiz is similar, but tests the prerequisites for the next week. It is due for completion before the first lecture that week.

The Conceptual quiz is due Thursday evening, before the Students' session. They test the main ideas and connections of the week's concepts, which are presented in pre-lectures. The timing of this quiz is intended to enhance the Students' session to make general theory more present and available during the problem solving activity.

3.3 The mini exams

Mini exams occur in course week 3, 5 and 7. They concern content of week 1 and 2, 3; weeks 4 and 5; and week 6, respectively. Each mini exam is 45 minutes long and takes place the first hour of Tuesday, and includes two or three problems. Mini exams test the new concepts, but require very little calculation, due to the very short time.

3.4 Assessment of the project

For each quiz that is completed before the deadline, a student gets one point regardless of the number of correct answers. Quiz points and mini exam points are added, and this sum must be high enough to pass the project. A student who has all quiz points needs 40% of the available mini exam points to pass the project, while 80% is required with no quiz points.

3.5 The Student's sessions

On Friday mornings, students solve problems before the teacher and the entire class during lecture. They have six problems to choose between, and each student can choose either a problem that can easily be solved in class, or a harder example that may provide more learning. The Teacher's problems, solved by the teacher a few days earlier, are intended to provide inspiration. For each problem the teacher simply asks if any student want to solve it. It there is no one, the teacher solves it without disappointment, which underlines the voluntary character of the activity.

Circa 80% of the Student's problems were solved by students, by approximately 15 different students. About 60% of the circa 100 students registered were present during the lectures.

An alternative organization, that has not been tested, is to divide the room into six sections, enumerated 1-6, and then ask students to sit down at a number corresponding to the problem that one has prepared to solve. Then the teacher can ask a smaller group for a problem solver, and students are probably more focused on one particular problem.

3.5 Intentions of the course design

One intention with pre-lectures and post-lectures and problem solving in between, is that proofs are more meaningful for students after they have been active themselves. The style of a pre-lecture is indeed intended to make it possible for students to become mathematically active with the new concepts as soon as possible, and also to engage weaker students.

True-false quizzes are challenging for many students, since it is uncomfortable to provide many wrong answers, particularly if you expect the course to be easy. This is made milder (and perhaps more inviting) by the fact that the number of errors presented at the quizzes do not affect the assessment. Since the correct-incorrect feedback is unavoidable, the quizzes provide learning about particular mathematical difficulties.

This means that the choice of quiz questions is important – they must really formulate and test basic ideas. It was challenging as a teacher to produce false statements about mathematics. This is defendable since students immediately obtain a complete explanation about what is actually true, and why. An example of a false question is "*A tangent is always a polynomial of first degree*", which of course is false since some tangents are constants – polynomials of zeroth degree. Such false statements provoke students before answering to consider "*Is there some special case I am missing now?*". This is a highly appropriate scientific question, and it is desirable if it becomes habit.

The purpose of the quizzes is to make students more familiar with the theory and show how theory improves problem solving techniques. The Teacher's tasks are intended to pave the way for students to solve their Student's tasks during the Student's session a few days later. The Student's session is obviously assessment-free, characterized by the basic idea 'It is better to do the mistakes now than during examination'. Hence, the Student's sessions and the quizzes attempt to provide an atmosphere of openness about mathematical issues, deeper discussion and mathematics learning, which can be seen as a way of communicating that is opposite to that of the examination. This points towards a central issue to succeed with a course of this type: the quality of teacher-student dialogue and interaction (see next section.)

All students quickly learn that during course week 3, 5 and 7, (mini) examinations take place on the same topics that appear in Student’s tasks and in the quizzes. In this way, mini-exams on one hand and quiz-supported open Student’s sessions on the other, alternate during the course. Students are activated by mini exams, and provided extra opportunities to learn before them, partly via feedback to the teacher during the non-assessment phases and from quiz feedback. This alternation of examination and mathematical openness is a main underlying design feature.

After the course, in the written exam, the same knowledge is tested, but then requiring not only familiarity but also more complex calculations and combinations of several ideas in the same task.

The diagnostic quizzes are beneficial for a few different reasons. First, students refresh what they need to know to understand the lectures of the coming week. Typically, quiz-items will reappear during lectures since they are fundamental. Second, the teacher can, before the lecture, look at the number of correct answers on different questions and prepare to be more specific during the lecture on questions with many incorrect answers. The teacher thus obtains feedback from the student group with very little extra work.

Another reason not to assign points to Concept quizzes according to the number of correct answers is that they represent new facts that cannot be expected to be understood and examined yet. However, the students are informed that some quiz questions will appear at the final examination.

4. Comments about the realisation

Crucial for the result of this particular course design is the quality of teacher-student dialogue and interaction, since it is designed to inspire such interaction. From a course result point of view, most important is that feedback provides relevant and correct mathematics, but also that the feedback increases the student’s interest and willingness to work, i.e. increase efficient student work time during the course weeks. This latter goal involves in turn many different aspects. I argue that fundamental here is the teacher’s ability to listen: to find the real issue for the student. This issue may be trivial from the teacher’s perspective, and if so impossible to spot for a teacher who does not try to take the student’s perspective.

It is important for such feedback to communicate general properties of problem solving in mathematics. One such property is that mathematical problem solving typically is tentative: we try to use rules to change our expressions, but during calculation we cannot be sure if the calculation will actually lead to the result, nor what that result will be. This is normal, and part of the creative aspect of mathematical problem solving. This and other general observations about mathematical problem solving are rarely found in text books, and this is why it is even more important that it is communicated by the teacher. Classroom dialogue provides an excellent opportunity. Students may find a more realistic and practical view towards their own mathematical work.

Another general comment is that some errors during Student’s sessions are probably similar to errors seen during grading of exams, but which a teacher perhaps never mentions during a lecture. Teacher attention and explanation to this is certainly helpful for some students.

A central experience during the realisation was that feedback to students in the students’ session almost always can be done both mathematically accurately, relevantly, and in a way that is constructive and encouraging for students. This helps students’ participation in the course. Below are some examples of such feedback:

- Student: Makes a common student error.

  Teacher: "*You are really not alone in this error. And it is very good that it appeared before the exam!*" (Followed by a full explanation.)

  Implicit conclusion: The student has indirectly helped other students in class.

- Student: Misses important argument in the calculation.

  Teacher: "*You may think this, but you need to write it down also, otherwise you lose points unnecessarily. When grading I can only consider what you have actually written down*".

  Implicit conclusion: It becomes clearer for a student how an exam is graded, and what needs to appear in a complete solution.

- Student: Hesitates significantly halfway in to an attempted solution.

  Teacher: "*You have made a good start, if you want I can finish?*"

  Implicit conclusion: A course is process towards knowledge, and it is only natural that the knowledge is not immediately complete.

For the sake of the listening students during a Student's session, it is important that the teacher also provides relevant feedback for them. The typical question: "*Are there other ways to solve this problem?*" after a student's solution may reveal the degree of problem solving activity that has been going on.

Access to quizzes can also be released in the end of the course after all quiz deadlines have passed, for repetition and practice. This was asked for by students.

Is there a large extra teacher workload for this design? No, not really. Once produced, the quizzes can be used for many years, with successive minor improvements. The main extra work is grading the mini-exams. Since mini-exams do not require long calculations, this grading is a matter of a few minutes per paper. Similarly, time required for the teacher to study the results on the quiz questions is not large. However, to make the first design of the course is time consuming, particularly to find good quiz statements that cover the essentials of the course, and explanations to them.

5. Measurable course results

In the written exam, 98% of the registered students participated, and 52% passed. The typical percentage pass rate for the Analys 2 course is around 40%. Colleagues regard the written exam as slightly more difficult than usual (certainly not easier). While at the student's course evaluation, the course was rated good or very good in all respects.

The university teacher, Johan Silvander, was present during one Friday lecture as a part of a university pedagogical project, and he evaluated the activity as follows:

"*A student from each group showed the solution of their specific exercise on the whiteboard. The teacher encourages students who have not been in front of the whiteboard before to show the solutions. The teacher is helping the students by putting leading questions, if needed, and explains to the audience what is taking place on the whiteboard. If some steps are not included by the student, these steps are filled in by the teacher at an appropriate time. The students in the audience were active and asked questions or proposed solutions. Questions and proposals that could have been considered as 'stupid' were handled in a way that enlightened the student and saved the student's* *face. Since the same basic methodology was used to solve all of the exercises the teacher pointed that out during all the exercises. The teacher took care of informing the students about its relevance by always showing all the steps of the methodology even if more advanced students have omitted a step due to 'This can easily be understood'* ".

The take-aways from this session were:

- Letting the students show and explain their knowledge;
- Guiding the students to the answer and handling the students with respect;
- Using different abstraction levels when introducing the students to a new concept/methodology.

6. Conclusions and discussion

Mathematics is not an empirical area of research – it relies only on proof. However, a mathematics course is a highly empirical enterprise. Students' learning is the overall goal, and the quality of learning depends crucially on how well the design of lectures and activities match the prerequisite knowledge of the students that are registered to the course. This prerequisite student knowledge is the crucial empirical knowledge.

In order for a course to be scientific, a basic requirement is to collect relevant empirical knowledge, and use it to improve the result. I thus argue that for a course to be scientific it is not enough that the theory described is correct and relevant.

The course design presented in this paper aims at systematically collecting and using knowledge about students' prerequisite knowledge as well as challenging the student's thinking in mathematics, in order to reach further in mathematics learning.

Furthermore, it attempts to highlight types of mathematical knowledge that is important but rarely present in a traditional mathematical accounts. One important such type is how to find a workable idea to a mathematical solution of a problem, and to construct a solution from that idea. This is obviously fundamental for students during an examination and when working as an engineer, but not present in a calculus textbook that presents theory and correct solutions only. A correct solution usually does not contain a discussion of different possible solution methods, despite this being very important for mathematics problem solving.

The intention of the feedback is not only to learn about students' mathematical knowledge. It is also to reflect upon mathematical solutions, to communicate aspects of mathematical problem solving that is not easy to define in a clear way. This raises questions about mathematics teacher competencies and properties of mathematics that is outside mainstream mathematics teaching. Which kinds of creativity and mathematics knowledge enable students to formulate relevant basic ideas in order to construct mathematical solutions? How do mathematics teachers create a positive atmosphere so an exploratory discussion about mathematical topics takes place that problem solving students find relevant and engage in? How are such competencies educated?

An experience from this feedback course design is that it favours students that are engaged in the course, which is not always the most successful students. The feedback also tends to make the course more inspiring and rewarding to the teacher, which indirectly favours students' studies and results.

For a teacher, there is an easy test concerning empirical course knowledge. Before grading a written exam, it is possible and fruitful for a teacher to try to predict the mean number of points that the students receive for the different tasks in the exam. Slightly more advanced than this is to predict which kinds of errors are most common and which will invoke the largest loss of points before grading, and subsequently check this during the grading. This is a scientific teacher attitude – taking advantage of existing empirical knowledge.

7. References

Ernest, P., 1994. The Dialogic Nature of Mathematics. In: P. Ernest, ed. *Mathematics, Education and Philosophy*. London: Falmer Press, pp. 33-48.

Kindberg, T., 2013. The Significance of Emulation in the Oral Interaction Between Teacher and Students. *Journal of Philosophy of Education*, 47(1), pp. 99-111.

Lennerstad, H., 2008. Spectrums of knowledge types – mathematics, mathematics education and praxis knowledge. In: C. Bergsten, B. Grevholm, T. Lingefjärd, eds. *The 6th Swedish Mathematics Education Research Seminar*. Stockholm.

Rowland, T., 2000. *The pragmatics of mathematics education, vagueness in mathematics discourse.* London: Falmer Press.

Shön, D., 1983. *The Reflective Practitioner: How professionals think in action*. London: Temple Smith.

Wittman, E. Ch. 1995. Mathematics education as a 'design science'. *Educational Studies in Mathematics,* 29(4), pp. 355-374.

RESOURCE REVIEW

An Overview of the Pingo Audience Response System in Undergraduate Mathematics and Statistics Teaching

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Abstract

Obtaining interaction and engagement in large group teaching can be challenging, in particular in STEM subjects where it is essential to be able to efficiently present symbols, equations and formulae. Pingo is a free, web-based system that provides an excellent way of introducing interaction in large group teaching, as there are no restrictions on the number of participants. It is particularly useful in Mathematics-based subjects as it is TeX compatible. Such interaction also provides feedback to students and lecturers and it is a straightforward, but creative way to enhance student engagement. The author has tried and tested Pingo weekly in Mathematics and Statistics modules. This article provides all the information and resources required to start using Pingo along with some reflections on its use in the Mathematics Department at Swansea University.

Keywords: Large group teaching, audience response system, Pingo, TeX, LaTeX, STEM.

1. Introduction

For the last two academic years I have trialled the Pingo audience response system with the aim of increasing participation, providing better, and more useful, feedback, teaching in a more inclusive way and for generally providing a better student experience. The main reason I wanted to do this was because I felt that there was a lack of interaction in lecture style teaching. Students often would be reluctant to ask questions or provide feedback during large group teaching activities. I believe that this is to the detriment of students' learning experience and I therefore decided to research techniques to overcome this. Initially, I was led by ideas and evidence in Bruff (2009) that interactive systems like Pingo help achieve this. Furthermore, I found the work by Bates, Howie and Murphy (2006) interesting, in particular the statement "*The challenge is, therefore, to try to actively engage the students in the lecture, to develop it to be something more akin to a two-way conversation than a one-way transmission of information.*"

There are a number of audience response systems available to facilitate interaction in large group teaching environments, in particular Shon and Smith (2011) give an interesting account into the use of Poll Everywhere. Other systems include Zeetings, Socrative and Mentimeter. After researching many such systems, I decided to trial the use of Pingo due to a combination of the following reasons: it is TeX compatible, it is fee-free and there are no restrictions on the number of participants. TeX, or in its more advanced form including macros, LaTeX, is an open-source programme used by many academics in STEM-based subjects to typeset mathematical documents. In fact, many universities now require Mathematics undergraduates to be able to produce documents in LaTeX. Furthermore, the London Mathematical Society states in their Position Statement (London Mathematical Society, 2011), that "*We need to reject ICT products not suited to writing, presenting and processing mathematical texts. LaTeX is a recognised solution, and students should be encouraged, and where possible taught, to use LaTeX to present mathematics in reports and projects.*" Pingo can also be used in a text-only way, however, one of its main advantages, in my opinion, is that it can be used to create advanced, mathematically-based questions due to its TeX compatibility.

Pingo was created and is run by the University of Paderborn in Germany. It is a web-based system and therefore only requires the lecturer to have access to an on-line computer and the audience to have access to an on-line mobile phone, tablet or computer. A further benefit of Pingo is that members of the audience do not need to sign up to use the system; only lecturers have to open an account in order to create questions. Questions may be asked in several ways in Pingo, varying from single and multiple-choice questions to open-ended questions requiring a text or TeX-coded answer. Similar to other audience response systems, the audience is given a set amount of time to answer a question, and once this time has elapsed the anonymous results appear on the screen for all to see. This provides instant feedback to both the student and the lecturer, and, where necessary, further information may be provided by the lecturer to explain any troublesome questions. Pingo also has a *repeat* feature which is useful to check that the audience now does understand a particular question. In my opinion, the anonymous nature of answering questions is important, as the audience is more likely to participate.

I have used Pingo in a third year module in complex analysis and a first year module in applied statistics. In both modules, I used the system during the final lecture of each week as an interactive method to test and refresh the topics taught that week. The majority of the questions I used were either multiple choice or single choice and mathematical or statistical in nature. However, I would also ask the following open-ended question every week: *Are there any topics that you would like to go back over? If so, please state them below.* For any comments listed, I would spend some time during the next lecture to revise these topics. Not only does this provide instant feedback to the lecturer, but it also facilitates prompt feedback - rather than having to wait for module questionnaire or survey results. I believe the overall learning and teaching experience has been enhanced by this intervention, reflecting the statement "*The use of interactive lectures can promote active learning, give feedback to the teacher and the student, and increase satisfaction for both*." in Steinert and Snell (1999).

The system has been well received by students, as evidenced by module questionnaire comments and comments in staff-student consultative committee meetings which are discussed in Section 3 below.

2. Guide to Using Pingo

This guide is written from an academic perspective with the intention of lecturers/teachers using Pingo to create an interactive learning environment for their students. Clearly, this could be adapted to other scenarios where interaction is desirable. More detailed information about the features of Pingo can be found on the developer's website or their YouTube channel.

- As mentioned in the Introduction, lecturers are the only users who have to sign up for a Pingo account, and this is achieved by visiting http://trypingo.com.
- Once signed up, the website that both staff and students use is http://pingo.upb.de. On this page, lecturers sign in to create questions and sessions whereas students input the access number (see later) for a particular lecture to participate in the session.
- Questions may be created on-the-spot, or a catalogue of questions may be created in advance. To do this, select **Survey Design** followed by **New Question** to get the following (Figure 1):

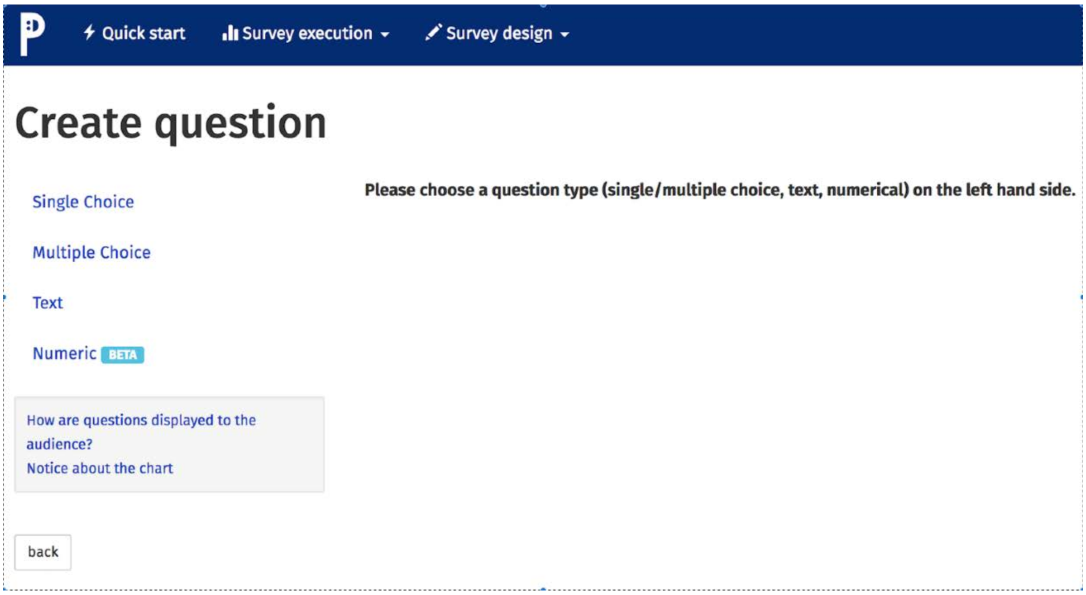

Figure 1. Creating a new question in Pingo.

- There are options to create *Single Choice, Multiple Choice, Text* and *Numeric* type questions. As an example, the image below demonstrates the **Single Choice** option (Figure 2):

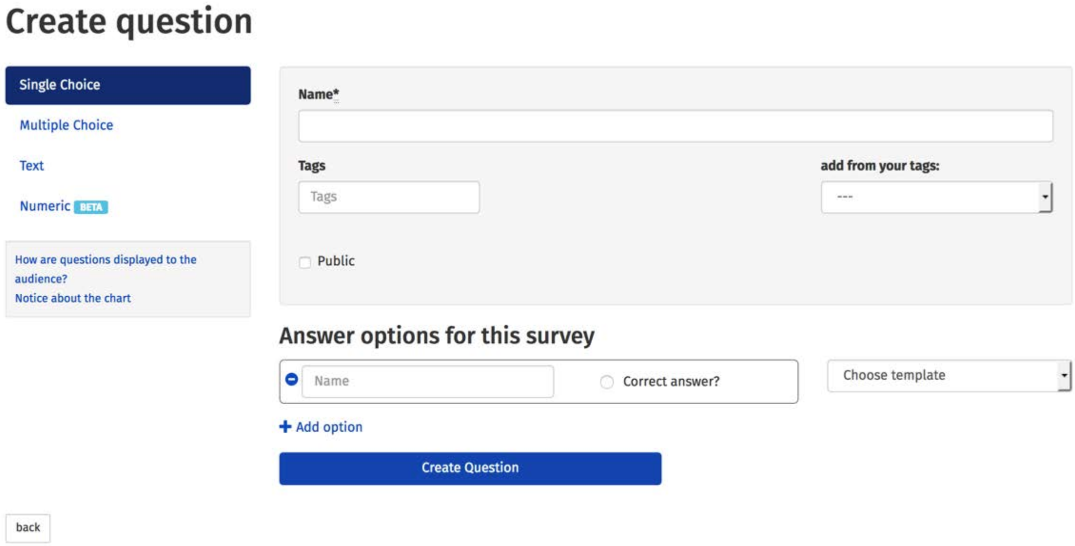

Figure 2. *Single Choice* question option in Pingo.

- Questions are typed into the **Name** box above, whereas possible answers are typed into the option boxes below. The correct answer is indicated by ticking the appropriate box. Any required TeX code within questions or answers must be encapsulated within dollar symbols in the usual way. The TeX code is compiled once the question is created. There are also a number of useful templates available from the drop-down menu **Choose template**. The **Tags** option is particularly useful if Pingo is used in more than one module/course as it provides a way of separating questions into categories making them easier to manage. The final step is to select **Create Question**.

- The system is very intuitive which makes creating the other types of questions equally straightforward. Repeating this procedure creates a catalogue of questions saved onto your

Pingo account. Once a catalogue of questions has been created, a **New session** must be created to use Pingo in lectures. This is achieved by selecting **Survey execution** followed by **New session.** A name must be created for the session and if TeX code is going to be used (including within previously created questions) then the **Formula support** box must be selected, see Figure 3 below:

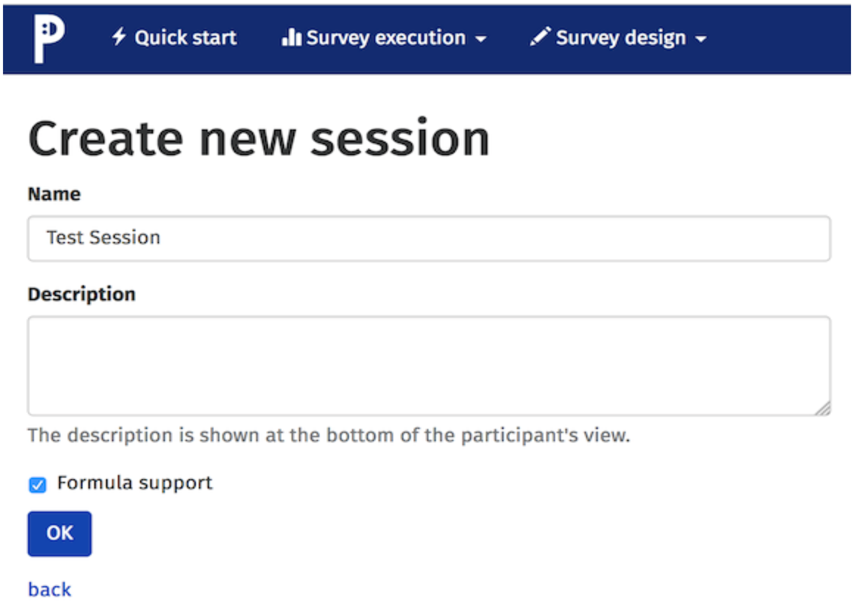

Figure 3. *Create new session* option in Pingo.

- Once a session has been created, it will be assigned a unique number and questions can be set by selecting from your previously created catalogue or by creating questions on-the-spot. These options are accessed on the right-hand side of the following screen (Figure 4):

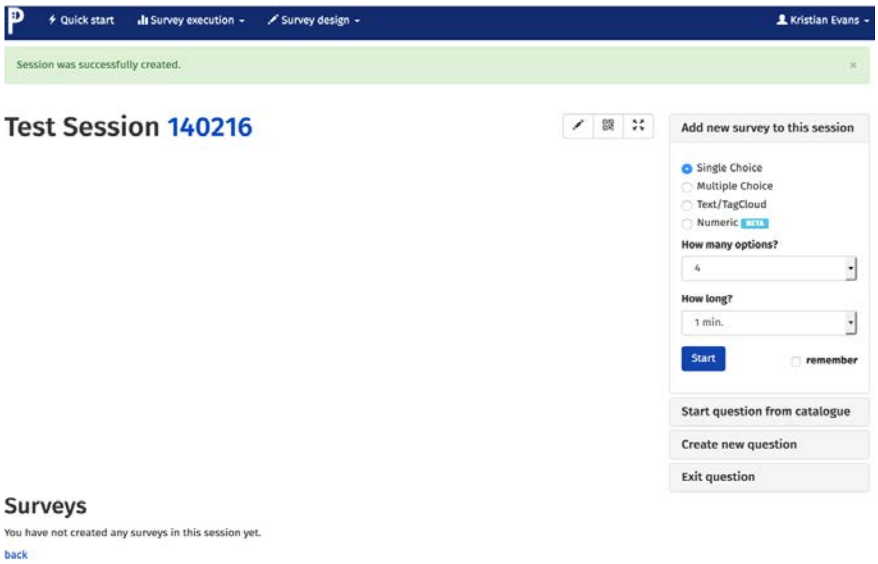

Figure 4. New session page in Pingo.

- Students enter the unique number into their devices to access the session. Once a lecturer sets a question, it will appear both on the screen in the lecture theatre and on the students' devices. There is also a countdown timer to ensure that students know how much time they have.

- After each question, the results are displayed and lecturers can select the correct answer(s) using the **Highlight correct answers** option. Figure 5 below shows an example of a Pingo output, along with the highlighted correct answer, for a question asked to a group of third year students studying a complex analysis module:

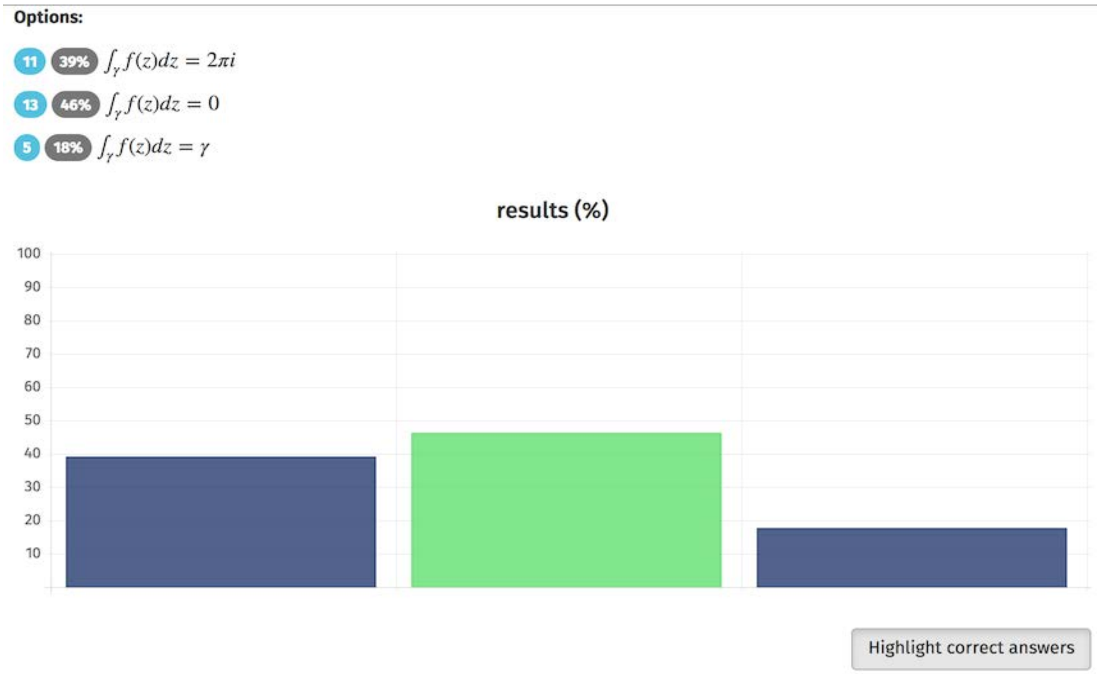

Figure 5. Example question output in Pingo.

- In addition, questions may be imported into Pingo and results of surveys may also be exported. Another useful feature of Pingo is **Pingo Remote** - this allows the system to be incorporated into presentation software. This option is available from http://trypingo.com

3. Reflections on Using Pingo

From a lecturer's perspective, I found Pingo to be extremely useful in obtaining interaction in large group, lecture-style teaching. The results of the surveys also provided instant feedback on what topics were well-understood by the group and which topics required further explanation. This meant that prompt feedback, and further relevant information, could be provided to the students based on this information. I found the system to be user-friendly and it provided an additional tool to track the general progress and understanding of the group, allowing me to adapt my teaching delivery accordingly.

The system has also been very well received by students; to date, I have not received any negative feedback on Pingo. In fact, the questionnaires for the modules in which I have used Pingo have included many positive comments about the system, including:

- "*Pingo! I love learning this way, it makes things so much easier for me to understand.*"
- *"Interactive example classes on Pingo are very helpful since you can anonymously answer the questions and consolidate your understanding."*
- *"Pingo surveys are very useful. I would like all modules to include them."*

Furthermore, in the final lecture of my first trial of the use of Pingo I polled the students on their opinions of the system. All responses (n=20) said that they enjoyed the Pingo sessions and all responses (n=21) said that they found the Pingo sessions useful in their learning. Students have also commented during staff-student consultative committee meetings how they value the fact that they can anonymously request feedback for specific topics by using Pingo.

I have used Pingo with up to 50 participants without any technical issues. Furthermore, I did not encounter a situation where a student did not have a device compatible with Pingo. If this situation were to arise, pooled university devices could be distributed to such students. In the sessions I used Pingo I never encountered any problems with the WiFi network, but I always checked the network prior to every session.

I have given a variety of presentations with live demonstrations on the use of Pingo, both within Swansea University (2017 SALT Conference) and nationally via the Higher Education Academy (2018 HEA STEM Conference). Following these, some colleagues have adopted the system, again with positive experiences. In particular, one colleague recently remarked that Pingo enables him to "*respond to student concerns and questions as soon as possible*".

4. Conclusion

In my teaching, the use of Pingo has greatly improved the issue of lack of interaction in lectures set out in the Introduction. Prior to using Pingo, I would ask similar questions verbally and rarely receive a response. Using Pingo I have received up to 50 responses. Unfortunately, there are still some students who do not engage and it would be useful and interesting to do study to investigate why this is the case. In my opinion, the main advantages of using Pingo are its TeX-compatibility and the increased interaction obtained, which provides both feedback for students and staff. This feedback can be used by students to improve their learning and by staff to adjust their teaching. As previously mentioned, the system is fee-free and there are no restrictions on participants. The system is simple to set up and use, in particular if catalogues of questions are created before teaching sessions. I think the anonymous nature of answering questions makes it more likely for students to participate, however, the disadvantage of this is providing targeted feedback. A further disadvantage of such systems is that they are clearly reliant on technology working efficiently. As mentioned in the Evaluation, I have not encountered any problems, however, it would be wise to have a non-technological activity prepared in case of any problems.

5. Acknowledgements

I would like to thank the University of Paderborn for creating the Pingo system and permission to write this article. I became aware of the system due to Dr. Mathew Pugh of the Mathematics Department at Cardiff University and I would like to acknowledge this. Finally, I would like to Dr. Gibin Powathil for his input and remarks in using the system.

6. References

Bruff, D., 2009. *Teaching with Classroom Response Systems: Creating Active Learning Environments.* San Francisco: John Wiley & Sons.

Bates, S., Howie, K. and Murphy, A., 2006. *The use of electronic voting systems in large group lectures: challenges and opportunities.* The Higher Education Academy, New Directions Issue 2. 10.11120/ndir.2006.00020001

London Mathematical Society, 2011. *Use and Misuse of Information and Computer Technology in the Teaching of Mathematics at HE Institutions: Position Statement.* Available at: https://www.lms.ac.uk/sites/lms.ac.uk/files/Mathematics/policy_responses/ICT_statement.pdf [Accessed 5 April 2018].

Pingo Sign-up Website, 2018. Available at: http://trypingo.com [Accessed 12 April 2018].

Pingo YouTube Channel. 2018. Available at: https://www.youtube.com/channel/UCoA4vCzvqmc3oRLeEpGfayQ [Accessed 24 July 201].

Pingo Web-based System, 2018. Available at: http://pingo.upb.de [Accessed 12 April 2018].

Shon, H. and Smith, L., 2011. *A Review of Poll Everywhere Audience Response System.* Journal of Technology in Human Services, 29:3, pp. 236-245. 10.1080/15228835.2011.616475

Steinert. Y. and Snell. L.S, 1999. *Interactive lecturing: strategies for increasing participation in large group presentations.* Medical Teacher, 21:1, pp. 37-42. 10.1080/01421599980011

Using LaTeX's moodle package and R's Sweave to easily create data-driven, up-to-date financial mathematics and statistics quizzes for Moodle

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Abstract

Preparation of Moodle quizzes which are data-based and contemporary tends to be tedious and time-consuming. By using innovative tools, this process can be simplified and automated, providing a substantial benefit to the teacher wishing to employ such quizzes, and ultimately improving student learning experience. The purpose of this article is to show how to create data-driven, up-to-date quizzes for Moodle in an easy fashion. The methodology is based on several popular, open-source, free tools, and its implementation details are demonstrated with an example. This makes the methodology readily-available to the practitioners.

Keywords: reproducible, dynamic, data-dependent, free software, Moodle, R, LaTeX.

1. Introduction

Data-driven and up-to-date examples are a good way of engaging the students in the learning process. Likewise, by reflecting the current state of affairs, such examples allow the students to remain informed about the world we live in. Exercises that potentially could benefit from being refreshed regularly are primarily those that depend on short-time-scale data (data observed on daily, monthly, quarterly, possibly annual frequencies), data whose movements follow trends, though arguably some data will remain relevant over longer time scales. In financial mathematics most exercises would lend themselves to being updated periodically as they are concerned with prices of assets (stocks, bonds, exchange rates), often recorded on a daily frequency, prices of derivatives (financial instruments dependent on the underlying assets), risk metrics computed from prices of assets and derivatives, financial statements of companies, credit ratings, levels of interest rates. For example, it is likely that students solving exercisers where interest rates are between 5-10% per year will find themselves somewhat surprised in view of current economic conditions where interest rates are close to 0%. In statistics, exercises that would benefit from being updated regularly are those that deal with time series data, especially when forecasting is of interest (for example, forecasting monthly electricity production or temperature in the UK for August 2017 in a course given during 2018 might seem somewhat artificial), but also exercises where cross-sectional data collected over time is used.

An implicit step in preparing examples of that kind and employing them for assessment of students' learning is the use of a recent stretch of data, but the time-dependent nature of the examples/exercises means that they cannot easily be re-used - or can they? When the course management system happens to be Moodle (Moodle HQ and Moodle Community (2018)) and the activity that incorporates examples with current data is Moodle's Quiz, then it turns out that the examples can be easily produced and re-produced. One important tool that facilitates data-to-quiz transformation is a recently-developed LaTeX package called moodle (Hendrickson (2016); LaTeX is a type-setting system, Lambert (1994), LaTeX3 Team (2018)). Thanks to this package quiz questions can be embedded into a .tex file and once compiled, the resulting .xml file, containing the quiz questions, can be imported to Moodle. There are several benefits of the moodle package when used as a stand-alone tool, and these are outlined in Hendrickson (2016): a) no web-based interface is needed; this feature eliminates the necessity of switching between menus and boxes, making the process of editing questions faster, and it also eliminates the need to rely on the Internet access/speed; b) mathematical and scientific notation can be type-set fast because `LaTeX` source code is used to produce them, and users have the flexibility of defining their own macros, again to speed up the editing process; c) the `.tex` file containing quiz questions can be examined, modified, browsed and archived; this is in contrast to editing each question separately in a web-based editor; d) the `.pdf` file (created at the same time as the `.xml` file) can be previewed; this allows the user to proof-read the quiz and correct the errors more efficiently compared to the online option. But how does the data-element come into the picture? This step is achieved through `R`'s function `Sweave` (Leisch (2002), Leisch (2017); `R` is a computing software, R Core Team (2018)), because `Sweave` allows one to mix `LaTeX` syntax and `R` code (embedded in an `.Rnw` file) to produce dynamic reports, and of course `R` can be used to acquire contemporary data as well as to carry out various analyses.

The goal of this article is to demonstrate how several free, open-source and popular tools can be combined to easily produce data-driven, up-to-date quizzes for `Moodle`, using examples from economics. Because the exercises involved are produced dynamically, quizzes can be re-cycled, alleviating the problem of time-consuming and painstaking edits, whenever the input data changes.

We give a brief description of each of the above-mentioned tools, though we assume that most of the readers will be familiar with `Moodle`, `R` and `LaTeX`.

- `Moodle` and `Quiz` (activity on `Moodle`): `Moodle` is a learning management system employed by many institutions world-wide. One of `Moodle`'s activities is a quiz (`Quiz`). This module allows one to build quizzes made up of a variety of questions: True/False, multiple-choice, short-answer, matching, numerical, embedded-answers. The last type, termed `Cloze` in `Moodle`, consists of a passage of text that has various answers (pertaining to question types just mentioned) embedded within it, and due to its flexibility is a very useful category;
- `LaTeX`: is a document-preparation system for high-quality typesetting, which allows one to produce professionally-looking mathematical expressions, which can be typed fast using plain text;
- `moodle` (package for `LaTeX`): `moodle` is a package that offers more efficient implementation of `Moodle`'s `Quiz` compared to the web-based editor. All question types mentioned in the first point (plus an essay) are supported by this package;
- `Sweave` (function in `R`): `Sweave` is used to weave together chunks of `R` code (so-called code chunks) and chunks of `LaTeX` syntax (so-called documentation chunks) for reproducible analyses. The source file containing the mixture of both has extension `.Rnw`. The value of an `R` expression, `expr`, from the code chunk can be used in the documentation chunk via the command `\Sexpr{expr}`;
- `R` and `RStudio`: `R` is a programming language and software for statistical computing and graphics and `RStudio` (RStudio Team (2018)) is an integrated development environment for `R`.

The rest of the article is organized as follows. In Section 2 we describe the methodology, in Section 3 we give an example of its implementation, finally we provide a brief summary in Section 4. Supplementary materials (available from the author on request) contain input and output files used in the preparation of this article.

2. Methodology

The pre-requisites for executing the code of this article are: `R` including the `Sweave` function (and `RStudio`, for convenience); `LaTeX` compiler with the `moodle` package installed (distributions for various platforms, for example, Windows's `MiKTeX`, come with `moodle`). These programmes are available across different operating systems. If additional `R` and `LaTeX` packages are used (for example, `R`'s `Quandle` or `fOptions`) by the code and documentation chunks, these are assumed to be installed as well.

The work-flow in preparing the quiz consists of the following three steps. Step 1) In `R`, create an `.Rnw` file according to the `Sweave` guidelines, where the code chunks collect the data and perform analyses and where the documentation chunks elaborate quiz questions with the commands of `moodle`, as well as making use of values from the code chunks via `Sexpr`. Step 2) In `R`, run the `.Rnw` file using the `Sweave` function to obtain `.tex`, `.xml` and `.pdf` files. This can be done using `RStudio`'s menus or by creating an `R` script, which sources the `.Rnw` file (we present the 2nd option for a potential automation purpose). Step 3) Import the `.xml` file produced in the previous step to `Moodle`'s `Question bank` (all quiz questions are uploaded in this single step and are ready to be used for a creation of `Quiz` activity on `Moodle`).

Students attempting the quiz presented henceforth (Section 3) are assumed to come from Finance-type programmes at a BSc level and are assumed to have knowledge of `R` (and `RStudio`) and to be familiar with packages and routines used in financial mathematics and statistics, for example, such as those described in Stander and Eales (2011). The learning objectives associated with the quiz are: students are able to download data sets in `R`, transform the data, obtain numerical summaries and perform analyses. Such quizzes could form part of formative or summative assessment.

3. Example

We give an example of a quiz with three questions (file `example.Rnw` available in the Supplementary materials). In Question 1 (question type: matching) students are supposed to download two time series in `R`, transform them, find some numerical summaries of the latter and finally, match the type of a numerical summary with its value. In Question 2 (question type: embedded-answers with numerical) students use the data from Question 1, to find a rational price of a financial instrument called a European option. The price is given via the so-called Black-Scholes formula (well-known in finance), which is implemented in one of R's packages. Here, students have to fill-in-the gaps using numerical values used in the context of the exercise. In Question 3 (question type: embedded-answers with numerical and multiple-choice) students perform Principal Component Analysis on the transformed data from Question 1. Students have to fill-in-the gaps with numerical values or choose one of the options provided from the drop-down menus.

We begin (Step 1) with the file `example.Rnw`, which we save on our machine, remembering the path to the destination folder. We open `example.Rnw` in `RStudio` to view its contents (and optionally, to execute it using `RStudio`'s menus). To carry out Step 2), we save `runexample.R` (available in the Supplementary materials and shown below) in the same destination folder as `example.Rnw`, open it in `RStudio` and complete the line that specifies the working directory. We then execute the code, evoking function `Sweave` on `example.Rnw`, sending the output to `example.tex`. When the last file is compiled, `example.pdf` and `example-moodle.xml` are created. What remains to be done is to import `example-moodle.xml` to `Moodle`'s `Question bank` and to make the quiz (Step 3).

```
#'runexample.R'

library(tools)

#specify the working directory (no slash at the end)

mywd<-"C:/Users/..."

setwd(mywd)

myfile<-paste0(mywd,"/example")

Sweave(file=paste0(myfile,".Rnw"),output=paste0(myfile,".tex") )

texi2pdf(file=paste0(myfile,".tex"))
```

One possible extension of the above scheme is to create many variants of the same quiz by changing the input data slightly, for example, by shifting the start and the end dates of the data samples. This can be achieved by embedding the last two lines of `runexample.R` into a `for` loop, say `for(counter in 1:5)`, modifying the lines accordingly (`output` in `Sweave` and `file` in `texi2pdf`), and then using `counter` to alter the selected lines of `example.Rnw` appropriately.

To conclude this section we mention several aspects of using the software. One of the known limitations of the `moodle` package is the lack of shuffling of answers in multiple-choice questions when used inside `Cloze` question (Section 6 of Hendrickson (2016)), but that feature works correctly when multiple-choice is used as a stand-alone question. The combination of multiple-choice and `Cloze` questions was employed in Question 3 of the quiz example of this section. One can easily correct for this by editing Question 3 in the web-based `Moodle` interface. Also some users of `moodle` might find it difficult to incorporate images into the `.xml` file (Section 5 of Hendrickson (2016)). Again, this can be adjusted for by inserting the image via the web-based interface of `Moodle`. There might be some additional effort required when employing `LaTeX`'s `verbatim` environment in `moodle` (potentially-useful environment for code-listings). It is worth pointing out that the `moodle` package can be used without the involvement of `R` or `RStudio` (and `Sweave`), by simply creating a `.tex` document with quiz questions and then by compiling it (see also points a)to d) in the introduction of this article), but this does not allow for data retrieval/analysis and creation of dynamic reports. Finally, access to some of the data sets freely-available through `R`'s package `Quandl`, require the so-called API key. This key can be obtained at no charge from www.quandl.com upon registration at the website.

To finish off, we provide the information about the software type and versions used in the preparation of this article: Operating System: Windows 7 Enterprise; `Moodle` version 3.3; `LaTeX`: `MiKTeX` version 2.9; `moodle` version 0.5; `R`: Microsoft R Open version 3.3.2, `RStudio` version 1.1.383, `Sweave`: from `R`'s package `utils` version 3.3.2. Mac users may wish to consult French (2013), when setting up `Sweave`.

4. Summary

In this article we showed how to create a quiz for `Moodle` in such a way that it is based on data downloaded and transformed in `R`. The results of computations and analyses are incorporated into a `.tex` document using `Sweave`. Additionally, the `.tex` document sets quiz questions by employing the `LaTeX` package `moodle`. The possibility of automation of the process through exploiting `R`'s capabilities such as loops, allows the user to create different variants of the same question/quiz. This could be useful in formative assessments, where students repeat a similar exercise, or in summative assessments, where different students solve different exercises, to ensure academic integrity. We hope that the methodology of this article will benefit practitioners who would like to implement similar quizzes for their courses.

5. References

French, J. A., 2013. *Easy Sweave for LaTeX and R*. Available at: http://www.jason-french.com/blog/2013/08/16/easy-sweaving-for-latex-and-r/ [Accessed 20 July 2018].

Hendrickson, A. O. F., 2016. *The moodle package: generating Moodle quizzes via LaTeX*. Available at: https://ctan.org/tex-archive/macros/latex/contrib/moodle?lang=en [Accessed 20 July 2018].

Lambert, L., 1994. *LaTeX: A Document Preparation System*. Reading, MA: Addison-Wesley.

LaTeX3 Team, 2018. *LaTeX*. https://www.latex-project.org/.

Leisch, F., 2002. Sweave: Dynamic generation of statistical reports using literate data analysis. In W. Härdle and B. Rönz, eds. *Compstat 2002: Proceedings in Computational Statistics.* Heidelberg: Physica Verlag. pp. 575-580.

Leisch, F., 2017. *Sweave User Manual*. [Available at: https://stat.ethz.ch/R-manual/Rdevel/library/utils/doc/Sweave.pdf. [Accessed 20 July 2018].

Moodle HQ and Moodle Community, 2018. *Moodle: Modular Object-Oriented Dynamic Learning Environment*. Moodle project. https://moodle.org/.

R Core Team, 2018. *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. http://www.R-project.org/.

RStudio Team, 2018. RStudio: *Integrated Development for R*. Boston, MA. http://www.rstudio.com/.

Stander, J. and Eales, J., 2011. Using R for teaching financial mathematics and statistics. *MSOR Connections*, 11(1), pp.7-11.

CASE STUDY

Developing *STACK* practice questions for the Mathematics Masters Programme at the Open University

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Abstract

The design and implementation of Masters level e-assessment in *STACK* is described for the introductory module *M820 Calculus of Variations and Advanced Calculus* in the Open University's Masters Programme in Mathematics. Some basic design principles are described and illustrated for an online practice quiz on the use of the Jacobi equation to classify stationary paths.

Keywords: e-assessment, *STACK*, design, implementation.

1. Introduction

Over the past 15 years there has been an explosion in the use of computer-based/online assessment in university mathematics, with several systems pioneered by UK practitioners. In addition to commercial systems such as *Maple TA* (Maplesoft.com, 2018), we have seen, for example, *Maths E.G.* at Brunel (Mathcentre.ac.uk, 2018), *Dewis* at UWE (Dewisprod.uwe.ac.uk, 2018), *Numbas* at Newcastle (Numbas.org.uk, 2018), and *STACK* (Stack.ed.ac.uk, 2018), originally at the University of Birmingham, but now adopted by several institutions, including The Open University (OU), where it has been demonstrated that *STACK* can work well at large scales[1] (Heacademy.ac.uk, 2018). See Figure 1 for the implementation of *STACK* at the OU.

Despite the theoretical basis, success, and growing adoption of these e-assessment systems in Mathematics (Sangwin, 2013), their use has largely been confined to undergraduate mathematics, typically at levels 1 and 2. Anecdotally, at least, there has been a feeling that these systems would be less useful for higher-level mathematics and that they are ill equipped to deal with mathematical argument and proof, especially given the wide variety of student responses and approaches. Such worries are understandable and the jury is still out on whether e-assessment can be effectively implemented at level 3 and above.

However, all mathematics involves some degree of calculation and students benefit from practice in the necessary techniques, even if such calculation is supplemented by nuanced argument, especially at higher levels. So, when we decided to implement practice questions for the Open University's MSc module *M820 Calculus of Variations and Advanced Calculus,* we were confident that we could devise questions that would be of genuine help to students. We also knew several formative e-assessment questions had already been successfully implemented by our colleagues on the module *M823 Analytic Number Theory 1,* the other entry module to the MSc programme. Indeed, if they had developed *STACK* questions for pure mathematics, then surely it would be straightforward to do so in the more conventional calculational world of calculus of variations.

Hence, armed with a grant from the university's programme to aid student retention, we set out to develop six significant *STACK* exercises to support students on M820 to develop the standard techniques in these key areas of the theory:

1. Solution of the Euler-Lagrange differential equation for quadratic functionals to obtain the stationary path.
2. Solution of variational problems through the first-integral, for those functionals permitting such an approach.
3. Local classification of stationary paths of functionals into minima, maxima and saddle points, through the analysis of the Jacobi differential equation.
4. Calculation of the Noether invariants (first-integrals) for functionals invariant under a scale change in the variables.
5. Diagonalisation of quadratic functionals involving two dependent variables, thereby allowing the stationary paths to be calculated by the solution of two independent variational problems.
6. The use of the Rayleigh-Ritz method to find an upper bound for the least eigenvalue of a Sturm-Liouville problem.

Each of these problems involves fundamental techniques that might be included in the end-of-module examination, on which assessment for the module is based. In this article we focus on the third technique, on the Jacobi equation.

2. Authoring online practice quizzes

Our first task was to set the overall design of the e-assessment questions. In this we were fortunate that the OU has an extensive history of e-assessment in mathematics, from computer-marked 'objective testing' (i.e. multiple choice), in which the university was a pioneer in the 1970s/1980s; through an elegant but now largely superseded Java-based system OpenMark (now open-source but non-maintained) (Open.ac.uk, 2018), (Butcher, 2008); to widespread implementation of *STACK* as part of an enhanced *Moodle* Quiz Engine (Hunt, 2012). Since both of us were familiar with *STACK* and had authored *STACK* questions in the past (albeit at Levels 1 and 2), the choice of system was clear. Since the individual topics involved multiple stages of calculations, we opted for each topic to be a separate *Moodle* quiz, with each calculation stage a separate question, each question linked so that they had the same instance of each random parameter[2]. This was done so that students could benefit from feedback on early parts of the calculation before tackling the later, harder parts.

We adopted the following *modus operandi* for developing the *STACK* questions.

1. Ben (as M820 Module Team Chair[3]) scoped out the possible questions which he felt were appropriate for STACK implementation, taking into account the importance of the material and the prospect for tractable, randomised questions which illustrated the theory and provided multiple practice examples without involving the student in unnecessary algebraic and arithmetic complexity.

2. After an initial discussion, Grahame (as former M820 student and professional programmer) then took the outline questions and implemented them in *STACK*, ironing out the multitude of niggles and problems.

3. Ben then checked and commented on the implementation, suggesting amendments, usually on wording.

4. A final discussion finalised the implementation before moving on to the next topic.

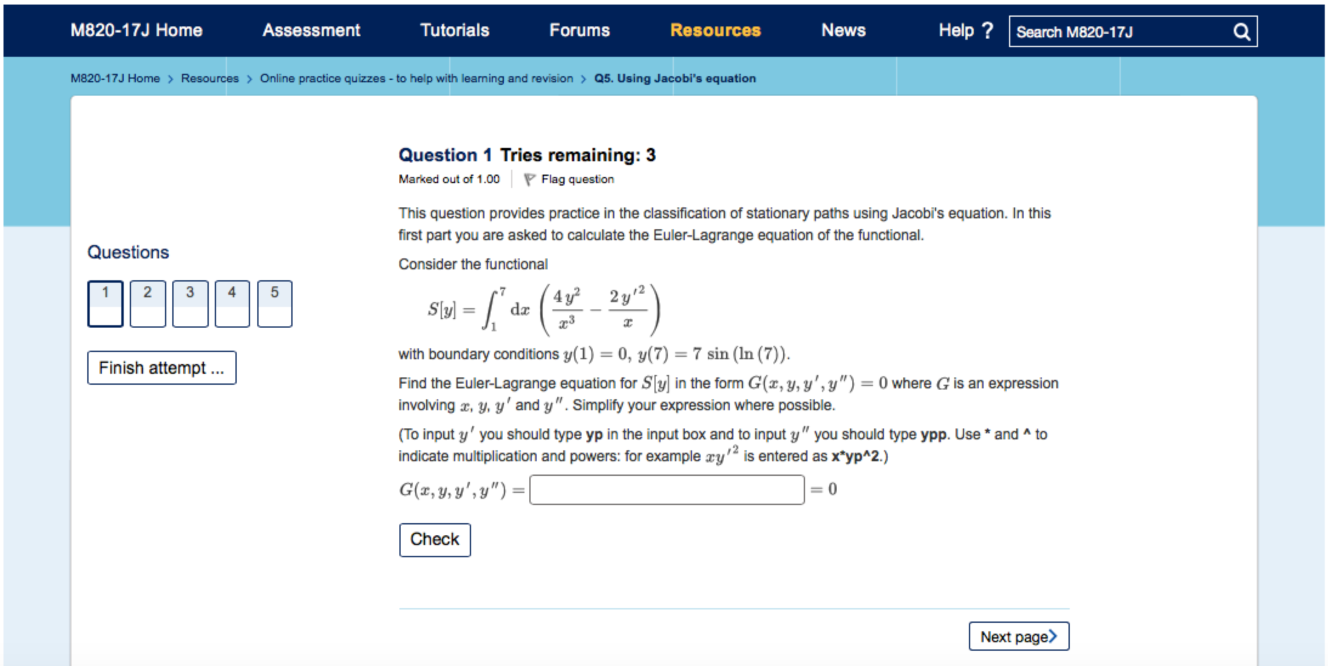

Figure 1. Implementation of *STACK* in the OU's *Moodle*-based Virtual Learning Environment. The example shown is the first part of a five-part practice quiz on the use of the Jacobi equation in the calculus of variations.

3. The calculus of variations and the Jacobi equation

Recall that the calculus of variations, dating from the 17th Century, studies extremal paths of functionals of the form $\int_a^b dx\, F(x, y, y')$ (and its many generalisations) together with assorted boundary conditions and constraints.

The candidate extremals are typically given by stationary paths, which are solutions of the Euler-Lagrange equation $\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0$. The theory, with its generalisations to higher dimensions and higher derivatives, has applications in many fields including physics (where it forms the basis of many modern physical theories), biology, control engineering, economics and chemistry. The modern theory incorporates singular and infinite functionals, but the focus of the M820 module is on hands-on calculation rather than on abstraction. See Gel'fand *et al.* (2000) and MacCluer (2012) for readable introductions, two of a plethora of excellent textbooks on this classical area of mathematics.

To provide the background for our case study, we now outline the use of the Jacobi equation to classify stationary paths.

4. Example: the Jacobi equation

Once the stationary paths have been calculated, the next step is to classify them into local maxima, local minima and saddles. Unfortunately, this is often a difficult task. One approach, a generalisation of the second derivative classification of stationary points of real functions, is to solve the Jacobi differential equation initial value problem

$$\frac{d}{dx}\left(P\frac{du}{dx}\right) - Q\,u = 0, \qquad u(a) = 0,\ u'(a) = 1,$$

where $P = \frac{\partial^2 F}{\partial y'^2}$, and $Q = \frac{\partial^2 F}{\partial y^2} - \frac{d}{dx}\left(\frac{\partial^2 F}{\partial y \partial y'}\right)$, evaluated on the stationary path $y(x)$. If $u(x)$ has no zeros ("conjugate points" in the jargon) in the half-open interval $(a, b]$ then $y(x)$ is a local minimum if $P > 0$ on $[a, b]$; a local maximum if $P < 0$ on $[a, b]$; and a saddle if P changes sign on $[a, b]$. Further analysis in needed in other cases. Unfortunately, despite being a linear equation, the Jacobi equation is frequently difficult to solve analytically. Therefore finding clean, tractable examples for students to practise the method is a challenge for educators.

Our aim for an online practice exercise on the Jacobi equation was to present the students with a functional, necessarily specialised in form, and to lead them through the solution of the Euler-Lagrange equation to find the stationary path and to then classify the stationary path using Jacobi's equation.

In order to reduce the calculations, we opted to work with a quadratic functional of the form

$$\int_a^b dx\ (\alpha_0\, x^m y'^2 + \beta_0\, x^{m-2} y^2)\,,$$

defined on the closed interval $[a, b]$, with fixed end-point boundary conditions $y(a) = A,\ \ y(b) = B$, where A, B are constants, chosen to reduce the algebraic complexity of the solution. The constants α_0, β_0 and m were also specialised as described below.

This type of functional has two distinct advantages. First, for functionals quadratic in y' and y, the Euler-Lagrange equation and the Jacobi equation turn out to be the same equation, with the important difference that for the Euler-Lagrange equation there are boundary conditions at the two endpoints a and b, while the Jacobi equation is an initial value problem.

Second, for this type of functional, the Jacobi equation is a second-order Euler differential equation which can be readily solved by converting to a homogeneous constant-coefficient differential equation. Specifically, the Jacobi equation is

$$A_2 x^2 \frac{d^2u}{dx^2} + A_1 x \frac{du}{dx} + A_0 u = 0, \quad u(a) = 0, \qquad u'(a) = 1,$$

where $A_2 = \alpha_0,\ A_1 = m\alpha_0$, and $A_0 = -\beta_0$. The usual transformation $x = e^t$, leads to

$$A_2 \frac{d^2u}{dt^2} + (A_1 - A_2)\frac{du}{dt} + A_0 u = 0, \quad u(\log a) = 0, \qquad u'(\log a) = 1.$$

Since our principal aim is to provide practice in the use of the Jacobi equation to classify stationary paths, we look for choices for which the classification varies with the solution of this initial value problem. Thus we choose the auxiliary equation to reduce to the form $\lambda^2 - 2\rho\lambda + \rho^2 + \omega^2 = 0$, where ρ and ω are 'nice' constants with $\omega \geq 0$ and ρ and ω not both zero. The solution (in terms of x) is $u(x) = \left(\frac{a}{\omega}\right)\left(\frac{x}{a}\right)^{\rho} \sin(\omega \log(x/a))$ for $\omega > 0$ and $u(x) = a\ \left(\frac{x}{a}\right)^{\rho} \log(x/a)$ for $\omega = 0$.

At this point several things are apparent. First, to avoid singular behaviour, we must take $0 < a < b$. Second, although $\omega = 0$ is an important special case from a pedagogical perspective, its inclusion would complicate the implementation, so that it would be better to restrict to $\omega > 0$ in the first instance, leaving the case $\omega = 0$ for future development.[4] Third, the solution for $a \neq 1$ is significantly more complex so that a restriction to $a = 1$ would simplify computations for the students, at the risk of not providing practice in the more complex cases. Compromises such as these are frequent in e-assesment and the trick is to balance the pedagogy and the programming.

So, restricting to $a = 1$ and $\omega > 0$, gives $u(x) = \frac{1}{\omega} x^\rho \sin(\omega \log x)$ and there are zeros of $u(x)$ in $(1, b]$ if and only if $\omega \log b \geq \pi$. By varying the choice of ω and b, we are able to flip between the two cases: there are no zeros and the Jacobi test may be applied, and there is at least one zero in $(1, b]$ and the Jacobi test fails. In the first case, the stationary path is a minimum if $P(x) = 2\alpha_0 > 0$ and is a maximum if $P(x) = 2\alpha_0 < 0$. Note that for this functional P is of fixed sign (for $\alpha_0 \neq 0$) so the stationary path is either a local maximum or a local minimum.

It follows that we can randomise the problem by making the following choices:

a. ω, a random small positive integer, e.g., in the range 1..3
b. m, a random integer, e.g., in the range -1..5 excluding 0
c. α_0, a random integer, e.g., in the range -3..3 excluding 0
d. b, a random integer greater than 1, e.g., in the range 2..9

and setting $A_2 = \alpha_0$, $A_1 = m\,\alpha_0$, $A_0 = \alpha_0 \frac{(m-1)^2 + 4\,\omega^2}{4}$, $\rho = \frac{1-m}{2}$, $\beta_0 = -A_0$.

However, these randomisations may not lead to a desired distribution of the three cases: minimum, maximum or test failure (conjugate points), and so it was necessary to restrict b further depending on the other parameters, randomising to some extent the choice of outcome, to ensure that the successful cases occurred sufficiently often. Certainly, development is an iterative process.

Now, the Euler-Lagrange equation is

$$A_2 x^2 \frac{d^2 y}{dx^2} + A_1 x \frac{dy}{dx} + A_0 y = 0, \qquad y(a) = A, \qquad y(b) = B,$$

Choosing, for simplicity, $A = 0$ and then setting $B = k b^\rho \sin \omega \log b$, where k is a small random positive integer, gives a simple formula for the stationary path $y(x) = k\, x^\rho \sin(\omega \log x)$.

In our implementation for the students, the problem was split into five sequential questions with the same instance of the randomisation (with extensive feedback, as shown in Figure 2):

1. Calculate the Euler-Lagrange equation;
2. Transform the resulting Euler equation using the transformation $x = e^t$;
3. Solve the resulting constant-coefficient differential equation and substitute back to get the stationary path;
4. Calculate $P(x)$ and $Q(x)$ and hence the Jacobi equation;
5. Solve the Jacobi initial value problem to find $u(x)$, investigate the existence of conjugate points and hence, if possible, classify the stationary path.

A great strength of STACK is the ability to tailor feedback to student input. At this development stage, our use of this facility was confined to providing hints for incorrect tries and to providing feedback if an expression is not in the right variables, if a correct answer can be simplified, if the appropriate differential equation or boundary/initial value is not satisfied, and if the final classification of the stationary path is incorrect.

Your answer is correct.

The Jacobi equation is, in this case, the same as the Euler-Lagrange equation. So the general solution of the Jacobi equation in terms of $t = \ln x$ is

$$u(t) = \alpha e^t \sin(t) + \beta e^t \sin(t - \ln(7)).$$

In terms of x, this becomes

$$u(x) = \alpha x \sin(\ln(x)) + \beta \sin\left(\ln\left(\frac{x}{7}\right)\right) x.$$

The initial condition $u(1) = 0$ gives $\beta = 0$.

So $u(x) = \alpha x \sin(\ln(x))$ and $u'(x) = \alpha(\sin(\ln(x)) + \cos(\ln(x)))$.

The condition $u'(1) = 1$ then gives $\alpha = 1$. So the solution of the Jacobi equation is

$$u(x) = x \sin(\ln(x)).$$

From the theory, in order to be able to use the Jacobi equation method to determine the nature of the stationary path, we require that there should be no point $\tilde{a}$ conjugate to $a = 1$ in the interval $1 < \tilde{a} \le 7$. In other words, there should be no point $\tilde{a}$ in this interval for which $u(\tilde{a}) = 0$. If this holds, then the stationary path is a weak local minimum if $P(x) > 0$ everywhere on the interval $1 \le x \le 7$, and a weak local maximum if $P(x) < 0$ on the interval. In our case $P(x) = -\frac{4}{x}$ and so $P < 0$ on the interval.

Now $u(x) = 0$ if and only if $\ln(x) = \pi k$, or $x = e^{\pi k}$, for some integer k. Thus the smallest zero of u with $\tilde{a} > 1$ occurs at $\tilde{a} = e^{\pi}$.

Since the smallest value of $\tilde{a} > 1$ for which $u(\tilde{a}) = 0$ is at $\tilde{a} = e^{\pi} = 23.140\ldots > 7$, there is no point $\tilde{a}$ conjugate to 1 in the interval $1 < \tilde{a} \le 7$.

Since $P < 0$, it follows that the stationary path is a weak local maximum.

Figure 2. Feedback giving a worked solution for a correct answer to the final part of the Jacobi equation quiz. Although not shown here, *STACK* facilitates feedback tailored to a student's individual input.

5. Our experience of authoring questions at MSc level

In this section we outline our experiences of authoring in *STACK* at the MSc level in a series of vignettes.

1. **Two developers working together.** Having a fairly complete outline of each question before embarking on the development was helpful. One benefit is that two people worked through the details of the questions, one at the pen-and-paper specification stage and the other at the *STACK* implementation. Given the complexity of the material, this much reduced the probability of major errors in the question logic. As it turned out, most of the amendments made in the later stages were presentational – clarifying wording and so on.

   One thing to be aware of when tackling questions of this complexity is that multi-part questions, where students are led through the question in well-defined stages, bring added work to the development process to keep all the parts *in sync*. So estimation is key – when the question outline is 20 handwritten pages it is likely that the development effort required is substantial.

2. **Avoiding arithmetically and algebraically complex questions.** One aim was to develop 'clean' questions which were not overly algebraically and arithmetically complex. From the experience of setting exam questions over the years, there was a belief that it was important to start with clean answers and to work backwards to generate clean questions. In previous projects, attempts

have been made to keep the answers numerically reasonable, since students may be put off by the need to input answers involving very large numbers, or odd-looking fractions, square roots and so on. In this case the problem was compounded by the multi-part nature of most of the questions.

It turned out that on a couple of the questions the best strategy was to pick “nice” values for intermediate answers rather than the final answer. Working backwards then gave reasonable values for the question parameters, and then the final answers were not too awkward because the intermediate values had been chosen to be clean.

In designing the quizzes, we were conscious both of the need for tractable examples with clean arithmetic/algebra so that students focus on the higher-level skills and for realistic examples of sufficient complexity to give students practice in a range of situations. Certain of our choices leaned heavily towards arithmetic/algebraic simplicity (leaving more complicated cases for future development), but we were also mindful of the need for sufficiently many randomised parameters to span a large enough slice of the space of possible questions. Ultimately, it must be a matter of judgement. Perhaps, in retrospect, restricting to $a = 1$ was too limiting, although a wide variety of questions was nonetheless available to the students.

3. **Skill set needed to develop *STACK* questions.** When developing questions like these one is drawing on skills and knowledge from previous experience. Having some background in software development is helpful, but almost more important is the mathematical intuition. As a former student of this module, Grahame was able to think from the student’s perspective about the logical steps required to develop the solution.

   It is more or less essential when developing these types of question that you should understand the material in enough detail to be able to work through the solution. Just as important is the ability to think about what can go wrong. What might confuse a student and how can I word the question or solution to be as helpful as possible? What errors are likely and can these be detected to give more helpful feedback than a generic “incorrect answer, try again” response?

4. **The most challenging problem during implementation.** These questions were at a level of complexity beyond anything we’d done before, so the usual minor issues were magnified. Often the *Maxima* system underlying *STACK* will decide on how to order the terms in a computed expression, and you have limited control over that. So sometimes you’re fighting the system a little and need specific *LaTeX* for formatting. The nature of the subject is that complex expressions involving partial derivatives and subscripted variables are common, so the *LaTeX* formatting job is non-trivial.

   Probably the most awkward part was to come up with the input values which led to reasonable answers. In addition to picking good values for intermediate results, sometimes the only way to do that was the brute-force method of running all possible question values in a reasonable range through a model of the question logic built in *Maxima*, and selecting those parameters which result in sensible results.

5. **Advice to others contemplating writing e-assessment using the *STACK* system.** The benefits of *STACK* for this kind of mathematics, especially involving algebraic manipulation and calculus, are considerable. There is good material available for learning the basics of *STACK*, aimed primarily at developing questions in the early undergraduate range. Gaining experience of the system on questions at this level would be sensible before embarking on developing the more advanced questions in our project.

One thing that worked very well in our case, and which we recommend, is the detailed outline of the questions that we had on paper before embarking on the actual build on the system. If you try to develop the question on the fly, then things will go wrong, especially since many of our questions had four or five linked parts. Planning is key!

6. Conclusion and some general principles

The six online quizzes were completed in a six-week period in December 2017 – January 2018, each taking about 4 person-days from design to implementation, including checking. The M820 students were offered the six practice quizzes as a tool for learning and for examination revision and many used them for those purposes. Each quiz has been accessed over 220 times and by at least 85% of the 93 students taking the examination, but a detailed analysis of their effectiveness, and of their reception by the students, will be made once the module results are known.

The focus in this article has been on the design and implementation phase and on our experience at higher-level mathematics. Certainly, we have found the creation of these online quizzes to have been both enjoyable and intellectually stimulating, albeit with a hefty dose of frustration when the system (usually *Maxima*) did not work entirely as expected.

We end with some pointers to those who are looking to design their own e-assessment material, whether for formative or summative assessment. These are naturally pretty high level and most have already been embedded by mathematics and statistics educators as part of their normal day-to-day practice.

1. Start small and work up. It’s best not to start with the most general case, but to deal with a basic case and add complexity afterwards, possibly as separate questions.
2. Avoid complicated answers. Use small integers/rationals where possible and avoid complicated intermediate algebra and arithmetic, the bane of computer algebra systems generally.
3. Specify the solution and work backwards to problem, or, possibly, work backwards and forwards from an intermediate step. Moreover, whenever you have an equation to solve (linear, differential, algebraic) specify the solutions first and then derive the equation, not vice versa, and be prepared to re-think your randomisation choices during development if the solutions are too complex.
4. Check the students’ answers directly, don’t try pattern matching with a model solution. The variety of student answers far outstrips our own imagination!
5. Often the hardest part is formatting the solutions.
6. Select randomly from a set of good parameter choices, rather than randomise each parameter independently.

7. References

Butcher, P., 2008. Online assessment at the Open University using open source software: Moodle, OpenMark and more. In: Khandia, F. (ed.). *Proceedings of the 12th CAA International Computer Assisted Assessment Conference*, 8th and 9th July 2008. Lougborough University, UK. pp. 65-78. Available at: https://research.moodle.net/149/1/Butcher_P_final_formatted_n1_2.pdf [Accessed 12 June 2018].

Dewisprod.uwe.ac.uk., 2018. *The Dewis e-Assessment System*. Available at: http://dewisprod.uwe.ac.uk/ [Accessed 6 June 2018].

Gel'fand, I. M., Fomin, S. V. and Silverman R. A., 2000. *Calculus of variations*. Mineola, N.Y.: Dover Publications.

Heacademy.ac.uk., 2018. *The Open University - STACK | Higher Education Academy.* Collaborative award for teaching excellence. Available at: https://www.heacademy.ac.uk/person/open-university-stack [Accessed 6 June 2018].

Hunt, T., 2012. *Computer-marked assessment in Moodle: Past, Present and Future.* In: D. Whitelock et al. (eds). Proceedings of the 2012 CAA International Computer Assisted Assessment Conference, 10th – 11th July 2012, University of Southampton.

MacCluer, C. R., 2012. *Calculus of variations: mechanics, control and other applications.* Mineola, NY: Dover Publications.

Maplesoft.com., 2018. *Maple T.A. - Online Assessment System for STEM Courses - Maplesoft*. [online] Available at: https://www.maplesoft.com/products/Mapleta/ [Accessed 6 June 2018].

Mathcentre.ac.uk., 2018*. Maths E.G.* Available at: http://www.mathcentre.ac.uk:8081/mathseg/ [Accessed 6 June 2018].

Numbas.org.uk., 2018. *Really versatile maths e-assessment | Numbas.* Available at: https://www.numbas.org.uk [Accessed 6 June 2018].

Open.ac.uk., 2018. *OpenMark Examples*. Available at: http://www.open.ac.uk/openmarkexamples/ [Accessed 12 June 2018].

Sangwin, C., 2013. *Computer Aided Assessment of Mathematics.* Oxford: Oxford University Press.

Stack.ed.ac.uk., 2018. *STACK*. [online] Available at: http://www.stack.ed.ac.uk [Accessed 6 June 2018].

[1] *STACK* services over a million e-assessment questions annually at the OU.
[2] The ability to serve multipart problems with consistent randomisations between the parts is a powerful feature of the integration of *STACK* within the *Moodle* Quiz.
[3] The Module Team Chair is the person in overall charge of the module and who leads the team of colleagues (which includes the OU's famed Tutors) who deliver the module to the students.
[4] Alternatives and special cases are often best developed as separate *STACK* questions, leaving the random selection of question type to *Moodle*.

CASE STUDY

Attendance at university lectures: A study into factors which influence it, and exam performance as a consequence of it.

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Abstract

This article investigates whether various factors affect attendance rates at university lectures, and whether there is any relationship between attendance and exam performance. Data were collected over 7 years from a 3rd Year statistics module. It was found that the time of day that the lecture occurred made no difference to attendance, and that there was no difference in attendance between the genders. It was discovered that an increase in tuition fees led to a rise in attendance. Additionally, it was observed that higher attendance rates led to better exam performance.

Keywords: Attendance, Gender, Timetable, Tuition Fees, Exam Performance

1. Introduction

Attendance records for a 3rd year undergraduate statistical software module at Coventry University (CU) were collected from 2011-2018. Each year, the module ran over 10 weeks, with a single two-hour lecture per week. There were 2 or 3 cohorts per year and approximately 30 students in each cohort, resulting in a total of 461 students included in this study.

In Section 2 of this article, it is investigated whether there was any significant difference in attendance between the males and the females. Section 3 explores whether the time of day of the class had any effect on attendance rates. Section 4 considers whether the increase in tuition fees resulted in an increase of attendance. Section 5 looks for any relationship between attendance and exam performance.

2. Gender and Attendance

Some previous studies have suggested that females tend to have better attendance at lectures than males. Woodfield, Jessop and McMillan (2006), in a study of 650 students at Sussex University, found that the mean female attendance rate of 88% was significantly higher than the male attendance rate of 84%. Kelly (2012), in a survey at University College Dublin found that, for those who live on campus, there is significant evidence ($p = 0.004$) that females have a higher probability than males of attending a lecture, although this difference was not found when all accommodation locations were included.

The CU module investigated in this article was attended by 245 males and 216 females in total over the seven years. The summary statistics for attendance according to gender are shown in Table 1:

Table 1. Attendance rates for males and females

		Attendance (%)		
Gender	No. of Students	Mean	St. Dev.	St. Error
Male	245	78.29	22.11	1.41
Female	216	80.97	18.61	1.27

On average the females attended slightly more often than the males (81% mean attendance rate compared with 78%). However, an independent t-test showed no significant evidence ($p = 0.157$) of a difference in attendance between the genders, as shown in Table 2:

Table 2. T test for difference in male and female attendance rates

t	df	p	Mean difference	95% CI
-1.416	458	0.157	-2.69	(-6.41, 1.04)

3. Time of Day and Attendance

There is a stereotypical image of a student that they are often up late partying. This picture may have been perpetuated by fictional television comedies such as The Young Ones (1982) and Fresh Meat (2011). Nevertheless, based on the author's own personal experiences when a student many years ago, it is not too inaccurate a representation. This perception could lead to the conclusion that students are less likely to attend early morning lectures than later ones. Also, Evans, Kelley and Kelley (2017) considered that, due to their circadian rhythms at that age, people in their late teens naturally feel more sleepy in the mornings. They consequently recommended that university lectures shouldn't occur before 11am.

For the module in this article, it was decided to investigate whether there was any difference in attendance between a morning class and an afternoon class. Four of the cohorts had their lectures starting at 9am, and 10 of the cohorts had their lectures starting at 4pm. The summary statistics for attendance according to start time are in Table 3:

Table 3. Attendance rates for 9am class and 4pm class

		Attendance (%)		
Class Time	No. of Students	Mean	St. Dev.	St. Error
9 a.m.	119	81.60	20.00	1.83
4 p.m.	342	78.83	20.74	1.12

The morning lectures were slightly better attended than the afternoon lectures (students on average attended 82% of the a.m. lectures compared with 79% for the latter). However an independent t-test showed no significant evidence ($p = 0.207$, two-tailed) of a difference in attendance between the two lecture times, as shown in Table 4:

Table 4. T test for difference in morning and afternoon attendance rates

t	df	p	Mean difference	95% CI
1.264	459	0.207	2.77	(-1.53, 7.07)

This contradicts the image put forward earlier which may well be outdated. Attitudes of students could be different in more recent times. Leeds Beckett University (2017, cited in the Yorkshire Evening Post) conducted a survey of 1,070 sixth-formers and found that only 9% said that the thing they are most looking forward to at university is the nightlife, whereas 29% replied that studying a subject they are passionate about was what they most anticipated.

4. Cost of Fees and Attendance

The suggestion in Section 3, that students are more likely to attend morning lectures than they did in the past, could be expanded to investigate whether they attend lectures in general more often nowadays.

Up until 1997, undergraduate students at English universities didn't pay any tuition fees. In 1998 a £1,000 per year fee was introduced. In 2006 this was increased to £3,000, and in 2012 to £9,000 (Hubble and Bolton 2018). It could be postulated that, when paying more to learn, a student will 'want to get their money's worth', and attend more classes.

For the module discussed in this article, the pre 2012 fee-increase attendance rate and the post fee-increase attendance rate were compared. This was a 3rd year module so the higher fee applied to those students from 2014 onwards in this dataset. Seven of the cohorts payed the new, higher fees of £9,000, and 7 of the cohorts payed the previous, lower fees of £3,000. The summary statistics for attendance according to fees paid are shown in Table 5.

Table 5. Attendance rates pre and post the 2012 tuition fee increase

		Attendance (%)		
Fees	No. of Students	Mean	St. Dev.	St. Error
£3,000	245	76.78	20.74	1.33
£9,000	216	82.69	19.96	1.36

The attendance rate (83%) was higher for those who paid £9000 tuition fees than for those who paid £3,000 (77% attendance rate). An independent t-test showed strong evidence ($p = 0.002$, two-tailed) of a difference in attendance, as shown in in Table 6. This infers that students have attended more after the 2012 fee increase was introduced.

Table 6. T test for change in attendance rates after the 2012 tuition fee increase

t	df	p	Mean difference	95% CI
3.107	459	0.002	5.91	(2.17, 9.65)

There seems to be a scarcity of previous published research on any changes in attendance patterns post the 2012 fee increase. Neves and Hillman (2017), in the 2017 Student Academic Experience Survey, which received responses from 14,057 undergraduate students, reported that student attendance has remained reasonably constant since 2013, but figures from before that date weren't included. Nevertheless, they did find a significant positive association between student satisfaction with the number of their timetabled sessions and whether they thought their course gave 'value for money'. Additionally, Neves and Hillman found that 71% of students who were scheduled for 10-19 contact hours / week said they were satisfied with the number of hours, whereas only 55% of those who were scheduled for 0-9 hours / week expressed satisfaction. These both highlight the current importance that some students place on having a sufficient number of lectures.

5. Attendance and Exam Performance

Of course, some students may need persuading on the worth of attending classes. They may argue that, as nowadays the vast majority of lecture materials and subject resources are put online by university lecturers, there is little need for them to attend lectures, as they can learn everything at home. Hence, they may contend that whether they attend or not, they would still get the same score in their assessment. In this section, it will be investigated whether there is any association between attendance and exam performance.

For this module, the author did put all of the lecture handouts online, usually a couple of days after each lecture. The assessment for the module took place a week after the final lecture, and consisted of a 90 minute exam. A scatter plot of exam mark against attendance % for all 461 students is shown in Figure 1.

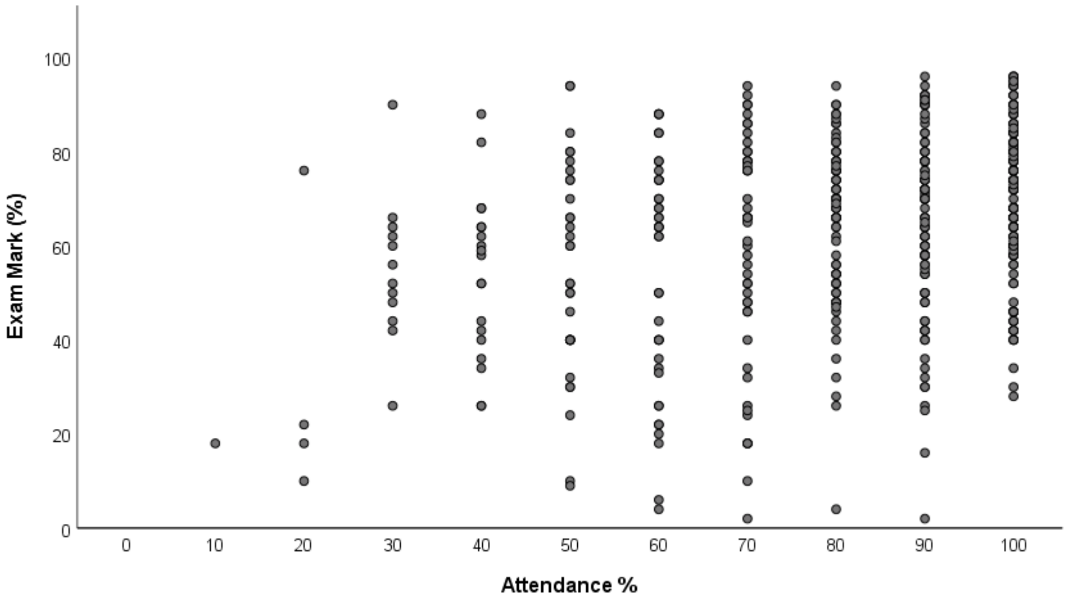

Figure 1. Scatterplot of exam mark against attendance %

The Pearson Correlation Coefficient was 0.316 with a p value < 0.001, which suggests very strong evidence of a positive relationship between attendance and exam mark. In other words, the higher the attendance, the higher the exam mark. This could be very useful at the beginning of future instances of this module as a tool to motivate students to attend well. The lecturer could possibly display the graph with the caption- 'the more lessons you attend, the higher your exam mark will be'.

The regression equation obtained was

$$\text{Exam Mark} = 38.4 + 0.311\ \text{Attendance}\,.$$

As there are 10 lessons in the module, missing one lesson would equate to missing 10% of lessons. Hence the above equation could be used as a warning to the students using the loose interpretation, 'Every time you miss a lesson, your exam mark will drop by 3% points'.

However, due to the variability of the points displayed in Figure 1, with many students with poor attendance actually doing well in their exam, the visual effect on the students may not be quite so convincing. A more effective chart to convey the intended message that it is better to attend, is a plot of probability of passing against attendance.

Hence a logistic regression analysis was carried out to predict the probability of passing the exam (i.e. scoring at least 40%) based on lesson attendance. The results are shown in Table 7.

Table 7. Logistic regression model for predicting probability of passing from attendance rate

	Estimate	St. Error	Wald	df	p	Odds Ratio	95% CI for Odds Ratio
Attendance	0.0345	0.007	27.406	1	<0.001	1.035	(1.022, 1.049)
Constant	-0.501	0.473	1.123	1	0.289	0.606	

The p value < 0.001 (based on a Wald Chi-Square test) tells us that attendance is a significant predictor for passing the exam. The model obtained was:

$$P(Pass) = \frac{1}{1 + e^{0.501 - 0.0345\ Attendance}}.$$

E.g. If Attendance = 50%, then $P(Pass) = \frac{1}{1+e^{0.501-0.0345\times 50}} = 0.768.$

Based on these figures, the students could be told, ‘for those people who miss half the lessons, there is approximately a 1 in 4 chance of failing the exam’. This announcement, along with displaying a scatter chart of attendance rates and their predicted probabilities (Figure 2) could form the basis of a good ‘scare tactic’ at the beginning of the course to encourage the students to attend.

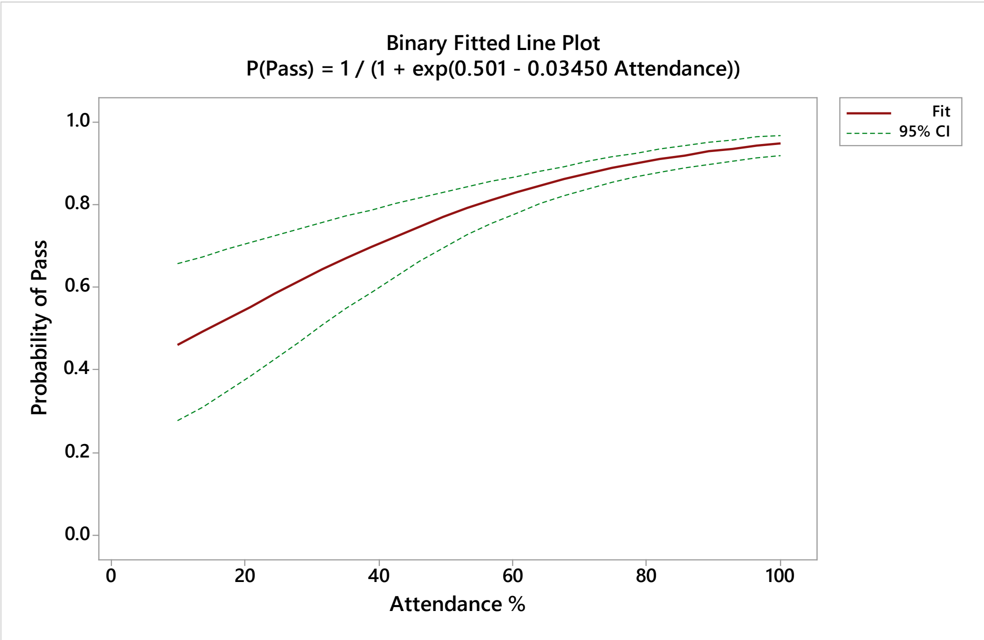

Figure 2. Probability of passing the exam according to percentage of lectures attended

From Table 7, it can be seen that the odds ratio is 1.035. This tells us that an increase of 1% in attendance increases the ‘odds’ of passing by a factor of 1.035. (Note that we use the definition of the ‘odds’ of passing as $P(Pass)/P(Fail)$). If we consider number of lessons attended rather than the percentage of lessons attended, the odds ratio is $1.035^{10} = 1.41$ (because there are 10 lessons). Hence the students could be further ‘warned’ with the loose interpretation ‘every time you attend a lesson, your odds of passing increases by around 40%.’

Similarly, a model to predict the probability of achieving a 1st Class mark (i.e. ≥ 70%) in the module was obtained using logistic regression. This is shown in Table 8, with the accompanying plot in Figure 3.

Table 8. Logistic regression model for predicting probability of achieving a 1st from attendance rate

	Estimate	St. Error	Wald	df	p	Odds Ratio	95% CI for Odds Ratio
Attendance	0.0259	0.0052	25.178	1	<0.001	1.026	(1.016, 1.037)
Constant	-2.323	0.433	28.846	1	<0.001	0.098	

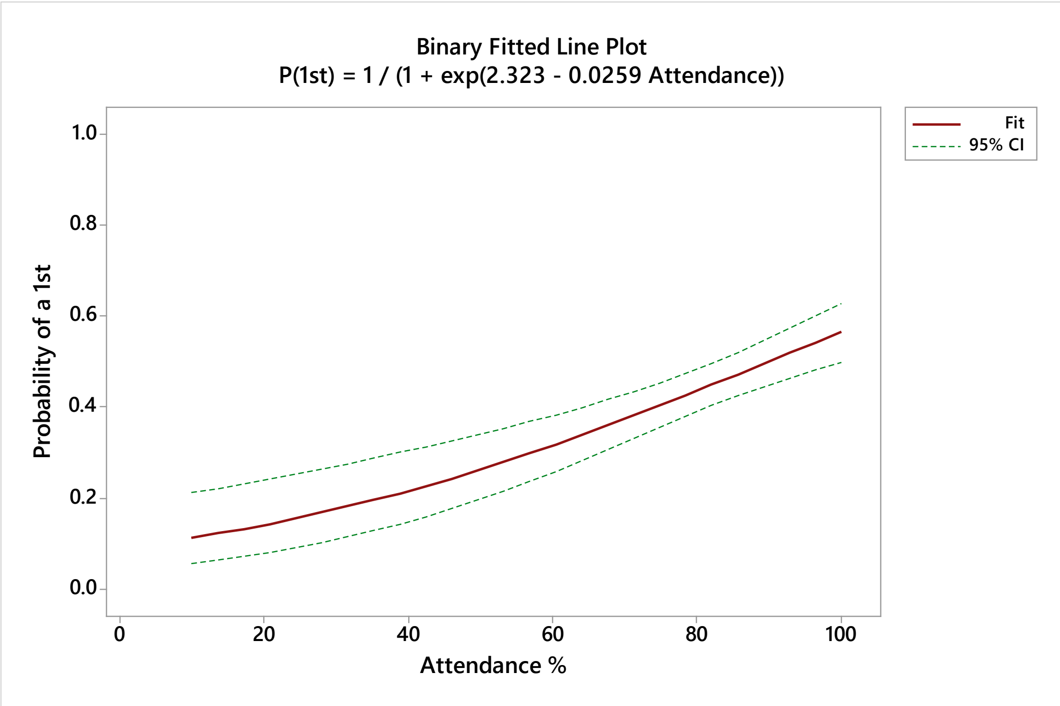

Figure 3. Probability of obtaining a 1st in the module according to percentage of lectures attended

From Table 8, p < 0.001 (based on a Wald Chi-Square test) so attendance is a significant predictor for achieving a 1st. The model obtained was:

$$P(1st) = \frac{1}{1 + e^{2.323 - 0.0259Attendance}}.$$

Considering 'number of lessons', the odds ratio is $1.026^{10} = 1.29$. Thus, this could be presented as 'every time you attend a lesson, your odds of achieving a 1st increase by around 30%'

The finding in this study, that increased attendance is related to higher attainment, is consistent with numerous other studies. Colby (2005), who looked at a 1st year module in the University of Central England, found that students who attended fewer than 70% of the lessons had a 2 in 3 chance of failing that module. Halpern (2007) investigated a 2nd Year module at London Metropolitan University and found significant evidence of a positive relationship ($r = 0.502, p < 0.001$) between attendance and coursework grade. Allen and Webber (2010) found that, for a module at the University of the West of England, every seminar the student attended increased their exam mark by around 4 % points on average.

6. Conclusions

In this study of attendance records from a 3rd year statistical software module, it was found that:

- There is no significant difference between males' and females' attendance rates;
- There is no significant difference between attendance rates for a 9am class and a 4pm class;
- After the 2012 tuition fees increase, attendance rates increased significantly;
- There is strong evidence that higher attendance is likely to lead to better exam performance.

Note that this is only a preliminary study, and further investigation would be beneficial. Interaction effects between the above factors could be explored, plus other factors such as prior attainment could be considered. Furthermore, data from classes from other subject areas would widen the scope of the conclusions.

Nevertheless, these findings give a useful insight into some aspects of attendance, in particular for this module. Furthermore, as discussed in Section 5, they can be used as a predictor for future cohorts, so can be used as a motivational tool to promote good attendance to incoming students.

7. References

Allen, D., and Webber, D., 2010. Attendance and exam performance at university: a case study. *Research in Post-Compulsory Education*, 15(1), pp. 33-47. https://doi.org/10.1080/13596740903565319

Colby, J., 2005. Attendance and Attainment - a comparative study. *Innovation in Teaching and Learning in Information and Computer Sciences*, 4(2), pp. 1-13. https://doi.org/10.11120/ital.2005.04020002

Evans, M., Kelley, P. and Kelley, J., 2017. Identifying the Best Times for Cognitive Functioning Using New Methods: Matching University Times to Undergraduate Chronotypes. *Frontiers in Human Neuroscience*. https://doi.org/10.3389/fnhum.2017.00188

Fresh Meat, 2011 [TV]. Channel 4. 21 September 2011.

Halpern, N., 2007. The impact of attendance and student characteristics on academic achievement: findings from an undergraduate business management module. *Journal of Further and Higher Education* 31(4), pp. 335–349. https://doi.org/10.1080/03098770701626017

Hubble, S. and Bolton, P., 2018. *Higher education tuition fees in England*, House of Commons Library Briefing Paper 8151. http://researchbriefings.parliament.uk/ResearchBriefing/Summary/CBP-8151 [Accessed 17 June 2018]

Kelly G., 2012. Lecture attendance rates at university and related factors. *Journal of Further and Higher Education*, 36(1), pp. 17-40. https://doi.org/10.1080/0309877X.2011.596196

Neves, J. and Hillman, N., 2017. 2017 Student Academic Experience Survey. Available at https://www.heacademy.ac.uk/institutions/surveys/SAES [Accessed 16 June 2018]

The Young Ones, 1982 [TV]. BBC2. 9 November 1982.

Woodfield, R., Jessop, D. and McMillan, L., 2006. Gender differences in undergraduate attendance rates. *Studies in Higher Education*, 31(1), pp. 1-22. https://doi.org/10.1080/03075070500340127

Yorkshire Evening Post, 2017. *Leeds Beckett study: More-sixth formers excited about university studying than nightlife*. https://www.yorkshireeveningpost.co.uk/news/leeds-beckett-study-more-sixth-formers-excited-about-university-studying-than-nightlife-1-8678015 [Accessed 17 June 2018]