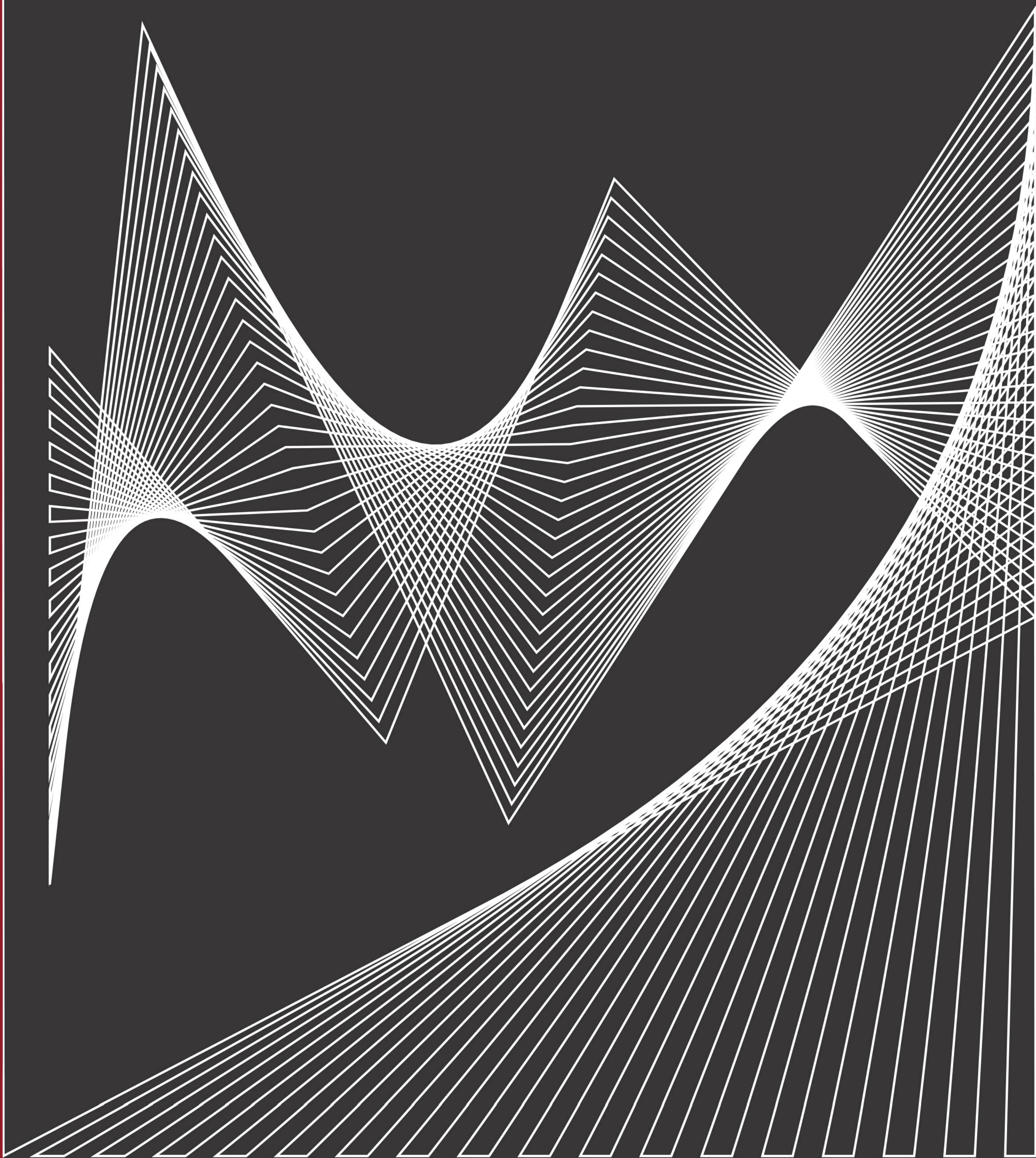
MSOR connections

Articles, case studies and opinion pieces relating to innovative learning, teaching, assessment and support in Mathematics, Statistics and Operational Research in HE.

Volume 17 **No.** 2

Special Issue: CETL-MSOR Conference



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This journal is published with the support of the **sigma**-network and the Greenwich Maths Centre.

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EDITORIAL

Editorial

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This special issue of MSOR Connections represents a collection of papers presented at the CETL-MSOR conference in Glasgow University, September 2018. The papers reflect the numerous ideas and stimulating discussions that took place at what was an excellent event, and I am very pleased that this journal is able to support the ongoing dissemination across the mathematics higher education community.

It is therefore fitting that the first paper in this issue originates from Glasgow University, where Ahmed et al. highlight how online and blended approaches to learning have been piloted to support the mathematical development of science based students making the transition to university study.

In the subsequent paper, Lishchynska et al. provide background and initial findings of the Transposition Project. An intervention developed to help better understand the difficulties faced by students when rearranging mathematical equations.

The next two papers present case studies on often under-reported aspects of mathematical support. Firstly, Richard considers provision specifically developed for postgraduate taught students. Then, Ahmed and Douglas discuss the provision of support for students undertaking numeracy tests as part of the employment process.

The following research article by Macdonald analyses five years of data from Glasgow Caledonian University in an attempt to quantify the impact of engagement with the mathematical support available to students.

Increased focus in subsequent papers is given to the learning support and development of mathematics undergraduates. Hilliam and Arrowsmith discuss the creation of a subject specific website that brings together existing resources and supports the development of a community of learners; McConnell provides details of a problem solving module piloted at Cardiff University, along with some initial impact on its inclusion in the curriculum; Jones and Megeney discuss an approach to teaching problem solving – a skill that is fundamental throughout all mathematics undergraduate programmes; Shukie et al. present a case study on the use of whiteboards – both large and small – as part of the overall approach to learning and teaching mathematics at Sheffield Hallam University; and Russell discusses the implementation of grade-based marking criteria for assessments as part of a final year undergraduate mathematics module.

The use of project based learning in Statistics, and its assessment is considered by Marshall in the following paper. An overview of this active approach to learning is presented along with some reflections on its effect on student engagement and understanding.

In the final paper of this issue, Marshall and Owen provide an overview of discussions from the sigma Network's Statistics Support special interest group. Points highlighted include statistical techniques, software, and evidencing the impact of statistics specific support.

It is hoped that MSOR Connections can continue to support this annual conference – and I am personally very much looking forward to the next CETL-MSOR conference taking place in Dublin City University, September 2019. For further details visit http://www.sigma-network.ac.uk/cetl-msor-conference-2019.

Mathematics support for science: a reflection of a blended and online development project

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Abstract

The diverse range of backgrounds that students bring to university has many advantages, but also some challenges such as a wide range of mathematical experience and ability. A particular issue identified by staff teaching mathematics to the Science Fundamentals cohort was lack of engagement due to the material presented being too easy or too difficult, with the main concerns directed towards students who are weak at mathematics and others who lack some of the basic skills necessary for a successful undergraduate experience at Glasgow.

Our experience from the Science Fundamentals course is that traditional lectures are a poor way to motivate the weaker students and the more collaborative models of teaching such as online and blended learning may be more appropriate. Following a two-year project, a suite of online resources was developed to supplement course material; students completed a 'Mathematics Confidence Test' to determine the level of support required and the number of mathematics lectures was cut down and replaced with tutorials aimed at the weaker students.

We will discuss our experiences of running this project, reflect on feedback and discuss further plans for supporting the Science Fundamentals cohort in subsequent years.

Keywords: Mathematics, chemistry, online learning, Moodle, QTI.

1. Background

Confidence and fluency in mathematics are necessary qualities for graduates across the whole range of science subjects. Mathematics is crucial to explore demanding new scientific concepts, to quantify observations accurately and for presenting results in science.

The University of Glasgow prides itself on accepting students from a diverse range of backgrounds into its Science programmes. Of particular concern are students who are weak at mathematics and others who lack some of the basic skills necessary for a successful undergraduate experience at Glasgow.

With the University operating on a college-entry system, undergraduates pursue three subjects in their first year of study. Currently, either Chemistry 1 or Science Fundamentals make up one of the compulsory course choices for Life Sciences students. Science Fundamentals is a two-semester course (administered by the School of Chemistry) consisting of Chemistry, Physics, Mathematics and Statistics. Students in this course bring a very wide range of mathematical experience with them: some may be studying level 1 Mathematics as one of their three subjects whereas others may have studied a minimal amount of mathematics some time ago. A particular issue identified by staff teaching mathematics to this cohort was lack of engagement due to the material presented being too easy or too difficult. Our experience has shown that traditional lectures are a poor way to enhance the mathematical skills of these students and the more collaborative models (Swan, 2014) of teaching such as online and blended learning may be more appropriate.

Following a successful application to the University of Glasgow's Blended Learning and Online Development (BOLD) funding scheme, this two-year project aimed to combine the School of Chemistry's ongoing plans for mathematical support provision to early years, whilst at the same time, addressing the ongoing challenges encountered with the teaching of mathematics to the Science Fundamentals cohort. The development of a suite of support resources was planned to be largely delivered online to support the learning and revision of suitable concepts. In addition to these resources, students requiring additional support were to be identified and provided with further focussed support through tutorials (O'Brien and Bedford, 2012).

2. Year one of the project

An initial consultation with Chemistry and Mathematics staff involved in the teaching of the course was undertaken, and a complete overhaul of the mathematics component of the Science Fundamentals course was planned. The Mathematics Support Adviser had an insight into common difficulties experienced by students, and these were considered when planning the course content.

A research assistant (RA) was recruited to create a new online course on Moodle (https://moodle.org). The existing course content was reviewed and much of it was reused but presented in a more user-friendly format. The Lesson function in Moodle was utilised to break the material up into manageable chunks. In addition, external links to other resources, mainly MathCentre (http://www.mathcentre.ac.uk), were included.

Advice regarding technical and design elements was sought from learning technologists, and it was recommended that we purchase a Wacom Cintiq graphics tablet to create Khan Academy-style video tutorials to supplement the Moodle lessons. In-house software, DTTPresent (Barr, 2017) and *Camtasia* were used to create the video tutorials.

Mathematics quizzes were created by the RA which provided the students with randomised questions, allowing them to practise and improve their skills. Technical information about the e-learning system can be found in the Appendix and links are available in the References below.

The course was presented such that students could either start from the beginning and work their way through all the material or dip into the topics they wished to work on. Each section consists of some exposition material, worked examples, video tutorials and randomised questions that could be used for practise as often as needed. Some feedback was provided by students regarding the content and layout of the course, which was used to further improve the Moodle resource.

3. Year two of the project

The new course was rolled out in September 2017. At the first lecture, the class was given a Maths Confidence Test and the scores were used to identify the level of support required by the students. Previously, two mathematics lectures a week had been timetabled for the six-week long Mathematics block for the Science Fundamentals class. As part of the new course, one lecture per week was replaced with an optional tutorial session. Based on the results of the mathematics confidence test, students were invited to either attend all tutorials or drop in to the sessions as required. For those students with a strong mathematical background, no attendance was required. The whole cohort was referred to the Moodle course for reference and revision as needed. In addition, mathematics support through LEADS was promoted and on offer throughout the year.

During the academic year, the resource was monitored regularly and iteratively enhanced as necessary. Any feedback received from staff or students was considered and used for making improvements to the resource.

4. Feedback

The optional tutorials were very well-attended all year, and feedback was collected at the end of both semesters.

4.1. Semester one feedback

In semester one, questionnaires were given *only to students attending the final tutorial* session. Good feedback was received overall. A selection of responses is listed below:

- I have found maths tutorials useful: 44% strongly agreed, 47% agreed, 9% neutral;
- I would prefer more maths lectures instead of tutorials: 0% strongly agreed; 3% agreed, 18% neutral, 49% disagreed, 30% strongly disagreed;
- I feel confident tackling the maths course content: 18% strongly agreed; 42% agreed, 30% neutral, 6% disagreed, 4% strongly disagreed.

Comments include:

- *"I found tutorials extremely helpful as they gave you a chance to work through material and ask for help one to one when you needed it"*
- *"Loved the tutorials great idea to do more practice and ask questions the confidence test is a great idea please have it for 2nd semester"*

4.2 Semester two feedback

In semester two, the *whole class was surveyed*, not just those who attended the tutorials, and the following feedback was received:

- I have found maths tutorials useful: 26% strongly agreed, 32% agreed, 34% neutral;
- I would prefer more maths lectures instead of tutorials: 7% strongly agreed; 13% agreed, 27% neutral, 35% disagreed, 18% strongly disagreed;
- I feel confident tackling the maths course content: 22% strongly agreed; 41% agreed, 23% neutral, 11% disagreed, 3% strongly disagreed.

Comments received included:

- *"Well rounded maths course that ticks all the boxes for this level of science"*
- *"I like the structure, I'm confident with the material so I liked being able to not go to tutorials"*

We were able to interview a student who was repeating the year, which meant she had experienced both versions of the course. She said *"It [the course] was a hundred times better. There was so much more support this year. The notes and examples were better, and the lecturer was very good. Last year two lectures in a week were too much sometimes. If I didn't understand material from the first lecture, I felt overwhelmed at the second one. Having the tutorial was good. It meant I could talk to other students, work through examples and get help if I needed it. The Moodle course was very useful. I liked being able to dip in and out. The questions were good because I could keep trying out examples as many times as I needed to. This felt like the course was maths for science rather than maths for the sake of it".*

4.3 Preliminary reflections from teaching staff

The number of lectures was halved, and the continuous assessment was more rigorous, but the results indicated no drop in performance. In fact, Physics scores have showed some improvement.

Staff were pleased with the level of attendance at the optional sessions: 40% attendance in semester 1 and 32% in semester 2. It was generally felt that the modified course had been successful. The students appreciated having a tutorial session where they could consolidate their knowledge and skills, and the more confident students were happy to attend just the lecture. The scores from the Mathematics Confidence Test were used more extensively to target support during the second year of running the course.

5. Appendix

The software used to deliver the QTI questions and tests is QTIWorks (Milne et al, 2013), an open-source system developed at the University of Edinburgh. QTIWorks supports IMS QTI v2.1 questions and tests, as well as some extensions to QTI specifically for Mathematics. The Maxima Computer Algebra System (http://maxima.sourceforge.net) is used to provide support for evaluation of algebraic expressions.

To provide a near seamless link between the Moodle Virtual Learning Environment (VLE) and QTIWorks they are linked using IMS LTI, a simple single sign-on protocol designed for educational tools, which also supports the return of marks to the Moodle gradebook. As LTI is a widely supported standard the same technique can be used to link to QTIWorks from other popular VLEs such as Blackboard.

Documentation and source code can be found at https://github.com/davemckain/qtiworks.

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CASE STUDY

The Transposition Project: origins, context and early findings

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Abstract

Within the Department of Mathematics at Cork Institute of Technology and at a wider level across the institute, the topic of transposition or rearranging equations has repeatedly been flagged as one of the most problematic. The transposition project aims to understand the reasons for students' difficulties and develop an effective intervention strategy.

Keywords: Transposition, algebra, rearranging equations, changing the subject.

1. Introduction

Transposition of formulae (also known as rearranging equations) is a skill vital in many industrial fields. It is hard to imagine a domain of science or engineering where a professional would not need to transpose a formula at some stage. The importance of the topic of transposition is reflected in Mathematics curricula at both second and third level throughout the world. In Ireland, the topic is covered at both the Junior and Leaving Certificate Examination level (the university matriculation examination) and yet it is apparent to lecturers at Cork Institute of Technology (CIT) that a large proportion of students that they encounter lack this key skill. CIT is a higher education college in the Republic of Ireland. The entry requirements at Institutes of Technology are often lower than those at the Universities and students attending CIT sometimes find the Mathematics content of their programmes challenging. The Mathematics Department at CIT is mainly a service department teaching on most programmes across the Faculties of Engineering and Science, and Business and Humanities. Approximately 75% of students within these Faculties take a mathematics module many of which have transposition of formulae on the syllabus. Remarkably, the mathematical deficiency seems to persist after the topic has been taught twice, in school and again at undergraduate level.

The difficulties surrounding the topic of transposition are frequently observed and discussed by lecturers from the Department of Mathematics, anecdotally in the staff room and more formally at staff meetings, seminars and programmatic reviews. Additionally, staff involved in the Mathematics Support Centre noted that many of the difficulties that students encounter in a wide variety of mathematical problems arise from a lack of proficiency in rearranging equations. A desire to understand and address this problem brought together twelve members of staff from the Department of Mathematics and over a series of meetings. As a result, the Transposition Project was formed. The main objectives of the project are summarised in Figure 1. This paper focuses on objectives 1 – 4.

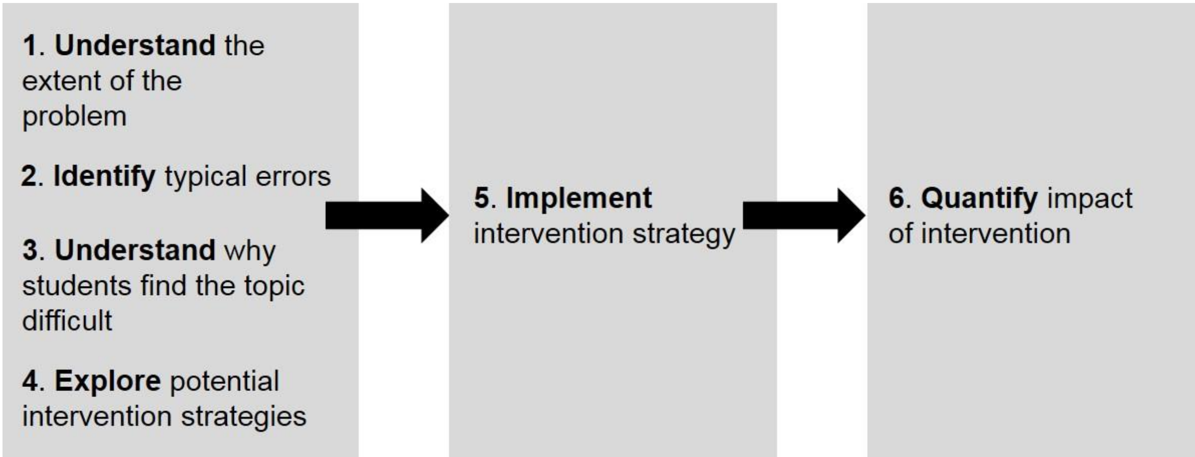

Figure 1: Summary of project objectives.

2. The extent of the problem

To understand the extent to which deficiency in the topic of transposition affects first year students at CIT, an online diagnostic test was given to over 350 students across the faculties of Science and Engineering and Business and Humanities. The test was designed to examine various algebra skills necessary in transposition as well as proficiency with transposition itself. A benchmark equation used to determine whether a student is proficient in the topic was as follows:

$$T = \frac{2v}{g} + 5$$

where the students were asked to isolate *g*. Alarmingly, over 75% of respondents were not able to obtain the correct answer.

In order to further assess the depth of the problem within the institute, two workshops for lecturing staff were held; one for Mathematics lectures and one for lecturers from applied disciplines (not Mathematics). Staff discussed the importance, context, teaching approach and understanding of the topic of transposition as well as potential interventions to improve proficiency. At the workshop for staff from applied disciplines, the discussion centred on the questions: '*Do students struggle with rearranging equations in applied courses*?' and '*How do these problems affect teaching applied topics*?' All participants agreed that students did struggle with the topic of transposition and this affected all the applied modules taught. Surprisingly, it was reported by workshop participants that in some modules the topic of transposition was so important to the understanding of applied concepts that it was taught (and assessed) again within the module, in the context of the subject material. Having to teach the topic again within a module reduces the amount of time available to cover the main (applied) material. Staff also reported that the assessment of key concepts related to their disciplines is confounded with the topic of transposition making it unclear whether a student is struggling due to lack of understanding of the concepts at hand or struggling to rearrange an equation.

CIT students are not unique in their aptitude for transposition of formulae. Our observations and subsequent interest in this echo efforts of mathematics teachers and educational researchers around the world. Multiple published works examining students' difficulties in algebra find that student' mathematical deficiencies in general and those pertinent to algebra are often multifaceted where, for instance, difficulties with basic algebra perpetuate into harder tasks like transposition. Most papers studying mathematical deficiencies invariably touch upon the topic of solving equations or transposition of formulae though various authors focus their work on diverse aspects of the problem such as poor algebra skills, reasons behind it, developing diagnostic tools, and intervention techniques. O'Brien and Ní Ríordáin (2017) and Lucariello, Tine, and Ganley (2014) developed tests for diagnosing students' difficulties in algebra. Several publications investigate students' misconceptions in algebra and agree on the fact that the misconceptions form barriers to further learning and need to be addressed and dispelled in class (Lucariello, Tine, & Ganley, 2014; Bush & Karp, 2013; Barbieri & Booth, 2016). The research suggests that, while necessary, focusing an intervention solely on cognition or motivation may not lead to improvement in algebra skills whereas a combined intervention approach that includes error reflection may be beneficial (Barbieri & Booth, 2016).

3. Typical errors

To understand the difficulties associated with the topic of transposition, an error analysis was performed on students' work. The deficiencies identified through the error analysis included: a lack of understanding of the equality sign and equivalent equations, incorrect use of inverse operations, mistakes expanding or simplifying equations, and inability to start a question. The types of errors identified by the error analysis are shown in Table 1, along with examples of students' work.

While all presented classes of deficiencies are major contributors to students' struggles with re-arranging equations, it is the lack of conceptual understanding that plays a major role here as without the deep understanding of what equations are, how they are formed and what the equality sign means other algebra skills become unproductive in the context of transposition. All non-conceptual deficiencies observed are algebraic in nature and constitute the basics necessary, but not sufficient, for manipulating equations and formulae.

Bush and Karp (2013), Stephens, et al. (2013) and Byrd, McNeil, Chesney, and Matthews (2015) concur the fact that lack of understanding of the equality sign and equivalent equations (i.e. concepts) is at the core of students' problems with solving equations. It is also worth noting that O'Connor and Norton (2016) while examining mathematical difficulties with quadratic equations analyse student error patterns and relationships between them. The findings suggest a strong connection between lack of procedural skills and misconceptions, and lack of conceptual understanding. Just as the students in the study by O'Connor and Norton did not have the tools to factorise quadratics, many of our students did not have the pre-requisite algebra skills to apply to transposition problems. Indeed, conceptual errors and procedural errors are often intertwined, Rittle-Johnson, Siegler, and Alibali, (2001) put forward the idea of an iterative model of the development of conceptual and procedural knowledge which we think may be especially relevant to the process of learning to rearrange equations. This means that while the emphasis on teaching for conceptual understanding is of major importance, procedural deficiencies must be addressed concurrently. These ideas feed into the intervention strategy currently under development.

4. Why is transposition difficult?

At the workshop for staff from the Department of Mathematics and over a series of project meetings, lecturers discussed the following question: *Why is the topic so difficult for students?* We also looked at students that are proficient in transposition.

A number of reasons were identified which are summarised in Figure 2. Crucially on the technical side, transposition of formulae is a demanding task as it is a culmination of many algebra skills and concepts coming into play at once and lacking one component of this 'portfolio' often results in a failure. On the non-technical side, there are several factors contributing to the problem. Staff observed that many students like to have a procedure involving a defined set of steps to follow also noted by Marjoram, et al. (2012); this is not the case for transposition. Though there are some 'rules', they can be applied in multiple ways generating different paths to the correct solution. A question on transposition often involves many steps, students do not like long problems and perceive them as “hard”! There is also the fact that the students have been exposed to the topic before therefore some already know it and are bored in class distracting others whereas some think they know it (when they do not) and are not paying attention. This does not create an optimum learning environment.

Table 1: Error classification

Common errors	Class of maths deficiency	Some examples
Lack of understanding of the equality sign.	Conceptual understanding	Given the equation $\sqrt{x^2+9}=5$, solve for x. $\sqrt{x^2+9}^{\,2} = 5$ $x^2+9 = 5$ $x^2 = 5-9$ $\sqrt{x^2} = \sqrt{-4}$ $\boxed{x = \sqrt{-4}}$
Incorrect use of inverse operations, including mistakes with signs.	Insufficient knowledge of the ‘rules’ of transposition	Solve the following equation for x: $2x+6=4x-2$ $2x+6-2 = 4x$ $2x+4 = 4x$ $4 = 4x-2x$ $4 = 2x$ $x=2$
Incorrect use of distributive law, factorising, fraction arithmetic and simplifying algebraic expressions. Including mistakes with signs.	Prerequisite algebra skills	If $\frac{y}{z}+1 = x^2$, rewrite this equation to find y. $\frac{y}{z} = x^2-1$ $y = x^2 - z$
Not knowing where to start.	Difficult to identify: possibly all of the deficiencies listed	Transpose the formula to make to make e the subject. $T=\frac{2v}{g}\left(\frac{1}{1-e}\right)$ Dont Know where to start

Our discussions with the lecturers from CIT and a survey of second and third level textbooks revealed an inconsistency in the technical language used which is not helpful and may be confusing to the students. Common terminology used to describe the manipulation of an equation includes: transpose entities, reverse operations, ‘move’ terms, ‘cross-multiply’ and ‘bring across equals sign’. This poses natural questions. Does the terminology matter? Does diversity in terminology cause confusion with the students? Are some terms more correct or appropriate than the others? Should using the terms ‘move’ and ‘bring across’ be avoided as it gives students an idea of moving entities around in a random fashion?

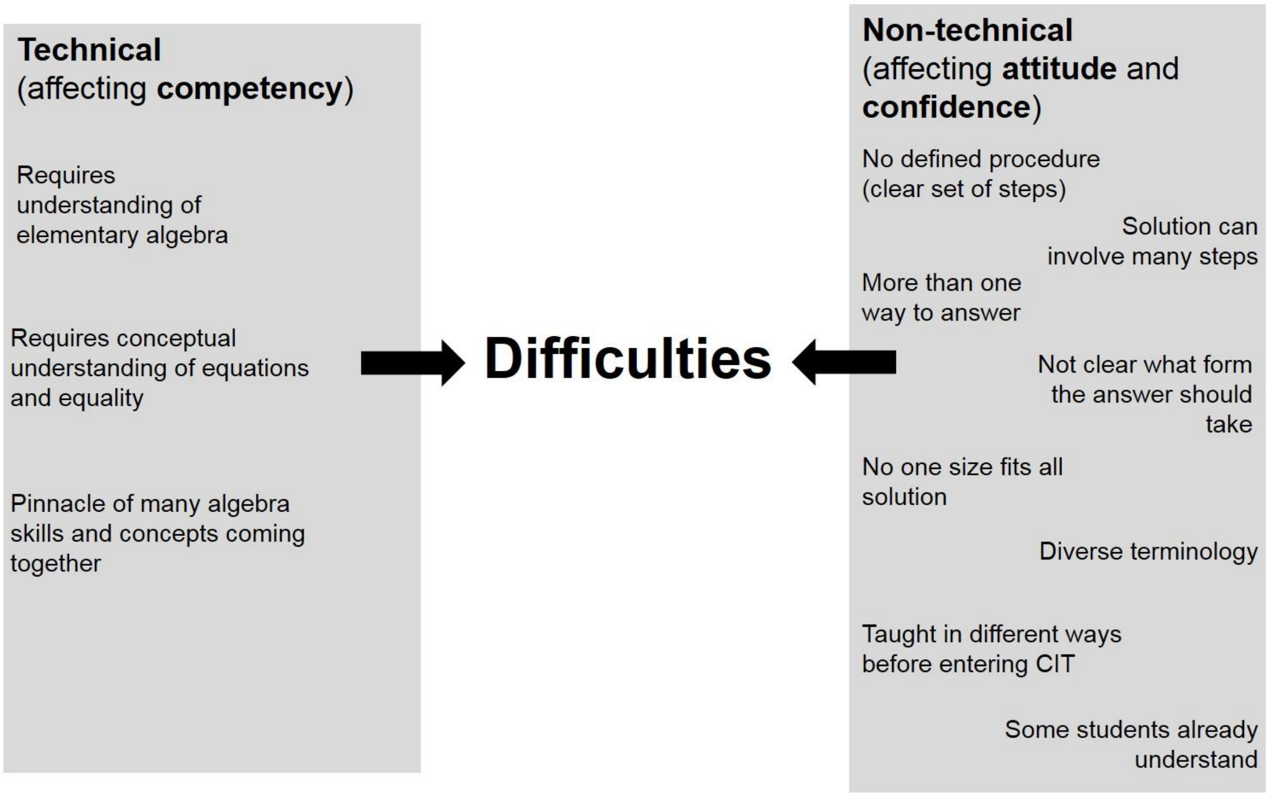

Figure 2: Summary of students’ difficulties with transposition.

Identifying the processes used by good learners is a powerful resource for designing educational interventions (Rittle-Johnson, et al., 2001). Therefore, in our search of ways to improve teaching the topic of transposition we also wanted to determine what it is that a student proficient in transposition understands that sets him or her apart from others. A cohort of second year level 8 engineering students, who are found to be competent with transposition, were asked to write down key ideas that they would use to explain transposition to their peers. Though the language of replies varied, the most common and relevant feedback included (a) “apply opposite/inverse operations” and (b) “do the same to both sides of the equation”. This is telling as the students showed understanding of the concept of transposition and stated exactly the two ideas that we will examine later as ideas *instrumental* to successful transposition.

5. Potential intervention strategy

Our early findings indicate that there are a number of technical and non-technical issues surrounding transposition that need to be considered in the development of an intervention strategy. On the technical side, the intervention must build up an understanding of the concepts of equations and equality, as well as address key algebraic deficiencies and misconceptions.

As educators, we always wonder if our students have developed a conceptual understanding. Conceptual understanding is knowing more than just isolated facts and methods. A successful student not only understands the ideas, but also has the ability to apply this knowledge to new contexts (Fosnot, 2018). In other words, conceptual knowledge is flexible and not tied to specific problems and is therefore generalizable (Rittle-Johnson, et al., 2001). Eric Mazur wondered about conceptual understanding too and proposed a peer instruction model to be used in conjunction with lectures (Mazur, 1997). Peer instruction is designed to engage students during class through activities that require each student to apply the core concepts being presented, and then to explain and discuss those concepts with their fellow students. Mazur's results show a significant increase in the percentage of students answering the concept question correctly. Mazur's successful findings have been replicated across disciplines at second and third level (Cummings and Roberts, 2008; Smith, Wood, Krauter, & Knight, 2011). Additionally, Smith, et al. (2009) found that combining peer discussion with instructor explanation increases student learning from in-class concept questions. The logic underlying the success of peer discussion based on concept questions is that it continuously engages students' minds and provides feedback to both students, and the lecturer, about the level of understanding. It also allows students to construct their own knowledge of the topic. There is a transformative effect of the shift from a lecturer-centred "transmissionist" environment to a more learner-centred constructivist classroom (Smith, et al., 2011). Engaging students in a learning activity in one class predisposes them to learn from a subsequent lecture.

If a teaching model based on peer-discussion is to be used in classes on transposition of formulae, what are the main ideas underlying transposition and what concept questions do the students need to be confronted with in order to foster a greater understanding? Physics, Biology, Engineering and other applied disciplines lend themselves well to concept based teaching and testing. It is not as straightforward to test conceptual understanding in Mathematics. There are several aspects to consider. What are the big mathematical ideas underlying a particular topic? What is the indication of one's understanding of such ideas? There are three aspects to such understanding: knowing which mathematical ideas are key and why they are important, knowing which ideas are useful in a particular context of problem solving and the ability to justify the approach to solving a problem. An ability to find an error in someone else's work and explain it is another hallmark of conceptual understanding. This prompts one to think about the right questions to ask when testing or discussing a concept: "why this works", "why can you do this", "how do you explain this", "justify your solution" and so on. When students have acquired conceptual understanding, they can give arguments to explain why some steps are consequences of the others.

What are the key ideas underlying transposition? The big key idea is that a*ny equation can be represented in an infinite number of different but <u>equivalent</u> ways.* Additionally, the following ideas are instrumental to successful transposition:

1. respect the equality i.e. do the same to both sides of the equation to obtain an equivalent equation, and;
2. simplify the equation/formula step by step by applying the inverse operations.

Note that the two ideas instrumental to transposition were exactly the ones that proficient students highlighted.

On par with fostering the correct conceptual understanding, it is vital to also dispel common misconceptions surrounding transposition of formula that persist in the student community. Therefore, the intervention strategy currently being developed within this project will focus on both building the right concepts while also highlighting and resolving misconceptions.

6. Conclusions and future work

The origins of students' deficiencies with transposition of formulae are complex, multifaceted and perpetuate far beyond a mathematics class. These difficulties have to be addressed in an efficient way to ensure graduates and future professionals are proficient at the technique. A peer discussion based teaching model may be a suitable format for cultivating better conceptual understanding in class.

Future work will mainly focus in two directions:

- development of an intervention strategy for teaching transposition, and;
- development of a diagnostic test that assesses students' conceptual understanding on the topic and measures the effect of the intervention.

We intend to develop and implement an intervention strategy which will involve peer-discussion based teaching with an increased emphasis on conceptual understanding while also dispelling the students' misconceptions relevant to transposition of formulae. The diagnostic test and post-test analysis will aim to quantify the impact of peer instruction on student attainment in the topic of transposition in a first-year service mathematics course.

7. Acknowledgements

The authors recognise the financial support of the Teaching and Learning Unit in CIT and thank all the staff of the Mathematics department for encouragement and contributions to workshops and seminars.

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CASE STUDY

Supporting postgraduate taught students through tailored maths workshops and Q&A sessions

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Abstract

Maths Support at the University of Aberdeen was established in 2012, and has been offered to all students, whatever their discipline and level ever since. Early on, interest was raised amongst the postgraduate population, which represents about 20% of the whole student population at the University of Aberdeen. Maths Support for postgraduate students, however, will necessarily take different shapes to Maths Support for undergraduates. Their time constraints are different; their timetable is often very full, with little opportunity to fit in potential extra sessions for Maths Support during the semester; they need to clarify their maths queries early in order to be able to keep up with the pace of study. In addition, a significant proportion of postgraduate students are mature students, coming back to study a Masters a number of years after completing their first degree, who may also be part-time, having to balance between study, work and family life. This paper will discuss the range of tailored Maths Support services developed at the University of Aberdeen for postgraduate students (PGTs) on Business, Engineering and Geosciences Masters programmes. Student and staff feedback on the usefulness of the service, gathered anecdotally, will also be presented.

Keywords: Mathematics support, university mathematics, postgraduate studies.

1. Introduction

Maths Support was established at the University of Aberdeen in August 2012. The post of Academic Skills Adviser (mathematics) was created as a 50% full time equivalent post, in the Student Learning Service. The remit of the post holder is to work with students from all levels of study, and all disciplines, but support is provided for maths only, and not for statistics.

There are no dedicated teaching spaces for Maths Support, and the Maths Adviser has a shared office, therefore, in order to accommodate for time and space constraints, a variety of delivery modes have been developed (Richard, 2016):

- One-to-one bookable appointments;
- Questions & answers sessions and workshops for specific courses;
- Online resources accessible through the VLE;
- Drop-in sessions at the University Library prior to the exams;

Maths Support is advertised to students via lecture visits at the start of term, emails, announcements on the VLE (Blackboard), campus plasma screen displays, and social media (Twitter, Facebook).

Postgraduate students showed interest in accessing Maths Support as early as 2012, however, they also raised concerns about their ability to use and make the most of the service. Anecdotal conversations with the students highlighted a number of factors specific to these cohorts.

Masters programmes run over a year for full time students, with two taught semesters, and a third semester for the project and dissertation. As a result, many of these programmes operate a very busy timetable, with little spare time to use Maths Support. A number of PGT students will be mature students with a family, possibly have a part-time job to generate income and cover for tuition fees and living costs, putting additional pressure on their availabilities.

The pace of teaching in the programmes can also be rapid, with the course progressing quickly to further topics. Consequently, it is important to offer Maths Support early in the year, to ensure that students have sufficiently consolidated the fundamental maths topics required for the understanding of their courses.

A significant proportion of students at the University of Aberdeen are postgraduate: 20% in 2016-17 and 21% in 2017-18. Yet, only 9% of students using Maths Support bookable appointments and drop-ins were PGTs in 2016-17 and 16% in 2017-18. This indicates that bookable appointments and drop-ins may not be the most suitable to offer to PGTs, possibly because of the factors highlighted above. Figures 1(a)-(c) show the number of appointments and drop-in visits by PGTs, in academic years 2015-16, 2016-17 and 2017-18. The largest numbers are recorded for PGT's in the Business School (Bus), with lower numbers recorded for PGTs in the School of Geosciences (Geo Sc) and the School of Engineering (Eng). In particular, in 2017-18, no PGT students from the School of Geosciences visited Maths Support, but 1 PGT student from the School of Natural and Computing Sciences (N&Comp Sc) and 1 PGT student from the School of Social Sciences (Soc Sc) did.

Some PGTs are seeking Maths Support because they have low-level maths qualifications (GCSE/National 5 Level qualifications) and need to gain confidence and practise to overcome the maths required in their degree. Other Postgraduate students, however, do have high-level maths qualifications (BSc, BEng), however, they return to studies after a number of years in the work place. Not only have these students forgotten some of their maths knowledge, but they are also no longer used to the formality of academic maths (notation, formal calculations, proof reasoning). This situation is becoming more prominent at the University of Aberdeen, due to the current economic climate of the Oil and Gas Industry, with a high number of staff registering on a Masters programme after being made redundant.

Over the past 6 years, some specific material has been designed and delivered to PGTs at the University in order to best answer the specific needs and constraints of these students. These projects have been developed in close collaboration with staff in three Schools of the University: the Business School, the School of Geosciences and the School of Engineering.

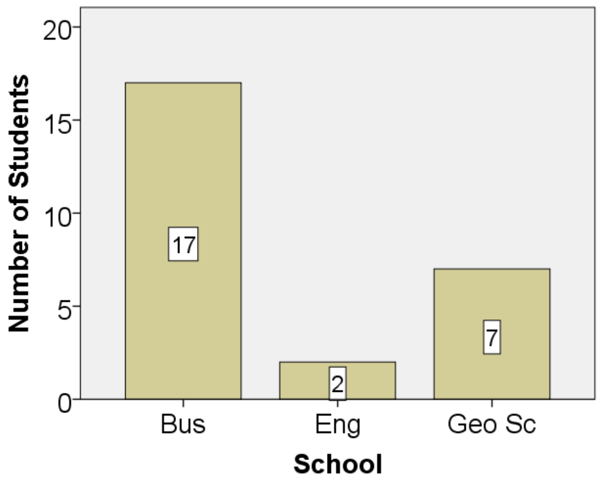

Figure 1(a): Number of PGT sessions (bookable appointments and drop-in sessions) in 2015-16

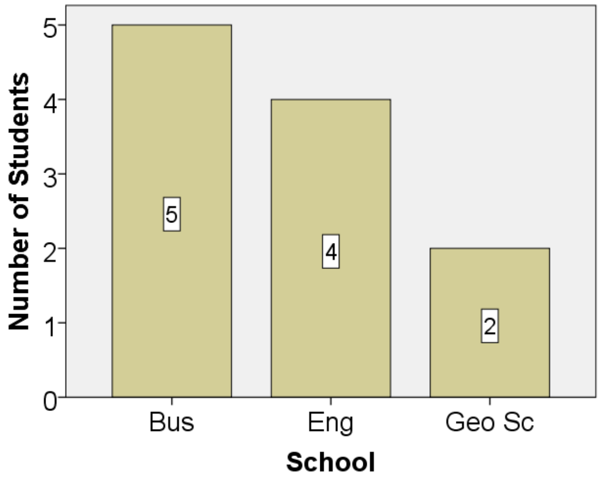

Figure 1(b): Number of PGT sessions (bookable appointments and drop-in sessions) 2016-17

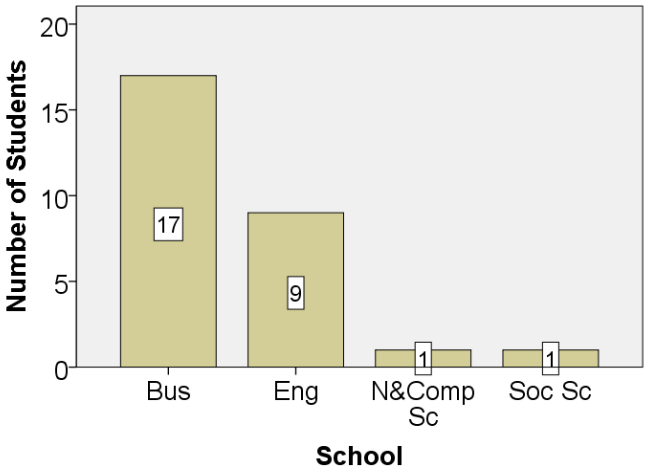

Figure 1(c): Number of PGT sessions (bookable appointments and drop-in sessions) in 2017-18

2. Masters programmes in the Business School

The Business School at the University of Aberdeen offers four MSc Programmes (MSc Petroleum Energy Economics and Finance (PEEF), MSc Accountancy and Finance (AF), MSc Finance and Investment Management (FIM) and MSc Finance and Real Estate (FRE)) for which there is a maths and statistics course, the Quantitative Methods course. The maths section of this course is taught over the first five weeks of the semester and covers three main topics:

- Optimisation of functions of one variable;
- Optimisation of functions of two variables;
- Lagrange optimisation method for constraint optimisation.

The students on the four different programmes have a wide range of maths qualifications, from GCSE/National 5 level qualifications to University level qualifications. Students with the highest maths qualifications tend to be the students on the PEEF and FRE programmes, while students with lower maths qualifications tend to be those on the AF and FIM programmes.

In close collaboration with the staff teaching on the Quantitative Methods course, a programme of two workshops has been created for students. The first workshop covers some basic Algebra that students will need for the course:

- solving systems of equations;
- factorising quadratics;
- rules of powers;
- exponential and logarithm functions.

These topics are not taught specifically to students in the Quantitative Methods course. The second workshop covers:

- differentiation and partial differentiation;
- optimisation of functions one and two variables;
- Lagrange optimisation method.

Students are taught these topics by the teaching staff as well. Sessions are a mixture of teaching and practise, and all exercises are set in the context of Business Studies (Mavron and Phillips, 2007; Renshaw and Ireland, 2012).

The workshops were first delivered at the beginning of the academic year 2013-2014, and have been run annually ever since. Students are split in three groups: two groups for the AF and FIM programmes and one group for the PEEF and FRE programmes. This allows instructors to work with groups of a manageable size, as well as to work with students of similar expected maths abilities. Workshops are delivered over the second, third and fourth week of the teaching semester.

Over the past three academic years (2015-2016, 2016-2017, 2017-2018) the average workshop attendance was 22 per group for the three groups, and the average total number of PGTs over the four programmes was 117. Many more students attend the workshops than those using bookable appointments and drop-ins. Figures 2(a)-(b) show attendance to the workshops across the three academic years.

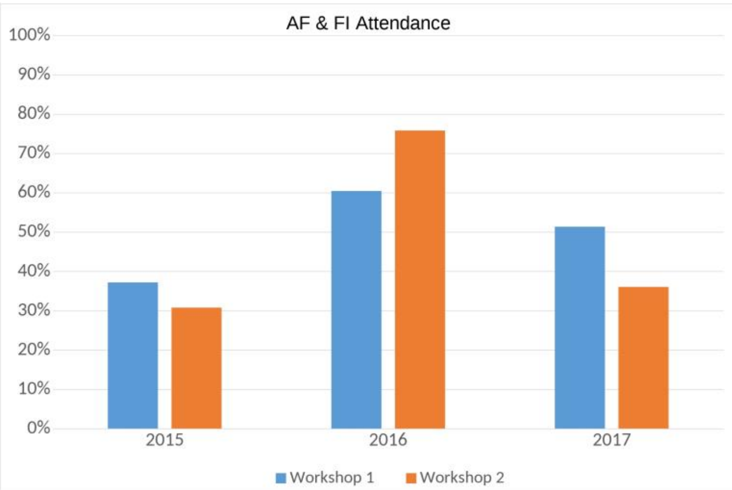

Figure 2(a): Attendance at workshops, as a percentage of the group total, for AF & FI programmes

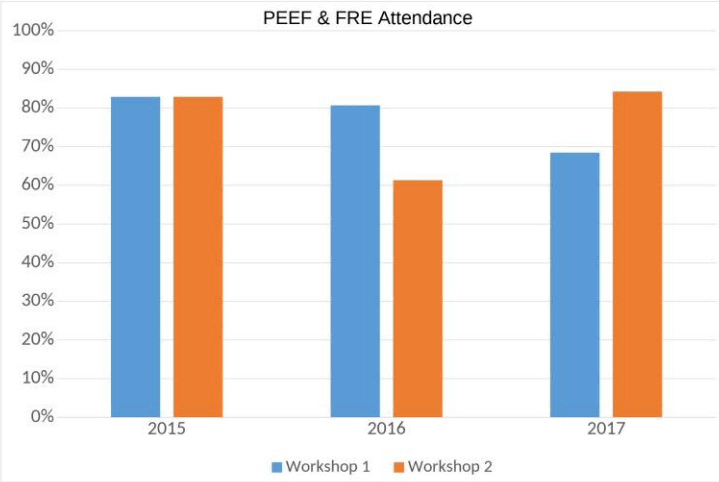

Figure 2(b): Attendance at workshops, as a percentage of the group total, PEEF & FRE programmes

Last academic year (2017-2018), some students commented that, in the sessions for AF and FIM, a number of students attended that did not experience difficulties with the maths topics. This was possibly due to the fact that the workshops were timetabled in the students' VLE timetable, resulting in students thinking that there were mandatory sessions, in spite of numerous messages from all staff. As a consequence, the students who had real difficulties with the maths component felt overwhelmed and intimidated. To respond to this, additional very small drop-ins were organised for those students (no more than five students), and from this academic year (2018-2019), maths workshops will no longer appear in the students' timetables.

Teaching staff, as well as students, have expressed the wish that the workshops be delivered earlier, possibly during Fresher's Week, as preparation for the course. Unfortunately, this is not easy to set up for this cohort of PGTs, as many students are overseas students who often are not able to come early due to visa issues, and sometimes are not even able to arrive after term has started. However, this was successfully implemented for a different cohort of students, namely, Postgraduates in the School of Geosciences.

3. Masters programmes in the School of Geosciences

In 2013, a small group of Postgraduate students in the School of Geosciences accessed Maths Support. These students had very little spare time, with lectures timetabled daily from 9 am - 12 pm and 2 pm – 5 pm, and thus could only attend Maths Support over lunch time. They had GSCE/National 5 maths qualifications, and had not done any formal maths since then. Therefore, they felt quite rusty, and had to get quickly familiar with a wide range of mathematical notations and topics which they had never encountered before: from calculating angles in radians rather than degrees, to studying partial differential equations.

Subsequently, discussions were held with staff in the School of Geosciences, and a Maths Induction course was devised, to be delivered to students during Fresher's week, prior to teaching commencing. The School of Geosciences offers four MSc programmes, enrolling students with different maths qualifications, and covering different maths topics:

- Integrated Petroleum Geophysics (IPG): students on this programme hold GCSE/National 5 maths qualifications, and the programme will require them to do some geometry, trigonometry, algebra and differential calculus;

- Reservoir Engineering (RE) and Geophysics (GP): students on this programme hold A Level or Higher maths qualifications, and must have completed at least a Level 1 University maths course. The two programmes will require students to do some geometry, trigonometry, algebra, and differential and integral calculus;
- Oil and Gas Enterprise Management (OGEM): students on this programme hold GCSE/National 5 maths qualifications, and will be required to do some geometry, trigonometry, and algebra.

The Maths Refresher course runs over two days, with three taught and practise sessions, and one question-answer session. The course is not mandatory, not assessed in any way, and the aim is to give students a first exposure to the kind of maths they will work with in their courses. The first two sessions are intended for all four programmes and cover:

- Algebra: Linear Equations and Straight Line, Quadratics and Parabola, Exponents, Exponential & Logarithm;
- Trigonometry: Radians and Angle Properties, Trigonometric Functions and Identities, Wave Functions.

The third session covers: Differentiation of Functions of one Variable and Graphs of Functions, Partial Differentiation, and Integration, and is intended for all programmes except OGEM. All sessions are a mixture of teaching and practise, all exercises being written in the context of Geology as much as possible (Waltham, 2000; Ferguson, 1988). During the last session, students have the opportunity to ask further questions, or get help with any tutorial exercises not covered during classes.

The course was first delivered in September 2015, and has been delivered every year since. The average attendance over the past three academic years at sessions was 35 where the average over all four programmes was 71 students. Figure 3 shows the breakdown of attendance over the past three academic years for each session: as for PGTs in the Business School, the number of students attending the Maths Refresher Course largely surpasses the number of students using bookable appointments and drop-ins.

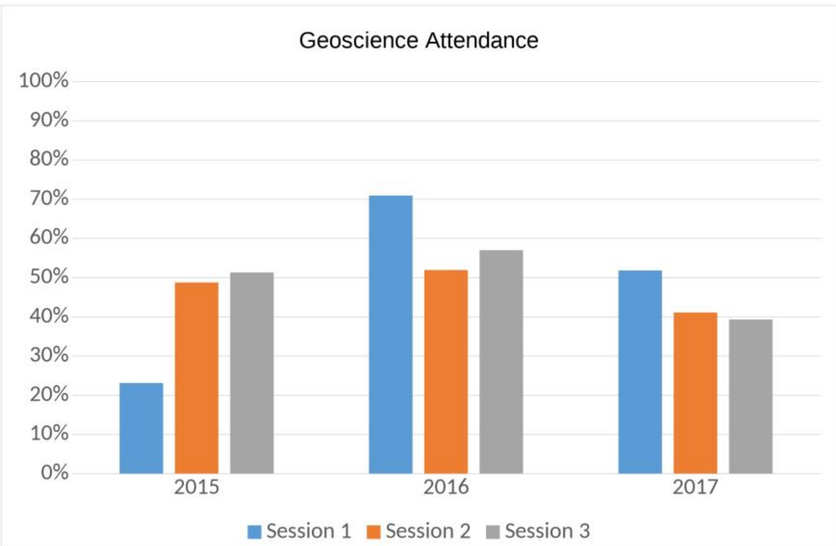

Figure 3: Attendance at the Maths Refresher course, as a percentage of the total of students on 4 MSc programmes for the School of Geosciences

The staff feedback is that the course is useful in preparing students to learn mathematical notation and topics, and that students have been less overwhelmed by the mathematical content of the programmes since the course has been running. No systematic feedback has been collected from students on their perception of the usefulness of the Refresher course. Very few of these students subsequently visit Maths Support during the academic year (seven visits from PGTs in the School of Geosciences in 2015-16, two in 2016-17 and none in 2017-18, see Figures 1(a)-(c)), but anecdotal comments collected at visits indicate that the course does help in tackling the maths content of the programmes.

In 2016, postgraduate students from the School of Engineering started attending the Maths Refresher course for Geoscientists and the Maths Workshops for the Economists. As these sessions were designed neither at the appropriate level, nor in the appropriate context for Engineering, it was decided to organise sessions tailored for the School of Engineering.

4. Masters programmes in the School of Engineering

The first PGT students in the School of Engineering seeking Maths Support were students on the MSc Subsea Engineering programme. In discussion with the programme co-ordinator and the students, three workshops were organised on Calculus, Matrices and Complex Numbers in 2016.

During the course of the semester, students from another programme, MSc Renewable Energies, approached the Maths Adviser, and additional ad-hoc sessions were organised for this cohort. Although the sessions were first delivered as workshops combining teaching and practice, students quickly changed the format to question-answer sessions on their lecture material. Although topics remained unchanged, this ensured that the help provided was most effective, as it was given directly in context.

These sessions were delivered in the following academic year again (2017-2018) to both programmes (three sessions for each programme). Once again, the first workshop was delivered as a lecture (Calculus Refresher), and subsequent sessions were driven by students' questions on their lecture material. Mature students from a third programme, MSc Petroleum Engineering, also approached Maths Support at the beginning of that year, and further Q&A sessions were organised for them (additional three sessions).

Recurrent topics in these sessions were:

- Partial derivative in the context of Thermodynamics;
- Integration in the context of permeability coefficient calculation;
- Numerical methods: Trapezium method and Newton-Raphson method;
- Probability;
- Complex numbers in the context of calculating complex impedance, intensity and potential.

Last academic year, 2017-2018, three students out of a total of six, attended the sessions from the MSc Subsea Engineering, seven students out of 24 from the MSc Renewable Energies and six students out of ten from the MSc Petroleum Engineering. These are much smaller numbers than for the other two Schools, and, in fact, comparable to the number of students making use of appointments and drop-ins. The cohorts are smaller than the cohorts in the Business School and School of Geosciences, and the proportion of students attending Maths Support sessions is also smaller (see Figures 4(a)-(b)), so it may be that less Engineering Postgraduates need Maths Support.

No systematic feedback was collected, but one student explicitly stated that they found the Maths Support sessions very helpful, because they had graduated over a decade ago, and felt that they could hardly remember any mathematical notions. Maths Support and the School of Engineering are now considering opening Maths Support sessions to all MSc programmes in the School of Engineering (total of 16 programmes).

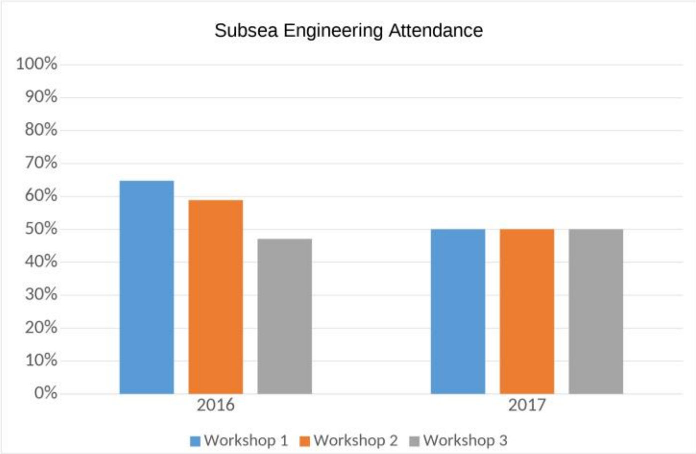

Figure 4(a): Attendance at workshops, as a percentage of the class total, for the MSc Subsea Engineering, School of Engineering.

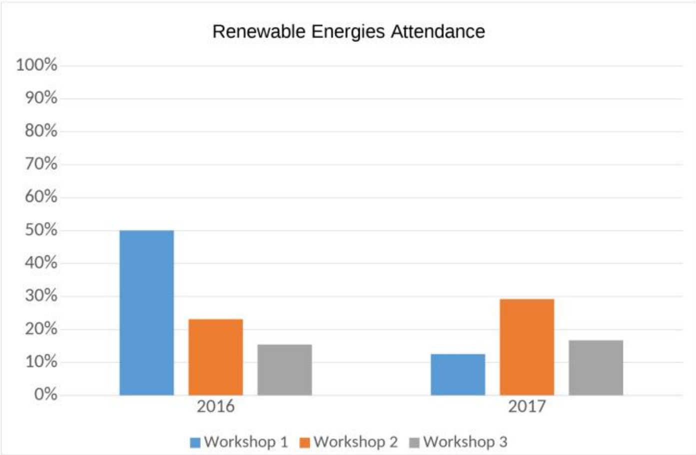

Figure 4(b): Attendance at workshops, as a percentage of the class total, for the MSc Renewable Energies, School of Engineering.

5. Discussion and conclusions

Maths Support at the University of Aberdeen started in 2012, and the remit of the Maths Adviser is to work with all students, at all levels. Given that 20% of the student population consists of Postgraduate students, this should be reflected in the usage of Maths Support. However, during the course of the semester, bookable appointments and drop-in sessions tend to be predominantly utilised by undergraduate students.

Discussions with PGT students have shown that this is perhaps because bookable appointments and revision drop-in sessions are not the most appropriate for this cohort. Consequently, over the past six years, we have developed Maths Support sessions tailored to each of the three Schools where PGTs have approached the Maths Support service (PGTs in the Business School, the School of Geosciences and in the School of Engineering). Each solution has been designed in response to students' requests, and in collaboration with teaching staff, to ensure that appropriate content was used, and with administrative staff, to ensure that sessions were timetabled appropriately, and advertised efficiently to cohorts.

Over the years, a large number of PGT students from all three Schools have accessed Maths Support only through these sessions, indicating that this may be a more suitable way to address their maths issues. This is particularly true for students in the Business School and the School of Geosciences. A limited number of group sessions are also offered to some specific cohorts of undergraduate students, however, it is interesting to note that, based on experience teaching PGTs, they seem to really enjoy working as a group with the Maths Adviser, and interact with each other as well as with the Maths Adviser. In the School of Engineering, particularly, less PGT students are attending the sessions, and yet, those students attending have taken 'ownership' of the sessions, and have been very proactive in changing the structure of the workshops into Q&A sessions.

The University of Aberdeen has recently opened a number of its postgraduate programmes to online learners, and in particular, all Masters programmes in the School of Engineering. At the moment, Maths Support for these distance learning students is limited to directing them to our online resources located in the university's VLE (Blackboard at the University of Aberdeen). The maths resources contain the handouts of workshops as well as a selection of HELM workbooks (Harrison et al., 2007) and Facts and Formulae leaflets (Richard, 2015). In addition, we are currently investigating how to open face-to-face Maths Support to online students, using Blackboard Collaborate together with Smartboard and Graphic Tablet technologies. However, given that Maths Support is only 0.5 FTE, support for a potentially growing population of off-campus students, who will probably experience challenges particular to their situation, will necessarily have to be limited.

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CASE STUDY

Success in employers' numeracy tests

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Abstract

Final year students applying for graduate jobs in industry are often required to sit numeracy tests as part of the recruitment process. Students in the College of Arts can be disadvantaged in this area as, often, the last time they will have come across any Mathematics will have been at National 5, GCSE or equivalent level (at the age of 15 or 16). This project was a collaboration between the Mathematics & Statistics support within the Learning Enhancement and Academic Development Service (LEADS) and the Careers Service to create a repository of resources designed to help students refresh their basic mathematical skills and give them the confidence to tackle Employers' Numeracy tests successfully. A Moodle course including videos and tailored learning materials was created to enable students to improve their confidence and ability in relevant mathematical and statistical skills, as well as sample tests (both timed and untimed) to give students an idea of what to expect in a real assessment.

Keywords: Numeracy, employability tests, mathematics, statistics.

1. Background

As part of the recruitment process, employers require students to use their mathematical, statistical and problem-solving skills to complete timed tests (Durrani, 2012). Due to their lack of mathematical experience and confidence in this area, final year students from the College of Arts were identified by Maths Support staff in the Learning Enhancement and Academic Development Service (LEADS), as the group demonstrating the greatest need for additional support (Gillespie, 1998).

Following discussions between the Maths Advisers and the Careers Manager, it was felt that the most effective way to support students in this area would be to create and collate relevant mathematical resources, especially as no such resource was currently in existence at the University of Glasgow. The authors were successful in securing a grant from the University Services Innovation Fund, a fund which is available for supporting cross-service initiatives, allowing a collaboration between Maths Support, Media and Careers Service colleagues.

The aim of this project was to deliver a centralised online repository of mathematical resources designed to guide and support students with (re)acquainting themselves with basic numeracy and mathematical skills and give them the confidence to tackle Employers' Numeracy tests successfully.

The resource was piloted with students from the Arts in mind, but with the outcomes of the project being easily transferable, it is intended to offer this resource to students across the university. Further examples of models used to support graduate employability are also presented by Rowlett and Waldock [eds]. (2017), and Pool and Sewell (2007).

2. The Resource

A part-time Research Assistant (RA), recruited for six months, undertook the following tasks: research online numeracy tests (both free and paid), find and collate existing available resources and build a resource for students on the university's VLE, Moodle.

There had been some anecdotal evidence suggesting that freely available tests online were ‘easier’ than ones that were commercially available. The RA spent some time comparing these so that a few sample tests could be created in-house which would bridge the gap between the two.

The resulting resource consists of: lessons on key aspects of numeracy, with specific reference to Percentages, Ratios and Currency Conversions and including examples from this style of testing; practice multiple choice questions and timed tests; links to other online resources; and videos of worked solutions to a selection of numeracy test questions. Most of the resources were created on Moodle itself, using both the Lesson and Test functionalities. The videos, however, were created using a Cintiq Creative Pen Display alongside Camtasia and software which was built in-house. The resulting videos show screenshots of real test questions with the solution written over the top and a voice recording of the explanation of the working as illustrated in Figure 1 below.

Examples of worked examples in video tutorials

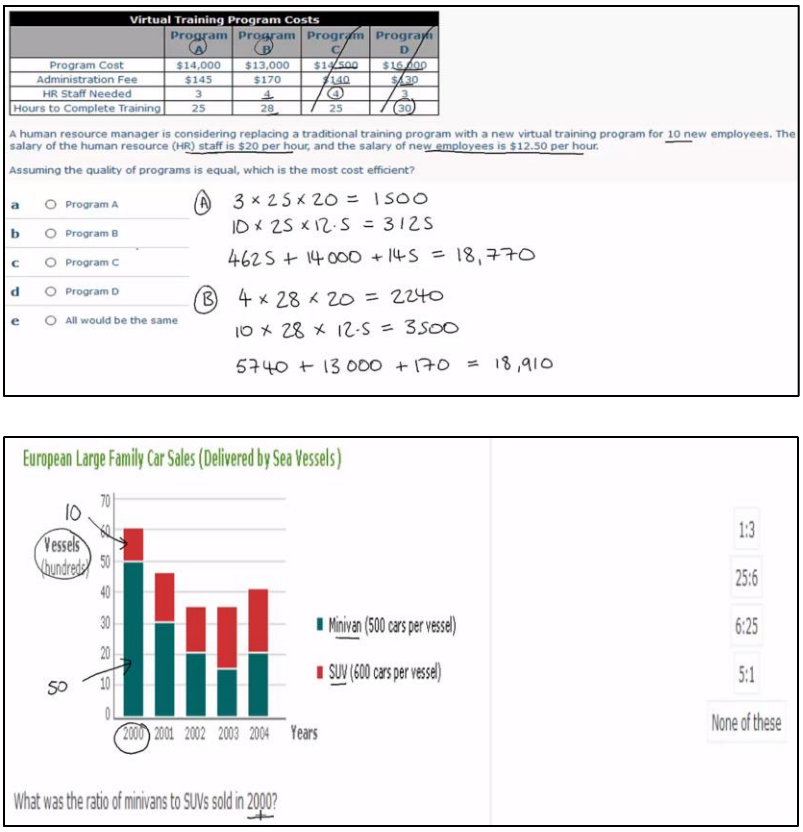

Figure 1: Screenshots of video tutorials

As well as these resources, the Careers Adviser invited some of her contacts in industry to participate in the creation of a video. This gives students an insight into the graduate recruitment process and the place of numeracy testing within it. The video also features the Careers Adviser giving students some tips about applying for graduate jobs and the Maths Advisers introducing themselves and the support available through LEADS.

3. Feedback

Initial feedback was obtained from colleagues in both LEADS and Careers, particularly focussed on those from non-numerate backgrounds. Colleagues tried out timed and untimed tests, as well as working through some of the numeracy lessons. The feedback obtained was used to make initial changes to the resources on Moodle.

The resource was then taken to a focus group of students. The focus group was two hours long and involved trying out elements from every aspect of the resource. Six students participated: four from the College of Social Sciences and two from the College of Science and Engineering. There was no uptake from the College of Arts. One student was a postgraduate, the rest were undergraduates at level 3. Interestingly, all the participants had sat a numeracy test before which meant they were able to give informed opinions on the usefulness and relevance of the resource.

The students were asked how they felt about sitting these tests and all admitted to feeling nervous or anxious despite the having some level of numeracy as part of their course at university. The students were then asked to give their views on:

1. The Moodle lessons on Percentages and Ratios – these were found to be ‘useful and well explained’. In particular, they liked the handy techniques such as finding 80% of a quantity when a 20% reduction was required. As the two lessons were found to be so helpful, it was suggested we add a lesson on Currency Conversions;
2. The video tutorials – the students liked these and found it more useful to watch a question being worked out and found these easier to follow compared to reading through a worked solution. It was felt that a short introduction to the question prior to starting the calculation would be helpful. This was a very good suggestion but as it was impractical to record all eighteen videos again, a short textual description of the question featured in the videos were added to the Moodle page featuring the video clips;
3. The practice tests/questions – the participants generally found these reasonable, however several remarked they were surprised at how many of their answers were wrong and were keen to try more to improve their score. The content and level of questions in this resource were found to be comparable to the employers’ tests previously sat by the student;
4. General comments – the students said they would recommend the resource to friends and would use it to help prepare for any further numeracy tests they were require to take in future. They were also interested to hear about the support on offer through LEADS and expressed an interest in attending workshops run by Maths Support staff. When asked about ways this resource could be advertised, suggestions included the use of social media, Careers website and through Schools.

The comprehensive feedback obtained was used to further shape the content on Moodle, most notably by the addition of a new lesson on Currency Conversions. We had hoped to carry out more focus groups, but found it challenging to recruit students to participate due to the time of year.

4. Further plans

The Maths Support team and Careers Service are planning on collaborating to deliver ‘Graduate Numeracy’ workshops throughout the academic year. The workshops will be one hour in length and will cover some key numeracy skills as well as going over some example test questions. Students will also be pointed towards the Moodle resource and further numeracy support available from LEADS.

There are also plans to promote the resource to students through College Employability officers and the Careers Service in general. A regular series of workshops is currently under discussion with one of the Colleges. There is also scope for collaboration with the Alumni Service in the near future.

Further plans include extending the resource further by adding sections on verbal and non-verbal reasoning.

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RESEARCH ARTICLE

Quantifying the impact of mathematics support on the performance of undergraduate engineering and computing students

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Abstract

The School of Computing, Engineering and Built Environment at Glasgow Caledonian University has provided mathematics support for all students in the school since 2012/13. This paper compares the performance of two groups of undergraduate students; those who engaged with mathematics support, through attending targeted lectures, and those who chose not to engage. Data collected over the five years from 2013/14 to 2017/18 for 4,690 engaging and non-engaging students, enrolled on modules with a substantial mathematical content, were analysed. Module marks achieved at first diet for the two groups of students, at all undergraduate levels, were selected as a measure of how mathematics support impacts on student performance. The study showed that there was a substantial and significant difference between the marks achieved by students that engaged with support and those that did not. Students who engaged on average showed an 8% increase in their module mark compared with those who did not engage.

Keywords: Mathematics support, targeted lectures, module marks, quantitative analysis, impact.

1. Introduction

During the early 1990s it became apparent that the mathematical skills of undergraduate students entering higher education (HE) were not at the level expected by a range of disciplines. In science and engineering a significant number of new students to university were identified as being underprepared for the mathematical demands of their degree programmes. In response to this shortfall Coventry Polytechnic (now Coventry University) introduced an early form of mathematics support (MS) in 1991 which was mainly targeted at engineering students. A number of institutions in England followed Coventry's lead as detailed in a 1993 survey into the provision of mathematics support in further education (FE) and HE (Beveridge and Bhanot, 1994). The authors sent 800 questionnaires to FE and HE institutions across the UK and of the 142 that replied most offered some form of MS. From the mid-1990s an increasing number of institutions, including the University of Hull (1995) and Loughborough University (1996), launched mathematics support centres (MSCs). Lawson, Halpin and Croft (2001) reported that of the 95 UK universities who replied to their survey 48% provided some form of MS. The survey was updated in 2004 (Perkin and Croft, 2004) and 2012 (Perkin, Lawson and Croft, 2012) by which time 65% and 85% respectively of the institutions that replied offered mathematics support. In 2005 a collaborative bid to the Higher Education Funding Council for England (HEFCE) resulted in Coventry and Loughborough Universities being jointly awarded Centre for Excellence in Teaching and Learning (CETL) status. Following this award the sigma CETL project was launched to promote collaborative work in mathematics and statistics support in England and Wales. The funding for the project ended in 2010 and in the ensuing years it has evolved into the sigma Network. The Network continues the work of sigma CETL and is managed by volunteers, including representatives from Scotland and Ireland, who are involved with mathematics and statistics support in H.E.

The first MSC was opened in Ireland in the University of Limerick in 2001 and the Irish Mathematics Learning Support Network (IMLSN) was formed in 2009. A survey in 2015 by the IMLSN (Cronin et al., 2015) into the provision of MS in Ireland recorded that 83% of the institutions that replied offered mathematics and/or statistics support. In Scotland; one of the country's first mathematics drop-in centres was established in 1988 at Edinburgh Napier University (then, Napier Technical College). Over the years MS continued in various forms, on a largely informal basis, at universities across the country. In 2008 however the support community was brought together through the formation of the Scottish Mathematics Support Network (SMSN). A recent report (Ahmed et al., 2018) by the SMSN found that 78% of the Scottish institutions that responded provide mathematics and/or statistics support.

While it is somewhat unrealistic to claim that MS is the sole contributing factor to improvements in student grades and retention there is significant evidence that it plays a major role in enhancing the performance of those who engage with it. Across the MS community, at both a national and international level, it is therefore highly desirable to identify parameters that can provide a quantifiable measure of the impact of MS on student performance. In the past various studies have been carried out in an attempt to measure the effectiveness of MSCs. Pell and Croft (2008) observed that while data will generally show that MSCs are well-used and valued by students a much more powerful measure of their effectiveness would be provided by quantitative data comparing how students who use these centres perform in their mathematics examinations and whether failing students are those who do not use the centres.

Matthews et al. (2013) reviewed the literature to date and highlighted studies that provided a quantitative analysis of MSC usage as well as studies into the impact of MS on students' mathematical confidence and progression and retention rates. Gill and O'Donoghue (2013) analysed the grades of students attending mathematics service modules at the University of Limerick between 2000/01 and 2003/04. Based on their performance on a diagnostic test, taken early in the first term, students scoring below 50% were categorised as 'at risk' and offered support tutorials. To measure the effectiveness of this support end of term examination results were compared for students in the 'at risk' group, who attended the tutorials, with those in the group who did not participate. For each year of the study 'at risk' students who attended the support tutorials were observed to out-perform their counterparts who did not attend. Additionally, the opening of the Mathematics Learning Centre (MLC) in late 2001 resulted in a dramatic fall in failure rate providing supporting evidence for the benefits of MS.

In their work Gallimore and Stewart (2014) categorised 42 first year engineering students at the University of Lincoln into three groups according to their performance on a diagnostic test and the use they made of follow-up support tutorials. The three categories included:

- students needing minimal support;
- students needing support who made use of the tutorial sessions;
- students needing support who did not use the tutorial sessions.

The impact of the support was measured by analysing the correlation between diagnostic test marks and final exam marks. Analysis of the results indicated that while those in the second and third categories had similar diagnostic test marks there was a significant difference in their exam results. The students who used the tutorial sessions performed significantly better in the exam than those in the third category whose lack of understanding and basic skills became apparent.

Pell and Croft (2008) analysed data for 644 first year engineering students taking five mathematics modules at Loughborough University in order to measure the impact of the Mathematics Learning Support Centre (MLSC). The authors found evidence that MS had improved the pass rate by approximately 3% and some students obtained passes in their final exams and remained at the university when they may have otherwise failed and withdrawn. Surprisingly perhaps it was also observed that 35% of the highest achieving students had used the MLSC indicating that a considerable number of mathematically stronger students use the centre to improve their grades.

These results were supported by a study carried out at the National University of Ireland, Maynooth by Mac an Bhaird et al. (2009). The authors found strong evidence to suggest that support played an important role in the retention of ‘at risk’ students as well as being used by strong students looking to improve their chances of achieving top grades.

2. Background

Glasgow Caledonian University is a post-1992 university, formed in 1993, through the merger of Glasgow Polytechnic with Queen’s College. The student population in 2017/18 was approximately 15,000 with over 4,000 mature students and the largest number of part-time students in Scotland. International students from more than 100 countries study at GCU. The university is committed to the Scottish government’s widening access initiatives which include advanced entry to university through articulation from the college sector and collaborating with schools across Scotland with traditionally low progression rates to H.E. These factors result in increasingly more entrants from diverse educational, social, and cultural backgrounds with widely varying experiences and knowledge of mathematics and statistics. Additionally, the introduction of new degree programmes, in areas such as computer games and audio technology, requires students to learn mathematical topics that they may not have been adequately prepared for in school or the college sector.

In 2008, in response to the increased numbers of students from non-traditional backgrounds entering university, GCU established a Learning Development Centre (LDC) in each of the university’s three schools with funding coming from both the university and the Scottish Funding Council (SFC). The role of the LDC was to provide academic support to students with the initial focus in all three schools being on support for academic writing. In the School of Computing, Engineering and Built Environment (SCEBE) however it soon became apparent that the mathematical content in a significant number of the degree programmes was influencing student performance and hence retention, progression and completion rates. In 2012/13 SCEBE appointed a dedicated member of staff to the role of providing support for mathematics and statistics.

2.1. Mathematics support in SCEBE

In 2017/18 the student population in SCEBE was approximately 4,330 with an 80%/20% male/female split and a 72%/28% full-time/part-time split. The structure of SCEBE includes three departments: Engineering (ENG); Computer, Communications and Interactive Systems (CCIS); and Construction and Surveying (C&S). Figure 1 shows the overall engagement with MS by students from these departments between 2013/14 and 2017/18. Clearly students from the Department of Engineering, accounting for 70% of all students, are the most prominent users of MS reflecting the significant mathematical content of their degree programmes when compared with programmes from the other two departments.

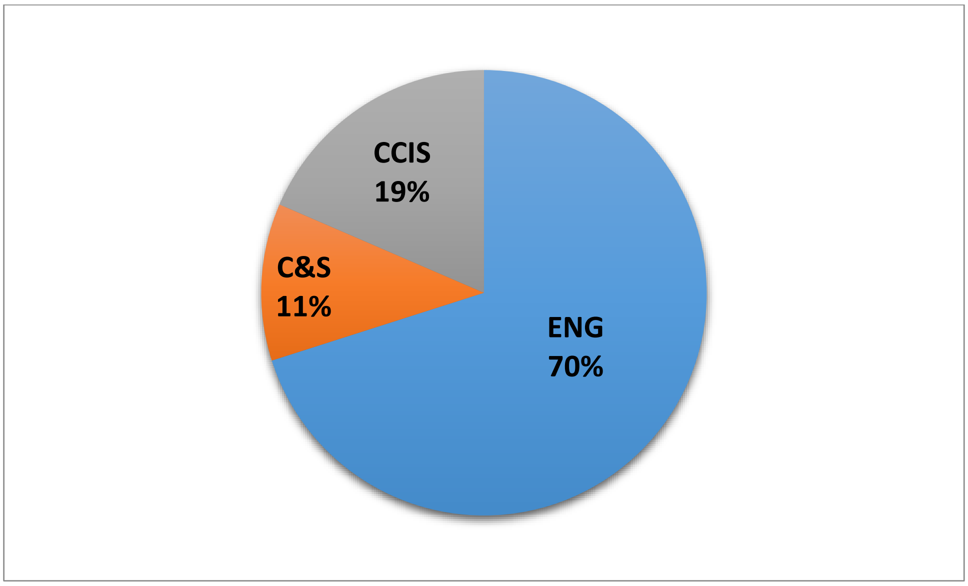

Figure 1: Engagement with MS by SCEBE department, 2013/14 – 2017/18

The main forms of support available in SCEBE include a mathematics summer school at pre-entry to university, diagnostic testing during induction and ongoing support from entry to completion. The ongoing support takes the form of:

- one-to-one and small group meetings – arranged on an appointment basis;
- drop-in sessions – students can visit the LDC between 9am and 5pm, without an appointment, to discuss their mathematics and statistics problems;
- targeted lectures – agreed with module and programme leaders to provide students with mathematics and statistics lectures on specific topics particular to their course;
- email – students can contact MS to discuss any difficulties they are experiencing. If these issues are not resolved a face-to-face meeting is arranged.

Figure 2 shows the distribution of contacts with MS, according to the type of support, between 2013/14 and 2017/18. Over the five year period there were over 2,000 contacts involving nearly 12,000 students. Targeted lectures are clearly the most popular mode of contact accounting for 40% of the total number. These weekly lectures are typically one hour in duration and appear on student timetables but attendance is on a voluntary basis. The material covered in class can include a revision of topics from previous years and/or providing a more in-depth treatment of new mathematical techniques. Students are issued with printed lecture notes that provide full explanations of relevant mathematical techniques, contain contextualised examples and extensive tutorial questions with full solutions. The notes and slide presentations can be accessed via the Blackboard Virtual Learning Environment (VLE) and contain appropriate links to mathematics support websites such as MathCentre (http://www.mathcentre.ac.uk) and Khan Academy (https://www.khanacademy.org). For some modules students can also access multiple choice tests on Blackboard authored using Maple TA software (https://www.maplesoft.com/products/maple). The Maple exercises provide students with immediate feedback in the form of answers and solutions. Furthermore, feedback is offered through questions asked during, or after, lectures and at one-to-one and small group meetings.

The quality of the support offered by SCEBE is reflected in the member of staff being nominated by students in successive years for the GCU Students' Association Teaching Awards. Furthermore, the NSS in 2016/17 provided qualitative evidence of the high regard in which the service is held by students. A thematic analysis of student comments for the NSS identified several direct references to the value and effectiveness of MS in SCEBE.

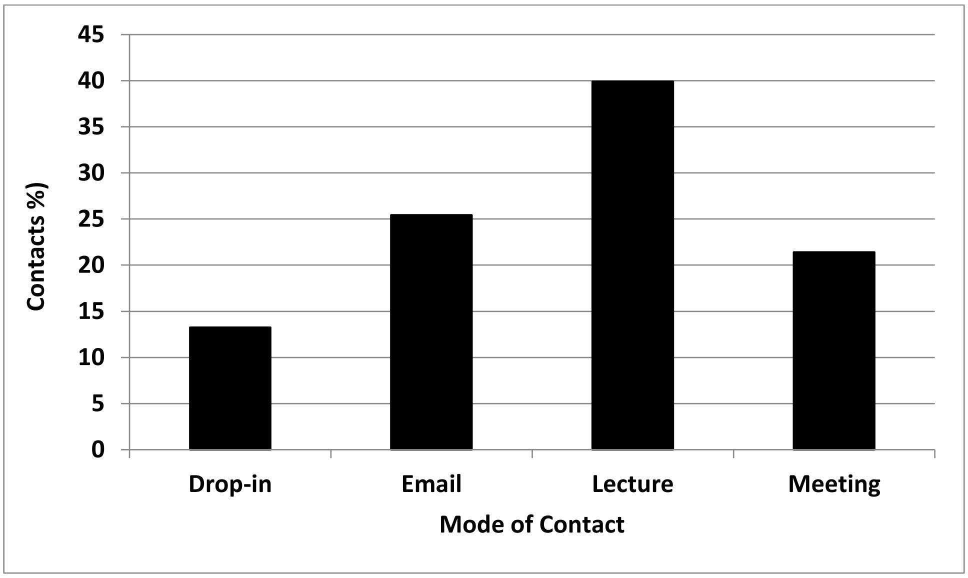

Figure 2: Modes of contact with MS for SCEBE students, 2013/14 - 2017/18

3. Methodology

The data used in this study were collected over the five year period from 2013/14 to 2017/18. Note that although MS started in 2012/13 a full data set was not available for that year. As discussed above the data relate to student engagement, and non-engagement, with MS through attendance at targeted lectures and the first diet module marks achieved by these students. Attendance at the targeted lectures was recorded using Student Attendance and Engagement Monitoring (SAEM) which is essentially a swipe card system. In order for a module to be considered for the study at least 10% of the cohort must engage with targeted lectures and only students attending 20%, or more, of classes are included. For example, for a module with 100 registered students at least 10 students must have attended 20% or more of the targeted lectures. Over a trimester the 20% threshold translates to engagement with two or more classes which, as observed by Lawson et al. (2001), indicates that the students consider their previous experience to have been beneficial thereby making a return visit to the class worthwhile. Based on these criteria between 2013/14 and 2017/18 a total of 47 undergraduate modules qualified for analysis and of the 4,690 registered students on these modules 1,142 (24%) engaged and 3,548 chose not to engage. The 47 modules range across all four levels of undergraduate study and some contribute to the data for each of the five years while others may only appear once due to factors such as being discontinued following a portfolio refresh. The study involved a total of 18 different modules as detailed in Appendix A. Figure 3 shows the distribution of the 1,142 engaging students and their parent department. The majority of engaging students (51%) are from the Engineering department as the modules these students study contain a significant mathematical element across all levels of study.

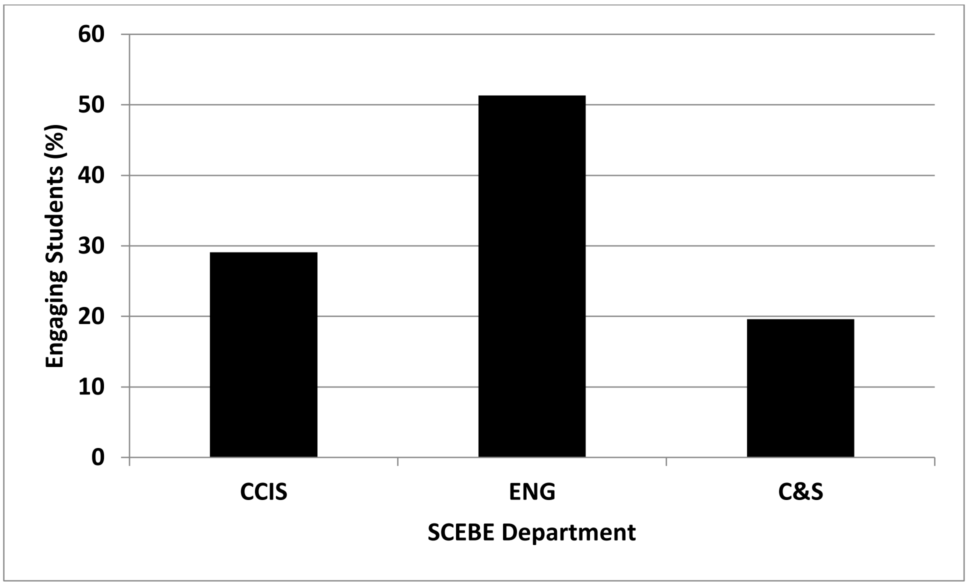

Figure 3: Distribution of students engaging with MS by department, 2013/14 – 2017/18

A point to note is that a small number of modules, especially in first and second year, are taught to students from more than one department. Engineering mathematics modules at Levels 1 and 2 are Department of Engineering modules but students from C&S also attend these modules. Hence, although C&S does not itself offer mathematics modules the department is included in Figure 3 as 20% of engaging students were registered with C&S.

Figure 4 shows how the 1,142 engaging students are distributed across the four undergraduate levels of study. Over 60% of these students are registered on modules at Levels 3 and 4. All Level 4 students come from Engineering programmes, while at Level 3 a total of 75% are from Engineering with the remaining 25% from CCIS. No C&S students participate in modules taught at Levels 3 and 4 for which targeted lectures are offered. The high number of students in their final two years of study engaging with MS can be attributed to several factors including; Levels 3 and 4 modules involving more advanced mathematics and module marks contributing to the final degree classification. Furthermore, a significant number of students in third year have articulated to GCU from the college sector and tend to engage with MS in greater numbers than continuing students. It is worthwhile noting that in 2017/18, 53% of third year students in the Department of Engineering were articulating students.

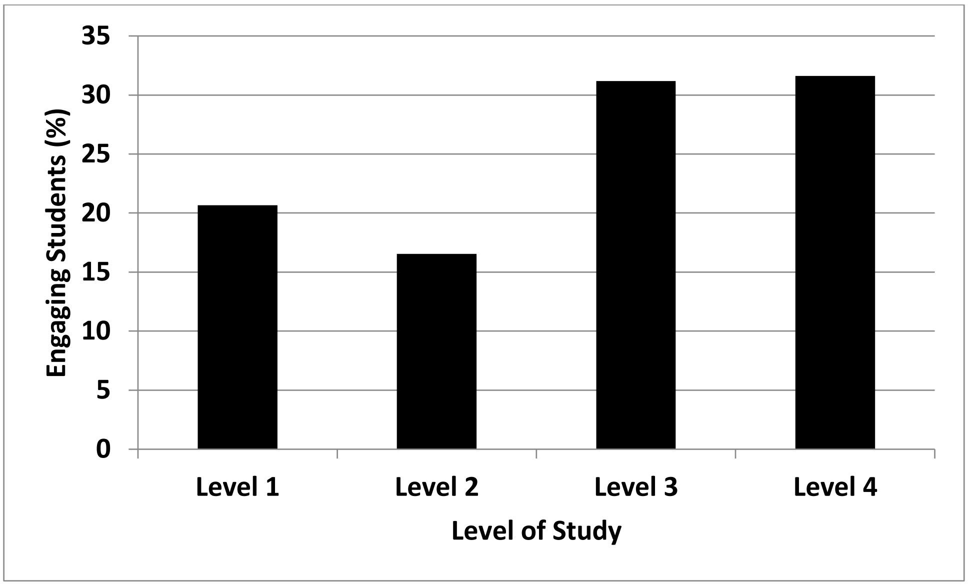

Figure 4: Distribution of students engaging with MS by level (year) of study, 2013/14 – 2017/18

4. Results

The majority of the modules featured in the study included an element of coursework and a final exam with the marks being combined to produce a module mark. In each year the average mark for each module was calculated for both engaging and non-engaging students. These average module marks were then themselves averaged to produce an average mark for that specific year for both groups of students. The results are plotted in Figure 5. It can be seen that for these modules students who engage with MS, by attending targeted lectures, clearly out-perform their counterparts who choose not to engage. When the marks for the 47 modules were averaged, to give a 5 year average, engaging students recorded a mark of 61% and non-engaging students 53%. Over the period engagement with MS therefore resulted in an 8% improvement in the average module mark. A point worthwhile noting is that of the 47 modules under consideration students who engaged with MS recorded a higher average mark for 39 (83%) of these modules.

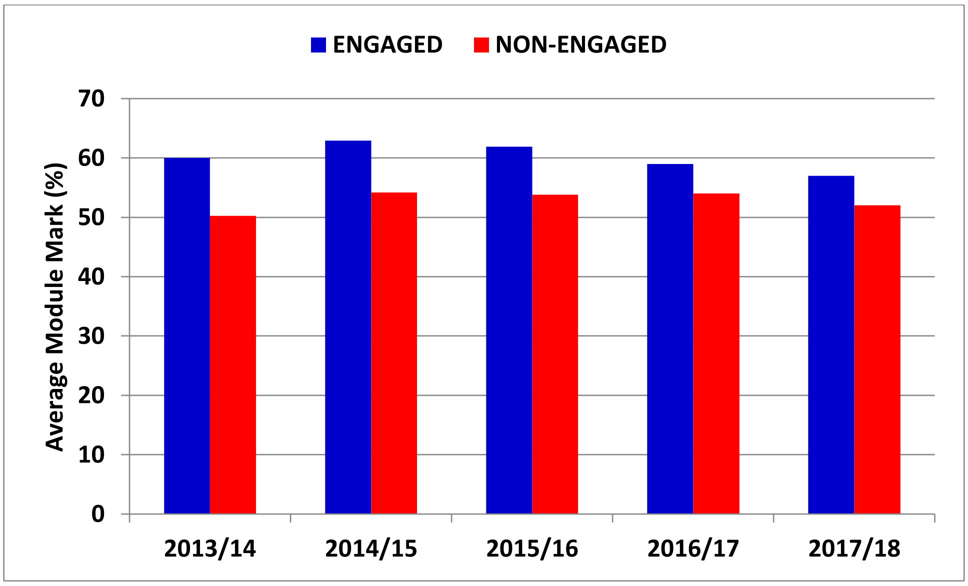

Figure 5: Average first year module mark for students engaging and not engaging with MS, 2013/14 – 2017/18

Figure 6 shows the average module mark achieved by engaging and non-engaging students over the period according to their level of study. It is clear that attendance at targeted lectures has resulted in an increase in module mark at all four levels. Over the first three levels the gap between the two groups closes from one level to the next which can perhaps be attributed to an improvement in students' mathematical skills and confidence growth as they progress on their degree programmes. At Level 4 however the gap increases to 9% emphasising the importance of engagement with MS in the Honours year. One influencing factor that has been identified through discussions with students is that of conflicting priorities where they spend a disproportionate amount of time on their Honours project at the expense of their other modules and attendance at targeted lectures.

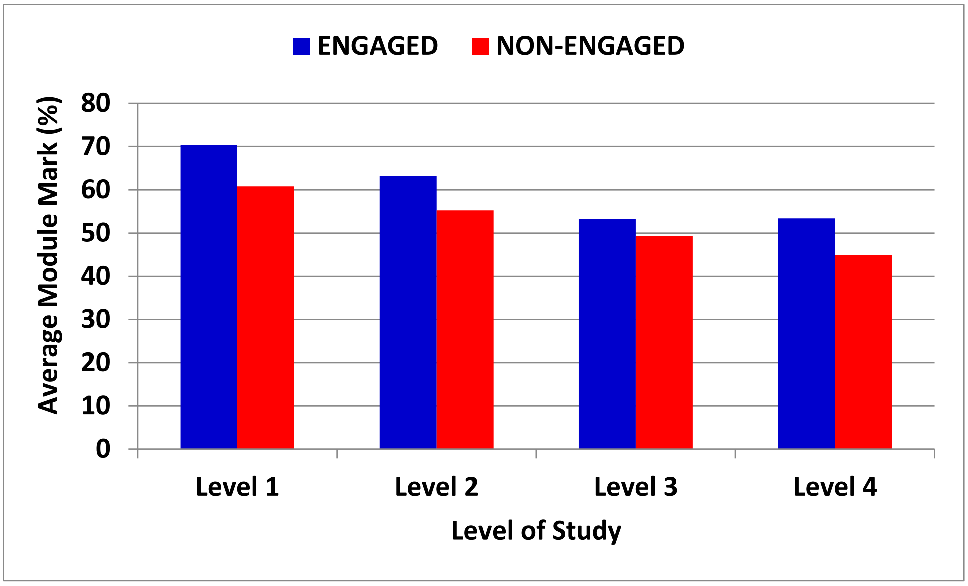

Figure 6: Average module mark by level (year) of study for students engaging and not engaging with MS, 2013/14 – 2017/18

5. Conclusions

First diet module marks for 4,690 undergraduate students, at all levels of study, were recorded over a five year period for those who engaged with MS, through attendance at targeted lectures, and those who did not engage. The data showed that attendance at targeted lectures resulted in an 8% improvement in the average module mark for engaging students when compared with the mark for non-engaging students. It should be noted that many of the students who attended the targeted lectures also used other forms of MS such as small group meetings and the drop-in facility. Clearly MS, and attendance at targeted lectures, cannot take all the credit for the results described here as other factors such as students' attendance at module lectures and tutorials influence their performance. Additionally, students engage with external support mechanisms such as tutors and online materials to enhance their understanding. Peer support also plays an important role with students forming study groups to work on aspects of modules, particularly mathematical topics, that prove challenging. Nevertheless, the results presented here are encouraging and provide a quantitative measure of the impact and effectiveness of MS on student performance and retention; a fact further supported by qualitative evidence from the NSS in 2016/17.

Looking to the future one aim would be to increase the number of students engaging with MS and ultimately improve performance and completion rates. Since MS was introduced in SCEBE in 2012/13 the number of student contacts has increased annually growing by 115% over the period and exceeding 3,000 in 2017/18. Incentives such as targeted lectures, which are included on student timetables, and offering contextualised mathematical material at module sites on the Blackboard VLE have contributed significantly to MS engagement. Numbers could perhaps be increased further by, for example, making attendance at targeted lectures compulsory for students who score below a pre-set threshold on the mathematics diagnostic test offered during induction. However, such a radical amendment to timetables is an institutional matter and in order to achieve these goals adequate resources such as staff and space, as identified by Cronin et al. (2015) and Ahmed et al. (2018) are essential.

6. Appendix

Table 1: School of Computing, Engineering & Built Environment modules qualifying for the study; level of module and number of years targeted lectures delivered.

Module	Level	No. of years targeted lectures delivered
Mathematics for Computer Games 1	1	4
Discrete Mathematics	1	1
Engineering Mathematics	1	3
Mathematics for Computing	1	4
Mathematics 1	1	2
Mathematics & Statistics of Experimentation	1	1
3D Mathematics for Simulation and Visualisation1	2	2
Data Communications & Transmission Systems	2	3
Mathematics 2	2	3
Mathematics 2A	2	1
Mathematics 2B	2	1
3D Mathematics for Simulation and Visualisation	3	2
Control Engineering 3	3	5
Plant & Electrical Distribution Systems	3	3
Quantitative Modelling and Cryptography	3	5
Control Engineering 4	4	4
Digital Signal Processing	4	1
Power Systems Technology	4	2

Note that in 2016/17 the Level 3 module, 3D Mathematics for Simulation and Visualisation was revised and re-assigned as a Level 2 module.

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CASE STUDY

Enhancing the student experience with the use of a dedicated subject website

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Abstract

It is crucial not only to support students at all stages of their student journey, but also to create a space where they can benefit from peer support and interact with the wider mathematics and statistics (M&S) community. In a society awash with social media, it is possible to create online spaces that complement and enhance existing communities available in traditional face-to-face courses, or to provide such an environment for students who learn at a distance. The School of Mathematics and Statistics at The Open University (OU) has recently consolidated existing resources into a website resulting in an active and vibrant community of learners. The site contains resources, which students access at appropriate points in their student journey through M&S modules and qualifications. These resources are complemented by a number of dedicated and well-used online forums. In particular, a forum providing course choice information. Discussions in the forum have ultimately led to improvements in the structure of M&S qualifications, influencing the content of new modules, more effective assessment strategies, and better ways of supporting students. It is a true community of learners, where everyone - students, academics and educational advisors - all contribute, learn from each other, and shape the student experience.

Keywords: Student experience, community of learners.

1. Introduction

Higher education has changed dramatically during the lifetime of the Open University (OU), resulting in the need for an increasing awareness of the entire student journey from first contact with the university through to becoming an alumnus. Morgan (2011) sets out the Student Experience Practitioner Model, which covers the six stages of the student lifecycle: first contact and admissions, pre-arrival, arrival and orientation, induction to study, re-orientation and re-induction, and out-duction. She argues that there are five themes to consider for each stage of the lifecycle: curriculum and assessment, pedagogy, support, finance and employment. This requires staff from across the institution to be actively involved in initiatives and activities, to support the student experience. The OU has its own version of the Student Experience Model, known as the Student Seamless Journey, which encompasses all phases from the first point of contact with the OU to the point where the student achieves their qualification.

The School of Mathematics and Statistics has always provided opportunities for students to engage at all stages of the student journey. In the early years of the OU this happened at annual well-attended and lively meetings. These were held in numerous centres across the country, giving students the chance to talk to academics, support staff, and each other. Students had the opportunity to discuss their career aspirations and relevant module choices. Over time, attendance at these meetings started to dwindle. During the 1990s, as use of the internet increased, the School of Mathematics and Statistics capitalised on this shift by setting up an online module advice forum to both augment and to provide an alternative to the face-to-face events. The forum aimed to offer peer support and advice, in conjunction with expert input from members of the School and subject-specialist educational advisors. In addition, resources that assisted students with their module choices were assembled and developed. For example, tables of examination results and student satisfaction ratings on individual modules, accumulated over several years, were provided so students could observe trends and inform their study planning. In order to deliver such information to students in an efficient way the School created a bespoke website (the M&S programme site) which ran for many years.

In 2017, the University introduced Subject Sites with the aim of providing a home website for students throughout their qualification. The School of Mathematics and Statistics took the opportunity to develop its site as a one-stop-shop to support the student journey. The resources, which the School had been delivering for many years through the M&S programme site, were updated and migrated to the M&S subject site. These were arranged in such a way as to mirror, support and enhance the student journey. This report outlines many of the resources on the Subject Site and, in some cases, how these have evolved over time.

2. The mathematics and statistics subject site

Each Subject Site is split into six sections: **Study Home, Connect, Discover, Skills, Plan** and **Succeed**, which are generic to all Subject Sites (Figure 1). The content that populates each section is at the discretion of the owning curriculum team, except for **Study Home,** which conforms to a standard template. This includes links to appropriate media sites; in the case of M&S the School's twitter and the Faculty's Facebook pages. The **News** and **Upcoming events** areas are also standard areas on every Subject Site, but the School of Mathematics and Statistics uses them in very specific ways. **News** provides students with news and notice of relevant events, both within the OU and in the wider mathematical and statistical community. **Upcoming events** is used to signpost students to relevant parts of the Subject Site at particular points of the student journey. Although the old programme site contained the supporting resources, it did suffer from a lack of facility to direct students to those resources at the particular time they needed them. **Upcoming events** provides this opportunity for time-dependent nudges. One such example is the reminder to students to consult **Plan**, which includes information about study pathways through qualifications and pre-requisites, when registration opens. Once registration is completed, students are prompted to prepare themselves for the individual module(s), under **Discover**, together with the **Skills** needed for successful study. In M&S, as is the case for many other subjects, it is important that new students have induction that is specific to their subject. Such material is on the **Discover** tab, together with **Are you ready?** module quizzes, and **Revise and refresh** module preparation websites. The site not only provides a place for student engagement between modules, but also between qualifications and beyond, with each of the five main content sections having a corresponding postgraduate subsection.

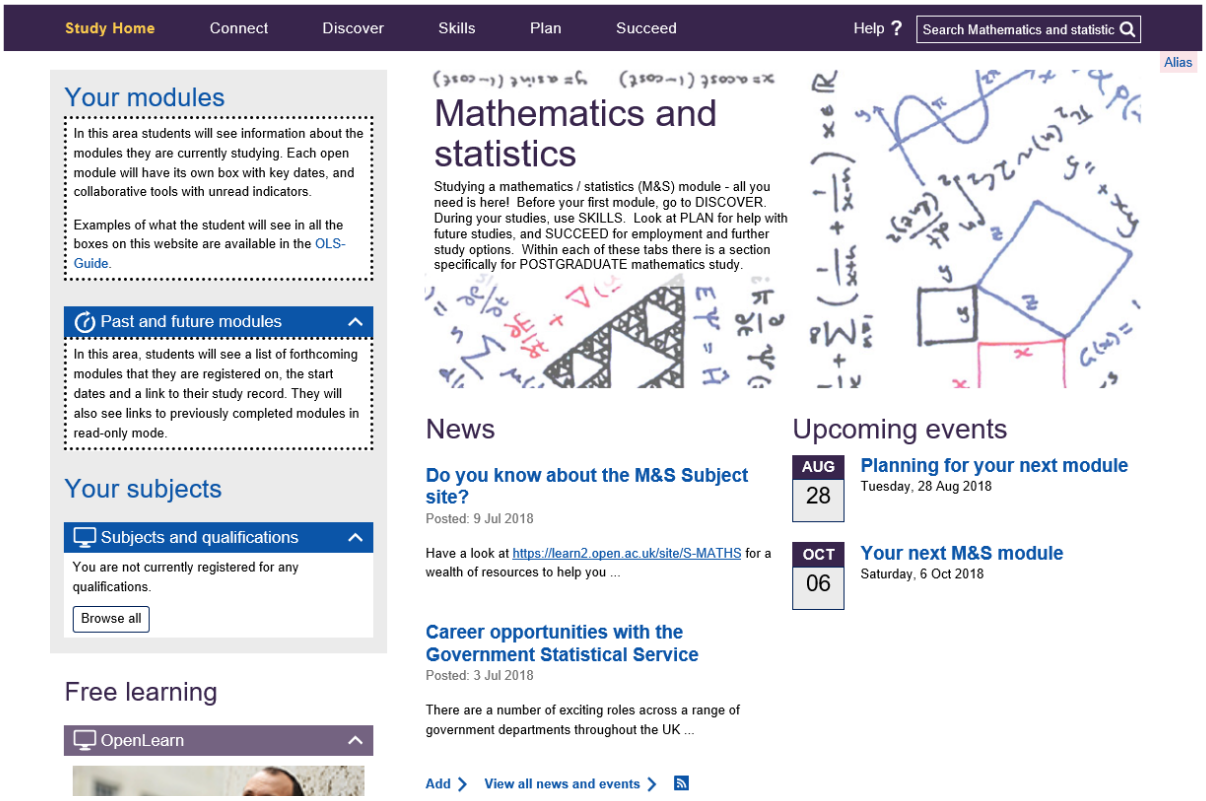

Figure 1: The Mathematics and Statistics Subject site.

2.1. Discover

As mathematics is a cumulative subject, the open entry policy of the Open University makes it imperative that each M&S student starts at a point appropriate to their own mathematical expertise. There are two initial modules available to OU students. *Essential Mathematics 1* (MST124) is a core option in around 20 OU qualifications, but it does assume pre-requisite mathematical knowledge of roughly A-level standard. It is therefore possible, in a number of qualifications, for students to start with an alternative module *Discovering Mathematics* (MU123). Although, induction and preparatory resources for these modules are in the **Discover** section, new students must register onto one of these modules *before* they have access to the Subject Site and therefore relevant information, advice and guidance (IAG) needs to be publically available.

In 2014 a survey into the learning and teaching experiences of students, carried out by the Higher Education Policy Institute (HEPI) in conjunction with the Higher Education Academy (HEA), showed that one in five first-year students found information provided by institutions to be 'vague' (Soilemetzidis et al, 2014). It is clear that information alone is not enough to give adequate support, so it must be complemented by sufficient advice and guidance (Diamond et al, 2014). The School thus replicates and extends the IAG about these two modules on its publically available **MathsChoices** website, (Open University, 2018). **MathsChoices** sits outside the University's VLE enabling prospective students to work through the resources and ascertain their correct starting point before they begin the registration process.

In the **Are you ready for?** subsection of **Discover** there are links to self-assessed quizzes for the entire suite of undergraduate M&S modules, plus the diagnostic quiz for entry to the MSc in Mathematics. The aim is for students to develop an awareness of the prior knowledge needed for their next module, and to identify what they need to consolidate at the start of the next stage of their journey. Student surveys and consultations have provided clear evidence that they appreciate the ability to self-diagnose their starting point:

“I did find the diagnostic quizzes very helpful they made it very clear that MU123 was where I needed to start this course. I was free to have a go with MST124 and it was very evident from this experience that I didn’t have sufficient knowledge. Also I felt my results of the diagnostic quizzes married well with the information given on the website about where the OU thought you should start. This gave me greater confidence to subscribe and get on with the course”

Students also appreciate assistance with identifying appropriate level 2 and 3 modules:

“I have used the diagnostic quizzes (AYRF…) on a few occasions, especially the level 2 and 3 maths practice prior to doing a course, and what was required for the course. It is a good way to refresh areas prior to starting a course and also testing previous knowledge – if you don’t use it you lose it!”

One of the most effective ways to reduce the number of students dropping out is to identify and respond to students who are at risk (Foster et al, 2012). Evaluation has shown that each student’s M124 quiz score on the **Are you ready for?** quiz gives the best indicator of future success on MST124 (Calvert et al, 2016). Hence there is a proactive calling campaign based on the quiz scores; one outcome of these conversations is to direct students to the **Revise and Refresh** for MST124 resources described below. This is another example of wrap-around support centred on the academic unit, with input from all stakeholders to develop the resource and then utilise it to full effect; thus a true community of practice (Lave and Wenger, 1991).

Students often have a study gap between one M&S module and their next, either because they take a break in their study, and/or because they are taking M&S modules as part of a non-M&S qualification. In order for a student to brush up their mathematical knowledge and skills before embarking on the next stage of their journey there are **Revise and refresh (R&R)** resources in the **Discover** area. These resources give students the chance, before starting their next module, to consolidate their learning, re-visit and practice key topics, and improve in areas where they may not have fully engaged in previous study. In each section, the resources include quizzes, short explanatory texts, screencasts of worked examples, tutor-moderated forums, and boot-camp style tutorials, which are timetabled to run in the break between modules. Each **R&R** subsection, focusses on an individual topic. Students are able to self-diagnose what they need to revise by taking the quiz for particular topic (Figure 2). The quiz can be repeated multiple times to enable students to check their progress. There are brief refresher materials and cross-references to where the topic is in the pre-requisite module, and screencasts where a tutor talks through how they tackle a typical example. For each topic there are exercises which the students can use to practice their skills, and a tutor-moderated forum for them to ask further questions.

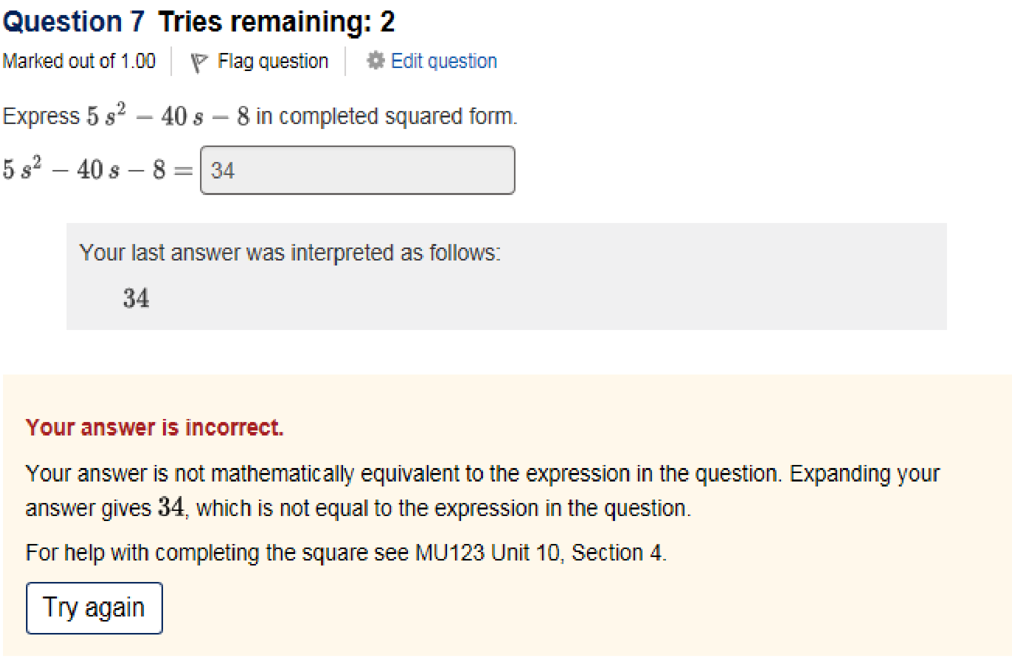

Figure 2: Revise and Refresh for MST124: Quadratic expressions and equations quiz

The provision of taster material for both undergraduate and postgraduate students is very important, and the **Discover your module** section of **Discover** has sample units from every undergraduate and postgraduate module. Students comment on the benefits of such resources:

"I am grateful that we can download the first two chapters of the next module ahead of time, so you can take a couple of hours here and there (on the train, waiting for a flight or just downtime you have at home) to look at things with no time pressure, ease yourself slowly back into it and get a feeling for the course".

The combination of **Are you ready?** quizzes, **Revise and refresh** resources and the sample content in **Discover your module** aims to ensure that students choose appropriate modules, and have the appropriate pre-requisite knowledge when moving onto their next module.

2.2. Skills

The **Skills** section contains advice about how to study mathematics and statistics modules effectively. There is a subpage entitled **Learning mathematics and statistics** which has resources covering a range of study skills specific to M&S these include: **How to study mathematics and statistics effectively**, **Problem solving**, **Writing mathematics and statistics**, and **Mathematical proof**, with plans for sections on **Mathematical modelling** and **Working with Data**. A further recommendation from Foster et al (2012) is to ensure there is good communication about, and access to, additional student support. In order to achieve this there is a dedicated section on overcoming accessibility issues when studying mathematics and statistics, containing detailed information for students who need additional resources and alternative formats. The **Skills** section also contains pages on **Good academic practice and Plagiarism**, **Assessment preparation and submission**, **Advice about examinations** and **Calculators**, because each of these aspects has specific connotations in the context of studying M&S. Finally, there are resources dedicated to helping students with **Typing mathematical notation,** and about the **Software** used in mathematics and statistics modules.

2.3 Plan

When registration opens each year, students are reminded, via the **Upcoming events**, about the information under **Plan**. The **Plan** resources include a collated table of the last four years of pass and completion rates for each M&S module, and a further table giving the last four years of student feedback from in-house student-satisfaction module surveys. Although this information is published elsewhere, it is not readily available and certainly not collated together in an easily comparable format. **Plan** also provides a location in which to outline any proposed significant changes to, or new, curriculum.

2.4 Succeed

The School of Mathematics and Statistics has run several subject-focussed careers events, and therefore has close links with the University's central careers service. There is evidence that only a small number of undergraduate students feel they have really developed the skills necessary to help them get a job, such as CV writing or career planning (Neves, 2016). Whilst a number of OU students are studying whilst in employment, many are looking to change career and a growing number are full time students yet to enter the job market. Therefore, the **Succeed** section includes; general careers advice, guidance around what employers are looking for in job applications; ways in which students can enhance their employability through developing specific expertise; advice and guidance on job-seeking and making an application. There is also advice on studying beyond an undergraduate degree. Most of these resources were produced through collaborative work between the central OU careers service and the School of Mathematics and Statistics.

2.5. Connect

Whilst each of the tabs on the subject site provides IAG for the student journey this does not in itself create a community. Crosling et al (2008) state that "*Students are far more likely to continue with higher education if they are engaged in their studies and have developed networks and relationships with fellow students.*" This is exactly what the **Connect** tab provides. There are some dedicated time-limited forums for **Revise and refresh**, **Early start** and other specific events. As many students are working towards an M&S teaching career whether that be Primary, Secondary or Further Education, there is a specialist forum for topical issues related to mathematics education. However, the heart of the entire subject site is the **mathematics and statistics advice forum**. In that forum the entire M&S community (students, tutors, academics and support staff) discuss future study plans, how different modules may help with future careers and what it is like to study particular modules. Essentially anything related to module choice and study planning. In addition, students provide first-hand feedback on all aspects of the student experience in terms of both curriculum development and general student support. As one student expressed, it is *"the most interesting forum provided by the OU"*. Its success lies in the wide range of contributors and the acceptance that everyone is listened to. It is held up as the blueprint all Schools should aspire to in terms of providing a space for student feedback. Internal publications and events delivered by the School have provided advice for other OU Schools when developing their resources to populate their own subject sites.

3. Conclusion

The School of Mathematics and Statistics has a long tradition of providing IAG, in a variety of forms, to help students make informed decisions regarding their own study. The value of creating an M&S community has always been important, especially when face-to-face opportunities diminished, and the School embraced new online ways of achieving this. The advice forum is a jewel in the School's crown, and is not only seen by students as the go-to place for asking questions about module choice and study planning, but as a mechanism for their contributions and opinions to lead directly to improvements in qualification and module delivery. The importance of making changes, based on their feedback, means that students feel that they are a valued part of the learning community

(Howson, 2014). The M&S Subject Site is a place where students, tutors, academics and support staff feel empowered and equally valued in their contributions. All contributors reference relevant areas within the site in response to questions, and those resources, and the way they are organised, mirror the student journey and enhance the student experience. The M&S Subject Site is a resource for a true community of learners in mathematics and statistics provided by an active community of practice.

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CASE STUDY

Piloting a problem solving module for undergraduate mathematics students

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Abstract

We report on a new problem solving module for second-year undergraduate mathematics students first piloted during the 2016-17 academic year at Cardiff University. This module was introduced in response to the concern that for many students, traditional teaching and assessment practices do not offer sufficient opportunities for developing problem-solving and mathematical thinking skills, and more generally, to address the recognised need to incorporate transferrable skills into our undergraduate programmes. We discuss the pedagogic and practical considerations involved in the design and delivery of this module, and in particular, the question of how to construct open-ended problems and assessment activities that promote mathematical thinking, and reward genuinely original and independent mathematical work.

Keywords: Problem solving, mathematical thinking, employability, transferrable skills.

1. Motivation

The ability to apply mathematical knowledge to tackle unfamiliar problems is widely recognised as an essential skill that all mathematics graduates should possess (QAA, 2015; Badger et al., 2012). However, there is widespread concern that for many undergraduates, 'traditional' teaching and assessment methods do not offer sufficient opportunities for developing these skills (Rowlett, 2011, p.19).

Given that exams are the main method of summative assessment in mathematics (Iannone and Simpson, 2012), it is natural that much of students' independent study time is spent preparing for them. However, in most exams, time constraints make it impossible to ask students to solve genuinely challenging problems, and further, there is limited scope for rewarding original and interesting mathematical work. While regular exercise sheets and textbook problems may provide an avenue for assigning more difficult work in many modules, not all students engage with these in the way we might like – anecdotally, it would appear that many view the problems as templates for future exam questions, and use them (together with model answers, if these are provided) simply as a revision tool closer to exam time. As a consequence, the problems that students encounter are often predictable and narrow in focus, and there is a risk that students may graduate with reasonably good grades without important mathematical skills being developed (Selden, Selden and Mason, 1994).

In addition to these mathematical considerations, the need to integrate more general transferrable skills into undergraduate mathematics programmes is also widely acknowledged (Challis, Houston and Stirling, 2003, pp.14-17; Waldock, 2011). For example, while good mathematical writing is encouraged, this is often overlooked when marking exams, and students are rarely asked to produce more substantial pieces of written work. The ability to communicate mathematical work orally has been identified as another area of weakness. Collaboration and group working, while encouraged by many lecturers, is also rarely formally embedded in modules and their assessment. In light of the current institutional and regulatory focus on employability in UK Higher Education, it was a challenge to not only develop these skills, but importantly, to do so in a meaningful way within the context of university-level mathematics.

2. Designing a new module

In response to the concerns raised in the previous section, a new module entitled *Problem Solving* was introduced in the 2016-17 academic year. This is a second year (level 5), 10 credit module, which has been piloted as an optional module, available to all single honours mathematics students on our undergraduate programmes (including integrated masters' students). .

The decision to introduce Problem Solving as a second-year module was partly motivated by research that has indicated that many students become significantly disengaged with their studies during the second-year of their undergraduate programme, and often do not recover from this (Croft and Grove, 2015). Moreover, we wanted to students to attempt problems that made use of the results and methods from their first-year modules, thus it was necessary for them to have encountered enough university-level mathematics beforehand.

In this module, there would be very little formal teaching, and no new mathematical content - students would spend the majority of class time working on unseen, open-ended problems using their existing mathematical knowledge. This work would be carried out in small groups, which would remain fixed throughout the module. Finally, the module would be assessed entirely through (group) reports and presentations, and the marking criteria would reward evidence of genuinely original mathematical thinking, rather than simply results.

Contact time for Problem Solving consists of a single three-hour block per week. The first week is an introductory session, explaining the aim and structure of the module. Most of the remaining classes are tutorial-style 'problem solving sessions' where students are presented with a choice of two unfamiliar problems, which they work on during class and submit a short summary of their work at the end.

A major part of the lecturer's role during these sessions is to advise students on how to work independently, rather than depending on the practice of imitating similar work. For example, when students are struggling, this may involve suggesting questions to investigate to gain insight into the problem at hand, as an alternative to searching for solutions on the internet. Equally important is to reassure students that when doing mathematics, much time is spent being stuck, confused, uncertain, or simply wrong, and that this is not a sign of failure, but a necessary step along the way.

Half-way through the semester, there is a peer-assessment session where groups exchange drafts of their work and give feedback. During this time, we also discuss how students might demonstrate evidence of intelligent problem solving strategies, with reference to the ideas in Polya (1945), and Mason, Burton and Stacey (2011). During the final two weeks of the semester, students give group presentations on a problem of their choice.

3. What makes a good problem?

The most challenging task when designing this module was creating a set of suitable problems, specifically problems that demand aspects of mathematical work that are typically overlooked in other modules, while at the same time make use of results from our undergraduate mathematics curriculum. Although problems from similar courses being taught elsewhere can be found in the literature, such as in Badger et al. (2012), we found that many of them did not offer sufficient opportunity to make use of university-level mathematics, particularly topics in pure mathematics. Based on some of the ideas discussed in Badger et al. (2012, p.27) and Mason (2000, pp.104-105), we derived properties that we suggest such problems should have, which we discuss here.

Unlike the questions that students encounter on exercise sheets and exams, most mathematical work, both in academic research and elsewhere, does not begin with a precise question and end with an elegant, complete, and correct solution, but instead involves an iterative process of examining examples, restricting to special cases, making, testing, and refining conjectures, and many other activities (Hirst and Biggs, 1969). By contrast, when working on exercise sheets, the student typically knows from the outset that the problem does indeed have a solution, and moreover, that the lecturer expects that they ought to able to find it. Rarely are students assigned problems that are ambiguous, open-ended, or where they could only ever be expected to arrive at a partial solution.

For this reason, we tried to design problems that both required some effort to translate into a precise mathematical question or questions, and admit many possible routes towards a solution, and ideally, some ambiguity as to what constitutes a valid solution. Some degree of evaluation of one's own work should be required, which may take many forms - testing examples, looking for a more elegant solution, seeking generalisations (or showing that they cannot be found).

Given that an exercise sheet is associated with a particular module, it is usually clear from the beginning that the necessary tools for solving these problems will be found mostly within the syllabus. Again, this is somewhat artificial, and we tried to address this by designing problems that were not tied to any particular topic, and where students were free to research new material as part of the process of solving it.

A major difficulty with this approach was the question of how to deal with students with different abilities. Indeed, a reasonably challenging problem for one student may be a trivial exercise for another, and completely impossible for a third. To account for this, we tried to design problems that would allow any reasonable student to reach a 'partial solution' (e.g. covering a simplified analogy or elementary special cases of the problem), and moreover, had significant scope for generalisation that a stronger student could undertake.

4. Example problems

In this section we give some examples of the problems assigned to students in this module. Broadly speaking, these fell into two categories - problems from pure mathematics, and coding problems (all students take a Python course in year one).

The first example of a purely mathematical problem was adapted from Mason, Burton and Stacey (2011).

Problem 1: Let S be the set of those functions $f: R \to R$ having the property that, for all intervals $[a, b]$, the mean value of f' over the interval $[a, b]$ occurs at the midpoint. Identify some suitable conditions that can be used to determine whether or not a given function belongs to S.

Problem 1 reflects a common theme in research mathematics - trying to obtain a useful classification of objects satisfying a certain property. There is significant uncertainty involved for the student in determining what might constitute a 'suitable condition that can be used' – indeed, it is necessary to experiment with many examples before making a conjecture about what a 'solution' to this problem might look like.

Once they have made a suitable conjecture, an average student should manage to answer this question completely for polynomials using some fairly elementary algebra, while going beyond this demands the use of more advanced tools (e.g. Taylor's Theorem). For a student who does answer this question more or less completely, many interesting generalisations may be prompted - for example, is there an analogy of this property in higher dimensions? Are there functions f for which the mean value of f' over $[a, b]$ always occurs at a point that divides the interval $[a, b]$ according to another fixed ratio?

The second example is a typically example of one of the coding problems in this module.

Problem 2: Approximating π. Using as many approaches as you can think of, try to construct some algorithms/formulae that can be used to approximate π using only integers and the operations of addition, subtraction, multiplication and division. Evaluate and compare any successful approaches you have found, and explain the difficulties that arose in your unsuccessful attempts.

At a first glance, this problem looks quite straightforward, until students realise that most obvious approximations of π make extensive use of irrational numbers e.g. using the perimeter or area of a polygon to approximate that of a circle. A significant amount of work is needed to find any suitable algorithms. Some interesting examples of student work on this problem used Taylor Series for inverse trigonometric functions, and various methods of numerical integration to estimate the area of a circle.

Some problems lead to some genuinely impressive work being submitted by students, such as the following, also adapted from Mason, Burton and Stacey (2011).

Problem 3: A point $(x, y) \in R^2$ is called rational if both x and y are rational numbers. For a curve C in R^2, let $N(C)$ denote the (possibly infinite) number of rational points that lie on C. Determine the possible values that $N(C)$ can take for various types of curve C (e.g. lines, circles, parabolas). For each value of $N(C)$ that you find, you should attempt to describe the set of all curves of this type that have exactly this number of rational points (e.g. a circle C has $N(C) = 1$ if and only if ...).

Not all problems assigned to students in this module have been a success. Our final example was adapted from the Industrial Problem Solving for Higher Education (University of Bristol, 2016) website.

Problem 4: As part of a manufacturing process, it is necessary to cut discs of radius 12.5cm, and discs of radius 5cm, from square sheets of steel of size $1m^2$. They need twice the number of small disks as they do large disks. You job is to identify the best arrangement of cutting heads to minimise the amount of wasted steel. Design and test (using Python) some algorithms for finding the optimal arrangement of the discs, and compare their performance.

This looks like a reasonable problem at first - there are many natural initial steps that can be taken, and a large number of potential approaches that students could undertake. However, on closer inspection, most strategies that a year 2 student could reasonably be expected to pursue either lack non-trivial mathematical content (such as using basic geometry and examining arrangements by inspection), or else are far too difficult. Problem 4 illustrates the need for the lecturer to test problems extensively for suitability before assigning them.

5. Assessment

In the 2016-17 academic year, Problem Solving consisted of 11 three-hour classes, one per week. During 9 of those classes, students were assigned a new problem at the start of each class, and spent the class working on that weeks' problem in groups. For the summative coursework, each group was required to submit a total of 18 pieces of work, consisting of:

- a short summary of their work, handed up at the end of each class, worth 5% of their grade, and;
- a more detailed report (typeset in LaTeX) prepared over the following week, worth another 5% of their grade.

Note that all reports were group reports, and each member received the same grade irrespective of contribution. The remaining 10% of their grade came from a group presentation on one of their reports.

In the written reports, rather than simply presenting their solutions, students are instructed to outline the steps that they took, and their reasons for taking these steps, as well as evaluating any results that they obtained. Credit is awarded for demonstrating an intelligent problem-solving strategy, rather than simply the progress made towards a complete solution. The hope is that by writing about the process of doing mathematics, students will recognise general themes in mathematical work, and strategies that one might undertake in attacking an unfamiliar problem. Moreover, they are encouraged to pursue any interesting related questions that are prompted by their work, even if not directly relevant to the problem statement – students are rewarded for any genuinely original work that contributes to their reports.

Identifying and preventing academic malpractice is a significant concern in written coursework in mathematics (Challis, Houston and Stirling, 2003, pp.23-25), particularly since solutions for almost any conceivable mathematical problem can be found on the internet. We did not aim to restrict the use of external resources, but instead to set assessment criteria that discourage the practice of imitating similar work. By assigning ambiguous, open-ended problems, and by requiring that students justify and evaluate their work, we believe that any unfair advantage derived from finding solutions to similar problems is minimised.

6. Evaluation and future outlook

Enrolment in Problem Solving was initially capped at 32 students in 2016-17, though it is now available as an optional module for all second year undergraduates in the School of Mathematics. A total of 25 students took Problem Solving in 2016-17, followed by 39 students in 2017-18, out of a total of approximately 160 second year students. In both years, just over 40% of those enrolled in the module have been female, which is broadly in line with the gender ratio across the School of Mathematics.

Some interesting patterns emerge when comparing those students who took Problem Solving to their cohort as a whole. In both 2016-17 and 2017-18, those students who chose Problem Solving in year 2 had had a significantly lower year 1 average than those who did not (see Figure 1). One possible explanation for this is that a module that is assessed entirely with coursework may appeal to students who under-perform in written exams. In the medium term, it will be interesting to study how these students perform over the course of their degrees, and in particular, to investigate whether or not choosing Problem Solving is associated with any improvements in academic performance.

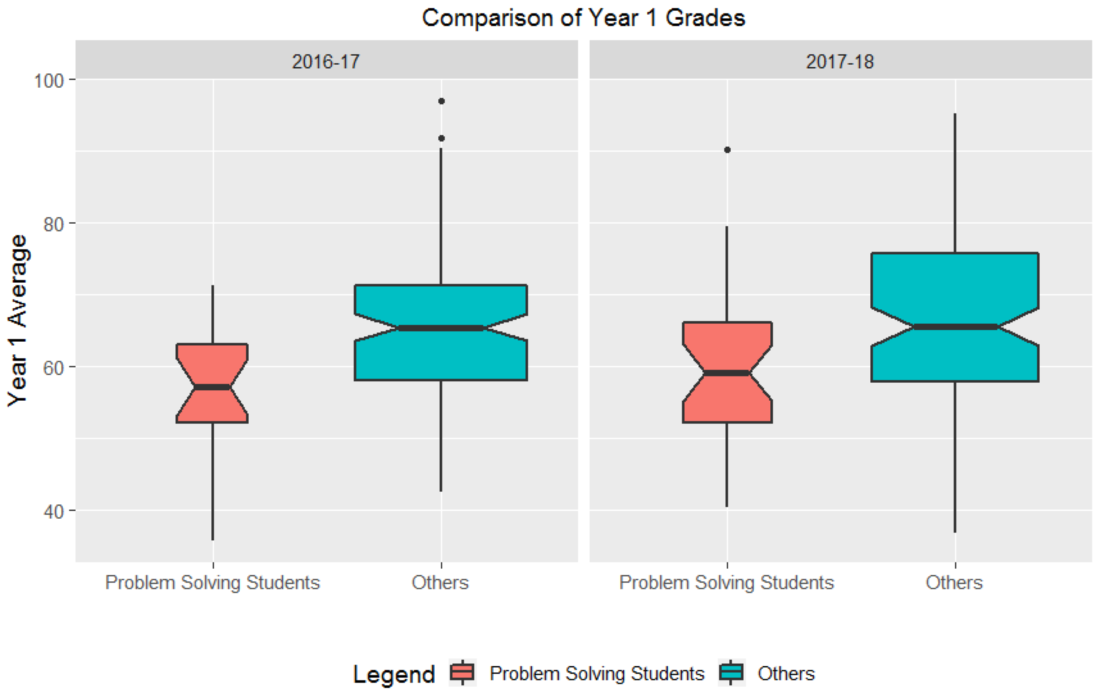

Figure 1: Year 1 grades for 2016-17 and 2017-18, comparing students who chose Problem Solving to the rest of their cohort.

The fact that a large number of apparently weaker students have been choosing Problem Solving was unexpected, though may present a useful opportunity as these students could potentially benefit the most from a module with a significant focus on mathematical thinking, independence and original work. However, there is a risk that if this continues, the module may be viewed by both students and academics as a 'soft' subject, that can be chosen in order to avoid taking a more challenging subject in year 2 - thus we may face the task of convincing students who perform well in traditional examinations to take a module that lies outside their comfort-zone.

Student feedback has so far been overwhelmingly positive (Table 1), though only a small number of students responded. We initially feared that open-ended nature of the problems and assessment tasks would be intimidating for students, however it would appear that many students enjoy having the flexibility to explore questions that they set themselves. The responses suggest that students also appreciate the value of a module that develops their employability skills.

Table 1: Module Evaluations for Problem Solving, selected questions

STATEMENT	% AGREE 2016-17 (N=10)	% AGREE 2017-18 (N=6)
The module has helped my personal development by improving my employability skills (e.g. Presentation skills, communication skills, reasoning skills, independent learning).	**100**	**100**
The module inspired interest and was intellectually stimulating.	**90**	**100**
I had a clear sense of what was required of me in the assessment.	**100**	**100**
Overall I am satisfied with this module.	**100**	**100**

Despite this positive student feedback, it quickly became apparent to the lecturer that the volume of summative coursework during the pilot year was excessive. While most groups could make a reasonable attempt at each week's problem, students were unhappy with the workload involved. More importantly, there was insufficient time for many of the activities that we wanted to promote, such as reviewing and evaluating solutions, or looking for generalisations, extensions and applications of their work.

Therefore, during the second iteration of Problem Solving, it was decided that the work submitted at the end of each class would be purely formative, and of the problems attempted during these classes, students would choose one from the first half of the semester and one from the second half, and prepare detailed written reports on each of these, worth 20% and 50% respectively. The group presentations would now be worth 30%. The quality of submitted coursework appears to have improved significantly as a result of these changes.

In summary, Problem Solving in its current form appears to be working as intended – some of the coursework submitted, particularly in 2017-18, has contained genuinely impressive, original mathematical work. Nonetheless, some aspects will need to be reviewed over future iterations of the module. In particular, certain problems need to be reworked or replaced, and ensuring that student work is being assessed reliably and consistently is an ongoing challenge.

We hope that eventually, all second-year mathematics students will take this module, though some changes would be needed to make this possible. The three-hour tutorial style classes could not be taught by a single lecturer, nor could these take place in a traditional lecture hall. Even with a relatively small number of students, a significant amount of time is spent on assessment and feedback, as well as meeting groups of students during office hours. However, we would prefer to see the current format of this module remain broadly unchanged.

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CASE STUDY

Thematic problem solving: a case study on an approach to teaching problem solving in undergraduate mathematics

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Abstract

Specialist mathematics, statistics and operational research (MSOR) programmes are recognised as intellectually demanding, and require students to formulate, abstract, and solve mathematical problems in a rigorous way. The process of developing the skills to do this well and communicate results can be challenging for learners as it requires a deep understanding of themes in mathematics as well as methods for solving problems. In this article we demonstrate how elements of Freudenthal's Realistic Mathematics Education can be applied to teaching problem solving in undergraduate mathematics programmes. We describe an approach that moves away from standard practices and goes beyond problem solving methods to develop an understanding of common themes in mathematics.

Keywords: Problem solving, mathematical education, realistic mathematical education, cognitive process.

1. Introduction

In this paper we discuss an approach to teaching higher level mathematical problem-solving skills to specialist undergraduate mathematics students. Our approach introduces students to the step-wise problem solving methodology found in Polya (1957), Mason et al. (2010) and to a lesser extent Bransford and Stein (1993). However, our procedure goes beyond these kinds of cognitive training programs and aims to develop students' familiarity and confidence in usage of themes in mathematical proof. We define a theme as an argument or portion of an argument that is common to a number of proofs that students encounter in their studies. This definition stems from the observation that most arguments in mathematics are made up of smaller, common reusable arguments. For example, the standard proof of the uniqueness of the identity element in a group remains essentially unchanged if one replaces a group with almost any algebraic structure. Similarly, in number theory a common theme when studying integers is to rephrase a problem in modulo arithmetic where only finitely many cases need be considered. When dealing with convergent sequences in analysis it is often useful to split the sequence into a finite part and the tail of the sequence.

Each of the themes discussed in the preceding paragraph, and many others, are typically encountered early in undergraduate mathematics degrees. In this article we argue that for students to develop their problem-solving skills they not only need to train their cognitive thinking processes but also need to recognize and collate a library of themes and techniques that are applicable to a wide range of problems. This archive of higher-level themes in mathematics is often overlooked in problem solving literature where the focus is more on models for cognitive problem-solving methods. In fact, it is only in Polya (1957) that we find a substantive discussion of the notion of a theme.

We present in this paper a case-study of a second year specialist mathematics module taught entirely in problem solving workshops where activities are designed, using structures familiar in

Freudenthal's Realistic Mathematics Education (1968 and 1973) and Moore's method (Jones, 1977), to develop an understanding and appreciation of themes.

The paper is set out as follows. In sections 2 and 3 we discuss the prevailing discussion surrounding problem solving skills in undergraduate mathematics education. In section 4 we introduce and discuss some of the relevant aspects of Freudenthal's Realistic Mathematics Education – we argue that themes in mathematics, as introduced above, are as much to do with Freudenthal's notion of real-life mathematics as the concrete problems he was interested in. Finally in sections 5 and 6 we discuss our approach to developing students' understanding of themes and our initial findings from teaching these techniques.

2. Problem solving from Polya onwards

Polya's work on the systematisation of problem-solving methods in mathematics has long been hailed as one of the most important treatises on this topic, (Polya, 1957) – certainly one of the most comprehensive. Its influence on mathematics educators cannot be overstated. In fact its influence reaches beyond mathematics, see for example Bransford and Stein (1993). Polya was one of the first authors to suggest presenting mathematics as an experimental science. He observed:

"Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science." (Polya, 1957)

Importantly, Polya appreciated that these two sides of mathematics – the rigorous logical argument structure of the definition/theorem/proof inherited from Euclid, and the haphazard experimentation of mathematical discovery – rely one on the other. Implicit in his work is the notion that the student of mathematics cannot hope to progress in one without progressing in the other; indeed, many of Polya's heuristics rely on experience and advanced knowledge of previous work.

Polya's model for problem solving specifies four stages: Understanding the problem, devising a plan, carrying out the plan, and looking back. It is the second stage that Polya (1957) largely focussed on and, indeed, is the focus of the current article. A modern practical approach to Polya's work that deserves a mention is (Mason et al., 2010) that translates Polya's four stage approach to three "phases of work": Entry, Attack, and Review. Emphasis is made of the cognitive processes of mathematical problem solving. The authors discuss briefly themes in mathematics, however this is not pursued in detail. Other authors, Bransford and Stein (1993) for example, have argued that Polya's methodology can be applied to a broader range of problems. However, the authors discuss the importance of specialized subject-specific knowledge and note that *"[o]ur ability to solve problems is not simply equivalent to a set of general problem-solving skills."* We are particularly interested in this notion of subject specific knowledge. We have found that for mathematics, subject specific knowledge goes far beyond the definitions and proofs encountered, often didactically, in traditional mathematics courses. Indeed, it is our opinion that it is the themes alluded to earlier that are equally important.

Many authors have hypothesised cognitive thinking models for mathematical problem solving. In Mayer (1992), for example, the author proposes a model that specifies five types of knowledge that a student must demonstrate in order to solve a mathematical problem: linguistic knowledge, semantic knowledge (a student's general knowledge of mathematical facts), schematic knowledge (a student's knowledge of the topic of the problem and their ability to recognise different types of problem), strategic knowledge (a student's knowledge of how to use their available knowledge to *"develop a plan"*), and finally procedural knowledge (the student's knowledge of mathematical manipulation and argument construction). Here Mayer specifically isolates Polya's second stage. In Kintsch and Greeno (1985) on the other hand the authors' work on problem solving of arithmetic and algebraic word problems also highlights the importance of schematic knowledge and procedural knowledge. However less emphasis is given to the development of strategies to solve unfamiliar problems. In Reusser (1996) the author avoids the difficulties of solving problems in unfamiliar contexts, proposing a step-wise processing model including the five stages: constructing a propositional representation of the problem, creating a situational model, transforming the situational model into a formal mathematical representation, applying the operations to calculate the solution, and interpreting the solution in a meaningful way.

The current article presents a case study of an approach to problem solving that highlights the character of different subjects in mathematics (for example analysis, algebra etc.) and enables students to develop a library of techniques that provide insight into developing strategies. The proposed strategy utilises a stepwise approach, influenced by Polya's work, as well as Mason et al. but also aims to develop the notion of strategic knowledge (Mayer, 1992). However, our approach differs in the content of workshop sessions from the discursive model, described by Lakatos (1976) for example, and instead uses similar ideas from Realistic Mathematics Education, (Freudenthal, 1968 and 1973), discussed below to discover, study and reflect on themes.

3. Themes in mathematics: have you seen it before?

Proofs that students encounter in undergraduate mathematics, especially during the initial weeks of teaching, rely on what educators often call "tricks". For example, as mentioned in the introduction, to prove a group has unique identity one normally proceeds by assuming that it has two distinct identities, e, and e'. It is then argued that $e = ee'$ since e' is an identity, and therefore that $e = e'$ since e is an identity, thus producing a contradiction. This technique can be mimicked to produce similar proofs of unique identity elements for a number of algebraic structures, for example a vector space, or a field; it is often termed a "trick". The authors would argue that the notion of a trick in mathematics is at best a misrepresentation of mathematical arguments and at worst an untruth. The idea of a trick gives students the impression that it is something they would not be able to think of themselves. We instead recognise this example as a reusable theme in algebra. The different branches of mathematics include numerous examples of themes that students study routinely as part of their mathematics education, such as the examples in the introduction. It is these themes that Mayer (1992) terms strategic knowledge and Polya (1957) is describing when he asks the question: *"Have you seen it before?"* Furthermore, it is knowledge of these themes that professional mathematicians make use of daily to solve problems.

In order to enable students to develop as successful problem solvers it is important to highlight themes in mathematics and have students reflect on, recognise and make use of themes in their problem solving. In this way we depart from theories of problem solving (Kintsche and Greeno, 1985; Reusser, 1996) that prioritise the recognition of problems by their type. Instead we propose studying solutions to problems in order to develop a student's library of themes and tools that can be subsequently applied to solve problems more flexibly. It is from this point of view that we are proposing a technique to develop students' cognition of themes in mathematics and, in particular, their ability to reuse themes from one area in order to better facilitate the solution to problems in others.

4. Realistic mathematics education

In order to develop the library of themes discussed in the previous sections we have designed a number of activities that let students discover tools for themselves. We employ notions of scaffolded learning to enable students to develop their own internal reflective monitor that helps them study proofs and solutions to problems in order to isolate the main arguments employed. In this way activities are designed to teach students to reuse arguments and tools in problem solving.

The main influence in the design of these activities comes from Freudenthal's Realistic Mathematics Education (RME), (Freudenthal, 1968, 1973). Hans Freudenthal developed RME in opposition to the didactic approach to mathematics education that was being exercised throughout Europe and the United States in the 1960s and 1970s, and in particular of the 'new mathematics' of the 1960s. His approach emphasised the development of mathematics curricula, in the way Polya argued, as an *"experimental, inductive science."* For an interesting survey of Freudenthal's work see Gravemeijer and Terwel (2000).

Freudenthal took the point of view that although mathematics as an abstract subject is extremely flexible and hence applicable, it is *"wasted on individuals who are not able to avail themselves of this flexibility,"* (Freudenthal, 1968). However, he argued that simply teaching students what educators felt was *"useful mathematics"* would lead to a narrow knowledge of mathematics and in essence remove the flexibility inherent in mathematics. On the other hand, he argued that teaching pure mathematics and afterwards working through examples of applications was also *"the wrong order"* (Freudenthal, 1968).

Instead Freudenthal developed the notion of mathematizing, of doing mathematics as a human activity; he said,

"[Mathematics as a human activity] is an activity of solving problems, of looking for problems, but it is also an activity of organizing a subject matter." (Freudenthal, 1968).

This approach stood in contrast to other mathematics educators both then and now who often propose a more discursive approach to mathematics education, along the lines of Lakatos (1976) and Polya (1957). RME was therefore developed in order to facilitate mathematizing. In Gravemeijer (1994), (see also Gravemeijer and Terwel, 2000) the author clarifies the characteristics of mathematizing, or *"making more mathematical"*, as techniques:

- for generality: generalizing (looking for analogies, classifying, structuring);
- for certainty: reflecting, justifying, proving (using a systematic approach, elaborating and testing conjectures, etc.);
- for exactness: modelling, symbolizing, defining (limiting interpretations and validity), and;
- for brevity: symbolizing and schematizing (developing standard procedures and notations).

It is notable that these overlap with (Polya, 1957) and also 'specialising' and 'generalising' in (Mason et al., 2010). In Treffers (1987), the author further distinguishes different activities as horizontal and vertical mathematizing. Horizontal mathematizing involves taking a problem and converting it to a mathematical problem, whereas vertical mathematizing involves taking a mathematical problem and reformulating it or understanding it in a deeper way, similar to Polya's understanding the problem, or Mason et al.'s entry phase. In the words of Freudenthal,

"Horizontal mathematizing leads from the world of life to the world of symbols. In the world of life one lives, acts (and suffers); in the other one symbols are shaped, reshaped, and manipulated, mechanically, comprehendingly, reflectingly: this is vertical mathematization. The world of life is what is experienced as reality (in the sense I used the word before), as is a symbol world with regard to abstraction. To be sure the frontiers of these worlds are vaguely marked. The worlds can expand and shrink – also at one another's expense." (Freduenthal, 1991)

5. Activity design

As an example, this case study describes an activity designed to last approximately two hours with the objective of revisiting modulo arithmetic and developing it as a theme in number theory to solve problems concerning the integers.

The session begins with a description of the intended outcomes without reference to modulo arithmetic, namely in terms of tangible knowledge of tools for problem solving. Students, over the two hours, intermittently work through the following activities in order. Having solved one problem students are asked to reflect on the solution and to highlight the main tool used to solve the problem. By the 5th and 6th problems the students are working comfortably with modulo arithmetic.

Activity:

1. Throughout p denotes a prime number;
2. Show that if $p > 2$ then there is a $k \in \mathbb{Z}$ such that $p = 2k + 1$;
3. Reformulate your previous answer in modulo arithmetic with respect to 2 (vertical mathematizing);
4. Examine what can be said if you replace modulo 2 with modulo $3, 4, 5,$ etc. (generalising);
5. Show that if $p > 3$ then $p = 6k \pm 1$ for some $k \in \mathbb{Z}$ (specialising);
6. Show that if $p > 5$ then $p^2 \equiv 1 \; mod \; 8$ (vertical mathematizing);
7. Deduce that if p, q are prime numbers greater than 5 then $p^2 - q^2$ is divisible by 24 (themed problem solving).

The solution to the 6th problem is an exemplar for the topic of this article. Having developed students' familiarity with vertical mathematizing in previous problems, they are then able to reformulate the 6th problem and solve it. The session concludes with a discussion of the intended outcomes: both in terms of the broader notion of themes in mathematics, and in terms of the specific theme of using modulo arithmetic to reformulate problems in number theory. Students are asked to reflect on other tools and themes they might have come across in other subjects.

6. Outcomes

This article is a case study of a second-year module that utilised the above approach to problem solving and no primary research has been done to determine its effectiveness. However, a number of positive outcomes have been seen on the programme as a whole. Having worked through a number of similar sessions involving activities such as the one described above, students demonstrate an increased confidence in approaching problems in areas of pure mathematics. Additionally, students develop clarity in their ability to reflect on proofs and solutions to problems that they see in their core modules.

By the time students see more complex and difficult proofs in their third year students are demonstrably able to break them down into their constituent themes and can highlight common techniques and tools from other areas.

Overall thematic problem solving is proving to be a success in its ability to de-mystify problem solutions and proofs in mathematics and it is expected that as we develop the range of activities in the future this will lead to a deeper understanding of core material.

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CASE STUDY

Student use of whiteboards in the classroom

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Abstract

This paper discusses the use of whiteboards – both small, individual boards and larger, wall-mounted ones – within a variety of classes within our undergraduate mathematics degree. Details of those classes, and how students use whiteboards within them are presented. There is a focus on practicalities, particularly regarding the formation of student groups for whiteboard activities and the role of the member of staff in such classes. Issues which should be considered if introducing these to the classroom are discussed.

Keywords: Boards, whiteboards, group-working, group facilitation, whiteboarding.

1. Background

It is not surprising that in a technological era there is a plethora of research on the advantages and use of interactive whiteboards, electronic voting systems and other advanced technologies for the classroom. For this case study we have taken a technological step back in order to discuss how students can use erasable boards within the classroom. We refer to 'whiteboards' because that is what we have, but chalkboards would be as effective for much of what is discussed here.

Students' use of whiteboards in both school and university has been reported in a number of studies. For example, Forrester, Sandison and Denny (2017) examine a case study following the experience of a teacher introducing the use of whiteboards into secondary school teaching of mathematics. The teacher reported that the use of whiteboards increased student engagement, confidence, discourse and collaboration. She was able to monitor students' mathematical thinking, intervene and encourage student abilities.

Antoniades (2013) credits Sean Kavanaugh as championing the use of whiteboards, in the junior classroom, by creating and implementing the 360 Degree Math System where "the teacher becomes the audience and the students become the performers".

Schaffner et al (2015) also found benefits of whiteboarding and reported on their use by high school students and concluded that they made student thinking visible, provided immediate and effective feedback, encouraged mathematical communication and resilience, and demanded more participation from students.

There is a lack of theoretical models to explain why the use of whiteboards may result in improved student learning. However, Carpenter's (2009) study suggests that getting students to access prior knowledge along with collaboration and discussion cultivates deeper understanding of complex subjects.

Seaton et al (2014) reported on the use, in three Australian universities, of 'whiteboarding' where students work collaboratively at a large board, with occasional tutor assistance. They identified numerous advantages to this: the process promoted more active learning; it improved the quality and timeliness of the help given by the tutor; it promoted peer learning; it improved connections between students and so helped retention; it improved relations between students and staff; it improved group work and communication; it provided students with a more authentic representation of the work of a research mathematician (particularly around collaboration and experimentation), and that it helped tutors to understand how students were progressing.

Inouye et al. (2017) used hand-held individual whiteboards within lectures in animal physiology classes, and occasionally required students to work in groups on the boards to answer questions. They found improved student performance in those topics where whiteboards were used. Megowan-Romanowicz (2016) describes whiteboard use in high school physics classes, and describes how whiteboards are an "important cognitive and communicative tool for both teacher and students". She discusses how students use the boards to discover ideas and negotiate with their peers, and says they "afford the teacher a valuable window on student thinking as it is happening."

This paper reports on students' use of whiteboards, but within that overarching idea there is a variety of different approaches used by different tutors, in different modules (five of which are included here), across all three levels of the undergraduate mathematics programme at our institution.

2. Modules

The modules covered by this paper are briefly described in Table 1. Four of the five modules take place in classrooms where the tables are arranged in groups of six-eight students, and a single two hour class combines elements of lecture and tutorial. The fifth has a more traditional 'lecture plus tutorials' format.

Table 1: Modules in which students use whiteboards.

Year	Module code	Core/ Elective	Number of students	Content	Delivery
1	1A	Core	~80-100	Revision and extension of A-level topics such as calculus, plus material new to most students such as complex numbers and Taylor series.	Parallel one-hour tutorial classes (~20 students per class) with one tutor and a two-hour lecture (full cohort) later in the day.
	1B	Core	~80-100	Topics such as set theory, proof, group theory, number bases, Euclidean algorithm. Most content new to most students.	Parallel two-hour workshop for half the cohort, with two tutors. A mixture of content delivery, interspersed with student exercises.
2	2C	Core	~80-100	Fourier analysis, analytical and (primarily) numerical solution or	As for Module 1B, but with an element of flipped learning; approximately half

				ordinary differential equations.	the material delivered via video lectures which students watch before the class
3	3D	Elective	~25	Partial differential equations: derivation from first principles, analytical and numerical solutions.	A single two hour session delivered by one tutor. The first hour is mainly delivery of new material, with the second hour for students to work on tutorial sheets.
	3E	Elective	~50	Group theory, formal languages and automata.	A single two hour workshop, delivered by two tutors; as with Modules 1B and 2C, a mixture of material delivery with student exercises. Students are asked to form support groups at the start of the year, and sit with the same group in each session.

3. Individual whiteboards

Initially, individual whiteboards were introduced in some classes as a simple, cheap, effective, low-tech means for a lecturer to pose questions and see student responses quickly (for example, asking students to sketch some graph, and then hold this up in the direction of the lecturer - Figure1). As with electronic voting systems, this is helpful for the lecturer to judge how well the class understand the material. However the whiteboards are more versatile as students can write mathematical objects and draw on the boards. Furthermore, the whiteboards are not subject to technological malfunction.

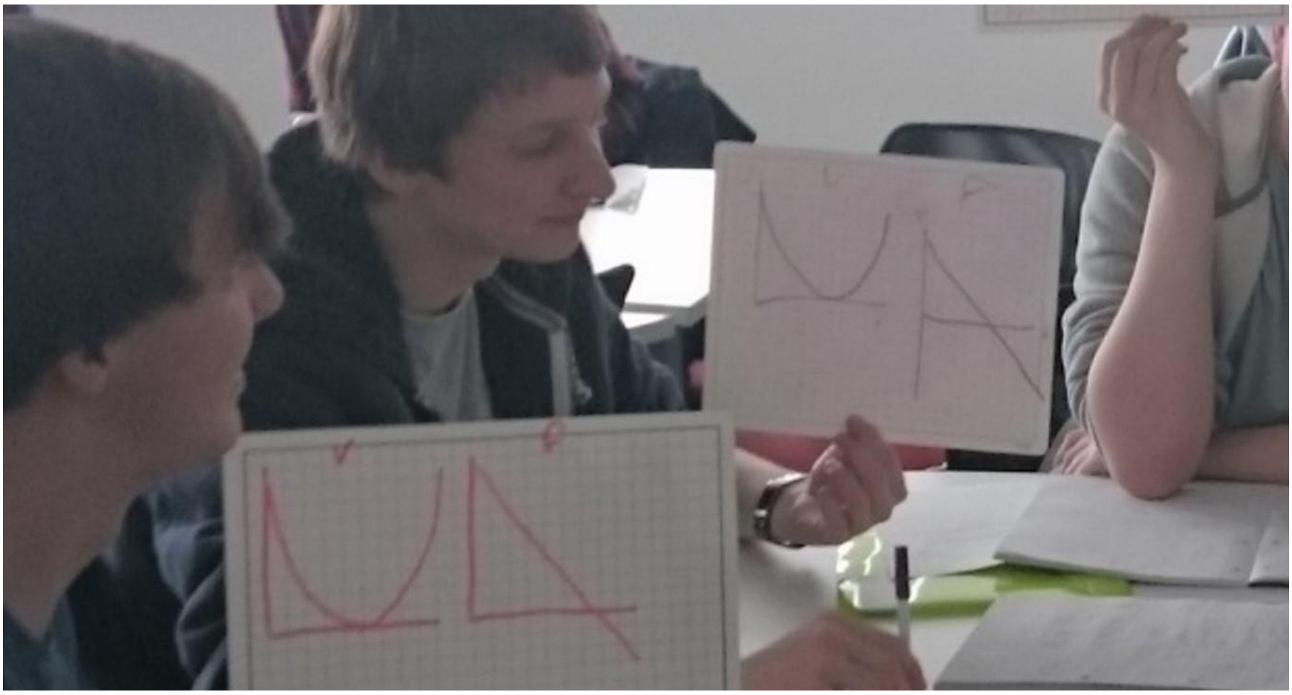

Figure 1: Students using individual whiteboards in response to a question posed by the lecturer.

It was observed that, once such individual boards were available, some students chose to use them in other ways, for example when working on set exercises or when sharing their ideas with other students.

As a result, in Modules 1B, 2C and 3E, the students are now given individual whiteboards in most teaching sessions, regardless of whether the lecturer *requires* their use. Their use is often recommended for exercises, especially when students are asked to try something before the correct answers are presented to the class. Many students opt to use the whiteboards available to work on exercises even without the recommendation. Some copy up the answer once it has been seen by a lecturer and some take a photograph of the work they produced.

4. Large whiteboards

Large, wall-mounted whiteboards are used in different ways in Modules 1A, 1B, 3D and 3E. The distinctions are discussed below, but generally, students are split into small groups (typically three-four students, but pairs and larger groups have been tried) and set exercises to complete collaboratively, whilst standing at a whiteboard. In some classes, every student in the group is given a whiteboard pen (in some cases, a different colour for each student) whereas in other cases, the group gets just one pen between them. The tutor moves between the various groups, observing, and intervening as they see fit. Students are encouraged to keep a record of the work by photographing it before erasing the board.

A key aim of introducing whiteboard work was to make students work together. It is worth noting that, in a more traditional exercise class, many students already choose to work in groups with their friends. There are advantages to students developing the skills to deal with having to work with unfamiliar peers. With this in mind, a variety of methods was used for forming groups:

- Students were asked to pair-up, and then the tutor joined together two pairs; thus students had one chosen peer, and two they had not chosen;
- Students were grouped according to some arbitrary factor, for example arranging themselves according to birthday, or last two digits of their phone number. This exercise works as an

icebreaker in Module 1A when the students have only just met each other; it also allows for students to ‘cheat’ if they really want to work with a friend;
- Students were allowed to self-select their groups completely;
- Students were formed into self-selecting study groups at the start of the year; when board work was used, they continue to work in the same study group. Thus the same group is very familiar with working together;
- Students were allocated groups by the tutor, with the aim of each group having a mix of personalities which would promote interaction;
- Students were asked to assess which exercises they needed to work on, and grouped with other students who wanted to work on the same questions.

Details concerning how whiteboards are used are given below:

4.1. Weekly board tutorials

In Module 1A, large boards were introduced in 2016/17 in most of the tutorial classes (21 out of 24 weeks), starting in the students' first week on the course. Students work at the board for most of the class; we therefore refer to this as a 'board tutorial'. The activity is introduced with care, with explanations about the purpose, discussions about working collaboratively and criticising constructively, and a recognition that this might be initially uncomfortable. These themes are reiterated regularly over the first few weeks.

The exercises set within Module 1A were modified, slightly, compared with the exercises from the previous year. In general, each set of exercises contained a combination of three elements:

- exercises based on the previous week’s lecture, enabling the tutors to check student understanding of previous material;
- exercises based on A level material which was to be revised and extended in the lecture later in the day;
- exercises designed to encourage students to ‘discover’ for themselves the results which would be formally taught later in the day.

The latter two elements in particular provided a means for the lecturer to modify the lecture plans, skimming over revision material which students clearly understood, going into more depth with material with which they struggled, and building on what students had 'discovered' when presenting the new material.

4.2. Occasional whiteboard use

The whiteboards are used infrequently in several modules, as discussed below.

In Module 1B, the students are *sometimes* (typically every three-four weeks) asked to use the large whiteboards instead of working at the tables. The exercise typically lasts around 10 minutes and then the students return to their tables to be taught or to continue with other exercises. The exercises are not re-written for whiteboard exercises, but the lecturers judge which questions would be suitable for such a task. They are used when group work and being able to easily erase written work would be beneficial.

In module 3E, there is just one board tutorial during the year, in addition to the usual two hour workshop. This takes place with about 25 students in the presence of two tutors. The students work on a question that involves proof by contradiction where they are given examples which are expected to lead to common mistakes. The large whiteboards are used for this session to encourage group discussion about the errors and because the students would only have to erase part of the solution when trying a different approach.

In Modules 2C and 3D, large whiteboards are used within revision sessions, which happen in the final three weeks at the end of year. This method is preferred to a tutor-led approach in order for the students to construct their own understanding and for the tutor to provide help where it is most needed. The students are split into groups and asked to work on questions from past papers.

Figure 2: Students working at large whiteboards during a 'board tutorial'.

5. The role of staff within sessions

5.1. During the use of individual whiteboards

Whilst the students are working on exercises, staff move around, looking at work in the same way as work done on paper. A big difference is that staff also use the individual whiteboards to talk through solutions with individual students or to give them a hint to get started with an exercise. This is particularly useful when several students are working together. The whiteboard can be held up to show more than one student at once.

5.2. Large whiteboard sessions

The approach taken by all staff within the large whiteboards sessions is roughly the same across different modules. Whilst students try the exercises, staff move between the groups, eavesdropping on student conversations, observing their working on the boards, answering questions, and intervening when they deem it appropriate.

Staff, within any style of classroom, may answer questions, or otherwise intervene, in a wide variety of ways. Sometimes staff may choose a direct method: answering a question fully, demonstrating a method, or addressing a misconception. Often staff employ more indirect approaches, designed for example to foster students' development of independent thought and mathematical strategies. Board tutorials, with their combination of group working, overheard conversations and easily visible mathematical working, facilitate effective tutor intervention in various ways.

Firstly, tutors can more easily assess students' comprehension of mathematical ideas and concepts, and their progress in applying these, which enables more effective and timely intervention. It is also possible to quickly see if students are disengaged from the work or failing to interact with other members of their group. Tutors can thus make more effective judgements about when and how to intervene whether in relation to mathematical ideas, engagement, or team working. This contrasts with a more traditional exercise class, where some students seek to hide their working (and with it, any problems), or avoid working at all, and this is less visible to the tutor.

Secondly, staff are able to visit each group of students frequently within a single session. This means that tutors are more able to, for example, provide a small hint, suggest that students think again about some part of their work, or solicit ideas from other members of the group and then, crucially, leave them to discuss this and work on the ideas, whilst being able to check on their progress either on the next visit to this group, or simply by glancing at their whiteboard. Within a more traditional exercise class, such approaches are restricted by the very limited opportunities for tutors to return to the same student 10 minutes later.

6. Conclusions

The student use of whiteboards has significantly changed the nature of some of our classes, to a degree which surprised the tutors, and made more modest changes in others. From a tutor's perspective, the most significant factors are:

1. Individual whiteboards are an effective way of questioning students during delivery of material; more flexible, cheaper, and simpler than electronic systems;
2. The use of whiteboards makes the student working - and so their thinking - much more visible to the tutors and other students. This is true to some extent with the individual whiteboards, but even more so with the use of large boards. As a result, the tutor interventions with students are more effective, and group working is facilitated;
3. In the board tutorials, it is much harder for students to be disengaged from the work; partly because their involvement with the rest of the group demands their attention, and partly because, whilst standing, any lack of engagement is more easily noted by the tutor.

We will be presenting a fuller evaluation of these ideas in an upcoming paper, including a detailed analysis of the students' perceptions of these approaches; however, for any staff considering using large whiteboards in their own classroom, we conclude with some issues for consideration:

- Classes need to be timetabled in suitable rooms with sufficient board space. This information is not always known to timetabling teams. Having a look around the university and providing the timetabling team with a list of suitable rooms makes it easier for all staff involved. If there are not any suitable rooms, you could request for additional whiteboards to be installed in some teaching rooms;
- Have a plan for how students with disabilities might be accommodated, particularly if this approach is to be used early in the course when students are not yet known to staff;
- Decide on the size of the groups and what might be appropriate for the activities which are planned. The experience of this work suggests three or four is ideal;
- Consider how the groups will be allocated. There is a tension between allowing students the comfort of working with their friends, and encouraging them to learn to work with others;
- Consider the nature of exercises to be set; some exercises may not be suitable and may need to be re-written for a whiteboard exercise.

7. Acknowledgements

We would like to acknowledge Dr Ros Porter, of Sheffield Hallam University, whose idea it was to introduce individual whiteboards, and Dr Heather Lonsdale who introduced the idea of board tutorials to us.

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CASE STUDY

Making the grade: supporting mathematics students in understanding the use of grade-based marking criteria for assessments

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Abstract

Grade-based marking criteria are used widely in humanities subjects, and also in some areas of the sciences. In many mathematics assessments, individual marks are clearly allocated for specific elements of working in calculations, computations or theoretical arguments. This case study will document the use of grade-based marking criteria for assessments in a final year mathematics module at a mainstream UK university. The module has the development of professional skills for mathematics as the central focus and the assessment tasks take the form of written reports and oral presentations. This paper will describe formative group tasks set in the initials weeks of the module to introduce students to the grade-based marking criteria, and to provide students with experience of using the criteria themselves. The success of these initiatives in developing student appreciation for the use of the assessment criteria will be discussed, along with ideas for the future based on student responses to the activities.

Keywords: Assessment, marking criteria, peer learning, group exercises, project work.

1. Introduction and background

Employability has been a key priority for universities over the past five years, with the contribution of student outcomes and graduate destinations three years after graduation now feeding directly into the next Teaching Excellence and Student Outcomes Framework (TEF) exercise (Office for Students, 2018). Universities have been keen to emphasise to students the importance of careers awareness and the need for students to develop and successfully articulate employability skills during their degree programmes.

The university in this case study introduced an institution-wide agenda focused on employability in 2012. Individual departments were instructed to introduce appropriate learning opportunities within undergraduate programmes for students to develop their employability skills. The Mathematics department established a skills strand of modules, with one skills module available to students in each academic year. These modules embed skills development and offer opportunities to interact with representatives from industry within credit-bearing taught mathematics modules rather than offering these activities in stand-alone sessions. It is recognised that stand-alone sessions are not generally as successful in engaging students as embedded sessions, and that students can place more value in advice from industrial representatives than academic or careers staff (Cranmer, 2006; Chadwick et al., 2011).

Each of the modules in the strand requires students to tackle unstructured problems and work in groups on extended mathematical investigations. The modules also require students to reflect on skills development as part of the module assessment. This is seen as a crucial part of employability training as individuals are able to articulate and evidence skills development clearer, and make appropriate plans for future development through sustained emphasis on reflection (Pegg et al., 2012).

This paper will consider the final year module in the strand, which has a specific focus on professional skills development. This final year optional module aims to simulate many aspects of working for a mathematical consultancy and requires students to work in groups on genuine problems faced by industry and business. The outputs in this module must be tailored to client audiences where there can often be no assumption that the audience has high-level knowledge of mathematics (i.e. beyond college level). The challenge in this module is for students to successfully communicate the results of extended mathematical investigations to these particular audiences, in both written and oral reports.

The final year module in the strand has been running since the 2012/13 academic year. The module is a 15-credit module which is available to both Single Honours and Combined Honours students. On average, fifty students select the module each year. The module is taught in a computer lab setting and the timetabled activity for the module is one three-hour lab session per week. The aim of this teaching structure is to allow student groups to spend extended time on the group projects.

2. Grade-based assessment criteria

The university in this study has a set of grade-based assessment criteria used across the institution. Examples of the criteria can be found in Table 1. The use of a standard set of grade-based assessment criteria across the university provides a common currency across different disciplines, which percentage scales don't always provide and can help to limit discrepancy with markers' individual judgements.

Table1: Examples of the university grade-based assessment criteria.

UG Award	Example Descriptors	Possible Outcomes (%)
First	• Knowledge of a sufficient number of core materials • Arguments are well constructed but do not develop sufficiently some significant issues • Clear style with satisfactory presentation	100,90,80,75,72
Upper Second	• Some knowledge of a restricted range of issues relevant to the assessment • Some development or illustration of points • Arguments are poorly constructed with weak / simplistic presentation	68,65,62
Lower Second	• Knowledge of a sufficient number of core materials	58,55,52

	- Arguments are well constructed but do not develop sufficiently some significant issues - Clear style with satisfactory presentation	
Third	- Some knowledge of a restricted range of issues relevant to the assessment - Some development or illustration of points - Arguments are poorly constructed with weak / simplistic presentation	48,45,42

Careful consideration is advised when applying assessment criteria to student work in general. The main points are summarised by Sadler (2005) who observes that criteria for assessed work should be clearly explained to students at the beginning of a course. In particular, *"students deserve to know the criteria by which judgements will be made about the quality of their work"* (p.127).

Sadler also advises that there should be clear communication of standards across a community of learners which should cover qualifying thresholds and agreeing standards. Sadler concludes that a major barrier to success when using grading criteria is that the key decisions are made in contexts where students do not normally have access to the standards which are applied, or the judgemental process of the assessor.

Several authors offer advice on avoiding the pitfalls highlighted by Sadler when applying assessment criteria to student work. Andrews et al. (2018) advocate bringing students into the conversation where marking criteria is concerned. Robinson (2015) notes that in-class exercises and informal discussions between students can help to understand marking and the associated feedback. On the theme of feedback, Schinske and Tanner (2014) argue that more time to should be allocated to self and peer evaluation and reflection in an active learning environment.

A major concern in the use of grade-based assessment criteria for the module in this study is that mathematics students have little experience of being formally assessed on report writing. Marking schemes are regularly provided in mathematics modules which account for each individual mark. This enables students to identify precisely where any marks were lost. The qualitative nature of marking to a grade-based set of assessment criteria is something which mathematics students need a formal introduction to. Subsequent advice and support in adapting to the use of such criteria is also needed.

3. Initiatives in the 2017/18 academic year

To support students in making sense of and using the grade-based assessment criteria, several initiatives were introduced in the 2017/18 academic year. These included class discussions on report writing, specifically the need to be aware of the audience or reader for a particular piece of written work. In this case, the differences in style required for client reports and more technical / mathematical reports was discussed in detail. The evolution of the criteria in line with the module aims was also explored as part of this initial set of class discussions. In addition to these class-wide discussions, a formative group exercise on applying the criteria to sample reports was set in the subsequent lab session.

For the group exercise, a sample project brief and three sample reports were provided to students via the Virtual Learning Environment. The sample brief was for a client-based project where the main output required was a client report written in a style appropriate for the given client. Students were asked to read through the brief and the sample reports. Each group was then asked to write down their initial thoughts on each report and responses were collected anonymously on an online form. This initial thought gathering was not intended to be formally linked to the assessment criteria at this stage, but was to collect first impressions.

The second stage in the group exercise was for the groups to formalise their critique of the sample reports by writing down a mark and formal feedback on each report, aligned with the marking criteria. This again was collected on an online form. For this activity, the groups were selected by the students themselves. Sixteen groups, each consisting of 3-5 members were formed. The activity took place in the second week of the module. This week was selected for the activity as this was before any assessed work was to be submitted and could therefore align with suggestions from Sadler that students must be clearly introduced to the methods by which they will be assessed before any formal assessment takes place.

Students were informed that the intention of the exercise was to familiarise themselves with the way in which work is assessed in the module using the assessment criteria, and how initial impressions on reading through a piece of work must then be justified and considered against the formal criteria. This approach aligns with the suggestions of Robinson (2015) and Schinske and Tanner (2014).

4. Results and analysis from the group exercise

Sixteen responses were received. Each of the sixteen student groups proposed a mark and provided initial thoughts and formalised feedback on the three sample reports. The sample reports had been marked by the lecturer separately (prior to the group exercise) and a comparison of the classifications, along with the main student comments on the reports can be found in Table 2. A group comment was classed as a ‘popular’ comment if it was mentioned in the feedback from four or more groups in the exercise. An open class discussion was held to cover the results of the exercise and to clarify any confusion or concerns about the use of the criteria.

Table 2: Results from group exercise on applying the marking criteria.

	Sample Report 1	**Sample Report 2**	**Sample Report 3**
Mean student group mark	Upper Second Class	Third Class	Upper Second Class
Lecturer classification	First Class	Third Class	Upper Second Class
Popular student comments	• Shows awareness of the client’s needs • Robust arguments	• Presentation is poor overall • Lack of structure and difficult to navigate • Graphs and figures are all	• Good overall presentation • Suitable sections • Evidence of original thought

	with justifications • Referencing style is incorrect and inconsistent • Some graphs are difficult to read • The report should be longer and more detailed	located in the appendix when these should be used to justify recommendations in the main body of the report • Too many long blocks of text	• Bit heavy on the mathematical content / jargon for a client report • Conclusion lacks detail • Good use of graphs and tables

Most of the points raised by the student groups in their feedback on the sample reports are relevant and are emphasised in the descriptions of the marking criteria. The lecturer awarded sample report 1 a First Class mark but the student groups classified this report as Upper Second Class standard. When this was discussed with students, it was acknowledged that the point raised by the groups on the length of this report was perhaps not entirely valid for this report brief. The class agreed that a longer report does not always indicate a better report. In this circumstance, the report was for a client reader and students were reminded about their briefing on report writing, specifically that lengthy mathematical calculations are not really appropriate for this type of report. Students were also reminded that client reports require the author to be more concise and to-the-point than formal mathematical research reports. The style guidelines for client reports were also highlighted again. In particular, the use of summaries and appropriate sub-sections was recommended to help with the readability of the report.

There was consensus that sample report 2 had very poor structure and layout. The class discussion was used to emphasise the importance of presentation and the organisation of mathematical results in a client report. Some students queried whether sample report 2 should be classified as a Fail. The class examined the marking criteria again and observed that the report aligned with the Third Class classification, but possibly the lower end of the mark scale.

Sample report 3 was widely appreciated for the many positive aspects of its layout and content. Evidence of original thought was highlighted but the student comments which were submitted acknowledged that the report was not written in a style which was suitable for the client reader. This was encouraging as it indicated that students were possibly developing an awareness of the structure and content which is suitable in a client report.

In this exercise there was a clear difference in the language used by student groups when articulating ‘initial thoughts’ and ‘formal feedback’. The formal feedback comments regularly made use of the formal marking criteria descriptions while the initial thoughts were brief, at times informal, comments such as “too long”, “too many tables”, “mostly a solid report, some grammar errors”, or “easy to follow”.

There was a tendency in the formal feedback for student groups to list many negative points about the given report without highlighting any positive features. This was discussed with the class as a whole and students recognised the importance of fixing the grade based on a clear link with the descriptors, stating features which are present in the work, while also identifying features from higher band descriptors which can be included next time as useful feed-forward actions.

5. Impact of the initiatives and future plans

Following the exercises in the first few weeks of the module, the lecturer took time to discuss the results and formal feedback from the first assessed report with each group individually. These discussions indicated that the groups understood their marks and saw how these were aligned with the assessment criteria. This first assessed report was on a project with significant time pressure and several groups acknowledged that the communication and presentation of their results had suffered. These groups admitted that the project planning stages had not allocated sufficient time to the development of the report and the communication of the mathematical results. Some groups were able to identify criticisms within their own feedback from the lecturer which they had previously highlighted themselves in the exercise on grading sample reports. These groups were frustrated by this fact, but the realisation seemed to enhance their determination to put actions in place for subsequent projects. In particular, the experience here seemed to lead to a greater appreciation of the project process as a whole rather than just the final report as an isolated element.

Each group was also able to see how the feedback could be channelled into successful feed-forward actions for the next project. This is encouraging and indicates that some students are able to see the process of receiving feedback as purposeful and motivational. The documented dangers of grading playing on students' fears or desires to outcompete peers (Schinske and Tanner, 2014) do not appear to be manifesting in this module. Instead, students are engaging in meaningful reflection and demonstrating enhanced appreciation for the clarity of the grade descriptors.

In previous years, there were occasionally a couple of student comments in end-of-module feedback forms raising some dissatisfaction with the clarity of the marking process when compared with other mathematics modules. The introduction of the initiatives and group exercises in the 2017/18 academic year resulted in the positive developments outlined above and also the absence of any comments in end-of-module feedback forms concerning the use of the grade-based assessment criteria. Student opinions and experiences of the marking process in the module should be explored further as the end-of-module feedback form does not specifically ask for feedback on the grading of assessments or the use of relevant criteria. A specific question on this could be included in future.

Future work could formalise this process of reflection on group projects and the documentation of feed-forward actions based on this. It could also be interesting to consider the use of anonymous peer marking to provide students with further experience of using the marking criteria themselves in practice. The process of reading the work of others could prompt groups to make improvements in their own work based on features identified in the work of their peers. Again, it would be interesting to document this somehow, possibly in the form of reflections after the peer marking exercise.

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CASE STUDY

Embedding and assessing project based statistics

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Abstract

Traditional approaches to learning and teaching in statistics often involves the passive absorption of information through lectures, a focus on mathematical theory and assessments which test mastery of procedures. This often results in students struggling to apply their statistics knowledge in practical and authentic contexts particularly within final year projects and in the workplace. For some time, statistics educational literature has recommended shifting the focus of teaching and assessment from theory to statistical problem solving, application based statistics using real-life scenarios, and effective communication of statistics. This research has led to the production of guidelines for statistics educators from the American Statistical Association.

This paper discusses how educational literature and guidelines have been used to implement changes in the teaching of a first year probability and statistics module for mathematics undergraduates at Sheffield Hallam University. Changing to project based learning with a focus on active learning, effective decision making and communication enabled students to successfully undertake an open group project by the end of their first year. In addition, attendance, engagement and understanding were noticeably improved. The rationale, challenges and benefits to changing the focus of the course and also the teaching style are discussed.

Keywords: Statistics education, project based learning, active learning, assessment.

1. Introduction

The main goal of an introductory statistics course should be to enable students to master the basics of statistical thinking with an emphasis on application based statistics, problem based skills and effective communication of results (Cobb, 1992), which is more relevant to employment. Students are less able to reason or think statistically when the focus is on theory, methodology and mathematical calculations (Snee, 1993) and struggle to apply their statistics knowledge in a practical and authentic context. Traditional courses and exams typically test mastery of procedures and using formulae rather than the ability to apply statistical thinking and problem solving so more authentic assessment techniques need to be developed (Chance, 1997; Garfield, 1994) to test statistical thinking. Garfield, 1994, suggests that using a variety of project based assessment methods can contribute to the learning process as well as to test statistical understanding. The American statistical association guidelines for assessment and instruction in statistics education (GAISE, 2016) emphasise the importance of teaching statistics as a problem solving and decision making process rather than a collection of unrelated formulae and methods.

This paper discusses changes to the teaching and assessment within a 1st year introductory statistics course for maths undergraduates using the six recommendations of the GAISE report:

1. Teach statistical thinking;
2. Focus on conceptual understanding;
3. Integrate real data with a context and a purpose;
4. Foster active learning;
5. Use technology to explore concepts and analyse data;
6. Use assessments to improve and evaluate student learning.

2. Methods

2.1. Teaching statistical thinking

The process of the statistical problem solving approach (Marriott et al., 2009) shown in Figure 1 was embedded from the start of the module. The process starts with a motivating research question, considers suitable data collection or manipulation and the appropriate techniques to address the research question. Students on this course learnt to use Excel or SAS to carry out suitable analysis in addition to the mathematical calculations and how to report the results effectively.

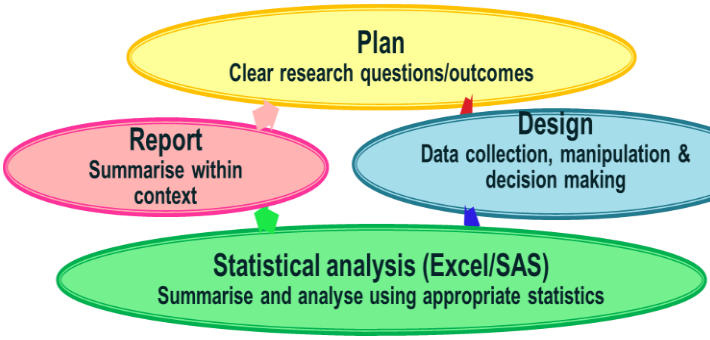


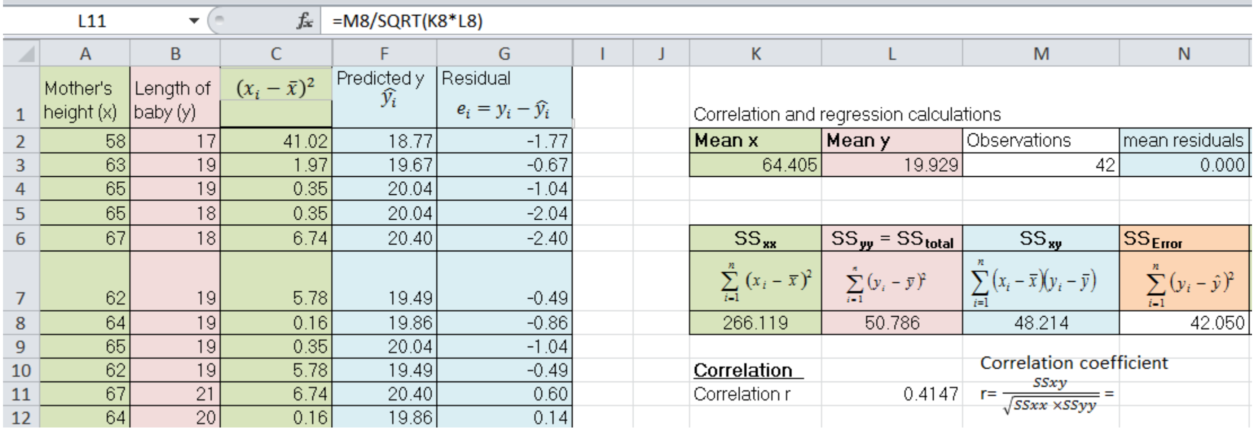
Figure 1: Statistical process cycle

Only simple summary statistics and charts were used in initial teaching and assessment which enabled students to master the terminology and the process for statistical problem solving before moving on to hypothesis testing. The assessments included individual reports and a group presentation which tested all aspects of the statistical process cycle including the selection of appropriate techniques.

2.2. Conceptual understanding and the use of technology

In the past, the content of statistics modules was limited to what could be computed mathematically and very little has changed in many institutions despite the advances in computing power and how statistics is used in the workplace. The general focus of statistics educational research centres on shifting the focus to conceptual understanding over mathematical procedures, (Garfield, 1995; Cobb, 1992, Snee, 1993). In addition, students generally find the core concepts of statistics difficult and need more time to develop conceptual understanding (Tishkovskaya, 2012; Garfield, 1995). To build in more time for conceptual understanding, the guidelines suggest reducing the time spent on probability theory, summary statistics, using the critical value method of hypothesis testing and large numbers of procedural calculations which do not enhance the students' conceptual understanding. The use of technology enables students to speed up calculations, carry out more complex tests and visualise concepts such as variability between sample means.

Students are taught to use SAS to perform statistical analysis, as many employers of our graduates use this package, and the maths taught alongside enables students to better understand the output. Excel is also used as a tool for speeding up mathematical procedures, demonstrating the impact of changes to data and to allow students to convert their mathematical understanding in a more practical way. For example, Excel is used to demonstrate the impact of outliers on means and standard deviations by changing one value to an outlier and to produce templates for statistical techniques. To improve their understanding of regression, students were asked to produce a spreadsheet for calculating regression equations and checking assumptions (see Figure 2 below). This demonstrated their understanding of the mathematical calculations without large amounts of time being wasted on calculator calculations in class or in an exam. Students reported that this was a useful exercise.



L11 =M8/SQRT(K8*L8)

	A	B	C	F	G
1	Mother's height (x)	Length of baby (y)	$(x_i - \bar{x})^2$	Predicted y $\hat{y}_i$	Residual $e_i = y_i - \hat{y}_i$
2	58	17	41.02	18.77	-1.77
3	63	19	1.97	19.67	-0.67
4	65	19	0.35	20.04	-1.04
5	65	18	0.35	20.04	-2.04
6	67	18	6.74	20.40	-2.40
7	62	19	5.78	19.49	-0.49
8	64	19	0.16	19.86	-0.86
9	65	19	0.35	20.04	-1.04
10	62	19	5.78	19.49	-0.49
11	67	21	6.74	20.40	0.60
12	64	20	0.16	19.86	0.14

Correlation and regression calculations

K	L	M	N
Mean x	**Mean y**	Observations	mean residuals
64.405	19.929	42	0.000
SS_{xx}	$SS_{yy} = SS_{total}$	SS_{xy}	SS_{Error}
$\sum_{i=1}^{n}(x_i - \bar{x})^2$	$\sum_{i=1}^{n}(y_i - \bar{y})^2$	$\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$	$\sum_{i=1}^{n}(y_i - \hat{y})^2$
266.119	50.786	48.214	42.050
Correlation		Correlation coefficient	
Correlation r	0.4147	$r = \frac{SSxy}{\sqrt{SSxx \times SSyy}} =$	

Figure 2: Using Excel to understand regression

Research suggests that the use of simulations in statistics education can be effective in improving student's statistical reasoning (Zieffler et al, 2008) and understanding of the core concepts of hypothesis testing. Students were given different samples of data to aide in their understanding but next year, Excel templates will be used to generate random samples and demonstrate distributions e.g. changing the parameters of the normal to observe the change in shape.

Although the focus of the module was on project based statistics, the number of formulae covered still resulted in the students feeling overloaded. In the next academic year, some of Excel and data summary aspects of the statistics course will be covered in a 'Maths Technology' module and less time will be spent on Probability so that the core concepts can be covered more slowly, in line with the suggested guidelines for reducing content.

2.3. Integrating real data with a context and purpose

Most HE students encounter statistics as part of their course or research but negative attitudes and anxieties can contribute to difficulties grasping the basic concepts of probability and statistics (Gal and Ginsburg, 1994). Using interesting research, media articles and data sets can make the learning process more positive (Zieffler et al, 2008; Chance, 1997), and demonstrate the relevance of course material in everyday life (Neumann, Hood and Neumann, 2013). Most statistics courses use different data for almost every tutorial question and often variables are referred to as x and y which doesn't help a student see the whole statistical process or how examples relate to real life. Where possible, this module used a few large key datasets to link the different parts of the course so that students could observe the whole process and start to see statistics as a set of related rather than unrelated techniques. .

One data set used is the Titanic data set which can be linked to several newspaper articles including the one in Figure 3 and allows a range of research questions centering on which variables impacted on survival. The introduction of this data set prompted more noticeable engagement within the lecture and allowed students to investigate claims made by papers with real data including whether or not we can ascertain that more British people died because they queued.

Which variables could you use to investigate this statement made in the newspaper article below and which summary statistics could be used?

INDEPENDENT News InFact Politics Voices Indy/Life Business

News › World › Australasia

More Britons than Americans died on Titanic 'because they queued'

Kathy Marks | @kathymarksoz | Wednesday 21 January 2009 00:00 GMT | 0 comments

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British passengers on the Titanic died in disproportionate numbers because they queued politely for lifeboats while Americans elbowed their way on, an Australian researcher believes.

Figure 3: Example of newspaper discussion topic

The Titanic data set which is primarily used for categorical analysis and the Birthweight data set used for correlation and regression are both available via the Sheffield Hallam maths support webpages along with examples of using these data sets to demonstrate statistical techniques. (See: https://maths.shu.ac.uk/mathshelp/Resources/index.html).

At the beginning of the course students had the opportunity to undertake a voluntary 'Data Challenge' which involved searching for data and articles which were more interesting than the lecturers' examples. Surprisingly, despite not being assessed or widely advertised, about a 3rd of students participated with the winners being decided by votes and the views of the statistics team. A summary of the articles and data, which could be used by lecturers within introductory courses, can be obtained from the author.

2.4. *Fostering active learning*

Many studies have shown that problem based learning using real life data and scenarios improves statistical thinking and engagement particularly when engaging with real data and scenarios (Marriot et al., 2009; Rossman et al, 2006; Garfield and Ben-Zvi, 2007; Garfield, 1995) and this motivation leads to better learning outcomes. Statistics educational literature also suggests that using active learning within the class is more effective than the passive absorption of traditional lecturing (Cobb, 1992; Freeman et al, 2014). Active learning is an aspect of constructivist teaching where students construct their own meaning by building on from existing knowledge and through the use of authentic tasks. Methods for active learning include problem based learning through case studies, simulation, the use of technology generally, cooperative or collaborative learning. Embedding brief activities to encourage students to think about statistical interpretation and understanding before being told the answer or method (Tishkovskaya, 2012; GAISE) is a simple way of introducing active learning within the classroom. Many useful teaching resources and explanations of different types of educational methods can be found on the CAUSEWEB (https://www.causeweb.org/cause) and STATS 101 toolkit (https://community.amstat.org/stats101/home) sites.

Prior to the author teaching on the module, attendance and engagement with statistics were very low. Students were given a full set of notes and many did not see the point in attending resulting in poor understanding of the topics. The author used simple applications of active learning such as gapped notes, interactive lectures and encouragement of collaborative learning to successfully improve engagement, attendance and understanding. The gapped notes included questions to be answered from the lecture material presented, rather than gaps to copy exactly what was on the board, to ensure students understood the lecture material. The interactive lecture activities were designed to either get students to think statistically before explaining a concept, interpret results or to apply what they have learnt. Students were able to construct their own meaning by building up from knowledge of more basic techniques which also allowed them to connect the different aspects of statistics and deepen their understanding. In addition to the benefits of breaking up lectures for students, this also enabled the lecturer to assess understanding by the ease at which students undertook the activities and questions being asked. Students said that they preferred the gapped notes to a more traditional lecture as it kept them engaged and enabled them to reflect on their understanding.

Students had access to written booklets and lecture slides but the average student attended 78% of statistics lectures which was a considerable improvement on the module attendance prior to the changes. It is well known that the relationship between attendance and performance is usually weak – moderate but Figure 4 shows a strong correlation ($r = 0.7$) between lecture attendance and overall coursework mark for this module which suggests that attending the lectures was beneficial to learning. It should be noted that attendance relates purely to the statistics lecture as attendance at tutorials and the probability lectures was not consistently recorded. The regression model suggests an improvement of approximately 6% on coursework for each 10% increase in attendance (approximately two lectures) and that 50% of the variation in performance can be explained by attendance. Students were generally positive about the style of the teaching and when asked to give advice to new students on the standard University module feedback form, the most common suggestions were “attend all the classes” and “fill in all the booklets which are really useful”.

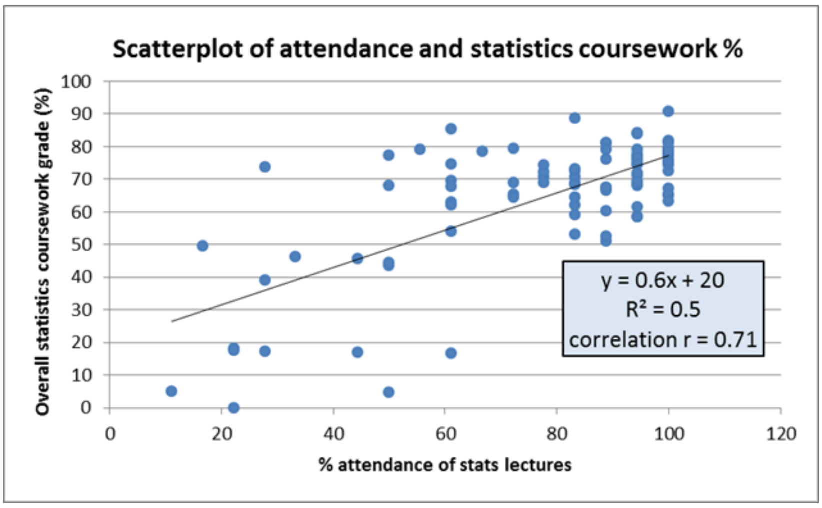

Figure 4: Scatterplot showing the relationship between attendance and performance

2.5. Using assessment to improve and evaluate learning

Statistics when taught to maths undergraduates has traditionally focused on the mathematical procedures of statistics and students learn to recognise when to use given formulae to compute final answers. These processes are usually tested through exams or closed questions in coursework with the emphasis on the correct answer rather than the interpretation. This reinforces the view that statistics is a set of unrelated mathematical topics and does not develop statistical thinking. Garfield (1994) sees assessment of problem based skills as an integral part of teaching and learning of statistics and adds that as students tend to value what is assessed, assessments testing statistical thinking and communication should be introduced. These aspects were tested through two written reports and a group presentation which this year all used the same data set. Some students were initially sceptical about the ‘non-maths’ aspects of the course but the use of the diagram in Figure 5 to explain the different skills they would need in the workplace helped them understand and appreciate the rationale behind the teaching and assessment.

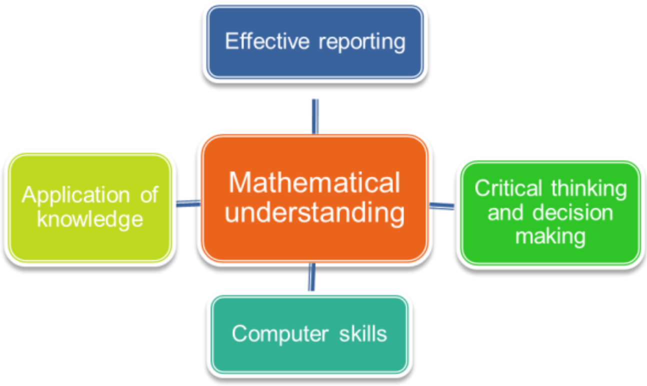

Figure 5: Skills needed by mathematician/statistician

They also struggled with the first project based assignment as it was different to the traditional maths assessment but by the last assignment they had a greater appreciation of what statistics really is and even found the collaborative scenario based presentation enjoyable.

In the first assessment, students were given a number of research questions and a dataset from a medical trial. Students were expected to select appropriate summary statistics and graphs for each question and communicate these results efficiently and effectively in the form a report. The understanding of data types and selection of appropriate statistics to address specific hypotheses was tested trough an online test and the visual representation, calculations and interpretation were assessed through a formal written report. Students generally performed well on the online test but did not necessarily connect the questions from the test to what was expected to be included within the more open report. The negatives were that the marking of the reports was very time consuming and that the students didn't feel that they had been adequately prepared for the writing of the report. In the next academic year, students will receive guidance on report writing; the assessment criteria will be more clearly explained and they will be given the opportunity to get feedback on formative sample questions within tutorials. To ease marking, only one individual written report will be assessed and two parts will be submitted; a formal report and an Appendix containing calculations and SAS output.

The final assessment, a cooperative group presentation involving analysis of data from a medical trial, enabled students to fully express statistical reasoning, thinking and communication in a more open project. Students seemed genuinely enthusiastic about working on a real life scenario and commented on how their understanding had improved through the cooperative learning. The team of staff assessing the twenty six presentations were impressed by the students' ability to apply statistical thinking and a wide range of techniques appropriately. Next year the students will be undertaking work related collaborative assessment for external companies who will provide data and ideas for statistical analysis and it is hoped that this will engage students further with statistics.

2.6. Reflections and feedback

The author's primary role is in statistics support and has subsequently observed the gaps in student knowledge when undertaking their own research. One of the main aims of the author was to integrate statistical skills particularly choosing suitable techniques and effective reporting into the first year module to adequately prepare the students with roles within industry alongside the traditional teaching. Whilst the author had some material, data and examples from other courses and notes from her predecessor, making fundamental changes to the structure, aims and content of any module is time consuming and ongoing particularly with a module that students traditionally dislike. Staff who teach on further stats modules were impressed with the understanding and statistical skills demonstrated by this cohort of students when watching the group presentations. They also observed a noticeable improvement in student understanding when the students started their 2nd year stats module.

Students currently in their 2nd year were asked to comment on the course and were very positive about the structure, content and style of teaching in general and still use the course booklets to help with their understanding of further statistics modules. They particularly liked the effective use of gapped notes and exercises within the lectures as a means of staying engaged and strengthening understanding. Many students started the course with negative views about statistics based on their A level experiences but a number of these students have now expressed an interest in careers in statistics and have applied for statistics related placements.

3. Conclusion

For many years, statistics educational literature and guidelines have suggested that a shift in focus from mathematical calculations to statistical problem solving and communication is needed to meet the research and employment needs of students of today. This paper discusses changes made to an introductory probability and statistics course to ensure students are able to understand and apply statistics in a real world setting and how the current guidelines and suggestions for statistics education (GAISE) can be implemented. Introducing and reinforcing the problem solving approach throughout a course, using large data sets and real life scenarios, using active learning within lectures, using technology to enhance learning and changing assessments to test decision making and communication were all successfully implemented within a first year statistics module which led to improved understanding and engagement. Making substantial changes to a module is an iterative process however and further adaptions to the teaching will be made in the next academic year including more simulation and constructivist teaching allowing students to create rather than memorise formulae.

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WORKSHOP REPORT

Statistics SIG: identifying and addressing issues within statistics support

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Abstract

The Special Interest Group (SIG) in Statistics Support maintained by the sigma Network for excellence in mathematics and statistics support held a discussion workshop at the CETL-MSOR conference 2018. This was focused on identifying and addressing issues within statistics support and facilitated a very interesting and valuable sharing of experiences and suggestions for good practice in the provision of statistics support. Discussions were primarily focused on the limitations and scope of statistics support, and in particular the following issues were discussed: supporting student with statistics software, understanding the remit of statistics support, supporting students with choosing statistical techniques, as well as a short discussion on raising awareness of statistics support and evidencing its impact. One key action arising from the workshop was the proposal to run a workshop on statistics software.

Keywords: Statistics, support issues, scope, remit.

1. Overview

The sigma Network for excellence in mathematics and statistics support (http://www.sigma-network.ac.uk) maintains a Special Interest Group (SIG) in Statistics Support. This SIG is focused on statistics support provided in Higher Education institutions primarily in the UK, but does have a wider global perspective since the same issues and challenges are faced worldwide. The SIG held a discussion based workshop at the CETL-MSOR conference 2018, which was focused on identifying and addressing issues within statistics support. This workshop report outlines the issues that were discussed during the workshop, as well as offering some thoughts on suggested good practice. The report is organised by reporting on the preparation and management of the workshop in Section 2, whilst Section 3 provides a commentary of the discussions at the workshop, which are then summarised in Section 4 to complete the report.

2. Workshop preparation and management

Prior to the workshop taking place at the conference, colleagues subscribing to the sigma-Network mailing list at (https://www.jiscmail.ac.uk/cgi-bin/webadmin?A0=SIGMA-NETWORK) were contacted, asking (irrespective of whether or not they were planning to attend the conference) to share questions they had or challenges they faced when providing statistics support. Sharing was done by posting on the workshop's padlet wall (https://padlet.com) at https://en-gb.padlet.com/owen_a3/nob4xbxkekgj. Colleagues were invited to consider any questions or challenges posted by others, and "like" them if they shared similar questions or challenges. This was to gauge how common particular questions or challenges are. Finally, colleagues were invited to add ideas or suggestions in response to existing questions or challenges posted by others.

It was clear from the questions and challenges posted on the padlet wall prior to the conference, and the subsequent discussion of these at the workshop, that many of these related to the theme of "Limitations and Scope of Statistics Support". Figure 1 shows a slide which summarises some of the questions and challenges posted prior to the conference that were included in this theme. This slide was displayed at the workshop and these questions and challenges related to this theme were discussed, along with other aspects related to this theme.

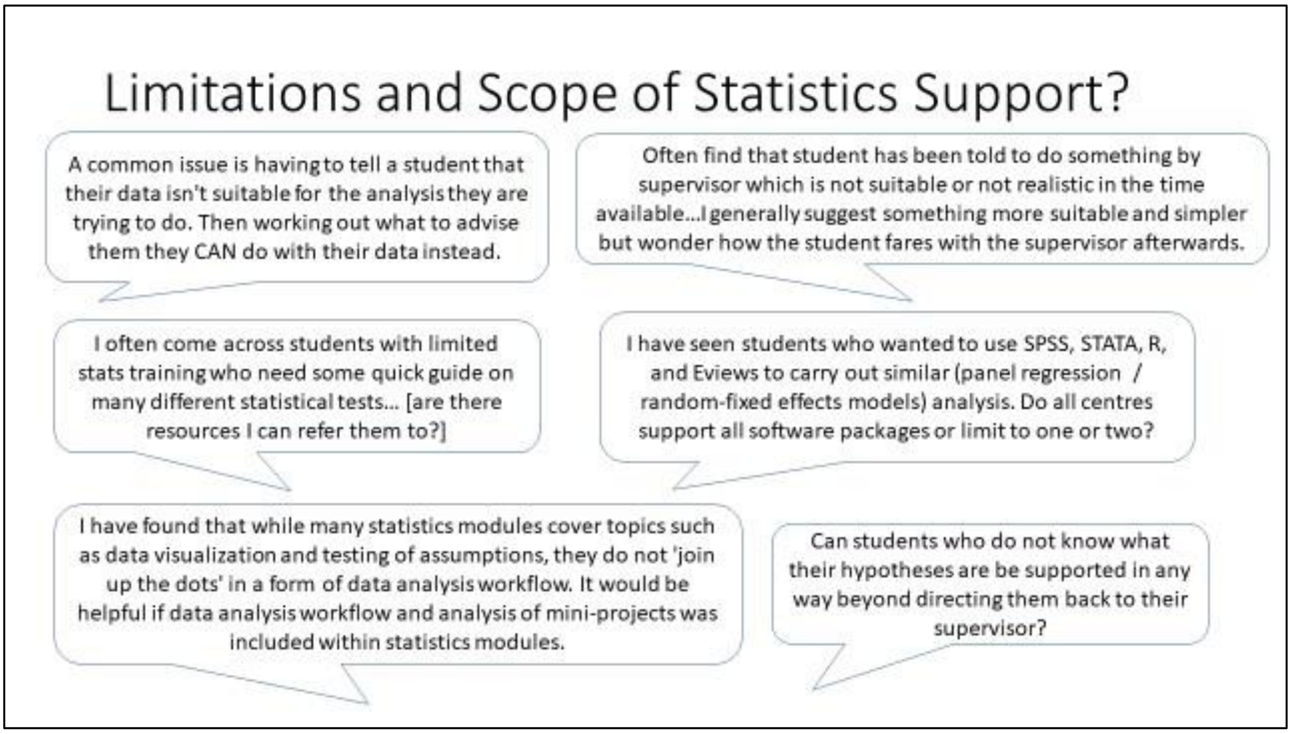


Figure 1: Questions and challenges posted prior to the conference that were included in the theme of "Limitations and Scope of Statistics Support".

A total of 10 colleagues attended the workshop who were organised into two groups, and were tasked with discussing any questions and challenges not only posted prior to the workshop, but also those that they themselves wished to discuss. The notes recorded by these two groups were captured on flipchart paper and reproduced in the appendix to this report.

The remainder of this report summarises the discussions that subsequently took place during the workshop. All of the views and opinions and recommendations expressed in this workshop report are therefore of those that attended the workshop or colleagues who contributed to the discussions prior to the workshop.

3. Discussion

3.1. Support with statistics software

A range of different statistics software is used within Higher Education (HE) institutions in the UK, but the main question shared at this workshop was "Should statistics support tutors be expected to offer support in all of them?". The most commonly used statistics software taught within H.E. institutions is still SPSS, although R is becoming increasingly popular. The workshop concluded that providing support with SPSS and R together with Excel could typically be expected in mathematics and statistics support centres in HE institutions. Statistics software other than SPSS or R, such as SAS, Minitab, STATA and eviews, is used in many HE institutions, and it was generally felt that, whilst statistics support tutors should be able to help with the interpretation of output from any statistics software or package, it should not be expected that tutors are proficient users of all of these.

The main suggestion for good practice in relation to this issue related to the need to manage the expectations of students, clearly stating the statistics software that support staff were proficient in on relevant support centre websites. In addition it was also agreed that maintaining suitable links on support centre websites to resources on how to use other statistics software would be very helpful to students.

The need for tutor training with the use of statistics software was considered and it was thought that basic training in R would be most beneficial, as many tutors are not familiar with R but its popularity is increasing. There are some R resources available via the community resources contributed to statstutor (https://statstutor.ac.uk), whilst additional resources are available through the Sheffield Hallam Maths and Stats support centre website at https://maths.shu.ac.uk/mathshelp/Resources/. However, a basic training giving an introduction to R was felt would be the easiest way for tutors to get to grips with the basics. Another software package, Nvivo (https://www.qsrinternational.com/nvivo/home), which is aimed at the analysis of qualitative and mixed methods data, was also identified as something statistics tutors would like to have training in. One outcome from these discussions was the proposal for a potential meeting of the SIG hosted at Liverpool John Moores University, focused on a training workshop on statistics software to include R and Nvivo and other relevant statistical software products.

3.2. What is in the remit of statistics support?

The provision of help and advice with statistical software is commonplace with the context of statistics support. However, it was agreed that in many cases, the software students are being expected to use is not always covered within departments' core course delivery. It was agreed that the basics of any software students are expected to use should be provided within the student's course, and that it shouldn't be an expectation that a mathematics and statistics support centre will offer the teaching and learning opportunities in this area. This is particularly true where packages that many statistics support tutors are much less likely to be familiar with, such as STATA and eviews. Indeed, it was agreed that the helping students with the basics in the use software could be seen as computing support, rather than statistics support. If the student has been told to use a specific package but has received no teaching or learning opportunities with the software, it could be argued that statistics support tutor should help the student develop skills with a package the tutor is familiar with.

One area of statistics support that it was agreed should not necessarily fall within the scope of statistics support is advising on the formulation of hypotheses or research questions in the context of students seeking help with projects. However, whilst statistics support tutors cannot be expected to understand the research context within all academic disciplines or help formulate research questions, it was agreed that sometimes asking probing questions of the student about what they hoped to learn from their study and/or the nature of the data they have or plan to collect could help the student clarify their aims. If after a session focused on these aspects, the student still wasn't sure of their aims or were unable to formulate a hypotheses or research questions, it was recommended that they are re-directed back to the supervisor.

In contrast, other aspects of support with student projects, such advising on study design, data collection and project management, it was felt could be considered as falling within the remit of statistics support. However, it was agreed that students' project supervisors should play a key with some of these aspects, particularly when there is a context of the project that related to supervisors' specialist areas. Another issue that was raised related to supporting Postgraduate Taught (PGT) students, where the demand for help peaks in the summer when many PGT students' supervisors take annual leave. In these circumstances it is not uncommon for the student to perceive the statistics tutor as a surrogate supervisor for their project, and so the statistics tutor should take care to explain their role and manage the student's expectations. It was reported that Chester University have trialled a scheme where statistics support tutors attend initial dissertation meetings with students with the student's supervisors present to offer guidance at the beginning. Whilst providing this sort of support/training for supervisors would clearly be beneficial in the long run, discussion centred on whether this was feasible in larger institutions. It was suggested that advising groups of project supervisors on study design and analysis, in a workshop context, could work but whether this should be within the remit of statistics support was questioned as well as whether it was manageable in larger institutions.

3.3. Supporting students with choosing statistical techniques

There were many issues raised relating to the expectations of students and project supervisors. These included students seeking help with techniques suggested by their supervisor or teacher that appear to be inappropriate, choosing the appropriate level of statistics to support a project student with and whether this choice conflicts with the suggestions from the student's supervisor.

Many students come to statistics support with an idea of the technique they want help with, or may have been suggested by their supervisor or teacher, but often the suggested technique is considered by the statistics support tutor to be inappropriate or perhaps too advanced for the student given their evident mathematical/statistical expertise or the time frame within which the student is working. A student obviously wants to do as their supervisor suggests, even if they have no idea what the technique is or why it is appropriate. It was agreed that it is important to check why the student or their supervisor thinks they should use a specific technique and how it relates to their research questions. In addition it is equally important for the statistics tutor to explain to the student (and their project supervisor) the justification for any suggestions the tutor makes for using a different technique to the one the student or their supervisor had suggested.

If a student's project supervisor has suggested inappropriate methods, the statistics support tutor should be cautious, since ultimately it is the supervisor who will assess the student's work. In addition, good close relationships between statistics support staff and the staff teaching students is are vital, to maintain the support of teaching staff in promoting the value of using mathematics and statistics support centres.

The general consensus was that the statistics support tutor talks through the research questions and possibilities for analysis with the student and suggests that the student then talks through the options with their supervisor. This also applies to techniques which the statistics support tutor feels are too advanced for the student given the students level of ability or time frame. Students often worry if using simpler techniques will reduce their grade, therefore it is important to stress that understanding and reporting the chosen technique in relation to the research question is preferable to doing more complex analysis badly or incorrectly. It was also agreed that the importance of ascertaining the student's prior statistical knowledge, clarifying expectations and the time available, before making any suggestions regarding techniques, are all elements of good practice in providing statistics support. If a student is new to statistics and has more time, basic hypothesis testing using a simple test should be discussed first before moving up to more complex analyses.

Statistics teaching in general typically follows a "cookbook" focusing on individual techniques one at a time, with little or often no attention paid to when or why any specific technique would be relevant. As a result it was agreed that most students struggle when choosing the most appropriate methods of data collection and analysis for their data. This is compounded by the fact that when project students first seek statistics support, they have often already collected their data, often using a poor study design. Choosing the correct technique is an integral part of a statistics support session, which should be carried out even if the student thinks they know the appropriate analysis. It was agreed that helping students understand which technique is suitable can be achieved using flow charts or tables based on the data types of their independent/dependent variables rather than just informing them with no explanations. Some useful resources that were discussed include those available through the Sheffield Hallam Maths and Stats support centre website at https://maths.shu.ac.uk/mathshelp/Resources/testchoose.html, or the whattest resource available from the Mathematics Learning Centre at Loughborough University via http://whattest.lboro.ac.uk/index.html

Given the number of students requiring help with the process of choosing the right technique, embedding these skills within taught modules would clearly be the most beneficial approach. However, it was evident that some mathematics and statistics support centres also run workshops on this aspect of statistics.

Finally, in relation to supporting students with the right technique, it should be acknowledged that there will be, and often are, limitations on the range of statistical techniques that staff within a mathematics and statistics support centre have expertise in, particularly where statistics support is offered by non-specialists. As with providing support with a range of statistical software discussed in the previous section, it was agreed that it should be possible to give an overview of the level of statistics taught within individual centres, via centre websites etc. As a closing remark in this section, in the case of less well known statistical techniques, one suggestion was to ask the lecturer who specified the technique to teach support staff about the topic.

3.4. Advertising and evidencing the impact of statistics support

There are many modes of advertising used by mathematics and statistics support centres to raise awareness amongst the student population that statistics support is available. Although advertising during induction/welcome weeks, through events, talks and emails is common, it was agreed that this was not the best time to advertise statistics support, as students were often overloaded with information at that time, and more importantly their awareness of the need for statistics support would not yet be evident. Word of mouth was generally seen as an effective method of advertising and increasing awareness through lecturers, students union and student ambassadors for more timely advertising helped with this. In particular, it was felt that developing relationships with staff that lead project modules across the institution would be beneficial to facilitate timely promotion of the availability of statistics support and the need to seek advice on study design and data collection early. Having a good website presence which is easy for students to find and networking at higher levels to spread the word about the service was also suggested.

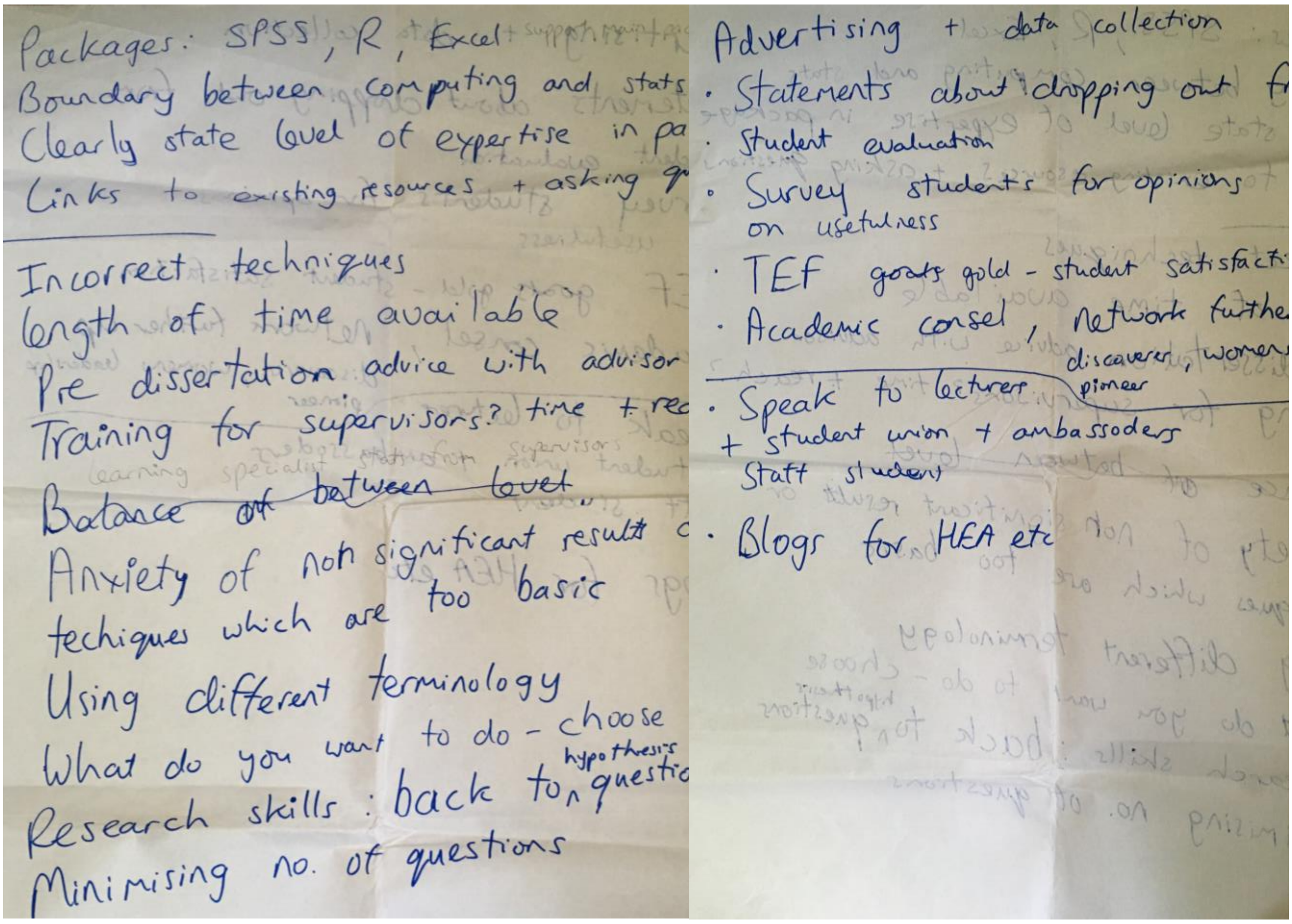
Discussion also took place about evaluating the impact of mathematics and statistics support, in order to raise the profile and demonstrate the value of the service to both students and the institution. It was agreed that surveying students who had used statistics support, particularly around the impact of the help they received in reducing their perceived risk failing or dropping out, would offer some evidence to support the development or continuation of statistics support provision in an institution. Finally it was agreed that networking with relevant committees and management across the institution, would also help in terms of promoting evidence of the impact that statistics support provision has on reducing student attrition, as well as contributing to positively to outcomes related to the NSS (and ultimately the TEF).

4. Summary

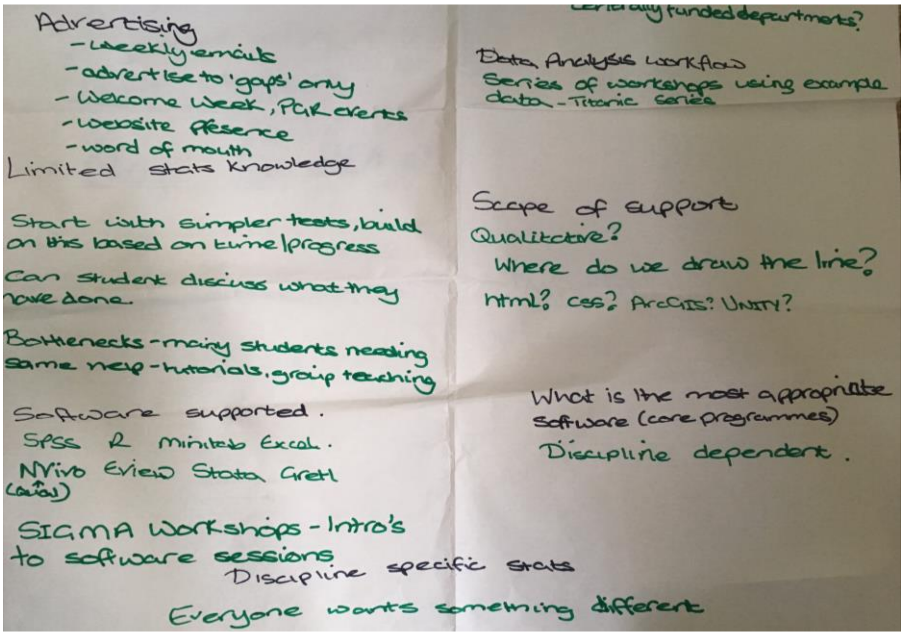
The Special Interest Group (SIG) in Statistics Support, maintained by the sigma Network for excellence in mathematics and statistics support, held a very valuable workshop at the CETL-MSOR conference 2018. This was focused on identifying and addressing issues within statistics support and facilitated a very interesting and valuable sharing of experiences and suggestions for good practice in the provision of statistics support. Discussions were primarily focused on the limitations and scope of statistics support, although some additional issues were a focus for discussion. Specific topics that were discussed included supporting student with statistics software, understanding the remit of statistics support, supporting students with choosing statistical techniques, as well as a short discussion on raising awareness of statistics support and evidencing its impact. One key action arising from the workshop was the proposal to run a workshop on statistics software.

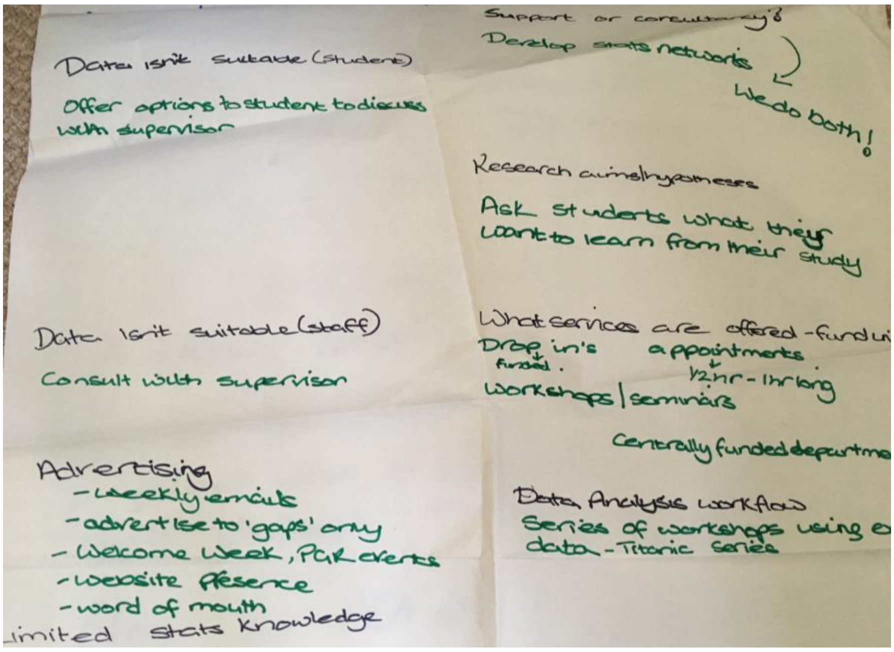
5. Appendix

Written comments recorded at the workshop: Group 1



Written comments recorded at the workshop: Group 2





6. Acknowledgements

The authors would like to thank and acknowledge all of those colleagues that contributed to the padlet postings prior to the conference and those that took part in the workshop. These include Alison Loddick, Carole Davenport, Cath P, Chetna Patel, Éva Szatmari, Jia Shao, Liam Brierley, Monica-Christina Hess, Morag McFadyen, Nick Goddard, Paul Rice, Peter Mitchell and Shirley J. Without their contributions the workshop and this report, and the subsequent sharing of good practice would not be possible. Our apologies to anyone we have forgotten to include.