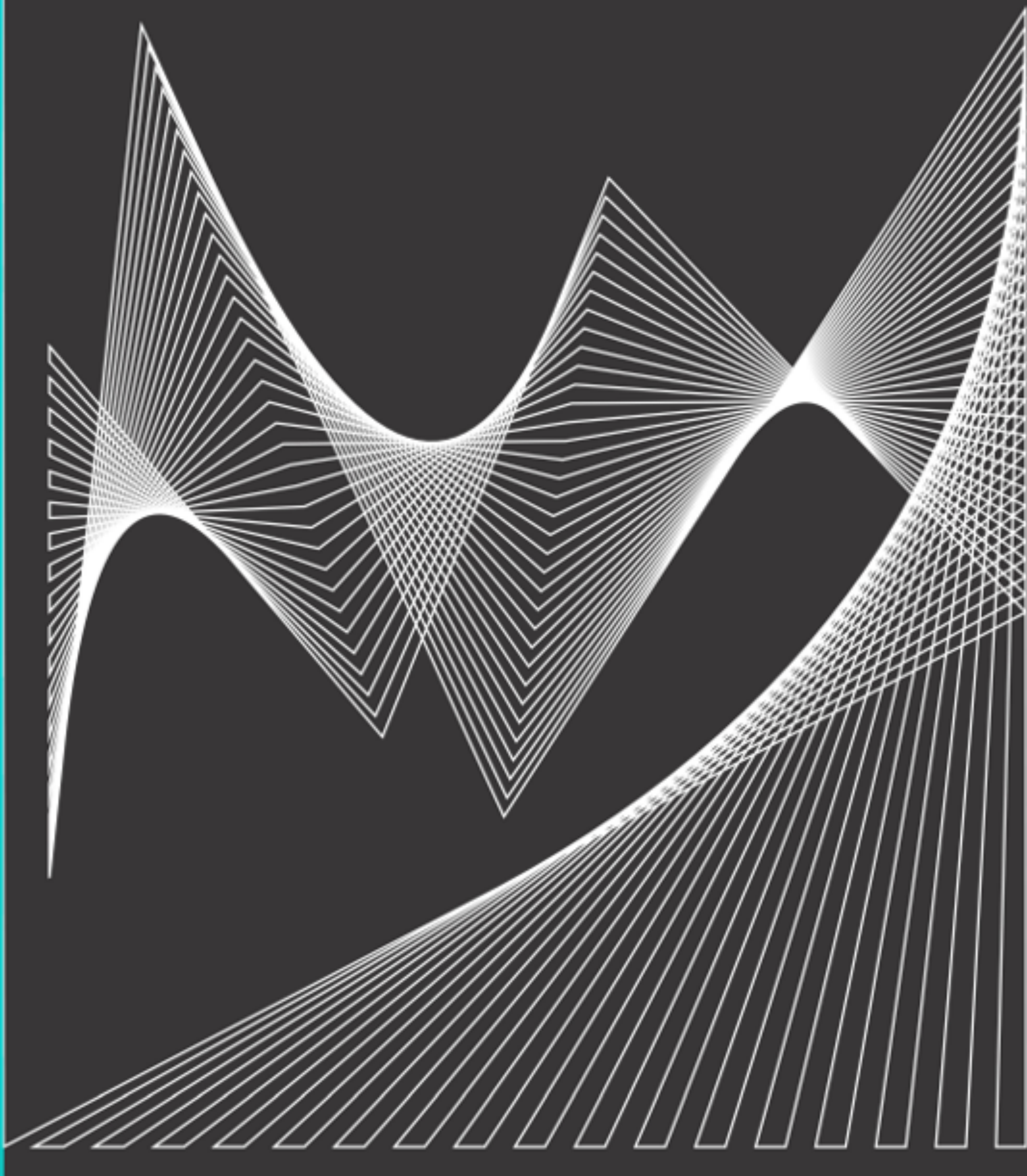
# MSOR connections

Articles, case studies and opinion pieces relating to innovative learning, teaching, assessment and support in Mathematics, Statistics and Operational Research in HE.

Volume 18 No.1



## Contents

For information about how to submit an article, notifications of new issues and further information relating to *MSOR Connections*, please visit https://journals.gre.ac.uk/index.php/msor.



This journal is published with the support of the **sigma** network and the Greenwich Maths Centre.

# Editorial

Peter Rowlett, Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield, U.K. Email: p.rowlett@shu.ac.uk.

This issue opens with an article from Grove, Mac an Bhaird and O'Sullivan sharing a wealth of experience of delivering professional development for tutors of mathematics learning support through case studies from the UK and Ireland, which hints towards a future where such professional development may be accredited. Sticking with mathematics support, Guerin and Walsh report an analysis of strategies for advertising mathematics support to students.

Next, Huntley, Middleton and Waldock provide interesting insights into active development of mathematical learning communities both physical (on-site) and virtual (distance).

Moving to consider undergraduate curricula, a research article from Ford, Gillard and Pugh attempts a classification of errors in undergraduate mathematics, noting that this is distinct from work which attempts to classify errors in school-level mathematics. Brignell, Wicks, Tomas and Halls are interested in marking criteria and student self-regulation, aiming for students who can attempt similar and unseen problems in the absence of expert help or model answers, and investigate the use of peer assessment.

Finally, Bortot and Coles present a case study using a modified version of the game Rock-Paper-Scissors (or Paper-Scissors-Stone) to teach concepts in statistics.

I would like to take this opportunity to express thanks on behalf of the editors to our outgoing editorial board members Tony Croft, Neville Davies and Paul Hewson for their years of service to the journal.

I believe *MSOR Connections* performs a valuable function for our community by providing a forum for sharing and discussion of ideas around teaching, learning, assessment and support. It can only act in this way if the community it serves continues to provide content, so I strongly encourage you to consider writing case studies about your practice, accounts of your research and detailing your opinions on issues you face in your work.

Another important way readers can help with the functioning of the journal is by volunteering as a peer reviewer. Many of the articles in this issue were reviewed by one experienced reviewer and a second who was reviewing an article for the first time with my support as needed. If you are interested to gain experience of being a reviewer, please get in touch.

To submit an article or register as a reviewer, just go to https://journals.gre.ac.uk/index.php/msor. When you register as a reviewer, it is very helpful if you write something in the 'reviewing interests' box, so that when we are selecting reviewers for a paper we can know what sorts of articles you feel comfortable reviewing.

I hope you enjoy reading this issue as much as I have putting it together.

CASE STUDY

# Professional development opportunities for tutors of mathematics learning support

Michael Grove, School of Mathematics, University of Birmingham, Birmingham, UK. Email: m.j.grove@bham.ac.uk.
Ciarán Mac an Bhaird, Department of Mathematics and Statistics, Maynooth University, Ireland. Email: ciaran.macanbhaird@mu.ie.
Ciarán O'Sullivan, Department of Mechanical Engineering, Technological University Dublin – Tallaght Campus, Dublin, Ireland. Email: Ciaran.OSullivan@TUDublin.ie.

## Abstract

Deficiencies in the mathematical skills of students entering university study are having a negative impact on their education, and more broadly have serious consequences for society as a whole. Research demonstrates that extra initiatives established to give these students an opportunity to succeed are making a difference, and that the staff who provide these supports play a fundamental role. Here we review two different models of structured training that were developed for these tutors, via two cases studies drawn from within the UK and Ireland. We discuss the key and transferrable skills that these tutors require, skills that are often not typically needed in a more 'traditional' teaching role. The majority of tutors remain in this crucial support role for only a short period of their careers, and so a fundamental question remains as to how they can receive appropriate recognition for their academic endeavours. Such recognition is important for both the institution, in demonstrating its commitment to teaching quality, and for the career progression of the tutors themselves.

**Keywords:** mathematics support, tutor training, accredited provision.

## 1. Introduction

Mathematics learning support (MLS) is now a widely accepted means of helping students address the difficulties they encounter with the mathematical and statistical components of their studies, particularly as they make the transition to university study (Lawson, 2015). Student evaluations of MLS provision identify, almost without exception, the crucial role of MLS tutors in their success (O'Sullivan, Mac an Bhaird, Fitzmaurice and Ní Fhloinn, 2014). This reflects the unique MLS student-tutor relationship where one-to-one support is provided in a relaxed and non-threatening environment. Sound mathematical knowledge, and the ability to apply it, are presumed for an MLS tutor. However, the diverse and challenging nature of the teaching involved and the many different situations they may potentially encounter (Croft and Grove, 2011) means that tutors need to be appropriately trained and subsequently supported (mentored) by those with experience of working in such a teaching environment. In recent years, there has been a significant increase in the levels of training available for MLS tutors (Croft and Grove, 2016; Fitzmaurice, Cronin, Ní Fhloinn, O'Sullivan and Walsh, 2016) and this coincides with national moves to increase the number of staff within UK and Irish HE with a recognised teaching qualification. For example Marshall, in her forward to the 2015 UK Student Experience Survey (Buckley, Soilemetzidis and Hillman, 2015), comments on the results (p.3):

> *When asked to rank the importance of three different characteristics of the people they are taught by, students in nearly half of all subjects rate staff having received training in how to teach as the number one priority. When asked last year about priorities for institutional expenditure, a significant number of students chose better training for lecturers.*

In this paper, we focus on the key role of tutors in MLS, and present two case studies of MLS tutor training successfully established (not entirely independently) in the UK and Ireland. These training models are the culmination of a journey from ad hoc (institutional specific) training to structured regional/national models of training.

## 2. Background

The postgraduate teaching assistant, has a well-established and vital role in supporting teaching and learning within HE and one that requires specific support (National Research Council, 1991, p.27):

> *Heavy reliance on the use of graduate teaching assistants, many of whom have limited experience or training for the responsibilities placed on them, has far-reaching consequences…Few graduate students, however, are ready to serve well the educational needs of first-year college students…*

Within the US, there has long been recognition of the importance of the postgraduate teaching assistant within the mathematical sciences. The Mathematical Association of America (MAA) within its *Guidelines for Programs and Departments in Undergraduate Mathematical Sciences* (MAA, 2003), makes it clear that "*since they are the future faculty members of our colleges and universities, it is important that graduate students have some instruction in teaching including serving as apprentice teachers.*" (p.3). Furthermore, it adds: "*Departments should provide long-term structured opportunities for acquisition and improvement of teaching skills by all who teach.*" (p.3). In comparison, perhaps the closest UK equivalent to this, the Quality Assurance Agency (QAA) for Higher Education Subject Benchmark Statement for Mathematics Statistics and Operational Research (QAA, 2015), a document designed to ensure the quality and standards for mathematics education in UK HE, currently makes absolutely no reference to training for those teaching or supporting the mathematical sciences within HE.

Within the UK, a 2005 survey involving approximately 60 postgraduate students from the mathematical sciences and exploring their training needs (Cox and Kyle, 2005) highlighted that most (70%) were involved in running seminars, tutorials, problem classes or other small group teaching of some kind in Mathematics or Statistics, 40% were involved in marking exams and about half marked coursework. Despite the far-reaching nature of their roles, postgraduates were increasingly reporting that they received little training or support for these duties: "*Before embarking on my teaching duties as a postgraduate it always struck me as somewhat unusual that PhD students are able to and expected to be capable of undertaking such duties with very little (or if given) suitable training*" (Lee, 2005, p.38). There are likely to be benefits for the undergraduates who receive the tutoring if the postgraduates have been appropriately trained. The postgraduates are not only better prepared to meet the needs and expectations of the undergraduate students they tutor, but they also develop their own skills, their confidence in teaching and receive a valuable preparation for a range of future academic and non-academic careers.

In more recent times the important role of postgraduate students working within a MLS environment has become increasingly apparent and this is coupled with the widespread growth of mathematics support observed across the UK and Ireland (Perkin, Croft and Lawson, 2013; Cronin et al., 2016). For example, in their analysis of recent surveys of MLS undertaken across the UK and Ireland, Grove, Croft and Lawson (2019) identified that out of 116 institutions, 53 were using postgraduate students as tutors within MLS. Further, amongst the 78 institutions they surveyed in 2018 within England and Wales as part of this work, there were seven institutions where mathematics support was provided solely by postgraduate students.

MLS tutors have diverse mathematical backgrounds (Cronin et al., 2016; Grove, Croft and Lawson, 2019), mainly encompassing postgraduate and undergraduate students, full-time and part-time staff, and staff who are external to the institution. While some of these tutors may have experience gained from traditional teaching roles within the institution such as lecturing or tutoring within a department, MLS tutoring requires an additional and enhanced set of skills. In a sense, it is not tutoring, it is not small group teaching, it requires communication skills, human empathy, and the ability to allow other people to work and fail but within a supportive and non-judgmental environment; it is the ability to guide independent learning. Within MLS, tutors work with some of the most vulnerable students, those lacking confidence, those with specific learning difficulties, and those most at risk of dropping out. Increasingly, MLS is also being accessed by the specialist and more-able student (Grove, Guiry and Croft, 2019; Croft and Grove, 2015), one who is seeking to enhance and develop their mathematical skills and knowledge even further and occasionally beyond the boundaries of their module(s) of study.

While many MLS tutors may have had some prior training as department tutors or lecturers, they are, as noted by Croft and Grove (2016, p.3), "*in the front line of tackling the lack of confidence and skill deficits of students who arrive at support centres looking for help*". In addition to both their content knowledge and their ability to think on their feet (since student queries are rarely predictable in MLS), tutors need to facilitate a social environment where mathematics learning can take place (Solomon, Croft and Lawson, 2010). The importance of providing MLS in a non-judgmental, non-embarrassing and non-threatening environment (Lawson, Croft and Halpin, 2003; O'Sullivan et al., 2014) is key to giving learners an opportunity to talk about their mathematical problems and concerns.

With students themselves now being required to contribute a greater proportion towards the costs of their education within England and Ireland (see for example *Higher Education and Research Act 2017* and Cassells (2016) respectively), there is evidence that their expectations in relation to their learning experience are changing. For example, the 2016 Student Academic Experience Survey (Neves and Hillman, 2016) concludes (p.5):

> *The student experience is still a positive one, but students as consumers are becoming more demanding. They are looking for evidence of value for money and are prepared to put in the effort themselves as long as they feel this is matched by being offered an involved experience with high-quality teaching, staff who continuously develop their skills, and appropriate levels of contact hours for the subject they choose.*

This, and the now well-documented challenges associated with student learning of mathematics within a range of disciplines, has clear implications for how HEIs can best support these learners to ensure that their programme of study not only meets their needs, in terms of future careers or further study, but also their expectations. Tackling this 'mathematics problem' is an area of priority for almost all HEIs within the UK and Ireland and one where MLS and its tutors have a critical role (Cronin et al., 2016; Grove et al., 2018). Many of these tutors are postgraduates, and for postgraduates who choose to work within MLS, while doing so is known to be challenging, there are many benefits. Since many MLS tutors do not remain in MLS provision for their entire careers, they develop an extensive range of transferable skills that should place them at an advantage when seeking employment or an academic career. However, in order for their experience to be a productive and positive one, they require training and ongoing support, with recent evidence demonstrating that they continue to develop as tutors by being part of a community of their peers (Grove and Croft, 2019).

We now move to consider training for staff involved in teaching/tutoring or supporting students in their mathematical learning. Doing so however, raises a more fundamental issue relevant to all disciplines: how can such training be structured and accredited so that there exists an institutional record that training has taken place and that the individuals who participate have a formal and transferable record of their commitment to their professional development? Through our work, we have identified that such training and support may take one of three forms: ad hoc and non-accredited; structured and non-accredited; and, structured and accredited.

Ad hoc training can perhaps be best described as MLS training designed within an individual institution as a one-off event. It would normally be given before a tutor begins in MLS, and provides essential information the tutor needs to manage typical situations that may arise. Structured training would be designed by a network of experienced MLS practitioners from a variety of institutions. It forms a programme, a regular series of activities, mentoring and support, which when combined offers a continued opportunity for tutors to develop and hone their skills. It would also allow for specific issues or challenges encountered to be discussed and advised on.

Non-accredited describes the situation where there is no formal acknowledgement or recognition that an individual has undertaken such training, and as such, there exists no record of their commitment to their professional development in a format which is easy for a tutor to evidence via a C.V. or a transcript of academic achievement. A record in this format is important for the institution and the individual: it demonstrates that training has not only taken place, but also that the individual possesses the required skills and abilities to perform their duties effectively. For the institution, this is important for demonstrating quality assurance while for the individual it can help in showcasing their skills as they look to make the transition to an academic career or employment.

The two case studies that follow, drawn from the UK and Ireland, illustrate structured and non-accredited approaches that evolved to address the initial professional development needs of MLS tutors but which in their present form are not easily amenable to enabling tutors to gain formal accreditation for the skills and expertise developed.

## 3. Case Study 1: Training staff who work in MLS within the UK

For many years there has been significant work within the US to ensure appropriate training, support and guidance for graduate teaching assistants within the mathematical sciences was available, see for example Rishel (1999). However, within the UK, before 2005, there had been very little. In autumn 2005, the Maths, Stats & OR (MSOR) Network, a national government funded disciplinary organisation with the mission of enhancing teaching and learning within the mathematical sciences in UK HE, set about changing this by introducing a series of one-day workshops aimed at postgraduate students who were teaching and supporting learning.

While we choose not to discuss these workshops here, further details of their structure can be found in Grove, Cox and Kyle (2006). They are important since they formed the subsequent model for a one-day workshop programme developed for those new to working in MLS. Through the National HE STEM Programme (Grove, 2013) a network was developed to assist those working in MLS across England and Wales. In 2010, this **sigma** Network (Croft et al., 2015), which sought to increase the extent of MLS provision and to share effective practice, identified there existed a pressing need to provide some form of initial training to the growing number of postgraduate students involved in the provision of front-line mathematics and statistics support to learners.

The training consisted of one-day workshops delivered by experienced members of the MLS community, who worked as facilitators rather than presenters. The focus for these events was clear, they were practical not theoretical. Further, the events were to be role and subject specific, that is, grounded in the reality of the duties tutors were likely to undertake when working in MLS and based firmly within the context of the discipline of mathematics.

Table 1. Format of **sigma** workshops for postgraduates working in MLS.

| Session | Short Description |
|---|---|
| 1. Welcome and introductions | With a view to establishing interaction and identifying common themes, delegates are asked to spend a few minutes considering what they want to learn/gain from the day. |
| 2. Mathematics support – what is it? | Participants are asked to explore ideas for what MLS is and how it is utilised by learners. The purpose is to develop interaction and help delegates obtain an understanding of the context of working in MLS. |
| 3. Problem solving | Participants consider a range of problems in small groups. The purpose is not to solve these, but to consider how they will help guide students using their MSCs. |
| 4. Principles of maths support – do's and don'ts | Delegates are asked to consider what they might do prior to, during or after working in the MSC. |
| 5. Offering statistics support | Offering support in statistics is different to offering support in mathematics (Croft and Grove, 2016). Advice is provided on how to deal with statistical queries for tutors who are not specialist statisticians. |
| 6. Tutoring in the mathematics drop-in centre – awareness of individual differences and needs | Students using MLS have a range of backgrounds, interests, and learning styles. Guidance is provided on the backgrounds of students who may use MLS with advice on how to support these. |
| 7. Group activity – exploring scenarios | Groups are given several wide-ranging scenarios that have arisen in existing MSCs and asked to discuss how they would respond. |
| 8. Resources and networking with others | A wealth of resources are now available for MLS. This session raises awareness of some of those that are freely available. |
| 9. Question and answer session | A final opportunity for delegates to explore aspects of the workshop in more detail or to have any questions they may have answered. |

Based on participant feedback, three core principles underpinned the delivery of these workshops, and were critical to their success: practice sharing; an informal environment; and, interactivity. Although workshops were open to all working in MLS from across the UK HE sector, basing them within a HEI meant that often the majority of participants were drawn from that institution. Bringing together new tutors, who were typically postgraduates, was an ideal networking and social opportunity, aiding their adjustment to what might be a new place of study and, in terms of their work in MLS, it was critical for building an environment where there were other like-minded individuals they could approach for advice and guidance post-workshop. The workshops were deliberately designed to be informal; it was deemed essential to engage the postgraduates fully in the activities offered, but also allow them the opportunity to shape the overall direction taken within sessions to respond to their particular needs and concerns. The trainers identified that building an informal environment was key to developing this interactivity, and achieving this involves developing the confidence of participants to freely share their views and ideas throughout the workshop. It was highly noticeable that, as a result of the interactive mathematical tasks, interactivity and the willingness of individuals to contribute views and ideas increased throughout the day.

## 4. Case Study 2: Training staff who work in MLS in Ireland

In Ireland, prior to 2009, the training of MLS tutors was developed and provided as a local activity within some of the individual MSCs. A 2008 audit of MLS provision (Gill, O'Donoghue and Johnson, 2008) found that only 2 of the 13 institutions who responded provided formal training for tutors. Due to the success of **sigma** within the UK, the Irish Mathematics Learning Support Network (IMLSN) was established in 2009. The newly formed IMLSN noted similar challenges and issues associated with the training of tutors amongst those institutions with MLS provision. Indeed, in 2011 the initial data analysis of the large-scale multi-institutional student evaluation of MLS (1633 first-year service mathematics students from nine HEIs) (O'Sullivan et al., 2014), the importance of the tutor role in MLS was so strongly identified by the respondents that the IMLSN gave immediate priority to the design of a structured training programme for MLS tutors on the island of Ireland. The training programme comprised of an amalgamation of tutor training materials designed by members of the IMLSN, whilst making suitable use of proven strategies and materials from within the **sigma** guide (Croft and Grove, 2011). A suite of four workshops to be run over one day was developed, as outlined in table 2. Extensive use of **sigma** materials was made for Workshops 1 and 3, and existing tutor training materials were largely used for Workshop 2. Workshop 4 was a combination of material from the UK along with bespoke material developed in Ireland so as to give a voice to the experiences of existing Irish tutors. As an example of the community approach to the development of these workshops, a document with five 'Do's and Don'ts' in MLS was written by an experienced Irish MLS tutor and used for Workshop 4 initially. Subsequently, a survey was developed and distributed across the IMLSN to try and establish the most common Do's and Don'ts in an Irish context, the findings from which were then incorporated into the workshop training materials.

In September 2013 and 2014 a selection of these workshops were piloted in individual HEIs and facilitators of the training commented that they had worked well and were very well received by tutors. However, this suite of workshops had not been implemented in a way that was accessible to all institutions within the IMLSN and so at that stage had limited impact across the IMLSN. In fact, data gathered in April 2015, as part a review of MLS provision within Ireland, showed that tutor training was provided in only 11 of the 25 of institutions with MLS (Cronin et al., 2016). This reinforced the importance of the IMLSN facilitating a co-ordinated tutor training programme, accessible to as many MLS practitioners as possible. Therefore, in 2015 the IMLSN undertook a national project to further build the capacity of MLS tutors in a structured pan-institutional format by implementing the four pilot workshop MLS tutor training programme in a coordinated way, open to all members of the IMLSN, and subsequently to evaluate the programme of workshops and their impact. The National Forum for the Enhancement of Teaching and Learning in Higher Education (National Forum) under its Disciplinary Network Funding funded the project.

In September 2015, the IMLSN conducted their structured but non-accredited tutor training programme in three institutions, with invitations extended to all MLS tutors across all HEIs on the island of Ireland. The training followed that of the previously piloted one-day four-workshop model. Forty-two tutors from six HEIs participated (Fitzmaurice et al., 2016). In September 2016, the training was offered again in a similar format and advertised across the IMLSN membership.

Table 2. Aims and objectives of the four IMLSN workshops for MLS tutors.

| Workshop title | Short Description |
|---|---|
| **1. Mathematics Learning Support**: Why is it important and how can we improve it? | Aim: To outline the typical mathematical ability of students who require MLS and how to interact effectively with them.<br>This workshop should enable tutors to:<br>(i) Recognise that many students enter HE with relatively poor levels of mathematical ability;<br>(ii) Note the mathematical topics where there are clear gaps in student understanding.<br>(iii) Develop ways of dealing with scenarios, which are commonly encountered during MLS sessions. |
| **2. Working with students:** Explaining, Listening, Questioning Skills | Aim: To equip tutors with the skills they need in explaining, listening and questioning so that they will be able to employ positive strategies of engagement with students who seek MLS. This workshop should enable tutors to:<br>(i) Use active listening techniques in working with students in MLS.<br>(ii) Select and use appropriate types of questions in helping students mathematically.<br>(iii) Employ strategies to engage positively with students seeking MLS. |
| **3. Individual differences and needs:** Scenarios you might encounter in mathematics support | Aim: To enhance tutors' awareness of both the implications of the non-academic differences between students and the range of situations that can occur as a result of diverse student approaches to learning. This workshop should enable tutors to:<br>(i) Recognise the importance of individual (non-academic) differences and needs amongst students, to understand how these can impact on student engagement and to be aware of appropriate interaction with the students in these situations.<br>(ii) Be aware of the range of motivational factors and approaches to learning that are adopted by students, and to be able to respond appropriately to the variety of situations that may occur. |
| **4. Developing as a Tutor:** The Do's and Don'ts. | Aim: To establish a framework of knowledge and techniques to enable tutors to develop as tutors in their future work in MLS. This workshop should enable tutors to:<br>(i) Identify the positive impact that MLS can have on students.<br>(ii) Assimilate the insights of experienced tutors of MLS into their practice.<br>(iii) Use techniques of reflection on key competencies to improve their skills as an MLS tutor on an ongoing basis. |

## 5. Discussion

### *5.1. Perception of training*

The workshops developed for postgraduates working in MLS were a natural evolution of those developed for postgraduates who teach. They share many common features in their design and implementation, but most significantly they were delivered at a national level through the funded MSOR and **sigma** Networks; the former no longer exists, and the latter now exists as an unfunded community of practice. Most significantly, both sets of workshops were an example of structured but non-accredited professional development provision.

For each of the MLS workshops that have been run within the UK, delegate feedback has been collected. This has not focused upon obtaining quantitative ranking scores, but instead on obtaining specific comments that can be used to develop the events through a feedback loop. Feedback particularly emphasises the three underpinning principles: practice sharing; an informal environment; and interactivity. For example: ‘clear explanation on things to do, not to do and things to expect - nice mixture of interactive sessions, lots of new resources’; ‘very dynamic, interactive and easy going. Helped me get a bit more confidence as to my ability to be a good tutor’; and, ‘it gave me a list of resources that should help with solving some of the…problems’.

Another key feature of the feedback was that delegates welcomed the opportunity ‘to network’ and ‘share ideas‘. This is a clear indication of the importance of providing such training provision through a network particularly as within a number of institutions providing mathematics support is known to be a solitary endeavour (Grove, Croft and Lawson, 2019). Typical feedback includes: ‘opportunity for group discussion on individual problems in own maths/stats support centre’; ‘sharing experiences with other people – knowing about what other universities are offering and the methods of support available’; ‘interaction with other would be tutors – advice from lecturers on how to aid others’; and, ‘the brainstorming part of the event was important in terms of knowing each other’s approach in dealing with different scenarios that may occur. Lateral thinking approach to solving questions that might be asked was equally important’.

### *5.2. Impact of training*

While this immediate feedback is reassuring, a key question remains as to whether the training continues to influence postgraduate tutoring practices when working with undergraduates in the medium to longer-term. For the UK model discussed above, there exists some evidence that those postgraduates who have participated in the training have used this to inform their approach to teaching. For example, the following quotes are from undergraduates who received MLS in the academic year 2016/17 within an institution where a compulsory training programme was run for all postgraduates involved in providing MLS; these quotes were taken between one and eight months after the initial training session and refer explicitly to the tutors who provided the support: ‘explained things clearly, talked through steps well, gave good examples to help understanding’, ‘good communication, started from my knowledge so I knew what was going on’, ‘very good help, got me to the answers without giving away too much and explaining theory’, and ‘very helpful, made sure I understood by giving me time to work through problem independently after thorough explanation’.

Within Ireland, focus groups were conducted with postgraduates 10 weeks after the September 2015 training programme. Full details of the results of these are available in Fitzmaurice et al. (2016) but some of their key insights follow here. Tutors reported that the training programme was beneficial for their tutoring practice, and many recommended making it compulsory. Tutors were asked for suggestions on ways the training process could be enhanced for all practicing and prospective tutors. They suggested that: more time be dedicated to the development of tutors’ questioning and assessment skills; they would like more training in teaching group/tutorial sessions. Furthermore, tutors requested greater use of role-playing of MLS scenarios be used in certain workshops.

As outlined previously, the Irish MLS tutor training events in 2015 occurred in three specific HEIs, and tutors from other HEIs were invited to attend. This system helped to establish a sense of community amongst all the tutors, and went some way towards addressing the feelings of isolation that some tutors reported while working in MLS. Tutors highlighted the importance of developing a mechanism to enable them to get to know other tutors, and allow them to feel as part of a team. The importance of these views are reinforced by the findings of more recent work (Grove and Croft, 2019).

Based on the evidence collected to date, the models described above would seem to address the substance of the initial training needs of staff who will work in MLS. Institutions should benefit significantly from employing such suitably trained tutors. Some evidence appears to indicate that tutors have benefited from the training, and as such have a range of skills that will greatly aid their progression to academic, or indeed a range of other diverse careers. However, accreditation is required to ensure that there is demonstrable validation to this effect.

## 6. Conclusions and next steps

The UK workshops were designed as stand-alone events which were non-accredited, however there is some limited evidence that HEIs are accepting these in lieu of more generic forms of institutional training for postgraduates involved in teaching and supporting learning. For example, at the University of Birmingham all postgraduates involved in teaching must receive appropriate training and support. From 2016, the School of Mathematics agreed with the Centre for Learning and Academic Development, the organisation that offers centralised training to postgraduates from any discipline, that participating in a one-day event of the nature described can be accepted in lieu of three of the four generic courses that postgraduates would otherwise attend. Establishing the equivalence of these mathematics-specific activities with their institutional counterparts has been achieved through the mapping of the provision against the UK Professional Standards Framework (UKPSF, 2011) and then establishing its equivalence with existing provision within the institution.

The Irish training programme of workshops for MLS tutors was devised by the IMLSN and as such offers the opportunity for delivery in a wide number of institutions across a network which brings many benefits. However, the IMLSN forms an organisation that is in essence a community of practice and not one that is aligned to any professional development standards – in itself, it has no mechanism for accrediting its own, or indeed anyone else's training provision. The training workshops were structured and based upon individual institutional tutor training materials (some of these formed parts of accredited programmes) and certain key aspects of the **sigma** training, but nevertheless, they were non-accredited.

For the benefit of tutors who are at the front-line of providing a vital student focused service, and indeed their institutions who are now required to demonstrate a clear commitment to furthering teaching excellence, the current lack of accreditation of their training is a situation that needs to change to ensure tutors get the recognition their endeavours deserve. In the context of more highly developed structures of professional development, how to evolve from the structured and unaccredited training to one that is structured and can gain accreditation for the participant forms the next key challenge.

To meet this challenge, the authors have designed a model of tutor development that may be amenable to accreditation which is currently undergoing piloting within Ireland. Work on this model began in 2016 arising from discussions regarding tutor training in light of a newly published framework for professional development in HE in Ireland (National Forum, 2016), which took place at the 10th IMLSN annual conference (IMLSN 10) which was held in Galway, Ireland (Pfeiffer, Cronin and Mac an Bhaird, 2016). The essence of the model is to provide formal recognition for the professional development undertaken by an MLS tutor in a way that the tutor can then seek accreditation from their institution or a professional body. The model provides the potential to move from existing structured and non-accredited models to a structured and accredited model via a formal recognition step which makes use of micro-credentials. At the heart of the model are four micro-credentials: MLS Knowledge and Skills; MLS Communication/Dialogue skills; MLS Professional

Identity Development; MLS Digital Capacity. The micro-credentials can be stored and shared digitally by the tutor awarded them. Each micro-credential contains a description of the abilities which the tutor must develop and a description of the evidence that they must provide to show that they have demonstrated these abilities. The micro-credentials have been designed to equate to 5 ECTs post-graduate credits once all four have been completed. The MLS micro-credential model is designed to allow for delivery through a community of practice which ensures the training is provided by those with the most appropriate teaching knowledge, experience and expertise in the discipline, something not all institutions will have access to. Simultaneously the model could allow a national organisation to offer formal accreditation for the training against a universally recognised framework, for example the UKPSF, whilst also facilitating institutions to understand the equivalence of the activities undertaken and much more easily allow them to be recognised as an appropriate form of prior learning. Piloting of the micro-credentials is ongoing and initial tutor feedback has been positive so the model may form the future basis for tutor training that is structured and can gain accreditation for the participant. We intend to report on the micro-credential based model and the outcomes of this pilot in the near future.

## 7. References


Buckley, A., Soilemetzidis, I. and Hillman, N., 2015. *The 2015 Student Academic Experience Survey*. York, UK: The Higher Education Academy.

Cassells, P., 2016. *Investing in National Ambition: A Strategy for Funding Higher Education. Report of the Expert Group on Future Funding for Higher Education*. Dublin, Ireland: Department for Education and Skills.

Cox, W. and Kyle, J., 2005. Supporting Postgraduates who teach Mathematics and Statistics – A preliminary report on recent workshops. *MSOR Connections*, 5(4), pp.5-6.

Croft, T. and Grove, M.J., 2016. Mathematics and Statistics Support Centres: Resources for Training Postgraduates and Others Who Work in Them. *MSOR Connections*, 14(3), pp.3-13. https://doi.org/10.21100/msor.v14i3.305

Croft, T. and Grove, M.J., 2015. Progression within mathematics degree programmes. In M. Grove, T. Croft, J. Kyle, and D. Lawson, eds. *Transitions in Undergraduate Mathematics Education*. Birmingham, UK: University of Birmingham and Higher Education Academy. pp.173-189.

Croft, T. and Grove, M.J. eds., 2011. *Tutoring in a Mathematics Support Centre: A Guide for Postgraduate Students*. Available at: http://www.mathcentre.ac.uk/resources/uploaded/46836-tutoring-in-msc-web.pdf [Accessed 15 May 2019].

Croft, T., Lawson, D., Hawkes, T., Grove, M., Bowers, D. and Petrie, M., 2015. **sigma** – a network working! *Mathematics Today*, 50(1), pp.36-40.

Cronin, A., Cole, J., Clancy, M., Breen, C. and O'Sé, D., 2016. *An audit of Mathematics Learning Support provision on the island of Ireland in 2015.* Available at: http://www.sigma-network.ac.uk/wp-content/uploads/2019/02/Audit-of-MLS-provision-Ireland.pdf [Accessed 15 May 2019].

Fitzmaurice, O., Cronin, A., Ní Fhloinn, E., O'Sullivan, C. and Walsh, R., 2016. Preparing Tutors for Mathematics Learning Support. *MSOR Connections*, 14(3), pp.14-20. https://doi.org/10.21100/msor.v14i3.307

Gill, O., O'Donoghue, J. and Johnson, P., 2008. *An audit of mathematics support provision in Irish third level institutions*. Limerick, Ireland: Regional Centre for Excellence in Mathematics Teaching and Learning, University of Limerick.

Grove, M., 2013. *National HE STEM Programme – Final Report*. Birmingham, UK: University of Birmingham. Available at: https://www.birmingham.ac.uk/Documents/college-eps/college/stem/national-he-stem-programme-final-report.pdf [Accessed 15 May 2019].

Grove, M. and Croft T., 2019. Learning to be a postgraduate tutor in a mathematics support centre. *International Journal of Research in Undergraduate Mathematics Education,* 5(2), pp.228-266. https://doi.org/10.1007/s40753-019-00091-8

Grove, M.J., Croft, A.C. and Lawson, D.A., 2019. The extent and uptake of mathematics support in higher education: results from the 2018 survey. Manuscript accepted for publication in *Teaching Mathematics and its Applications*.

Grove, M.J., Croft, T., Lawson, D. and Petrie, M., 2018. Community perspectives of mathematics and statistics support in higher education: building the infrastructure. *Teaching Mathematics and its Applications*, 37(4), pp.171-191. https://doi.org/10.1093/teamat/hrx014

Grove, M., Guiry, S. and Croft, T., 2019. Specialist and more-able mathematics students: understanding their engagement with mathematics support. Manuscript accepted for publication in *International Journal of Mathematical Education in Science and Technology*. https://doi.org/10.1080/0020739X.2019.1603407

Grove, M.J., Kyle, J. and Cox, W., 2006. The Weakest Link? Supporting The Postgraduate Teaching Assistant. In D. Corcoran and S. Breen (Eds.), *Proceedings of Second International Science and Mathematics Education Conference*. Dublin, Ireland: St Patrick's College, Drumcondra, Dublin, pp.116-123.

*Higher Education and Research Act 2017* (c.29). London: HMSO.

Lawson, D., 2015. Mathematics Support at the transition to university. In M. Grove, T. Croft, J. Kyle, and D. Lawson, eds. *Transitions in Undergraduate Mathematics Education*. Birmingham, UK: University of Birmingham and Higher Education Academy. pp.39-56.

Lawson, D., Croft, T. and Halpin, M., 2003. *Good Practice in the Provision of Mathematics Support Centres*. 2nd ed. Birmingham, UK: LTSN Maths, Stats and OR Network. Available at: http://www.mathcentre.ac.uk/resources/guides/goodpractice2E.pdf [Accessed 15 May 2019].

Lee, S., 2005. A Postgraduate's Experience of Teaching. *MSOR Connections*, 5(4), pp.38-39.

Mathematical Association of America (MAA), 2003. *Guidelines for Programs and Departments in Undergraduate Mathematical Sciences*. Washington, D.C.: Mathematical Association of America.

National Forum, 2016. National Professional Development Framework for all staff who teach in Higher Education. Dublin, Ireland: National Forum for the Enhancement of Teaching and Learning in Higher Education. Available at: http://www.teachingandlearning.ie/wp-content/uploads/2016/09/PD-Framework-FINAL-1.pdf [Accessed 15 May 2019].

National Research Council, 1991. *Moving Beyond Myths: Revitalizing Undergraduate Mathematics*. Washington, D.C.: National Academy Press.

Neves, J. and Hillman, N., 2016. *The 2016 Student Academic Experience Survey*. York, UK: The Higher Education Academy. Available at: http://www.hepi.ac.uk/wp-content/uploads/2016/06/Student-Academic-Experience-Survey-2016.pdf [Accessed 15 May 2019].

O'Sullivan, C., Mac an Bhaird, C., Fitzmaurice, O. and Ní Fhloinn, E., 2014. *An Irish Mathematics Learning Support Network (IMLSN) report on student evaluation of mathematics learning support: insights from a large scale multi-institutional survey*. Available at: http://mural.maynoothuniversity.ie/6890/1/CMAB_IMLSNFinalReport.pdf [Accessed 15 May 2019].

Perkin, G. Croft, T. and Lawson, D., 2013. The extent of mathematics support in UK higher education—the 2012 survey. *Teaching Mathematics and its Applications*, 32(4), pp.165-172. https://doi.org/10.1093/teamat/hrt014

Pfeiffer, K., Cronin, A. and Mac an Bhaird, C., 2016. The key role of tutors in mathematics learning support – a report of the 10th annual IMLSN workshop. *MSOR Connections*, 15(1), pp.39-46. https://doi.org/10.21100/msor.v15i1.367

Quality Assurance Agency (QAA), 2015. *Subject Benchmark Statement: Mathematics, Statistics and Operational Research*. Gloucester, UK: UK Quality Assurance Agency for Higher Education.

Rishel, T.W., 1999. *A handbook for mathematics teaching assistants.* Washington, DC: Mathematical Association of America.

Solomon, Y., Croft, T. and Lawson, D., 2010. Safety in numbers: mathematics support centres and their derivatives as social learning spaces. *Studies in Higher Education*, 35(4), pp.421-431. https://doi.org/10.1080/03075070903078712

UK Professional Standards Framework (UKPSF), 2011. *The UK Professional Standards Framework for teaching and supporting learning in higher education*. Available at: https://www.heacademy.ac.uk/system/files/downloads/uk_professional_standards_framework.pdf [Accessed 15 May 2019].

CASE STUDY

# Effective Marketing Strategies to Promote Engagement with Online Mathematics Learning Support

Aoife Guerin, Mathematics Learning Centre, Centre for Teaching and Learning, University of Limerick, Co. Limerick, Ireland. Email: Aoife.Guerin@ul.ie.
Richard Walsh, Mathematics Learning Centre, Centre for Teaching and Learning, University of Limerick, Co. Limerick, Ireland. Email: Richard.Walsh@ul.ie.

## Abstract

In this paper we report on the effects of a different marketing strategy on promoting engagement with the online Mathematics Learning Support (MLS) service (mostly screencasts) compared to our traditional advertising approach which was solely to send generic emails to students advertising the online services. The findings show that this new marketing strategy was far more effective than traditional methods of advertising in getting students to engage with the online service. This paper describes the approach taken and compares the engagement with the online services offered by the Mathematics Learning Centre (MLC) before and after utilising the new marketing strategy, the increased engagement from the trial group with the online services compared to the other groups, and the knock-on effects.

**Keywords:** Mathematics Learning Support, support services, online support, examination revision courses.

## 1. Introduction and Background

Mathematics Learning Support (MLS) has become a common service offered to students in most higher education institutions in Ireland and the UK. The establishment of Mathematics Learning Support was in response to the declining mathematical standards of students entering higher education, commonly referred to as the ‘mathematics problem’ (Symonds, Lawson and Robinson, 2008), which has remained present to the current day. Research has shown that students who used MLS services once were 1.63 times more likely to pass their examinations compared to students who did not attend at all, while students who attended on 15 occasions or more were 14 times more likely to pass (Jacob and Ní Fhloinn, 2018). The necessity of MLS is evident from the authors’ own service, where 38,644 student visits have been made to the Mathematics Learning Centre (MLC) over the past 5 academic years. This number does not include students’ accessing of online resources that the MLC also provides. However, the popularity of the authors’ MLC does not come without its issues. In the past number of years there has been a decline in the number of tutors that have been available to the authors’ MLC. This decline is primarily due to restrictions placed on postgraduate students by their funding sources on the number of hours they are permitted to teach per semester/year. The MLC had 17 hourly paid tutors in 2010 compared to 6 that it currently employs. To combat this, the MLC received an extra contract in the past 2 academic years in addition to the manager, both of whom now teach up to 40 hours per week between them. It is envisaged that if students engaged with online resources before visiting the MLC physically then the tutors would be under less pressure during their sessions, some of which can consist of 30 students looking for 1-to-1 help from the 2 tutors per session.

There is increasing awareness and concern over the amount of notifications that students in higher education are receiving regarding the services that are available to them (from sports clubs, support services etc.). In response to the growth in the volume of emails that students need to read through, they may choose to ignore or not read some emails fully (Sappleton and Lourenço, 2016). The authors, in their work in MLS, feared that students who need help in mathematics may not avail themselves of MLS due to them being unaware of what the MLC provides and how to access this service. Part of a programme at the university (where this research took place) which assists new students with various aspects of student life, focuses on raising students' awareness of the learning support services that are available to them. However with the range and depth of services offered by the MLC, it has proven very difficult to attract students to the correct resources/use of resources. Many students forget about the online support offered by the MLC and end up physically visiting the centre around examination time. This puts tutors under immense pressure to keep up with the volume of student traffic.

Approximately 22%, 21% and 18% of students use Khan Academy, YouTube, and Wolfram Alpha respectively to try to gain support with their mathematics studies. However student feedback on online resources reveals their desire to have videos/online support specifically tailored to the material that they are actually studying (Ní Shé et al., 2017). Empirically the researchers, through their combined 13 years experience of working in MLS, have observed many students who would benefit from online help before visiting the MLC. However it is apparent that students are not armed with the tools (e.g. they do not have a sufficient mastery of the mathematical terminology required) to search accurately for the resources that they need. Creating videos for particular modules allows lecturers/learning support staff to tailor the videos to the students' needs, even in the terminology that would be more reflective of their lecture notes.

The researchers sought to investigate the impact of an 'attractive' advertising campaign on students' engagement with our online support videos. A module (first year service mathematics module taken by science students, referred to hereafter as the 'trial group') with a historically high number of users of the MLC drop-in centre was used to investigate this. The authors compare these users' engagement with the MLC's online resources to the engagement of other groups who received the same advertising (generic email) as in previous years. The generic email advertised the online resources as mainly videos that cover concepts associated with the student's mathematics module which may be a useful revision tool. A comparison is also made between the trial group's physical attendances at MLC services in the current semester and their physical attendances in previous years where the new 'attractive' videos were not available to them. This comparison looks at the physical attendances in the weeks before and after the date at which the 'attractive' videos were made available.

## 2. Methods

In the semester when this study was completed (Autumn 2018) the MLC serviced 8 first semester mathematics modules on our online service. To avail themselves of this service a student needed to visit our website, click on the module that they wanted the online resources for, and join our Sulis page for their module (Sulis is the university's learning management system, used by lecturers as a way of communicating with their students and sharing resources). The MLC developed its own Sulis pages for certain modules. Each module has its own unique Sulis page for students to join so that data (e.g. the number of visits) is identifiable per module. The modules are described in table 1. There is a similar amount of video support available online on each of the Sulis pages for these 8 modules.

This study investigated the effectiveness of an attractive marketing strategy for the online resources for the 'trial group'. It was planned to trial this strategy with two modules (the second being a first year engineering module) however the second lecturer decided not to participate as he did not want students receiving solutions to past papers. The marketing strategy comprised of completing a past midterm examination paper for the trial group's module over a series of 11 short videos (45 minutes in total – see figure 1 for example of a screenshot). These videos were created on a tablet computer using the app ‘Educreations’ which allows the user to write on a ‘whiteboard’ with a stylus while recording audio simultaneously. Students in the trial group received the same generic advertisement email as their peers in the modules that the MLC have online resources for (listed in table 1) up until 16 days before their midterm examination, from which point the trial group received a different email advertising the online midterm revision course available to them (the email specified that these new videos would talk them through the 2016 midterm examination paper), while the students in the other seven modules continued to receive the original generic email. Students from these eight modules often frequent the MLC with past papers and tutors remark that they teach the same problem multiple times, even in a single session. After adding the online midterm revision course, the number of videos available to students of the trial group was slightly higher than the number of videos available to the students of the other seven modules.

The motivation for using this attractive marketing strategy was the expectation that this would lessen the pressure on tutors in the MLC drop-in sessions by potentially avoiding repetitively teaching the same problems during the weeks leading up to the midterm examination, and therefore avoid rushing with attendees during the sessions.

Table 1. Description of Modules and the number of students in each.

| Module | Description | Total Number of Students |
|---|---|---|
| Trial Group | First year science mathematics. Focus on trigonometry, vectors, matrices, functions and differential calculus. | 366 |
| 1 | Foundation mathematics for mature students pursuing an access foundation year (in humanities/business/science), following which they will be granted entry to university courses. | 52 |
| 2 | Foundation mathematics for mature students pursuing an access foundation year (in engineering) following which they will be granted entry to university engineering courses. | 13 |
| 3 | First year engineering mathematics. Focus on algebra, complex numbers, vectors, series, functions and differential calculus. | 296 |
| 4 | Second year engineering mathematics. Focus on linear algebra, Laplace transforms and Fourier series. | 287 |
| 5 | Business mathematics. First mathematics module taken by business students in their second year. Focus on algebra, matrices, differential calculus, curve sketching and basic financial mathematics. | 381 |
| 6 | Second year science statistics. Focus on fundamentals of probability, most widely used statistical distributions, and statistical inference. | 196 |
| 7 | First year technological mathematics for technology students. Focus similar to module 3, however with a lower difficulty level. | 296 |

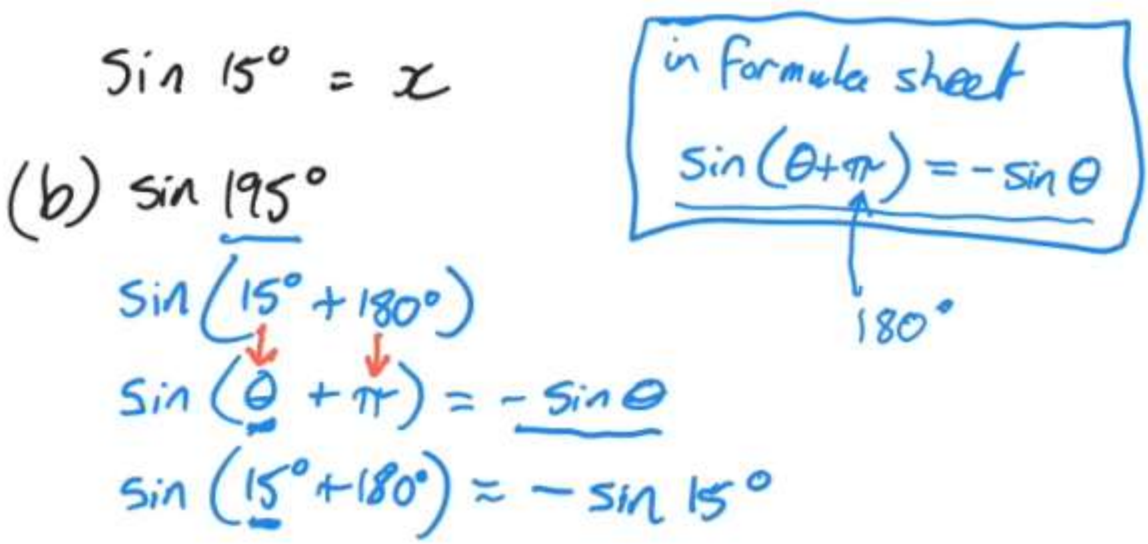



Figure 1. Screenshot from revision videos

Data was extracted from Sulis from the period 10th September 2018 until 1st November 2018, i.e. from the first day of the semester until the day of the midterm examination for the trial group in the eighth week of term. Data was extracted on the number of student members in each Sulis site, on which videos were viewed, and the number of times each video was viewed (the only option was to extract this data manually at certain times, there is no option on Sulis to inform the user of how many members a site had on a given day/time, the data available is restricted to the total number of members at the time of logging in). Data was also collected on students' usage of the MLC drop-in centre and support classes provided by the MLC.

## 3. Results and Analysis

The number of student members in each of the Sulis sites over time is given in figure 2 (the trial group received the email advertising the midterm revision videos on 16/10/18 during the sixth week of the 12 week academic term).

It is seen in figure 2 that the number of student members in the Sulis site for the trial group is approximately five times higher on 01/11/2018 than it was on 15/10/2018. The number of student members in the other Sulis sites remained similar throughout this time. The number of videos viewed per module over time from data taken at the end of each work day is detailed in table 2.

The number of total video views for the trial group increased by 160 percent from the day prior to this group receiving the email advertising the midterm revision videos (15/10/18) to the day after the trial group received this email (17/10/18). During the period 19/10/18 – 01/11/18, each time the data was collected there was an increase in the number of total video views by the trial group from the previous data collection time point. This increase ranged from a minimum of 14.5% to a maximum

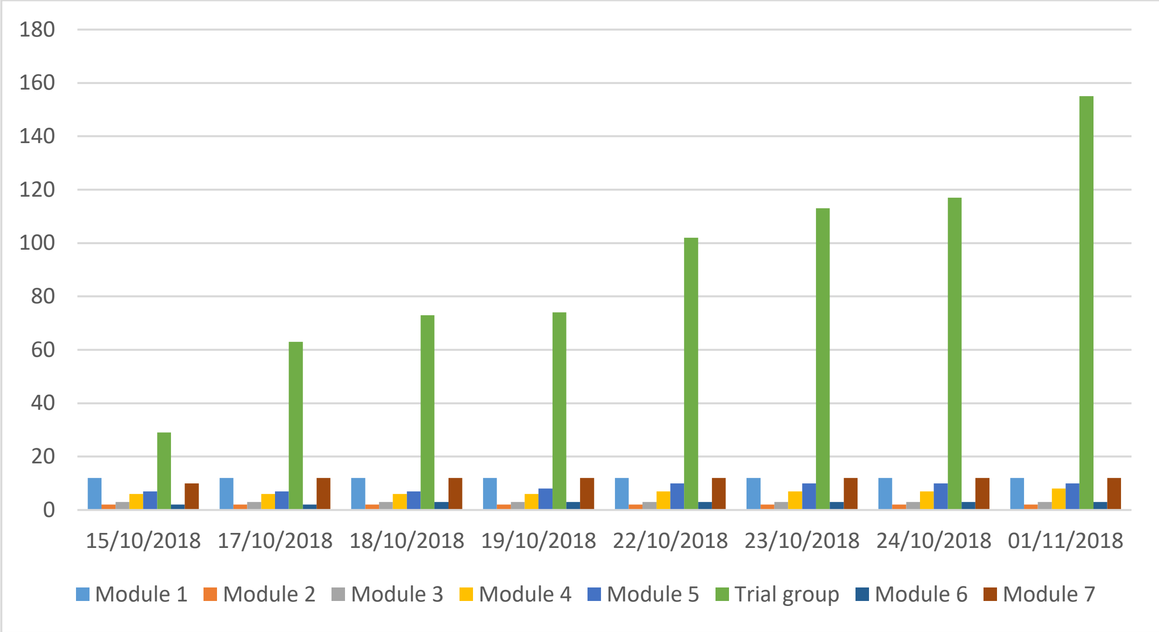



Figure 2. Number of student members in each Sulis site over time

Table 2. Number of total video views in each Sulis site over time

| | 15/10/18 | 17/10/18 | 19/10/18 | 22/10/18 | 24/10/18 | 26/10/18 | 29/10/18 | 01/11/18 |
|---|---|---|---|---|---|---|---|---|
| **Trial group** | 143 | 372 | 491 | 568 | 942 | 1079 | 1595 | 2597 |
| **Module 1** | 158 | 158 | 159 | 165 | 165 | 165 | 165 | 203 |
| **Module 2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Module 3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Module 4** | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| **Module 5** | 57 | 57 | 75 | 76 | 76 | 76 | 76 | 76 |
| **Module 6** | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 4 |
| **Module 7** | 142 | 188 | 188 | 188 | 188 | 188 | 188 | 188 |

of 62.8%. The highest increase in the number of total video views for the other modules was 32.4% for Module 7. However, this increase occurred at one time point only and the number of total video views for the module remained the same thereafter. There was little to no change in the number of total video views for the remaining modules over the time period 15/10/18 – 01/11/18.

The level of engagement with the online support was sustained by the trial group. The total videos views for each module at the end of the semester (after examinations) is shown in table 3. Note that 45 extra views of the midterm revision videos were made after the examination, therefore 1,555 views of other videos were made between the end of the midterm examination and the end of the semester by the trial group.

Table 3. Total number of total video views in each Sulis site at the end of semester

| **Module** | **Trial group** | **Module 1** | **Module 2** | **Module 3** | **Module 4** | **Module 5** | **Module 6** | **Module 7** |
|---|---|---|---|---|---|---|---|---|
| **Number of views** | 4197 | 247 | 0 | 0 | 124 | 787 | 13 | 562 |

## 3.1. Results on the trial group

The number of views per video for the top 20 viewed videos is shown in figure 3. Note that videos such as 1(a) or 4(b) (i.e. number(letter) format) are from the midterm revision videos. All other videos are not specific to the students' past midterm examination.

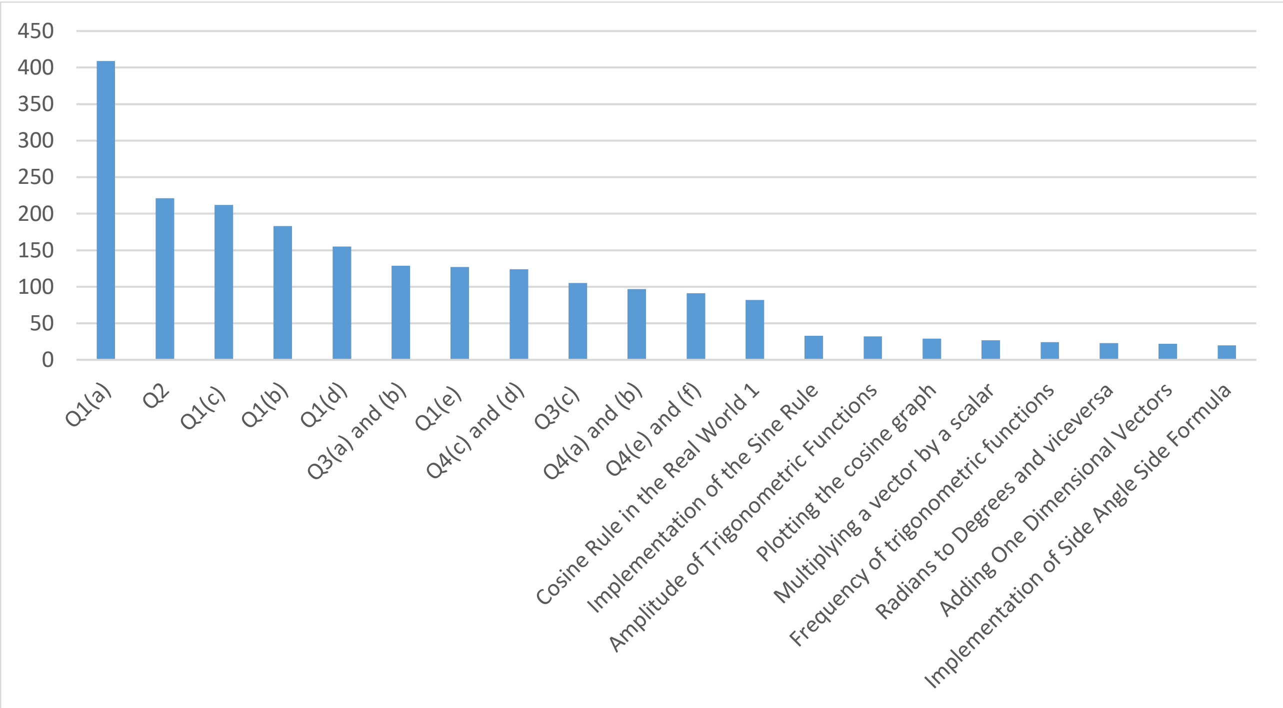



Figure 3. Top 20 videos viewed

As seen in figure 3, though the majority of videos viewed were from the students' past examination paper, 744 views of other videos (29% of total video views) not specific to the midterm questions were made, which is much higher than any of the other modules. This suggests that the attractive element of the midterm videos being made available has also led to students viewing the MLC's other online support.

The attendance figures at the MLC drop-in centre and at the MLC's evening support classes (up until the end of the eighth week of term) for the trial group's module compared to previous years of students from the same module (where they did not have the online revision videos) are shown in figure 4. Figure 5 shows the attendance figures for weeks 3-5 of term and for weeks 6-8 of term (before and after the advertising of midterm revision videos respectively).

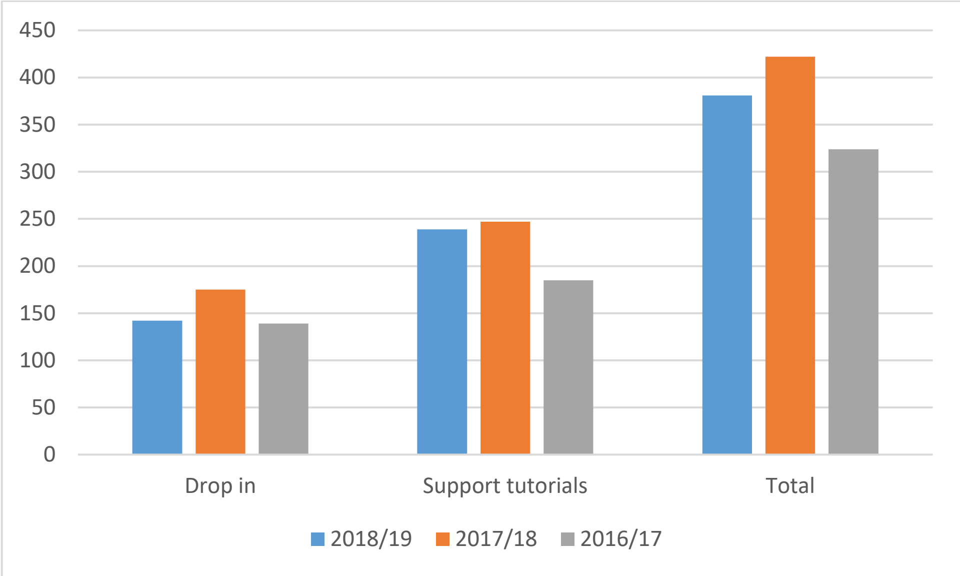



Figure 4. Total attendances made by students from the trial group (2018/19) and total attendances made by students from the same module for the previous two academic years.

The attendances at the MLC drop-in by students in the trial group (2018/19) were lower (~20%) than the attendances from students of the same module in 2017/18 but on par with students in this module in 2016/17. The attendances at the MLC support tutorials by the students in the trial group (2018/19) was slightly lower (~4%) than the attendances from students of the same module in the previous academic year but higher (~33%) than students in this module in 2016/17.

In each of the three semesters, there was a decrease in the total attendances at the MLC drop-in made by students from this module from weeks 3-5 to weeks 6-8. This decrease (~52%) was approximately the same for each of the three years. There was a decrease of 13% and 29% in the total attendances at the MLC support tutorials made by students from this module from weeks 3-5 to weeks 6-8 for the 2018/19 and 2017/18 academic years respectively. Although the new marketing strategy resulted in increased use of the MLC online support service by the trial group, it did not lead to a higher decrease in the number of physical attendances by the trial group at the MLC drop-in compared to the number of physical attendances by students from the same module in the previous two years.

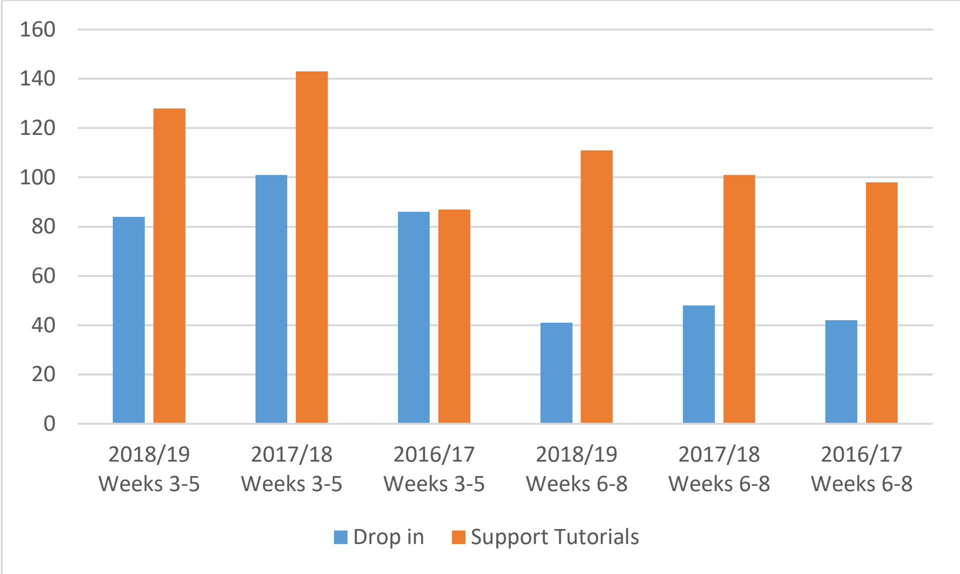



Figure 5. Total attendances made by students from the trial group (2018/19) in weeks 3-5 and weeks 6-8 and total attendances made by students from the same module for the previous two academic years in weeks 3-5 and weeks 6-8.

## 4. Conclusion

Tailoring online mathematics support to the specific mathematics that students are studying is an effective strategy to get more students to engage in online mathematics support. The lower decrease in the total attendances at the MLC support tutorials in the 2018/19 academic year, in addition to the substantial increased use of online support is suggestive of students availing themselves of an appropriate level of mathematics support. The familiarity of the online midterm videos as a hook to increase engagement with online support led to increased engagement with online support, which was not solely restricted to the midterm videos and this increase was sustained over time.

## 5. References


Jacob, M. and Ní Fhloinn, E., 2018. A quantitative, longitudinal analysis of the impact of mathematics support in an Irish university. *Teaching Mathematics and its Applications*, accepted. https://doi.org/10.1093/teamat/hry012

Ní Shé, C., Mac an Bhaird, C., Ní Fhloinn, E. and O'Shea, A., 2017. Students' and lecturers' views on mathematics resources, *Teaching Mathematics and its Applications*, 36(4), 183-199. https://doi.org/10.1093/teamat/hrw026

Sappleton, N., & Lourenço, F., 2016. Email subject lines and response rates to invitations to participate in a web survey and a face-to-face interview: the sound of silence. *International Journal of Social Research Methodology*, *19*(5), 611-622. https://doi.org/10.1080/13645579.2015.1078596

Symonds, R., Lawson, D., Robinson, C., 2008. Promoting student engagement with mathematics support. *Teaching Mathematics and its Applications*, 27(3),140-149. https://doi.org/10.1093/teamat/hrn011

CASE STUDY

# Using virtual and physical learning spaces to develop a successful mathematical learning community, both for on-site and distance provision

Belinda Huntley, University of South Africa (UNISA).
Andrew Middleton, Anglia Ruskin University, UK.
Jeff Waldock, Department of Engineering and Mathematics, Sheffield Hallam University, UK.
Email: jeff.waldock@shu.ac.uk.

## Abstract

This paper considers learning space and its relationship to student belonging and becoming. Student engagement, satisfaction and academic success are outcomes of a supportive learning community which can flourish in a culture of expectation and behaviour created by providing suitable support structures and by considering the effective use of physical and virtual learning spaces. We describe our innovative use of discipline-specific virtual and physical spaces to develop successful mathematical learning communities, in both a UK university where activities are principally face-to-face, and at a South African university where they are mainly virtual. By comparing our practices and spaces, we explore the 'equivalence of place' and the roles of academic staff in fostering the development of professional learner identities through each context. Based on evidence from our respective practices, we make recommendations for designing new learning spaces and for making effective use of existing learning spaces. Although this study focuses on mathematics, many of these suggestions can benefit all disciplines.

**Keywords:** physical and virtual learning space, mathematical learning communities, place, student engagement.

## 1. Introduction

Learning spaces, both physical and virtual, enable a wide range of course-related activities to take place, both those managed by staff and those led by students. In this paper, we present examples of successful course-related practices designed to foster informal learning communities in two very different institutions. This collaborative study considers experience from a UK campus-based university and a South African university where students are predominantly engaged through distance learning.

By examining our respective practices, a key objective has been to identify which of these practices may be transferable between our institutions. Informal learning spaces are defined here as any physical, virtual, or blended space in which unscheduled course-related learning can happen. In many cases such spaces may already exist, but in other situations they may need to be created or adaptations made to existing space. We argue that the design of learning spaces intended to foster a sense of belonging and course engagement should aim to,

- reflect a strong sense of the discipline;
- become a disciplinary 'home';
- reflect a staff-student partnership ethos;
- encourage peer support mechanisms to grow;
- have both a physical and virtual dimension;

- be co-constructed;
- engage students productively outside normal class contact time;
- be important in different ways according to the varying contexts, e.g. mode of delivery, or disciplinary culture.

Informal learning space sits between, and is largely defined by, its relationship to formal learning and non-academic experience. This idea of in-between space is reflected in the concept of third space in which Gutiérrez et al. (1999) describe a zone of development that is neither school nor home and where the teachers and students relate through a culture of highly productive collaboration, hybrid activities, roles and practices. We consider how informal learning spaces affect the learning experience, having a conjoining role that helps to facilitate the learner's boundary crossing and their need to manage and make sense of their competing and changing identities. We also consider how such third space affects the academic's role and attitude, challenging the attention they mostly give to formal delivery to the exclusion of complementary informal spaces which are often overlooked or felt to be beyond their control. Yet fundamentally, the relation of learning to space is about the student's lived experience and less about how space is conceived or perceived by others (Lefevre, 2005). Chism (2006) identifies the value of maintaining a holistic view of space as it relates to the student's experience of learning:

> *Environments that provide experience, stimulate the senses, encourage the exchange of information, and offer opportunities for rehearsal, feedback, application, and transfer are most likely to support learning.*

Nevertheless, space needs to be designed and provided by facilities managers and educators in order that teaching and learning can be experienced. In large organisations with many competing learning contexts and teaching philosophies, the design and organisation of spaces for learning is often reduced to generalised ideas of teaching and learning that reflect little of the lived learning experience which is often central to student engagement and success. Instead, attention is given to systematising the management of formal spaces based on simplistic Industrial Age notions of mass teaching (Scott-Webber, 2004), notions which work counter to a learner-centred paradigm, especially those where social interactivities are a valued part of the teaching philosophy and its methods.

Monahan (2002) refers to the effect of space on teaching and learning as 'built pedagogy'. For example, a lecture theatre is specifically designed for one-to-many lecturing. A consequence of this is that it makes many-to-many interactivity difficult. Equally, an online discussion board suggests a highly transactional form of discussion. While the inflexibility of such spaces is beneficial for clearly communicating what is expected of participants, it largely ignores contextual difference and the dynamics accommodated in other spaces. Inflexible, formal spaces require teachers and their students to adopt given formal behaviours.

The same logic applies to informal space: a café is primarily designed for catering, a corridor for moving between spaces, a wall for supporting ceilings and enclosure, but in each case the human will naturally attempt to impose themselves onto the space to make it their own according to their needs and desires: meetings, informal tutorials, and mounting representations of work respectively. Each of these examples shows how humans tend to experience space socially too. Mannarini et al. (2012) observe that "*people are more likely to feel satisfied with their social relationships when they identify as a spatial community*".

Using a user-centred and experiential perspective, a lecture theatre may be where friends sit together to learn and a café may be where a student has a coffee, notices peers, friends and lecturers, and thinks about how life fits together. A user-centred view leads us to appreciate space as place: space that matters to the individuals and their communal identity.

Looking beyond systematised, formal and transactional models of education to the student’s lived learning experience, it is imperative that academics appreciate and have strategies for creating a sense of place such that it accommodates the student’s need to become a mathematician. Place has a role in fostering their sense of belonging and underpinning the formation of their student and, eventually, graduate identity.

Oldenburg’s concept of Third Place (Oldenburg, 1989) provides a framework for defining space that matters. It reflects how space and the individual’s engagement with it are instrumental in shaping belonging, becoming, memory and identity. Oldenburg observed how some spaces become the locus for communal engagement and shared identity when space is experienced as being ‘homely’. Third Places are neither work nor home, but where people come together to socialise. His research identified how habits and rituals form around meeting points such as bars, coffee shops and clubs.

Third Place theory establishes the challenge addressed in this paper: how do we use space to create a sense of disciplinary ‘home’? Key ideas within Third Place theory and which support the agency of individuals and their self-identification as members of a community include space as,

- Neutral ground – individuals use it with little obligation;
- Leveller – rank and status are irrelevant and participation is open to all;
- Conversation – the main mode of participation is conversation;
- Accessibility and accommodation – the place is easy to access and use;
- Regular ‘customers’ - the identity is sustained by a core group of regulars;
- A home from home - where feelings of possession and of being at ease are fostered.

Educationally, Third Place theory highlights the value of memorable interactions involving communal partnership, peer co-operation and interdependency, and friendship. Thinking about educational placemaking in terms of memorable interactions indicates the relevance of informal space to student retention and success. However, the university campus as a locus for social cohesion is challenged by the fragmented life-wide experience of students who work while studying or who depend on technology for accessing content or communicating, especially where this involves non-traditional learners (Zepke et al. 2010).

A sense of place comes from place attachment and the creation of place identity, while belonging can be enhanced through the creation of social offerings where people can meet, and by creating a welcoming open environment (O’Rourke and Baldwin, 2016). Zoning and placemaking are important in higher education because they can positively impact on student retention and success. People are more likely to feel satisfied with their surroundings and what they do when they identify as a spatial community (Mannarini et al. 2012)

While Oldenburg’s thinking was informed by space experienced in the physical domain, it actually addresses the psychosocial context. For educators, this means we can use Third Place theory to analyse the engagement of learners, irrespective of whether they experience space as being physical, digital or blended.

Morris (2017) notes that “*campuses have libraries, coffee shops, cafeterias, quads, student lounges... few institutions pay much attention to recreating these spaces online*”. Morris’ comment focuses on the agency of the provider, nevertheless it does highlight the inflexibility of online built pedagogy: there are no corridors for serendipitous encounters within virtual learning environments.

The students' autonomous and habitual use of social media for learning may be affecting student expectations and behaviours however. Contiguous online social media spaces in which students naturally co-operate and support each other now augment institutionally provided learning spaces, whether they are physical or online (Middleton, 2018).

Harrop and Turpin in their research into students' use of informal learning spaces comment that, "*environments that provide experience, stimulate the senses, encourage the exchange of information, and offer opportunities for rehearsal, feedback, application, and transfer are most likely to support learning*". Ideal informal learning spaces promote active, collaborative and social processes that result in co-constructed knowledge. To achieve this, an effective space is one that allows students to engage productively outside normal class contact time by accommodating their preference to work "*in close proximity to friends or peers to create a sense of community, for co-support and for someone to take a break with*" (Harrop and Turpin, 2012).

## 2. Institutional contexts

### *2.1. The University of South Africa (Unisa)*

Unisa is South Africa's only comprehensive dedicated distance education university. The conceptualization of distance education and open learning (ODL) is central to achieving its vision. The majority of students are mature, completing their studies while working full-time, and come from various demographic and racial backgrounds. They are all distant online learners.

Open Distance Learning (ODL),

- bridges the time, geographical, economic, social, educational and communication distance between the institution, students, academics and resources (transactional distance);
- is focused on flexible provision, removing barriers to accessing learning;
- is facilitated by physical and virtual learning spaces.

Blended learning is accomplished by using multiple teaching and learning strategies, a range of technologies in combination with face-to-face interaction and the deployment of both physical and virtual resources. Unisa students operate in virtual environments, but physical resources are also provided in the form of contact centres where students can meet with face-to-face tutors or lecturers.

Unisa provides various physical facilities and services to cater for its diverse student population. Lecturers, face-to-face tutors, counsellors, the Unisa regional centres and Unisa libraries all play a part. These physical centres are dispersed regionally across South Africa and even abroad for international students. Lecturers/tutors travel to these regional centres when contact sessions are organised with the students. Staff are located in faculties at Unisa Main Campus in Pretoria and the Unisa Science Campus located at Florida.

A virtual online learning management system (myUnisa) provides various online tools: official study material, announcements, discussion forums, frequently asked questions and answers, glossary, podcasts, self-assessments, e-tutor site. Additional resources include online podcasts, vodcasts and videos of tutorials and lessons produced by the module lecturer. E-assessment strategies include online portfolios, journals and both timed and untimed quizzes.

Student surveys are conducted at the end of every semester; students are also asked to carry out a self-reflection on their performance in their compulsory assignments. These are generally positive and the feedback is quite constructive.

## 2.2. Sheffield Hallam University

Sheffield Hallam University began its existence in 1843 as the Sheffield School of Design, expanding during the 20th century and becoming one of the UK's largest 'new' universities in 1992. The Mathematics undergraduate degree course has around 300 students across 3 years of study, as well as those undertaking an optional work placement year. Its three principal characteristics are,

- the practical application of mathematics;
- successful graduate employment;
- strong student support.

The aim is that students graduating from the course are familiar with dealing with open ended problems and able to communicate the results in a variety of ways, for example, orally, in writing and through poster presentations. Students become adept at working in teams, proficient with technology, and confident in using their mathematical knowledge.

Appropriate learning, teaching and assessment strategies are designed so that students gain both generic and subject specific skills at the same time. In addition to mathematical skills, graduates should have generic capabilities that enhance their employability. The design and use of learning space, therefore, plays a role in the development of the students' self-awareness, self-confidence, creativity, their ability to communicate and apply existing knowledge and skills in new situations, and in developing their interpersonal skills.

Student support is a key element of the programme, but one that should not *always* be provided by academic staff. Graduates need to be able to operate independently, with the confidence to work problems through for themselves. New students are grouped into small project teams facilitated by a final year undergraduate – a Peer Assisted Learning (PAL) leader. PAL leaders are given appropriate training and volunteer for the role, which gains them valuable leadership, facilitation and people-management skills (Waldock, 2011a). The first-year students within each group quickly develop a strong bond and although the scheme only lasts for one semester, they tend to remain together throughout the course. It becomes a friendship group, representing a powerful source of peer support which often outlasts the course itself.

The physical and virtual learning environment can facilitate many activities designed to help students build relevant skills:

- **team-working skills** are supported by the provision of IT-enabled group working areas and small meeting rooms for student use.
- **reflection and action planning skills** – leading to enhanced levels of self-awareness and the ability to articulate and evidence capabilities - are supported by reflective on-line learning logs (Waldock, 2011b).
- **virtual support** is offered by a custom website, hosting amongst other things the learning logs mentioned above, access to custom software and a Twitter feed to which staff and students can contribute (providing up to date news). All students are encouraged to make use of LinkedIn, and to join the departmental LinkedIn group to remain part of the community after graduation.
- **peer support processes** – the group working facilities can host Peer Assisted Learning meetings, and the open learning space facilitates cross-level and ad-hoc encounters (Cornock, 2016).

- **social-professional attitudes** – informal learning spaces can also be used to support various other activities, which enhance their social-professional skills. In addition to the above, for example, they host the Maths Arcade - a scheme for developing logical thinking skills through playing strategy games (Cornock, 2015); graduation receptions; open day presentations; a regular Rubik's cube championship and a de-stress day for final year undergraduates. Through engagement in such activities, students begin to learn to be and become a professional mathematician.

To accommodate the above activities, the physical space

- is designed to be a working environment people want to use;
- is designed so that its 'look and feel' reflects its social, disciplinary, and professional identity;
- incorporates staff offices in close proximity;
- includes meeting rooms equipped with whiteboards that students can use;
- has a range of group working areas, also incorporating whiteboards for student use;
- is wi-fi enabled, supporting laptops, PCs and mobile devices;
- is accessible - to mathematics students only - outside of formal teaching time.

The decoration of the physical space, from the graphics displayed on the walls to the digital signage, helps cultivate a professional attitude and identity. This is further enhanced by advocating and maintaining a strong peer-support network. This holistic view creates a communal learning partnership enabling student engagement to flourish (Boys 2010, Healey et al. 2014).

The staff are aware of what makes a space feel like a place. Place is about environment, but also about people and what is going on inside. It also keeps learners engaged in course-related work between classes. The new physical space at Sheffield Hallam University was opened in December 2014 and, to determine whether the anticipated benefits were being achieved, a survey of students who had experienced both the old and new environments was carried out during 2015. The outcomes have been reported elsewhere (Waldock, 2015; Waldock et al., 2017) but some relevant results are reproduced below.

Key benefits were clear to staff:

- 'More inter-year communication. Conversations between year groups is happening more';
- 'Course cohesiveness. There is a definite feeling of belonging. Proximity between staff and students seems to encourage approachability'.

The following student comments demonstrate the achievement of key anticipated outcomes for the space and demonstrate their recognition of its value in supporting their engagement and learning:

- 'Having a home for the discipline makes the maths department seem more united';
- 'Working around people studying the same subject – [provides] a sense of home';
- 'Whiteboards and PC TVs promote group work and problem solving';
- 'I can also use gaps in the timetable to do work before going to lectures which may be right next to the main PC area';
- 'Before I only came into university for lectures and worked at home, which isn't always effective with the distractions of student life. Now I can spend all day in the maths department meaning that I work much more efficiently and get to spend more time on my studies';
- 'I'm more inclined to stay at uni (and be more productive) instead of going home after lectures'.

## 3. Discussion

Croft and Grove (2015), discussing reasons for the 'sophomore slump' – a common and well documented dip in achievement suffered by many students in their second year of study – stress the importance of a sense of belonging and inclusion in a peer or departmental mathematical community and the learning and teaching relationship between staff and students; alienated students refer to lecturers' lack of interest in them, existing on the margins and not being part of the learning community.

In the Student Experiences of Undergraduate Mathematics project (SEUM, Brown et al., 2005), feeling part of a mathematical community emerged as a crucial factor in the student experience; in SEUM this community focused on one physical space where students could work together and also engage with academic staff in an informal way. A critical factor identified was the opportunities provided for interactions with other students and staff.

A key issue for academics interested in developing (particularly physical) learning spaces is being part of the design process. Neary et al. (2010) stated that,

> *A central issue for Learning Landscapes in Higher Education is the extent to which the academic voice is engaged in the design of progressive teaching and learning spaces. This engagement includes the ways in which academics are involved with design decisions, the degree to which pedagogical principles are captured in the design of teaching and learning spaces, and, more fundamentally, the extent to which academic values are embedded within the processes and protocols through which universities are being refurbished and rebuilt.*

The difficulty is that in many institutions the decision-making process in the design and construction of new learning spaces bypasses academic staff altogether. It becomes important for academic staff with a keen interest in this area to join forces with staff in estates and other departments to gain some degree of influence over the creation of new spaces. This is particularly vital since new designs need to focus on what will be required over the next 10-20 years and possibly beyond as much as what is required now. Some evidence of what has worked successfully elsewhere – and why – is vital when writing a compelling business case for a new design.

### *3.1. Key features of successful physical learning spaces*

The rationale and design of the physical learning space at SHU is aimed at supporting achievement of key graduate outcomes, such as employability skills including communication and team working as summarised above. The resulting benefits of the informal learning space are that it,

- promotes peer interaction within and across year groups;
- encourages closer working relationships between staff and students;
- promotes a sense of belonging to a mathematical community;
- supports group work;
- supports virtual interactivity;
- promotes student motivation by working in a shared learning environment;
- leads to a disciplinary focus and a sense of a professional community.

### *3.2. Key features of successful virtual learning spaces*

In the Department of Mathematical Sciences in the College of Science, Engineering & Technology (CSET) at the University of South Africa (Unisa), the blended learning approach aims to create a quality learning environment using an appropriate combination of different media, tutorial support, online e-tutor, peer group discussion forums and face-to-face discussion classes. The blended learning approach in an ODL environment, in particular Unisa, allows students to access a variety of different resources to benefit their understanding of mathematics at a tertiary level. The e-learning environment allows students to work at their own pace and review solutions and procedures until they understand the concept.

Open Distance Learning (ODL), by definition, is a learning methodology that is learner-centred in its approach and aims to bridge the time, communication and geographical distance (transactional distance) between students and the institution (Dobbs et al. 2009). ODL has provided extensive opportunities for students who are unable to participate in campus-based, fixed time, face-to-face tuition to complete their studies. Higher education institutions that are moving towards an educational approach that includes the impact of technology and the flexible needs of learners, make the student the central focus in the design and development of curricula. This approach allows the student to study full-time or part-time, and offers a blend of contact tuition, electronic education and paper-based distance education. In such a flexible learning environment, there is a shift from conveying information to facilitating learning in accordance with appropriate modes of delivery.

Unisa has focussed its attention on using mobile technologies, collaborative learning using the institutional online learning and teaching (*myUnisa*) system, and blended learning to deliver effective learning and teaching (Borba et al. 2016).

In the delivery of the mathematics curriculum at Unisa, the internet-supported programme is utilised, providing for online participation for students which is both optional and supplementary. In the Department of Mathematical Sciences, the blended learning approach incorporates an appropriate combination of technologically enhanced media and digital student support: including tutorial support, online e-tutor, videos, vodcasts, podcasts, blogs, peer group discussion forums and face-to-face discussion classes. In the Unisa ‘blended’ mode, students study online using study materials that have been prepared using various ICT supported resources and using social media for communication among students themselves or with the lecturer. The study materials are uploaded online on the institutional learning and teaching system *myUnisa* and are also available in printed format. At Unisa there are Whatsapp groups with students registered for a particular module. In addition, students are also reminded via sms of upcoming residential face-to-face workshops.

In the Unisa model, academics are expected to facilitate learning and teaching by compiling all relevant teaching materials, and to administer and manage the online courses and the e-tutors themselves (Prinsloo, 2009). The lecturer also needs to provide additional resources to enhance the learning process.

Virtual learning spaces facilitate the changing roles of the lecturer, who:

- needs to adjust to the online communication environment;
- must assume the role of a facilitator, guiding the students and pacing the curriculum. This is achieved by means of a study guide which explicitly states the sequencing and pacing of the content on a weekly basis. The assignments which form part of the continuous assessment are spread out over the academic year, and are based on specific units as outlined in the study guide. In these virtual environments, lecturers can only guide the students through the content by pacing it with the work tested in the assignments.

- in the absence of face-to-face interaction needs to articulate questions and discussions in the online environment to guide the students' understanding;
- must provide resources to enhance the learning process.

Virtual learning spaces also facilitate the changing roles of the student, who:

- takes longer, balancing study with other commitments;
- needs to adjust to the online communication medium;
- is no longer a passive recipient of knowledge;
- actively engages with the content online, asking and articulating the right questions, communicating with their e-tutor and lecturer, and pacing their studies;
- takes ownership of their own learning through interacting in an online environment;
- must adopt a mature approach and positive work ethic to succeed when studying online.

There are also changes in the role of the curriculum and its mode of delivery, and in the nature of the assessment (Huntley, 2019).

When teaching by means of technology, such as in ODL environments, the foundations are laid for assessing online. If technology is incorporated in the presentation of the course, it makes little sense to avoid technology in the assessment part of the course. Assessment does not have to consist only of tests, assignments or computer quizzes. Waldock (2011b) introduced the idea of a *logbook* in which students have to write a few sentences on a weekly basis about each module they are registered for, indicating what went well, what did not go well and what plans and steps they intend taking to deal with problems that may have arisen. The objective is to develop students' planning and reflective skills and it has the further advantage for students that they are encouraged to face problems and commit strategies for solving these. To ensure that students participate in these activities, marks are awarded for regularity and quality of the logbook entries. Another way of assessing online is for students to develop an online *portfolio* of their work. During their progress, students accumulate an online collection of their work. This can include an ongoing resumé and separate pages for each module (Waldock, 2011b).

Students studying in the ODL environment typically take longer to complete their studies as they need to balance study workloads and other commitments. In an online environment, students are no longer passive recipients of knowledge. They have to take the initiative to actively engage with the content online, asking and articulating the right questions, communicating with their e-tutor and lecturer, as well as pacing their studies. By interacting in an online and asynchronous environment, the students benefit and take ownership of their own learning. The asynchronous nature of online courses requires students to make their own choice of actions, to reflect upon the course materials and their responses, and to work at their own pace (Suanpang et al. 2004). This individual and independent learning approach requires students to display both a level of maturity and a positive work ethic to succeed in an online environment.

## 4. Recommendations

As stated in the introduction, a key objective of our collaboration was to identify successful practice at each of the respective institutions with the aim of supporting further enhancement for our mutual benefit, and that of others. At both SHU and Unisa, as with all HEIs, there are physical and virtual aspects to the students' learning spaces, although the balance between them is markedly different.

The outcomes of the work reported above have led to a number of recommendations that may be helpful in developing a successful learning community:

- **Maintain a variety of channels of communication**
  In the physical world these are likely to be face-to-face; in the virtual world they can comprise email, live chat, blogs, discussion boards and online forums/logbooks. The use of small friendship groups (e.g. through Peer Assisted Learning schemes) has worked well in physical environments, but there is clearly scope to develop these in the virtual world also. Depending on the mode of communication, it may be synchronous or asynchronous.
- **Make careful use of social media**
  Tools such as Facebook, WhatsApp, Twitter and LinkedIn can enhance social cohesion within the student group. Even though these are based in the virtual domain they can also be complementary to the development of a physical learning community.
- **Utilise e-learning**
  Making use of a variety of electronic methods for the delivery and support of learning allows students to work at their own place and pace. Although such methods are paramount in environments such as Unisa, they provide vital further support for students everywhere. All HEIs use e-learning to some degree but clear opportunities exist to expand their scope.
- **Provide shared software tools**
  Students require access both on and off-site access to computer-based learning materials. Most institutions have processes that allow external access to software on the university-hosted system.
- **Include group as well as individual activities.**
  Students become professional mathematicians by undertaking authentic ‘real-world’ tasks; by doing so through group work they can develop interprofessional working skills. The onus is on the academic to set up an appropriate infrastructure for this. Many students at institutions such as Unisa study alongside work, and there are clear opportunities to use students’ existing experiences to deliver ‘authentic’ activities. It may be possible to share some of these with other institutions.
- **Encourage student-generated content**
  A sense of ownership can develop if students can co-create at least some of their learning materials and learning experiences, perhaps through contributing to online repositories of material.
- **The simulation of a physical ‘hub’ space**
  Students attending in person welcome the provision of a specialist ‘hub’ space for their discipline. They know they will find like-minded people sharing their own goals and likely to be working on similar things, so help from peers and staff is readily available, and appropriate resources are available. The look and feel of the space has a vital role to play in helping students develop a professional approach. In the virtual arena, a similar function can be fulfilled by an on-line learning hub, providing an array of facilities to help develop similar collaborative activities.

## 5. Acknowledgement

JW and BH would like to thank AdvanceHE for supporting – through JW's National Teaching Fellowship – their participation in the Delta conference on Mathematics Education in Gramado, Brazil in November 2017 at which this work was first presented.

## 6. References

Borba, M., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S. and Aguila, M., 2016. *Blended learning, e-learning and mobile learning in mathematics education – a survey for ICME-13*. [video] Available at: https://lecture2go.uni-hamburg.de/l2go/-/get/v/19773 [Accessed 25 May 2019].

Boys, J., 2010. *Towards creative learning spaces: Re-thinking the architecture of post-compulsory education*. London: Routledge.

Brown, M., Macrae, S., Rodd, M. and William, D., 2005. Full report of research activities and results: Students' experiences of undergraduate mathematics; King's College London Department of Educational and Professional Studies. Available at: https://www.researchgate.net/publication/258423334_Full_report_of_research_activities_and_results_Students'_experiences_of_undergraduate_mathematics [Accessed 25 May 2019].

Chism, N.V.N., 2006. Challenging traditional assumptions and rethinking learning spaces, In: D. G. Oblinger, ed. *Learning Spaces*. Louisville, CO: EDUCAUSE. Ch. 2.

Cornock, C., 2016. The evaluation of an undergraduate peer assisted learning scheme at Sheffield Hallam University. *Journal of Learning Development in Higher Education*. Special Edition: Academic Peer Learning (Part II). Available at: https://journal.aldinhe.ac.uk/index.php/jldhe/article/viewFile/360/pdf [Accessed 25 May 2019].

Cornock, C., 2015. Maths Arcade at Sheffield Hallam University: Developments made in a new space. *MSOR Connections*, 14(1), pp.54-61. https://doi.org/10.21100/msor.v14i1.253

Croft, T. and Grove, M., 2015. Progression within mathematics degree programmes. In M. Grove, T. Croft, J. Kyle, and D. Lawson, eds. *Transitions in Undergraduate Mathematics Education*. Birmingham, UK: University of Birmingham and Higher Education Academy. pp.173-190.

Dobbs, R. R., Waid. C. A. and del Carmen, A., 2009. Students' Perceptions of Online Courses: The Effect of Online Course Experience. *Quarterly Review of Distance Education*, 10(1), pp.9-26.

Gutiérrez, K. D., Baquedano-López, P. and Tejeda, C., 1999. Re-thinking diversity: Hybridity and hybrid language practices in the third space. *Mind, Culture, and Activity*, 6(4), pp.286-303. https://doi.org/10.1080/10749039909524733

Harrop, D. and Turpin, B., 2013. A Study Exploring Learners' Informal Learning Space Behaviors, Attitudes, and Preferences. *New Review of Academic Librarianship*, 19(1), pp.58-77. https://doi.org/10.1080/13614533.2013.740961

Healey, M., Flint, A. and Harrington, K., 2014. *Engagement through partnership: students as partners in learning and teaching in higher education*. York: The Higher Education Academy.

Huntley, B., 2019. Tertiary mathematics in a digital open distance learning environment. In: D. Singh and M. Makhanya, eds. *Essays in online education*. pp 156 - 169. World Conference on online learning. ICDE 2017.

Lefevre, M., 2005. Facilitating Practice Learning and Assessment: The Influence of Relationship. *Social Work Education*, 24(5), pp.565-583. https://doi.org/10.1080/02615470500132806

Mannarini, T., Rochira, A. and Talò, C., 2012. Identification processes and inter-community relationships affect sense of community. *Journal of Community Psychology*, 40, pp.951-967. https://doi.org/10.1002/jcop.21504

Middleton, A., 2018. *Reimagining spaces for learning in higher education*. London: Red Globe Press.

Monahan, T., 2002. Flexible Space & Built Pedagogy: Emerging IT Embodiments. *Inventio*, 4(1), pp.1-19.

Morris, S., 2017. *Critical Pedagogy and Learning Online*. Available at: https://www.seanmichaelmorris.com/critical-pedagogy-and-learning-online/ [Accessed 25 May 2019].

Neary, M., Harrison, A., Crellin, G., Parekh, N., Saunders, G. and Duggan, F., 2010. *Learning landscapes in higher education: Clearing pathways, making spaces, involving academics in the leadership, governance and management of academic spaces in higher education*. Lincoln: Centre for Educational Research and Development.

Oldenburg, R., 1989. *The great good place: Cafés, coffee shops, community centers, beauty parlors, general stores, bars, hangouts, and how they get you through the day*. New York: Paragon House.

O'Rourke, V. and Baldwin, C., 2016. Student engagement in placemaking at an Australian university campus. *Australian Planner*, 53(2) pp.103-116. https://doi.org/10.1080/07293682.2015.1135810

Prinsloo, P., 2009. *Modelling throughput at Unisa: The key to the successful implementation of ODL*. Available at: http://hdl.handle.net/10500/6035 [Accessed 25 May 2019].

Scott-Webber, L., 2004. *In Sync – Environmental behaviour research and the design of learning spaces*. Ann Arbor, MI: Society for College and University Planning.

Suanpang, S., Petocz, P. and Kalceff, W., 2004. Student attitudes to learning business statistics online vs traditional methods. *Educational Technology and Society*, 7(3), pp.9-20.

Waldock, J., 2011a. Peer Assisted Learning. In: J. Waldock, ed. *Developing Graduate Skills in HE Mathematics Programmes - Case Studies of Successful Practice*. Birmingham: MSOR Network. pp.22-23.

Waldock, J., 2011b. Progress Files. In: J. Waldock, ed. *Developing Graduate Skills in HE Mathematics Programmes - Case Studies of Successful Practice*. Birmingham: MSOR Network. pp.24-25.

Waldock, J., 2015. Designing and using informal learning spaces to enhance student engagement with mathematical sciences. *MSOR Connections*, 14(1), pp.18-27. https://doi.org/10.21100/msor.v14i1.235

Waldock, J., Rowlett, P., Cornock, C., Robinson, M. and Bartholomew, H., 2017. The role of informal learning spaces in enhancing student engagement with mathematical sciences, *International Journal of Mathematical Education in Science and Technology*, 48(4), pp.587-602. https://doi.org/10.1080/0020739X.2016.1262470

Zepke, N. and Leach, L., 2010. Improving student engagement: ten proposals for action. *Active Learning in Higher Education*, 11(3), pp.167-177. https://doi.org/10.1177/1469787410379680

RESEARCH ARTICLE

# Creating a Taxonomy of Mathematical Errors for Undergraduate Mathematics

Shauna Ford, School of Mathematics, Cardiff University, UK. Email: FordS6@cardiff.ac.uk.
Jonathan Gillard, School of Mathematics, Cardiff University, UK. Email: GillardJW@cardiff.ac.uk.
Mathew Pugh, School of Mathematics, Cardiff University, UK. Email: PughMJ@cardiff.ac.uk.

## Abstract

In this paper we develop a taxonomy of errors which undergraduate mathematics students may make when tackling mathematical problems. We believe that a taxonomy would be useful for staff in giving feedback to students, and would facilitate students' higher-level understanding of the types of errors that they could make.

**Keywords:** assessment, feedback, errors.

## 1. Introduction

There has been a considerable amount of research over the last century into mathematical errors (Radatz, 1980). Typically this research is in the context of learning mathematics in school (e.g. Radatz, 1980; Matz, 1982; Kieran, et al., 1990; Foster, 2007). Such studies tend to focus therefore on errors which are either arithmetic or algebraic in nature, such as errors in long division, or misinterpreting $(a+b)^2$ as $a^2+b^2$. An approach which seems to receive particular attention in the U.S. is error analysis (e.g. Ashlock, 2010; Idris, 2011). In this approach pupils' errors are systematically recorded by the teacher, and analysed for patterns so that teacher can then plan what potential remedial action will be necessary to correct any underlying misconceptions.

Whilst research into errors made by pupils in a school context can be of benefit to teachers in higher education, the contexts are also very different. School pupils will possess a wide range of mathematical abilities, and many will have a dislike or even fear of mathematics. On the other hand mathematics undergraduate students possess a strong mathematical ability and have chosen to study the subject further. Thus one would hope that many of the errors made by pupils, resulting from a misunderstanding of even basic concepts within mathematics, would not be made by mathematics undergraduates. The approach to mathematics also tends to be very different in the two contexts. In school the focus is almost exclusively on algorithms to solve problems, whilst undergraduate mathematics will also focus on understanding concepts and proving results. Therefore the types of error made in higher education will typically be of a different nature to those in primary or secondary education.

Much of the above research, as well as more general studies on mathematical errors, focuses on understanding the underlying cognitive causes of these errors, either in order to understand the cause of specific errors, or more generally to identify the mechanisms underlying these errors. It is argued that most mathematical errors are causally determined, and very often systematic (Radatz, 1980). Radatz (1979) identified five error categories: (1) errors due to language difficulties, (2) errors due to difficulties in obtaining spatial information, (3) errors due to deficient mastery of prerequisite skills, facts, and concepts, (4) errors due to incorrect associations or rigidity of thinking, and (5) errors due to the application of irrelevant rules or strategies. Ben-Zeev (1998) constructed a taxonomy of mathematical errors and attempted to identify the causes of these errors by integrating findings from different studies. The focus in this and other research is to understand why a student makes an error. For example, a student may over-generalize an algorithm which holds in one context to a structurally similar context where the algorithm no longer works, something Ben-Zeev calls syntactic induction (Ben-Zeev, 1998).

It will often however be difficult, if not impossible, to diagnose the underlying error in a student's reasoning or understanding solely from the student's written solution to a problem. Therefore this paper will not focus on this, but rather on classifying the particular types of errors students make when attempting to solve mathematical problems. Such a classification should provide enough details so that a student can identify what it is they have done wrong, whilst keeping the number of classes as small as possible. We believe that creating a taxonomy of errors is useful for the following reasons:

- it will be a useful resource for students to see which errors to avoid, some of which may not have been appreciated previously;
- it could be incorporated into a feedback tool for lecturers to enrich the feedback offered to students;
- it would allow for the consideration of relationships between different types of error.

Whilst this paper implicitly assumes that the students we consider are undergraduate mathematics students, the developments in this paper could also be applied to GCSE or A-level mathematics students.

This work was undertaken as part of an undergraduate summer project by the first author. The paper is structured as follows. In section 2 we consider the definition of error that we use in this paper, and what causes errors to take place, before relating this information to errors in mathematics. The taxonomy is given in section 3.

## 2. Human Error

Human error is a failure of a planned action to achieve a desired outcome. Errors can be made in one of two ways – either the plan itself may be inadequate, or else the execution of that plan may include actions that are unintentional and which do not lead to the desired outcome, as illustrated in figure 1.

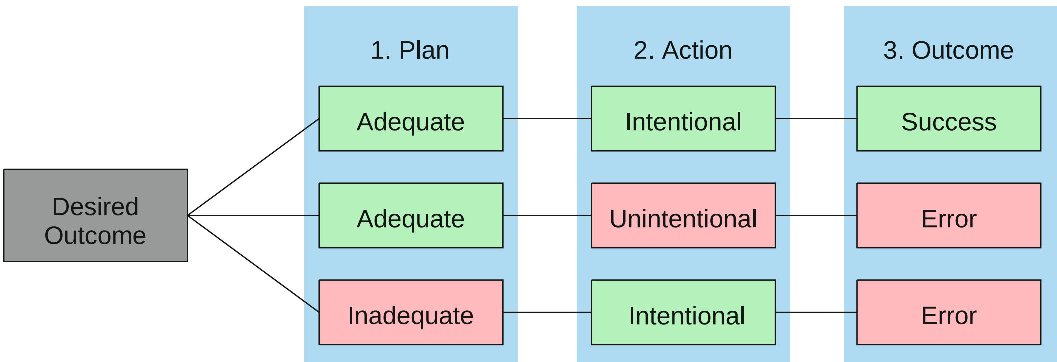



Figure 2. Occurrence of Human Errors.

Failures in planning are often referred to as mistakes rather than errors. There are two types of mistakes: knowledge-based and rule-based (Reason, 1990). Knowledge-based mistakes occur when an individual has an inability to reach an end goal because of a lack of knowledge. Rule-based mistakes occur when an individual wrongly modifies an established process. Such mistakes are more likely to go unnoticed when the outcome is not specifically known. The modification is likely to be informed by previous successful experiences (Rasmussen, 1986, p.102). Rule-based mistakes fall into two categories:

- Misapplication of a good rule: Occurs when an individual applies a rule which may be perfectly adequate in another situation, but which may not meet the conditions and demands of the problem being considered (Reason, 1990, p.75). Such errors are more likely to occur when an individual has applied the rule successfully for a previous problem.
- Application of a bad rule: A good rule may become bad following changes that an individual makes that are not thoroughly considered. This may be from the alterations not being managed appropriately, or the creation of a bad rule from incorrect knowledge. This can appear on varying levels; the rule could be entirely wrong, the rule may be clumsy or inefficient but still achieve the desired outcome, or the rule could be inadvisable since whilst leading to a good approximate solution, repeated use may worsen this approximation.

Unintentional actions are classed as skill-based errors. These often occur when implementing elementary or standard procedures, due to a lack of consciousness or control (Rasmussen, 1986, p.100). Skill-based errors fall into two categories: memory lapses and slips of actions.

- Memory lapse: These errors include losing place in a sequence of steps, forgetting to do something, or forgetting the overall plan entirely.
- Slip of action: An unintentional action that occurs at the point of execution. This error is often caused by a process being performed subconsciously, skipping or reordering steps in a procedure, or experiencing a distraction.

The skill-rule-knowledge framework described above only offers a partial account of possible deviant behaviour (Reason, 1990). Humans plan and execute their actions in social environments that may affect their performance. Whilst mistakes and skill based errors are defined as errors made in the individual's cognitive stages, their behaviour may also be altered by the situation's social context. Violations are deliberate alterations considered necessary by the individual to adjust to external influences (Reason, 1990, p.195). Hence the violator is not always entirely blameworthy for the decision made. The following three types of violation are distinguished:

- Routine violations: These occur due to natural instinct to take the process that requires the least amount of effort. This becomes habitual and forms a set pattern of errors in their behaviour.
- Situational violations: An individual alters their behaviour due to a change in their social surroundings. These changes can include excessive time pressure, stress, workplace design, and inadequate or inappropriate equipment.
- Exceptional violations: These occur when an individual adopts a cause of action known to be usually incorrect but determines that the current situation is an exception.

Violations and errors from the previous skill-rule-knowledge framework can coincide or appear alone. The classification of human errors described above is illustrated in figure 2.

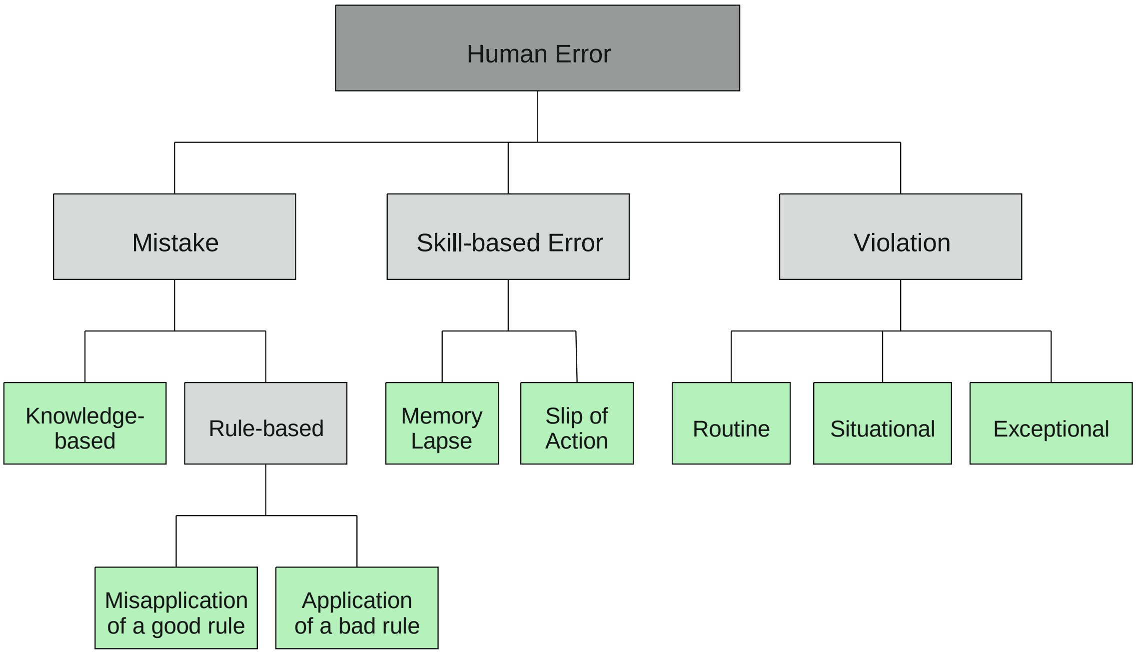



Figure 2. Human Error Types.

How might these types of errors appear in the context of a student attempting a mathematics problem? Suppose a student was answering the question:

Differentiate the function $f(x) = 3x \sin x^2$.

This requires using the product and chain rules to obtain an answer of $6x^2 \cos x^2 + 3 \sin x^2$. A 'slip of action' might be manifested as a numerical slip-up (such as writing the coefficient of the derivative as 5 rather than 6, possibly through subconsciously confusing $2 \times 3 = 6$ with $2 + 3 = 5$), or a careless error in writing the solution (such as writing $\cos x$ by mistake). If the student could not recall the chain rule then this would be a 'memory lapse', whereas if they did not know the chain rule then they would likely make a 'knowledge-based' error. If they had incorrectly recalled the chain rule, then the error would be an 'application of a bad rule'. On the other hand, if the student had (wrongly) integrated the function correctly, they are likely to be guilty of 'misapplication of a good rule'. A 'routine violation' could occur if a student had made the same error often enough so that they no longer realised it was an error, for example, writing $-\cos x$ for the derivative of $\sin x$. A 'situational violation' might be more likely if the student had to answer this question in an examination, perhaps due to the stress and time-pressures of the situation. An 'exceptional violation' may occur if a student is presented with the question 'Show that the derivative of $f(x) = 3x \sin x^2$ is given by $f'(x) = 6x^2 \cos x^2 + 3 \sin x^2$', where they might 'violate' a rule in a desperate attempt to arrive at a solution which matches the given answer.

## 3. Creating a Taxonomy of Mathematical Errors

A taxonomy is the "*theoretical study of classification, including its bases, principles, procedures and rules*" (Simpson, 1961, p.11). It is a way of classifying entities verifiable by observation (Bailey, 1994, p.6). A successful taxonomy will provide classes that are both exhaustive (an appropriate class for each entity) and mutually exclusive (only one suitable class for each entity) (Bailey, 1994, p.3).

There are a number of major styles of taxonomies used in research, but largely there is a strong relation between how research is conducted and the chosen taxonomy style (Senders and Moray, 1991). For a taxonomy categorising mathematical errors, there are two main styles that immediately seem most viable.

The first style is referred to as a taxonomy of cognitive mechanisms (Senders and Moray, 1991). Here, errors are classified into the stages at which information processing in humans occurs. Often these categorisations come under the following: errors of perception, errors of memory, errors of attention, etc. In other words, the errors of mathematics could be listed under the previously discussed error types. However, as specific errors can be linked to many error types, it becomes significantly harder to distinguish which errors come under which categories.

The second style of taxonomy is a phenomenological taxonomy (Senders and Moray, 1991). In this format, the categories refer almost directly to the errors as they are observed. Typical categories are labelled in the following manner: omissions, substitutions, unnecessary repetitions, etc. This technique of categorisation certainly looks to be the more appropriate choice, especially as there would be fewer discrepancies between the subgroups. Using this method, one would start by first identifying the errors before compiling the headings for each section.

The types of errors made in mathematical assignments can be very different from one another, giving a wide variety of possible mistakes. The majority of previous research was targeted more towards errors made by students studying for GCSE and A-Level, and therefore left many gaps for where undergraduate students may go wrong. Additionally, many areas are sub-discipline specific. For example, the types of errors that may occur whilst tackling an algebraic problem could be different from those which could arise tackling a statistics problem. Some errors are very general, such as communication errors and careless errors.

To be able to identify as many errors that can occur in mathematics as possible, two strategies were used. In the first instance, obvious errors that occur often were first recalled. Secondly, a selection of students' exam scripts from first year courses were analysed to identify other types of errors that had been missed.

The taxonomy that was proposed is given in table 1. Each error is given a code to allow for quick reference to the error when providing feedback to students on their work.

The first group (S) contain errors which are obviously slips of action, a common occurrence in students' mathematical work. Examples include changing the sign of a term from when step to the next, or evaluating $2 \times 3 = 5$. It could be argued that errors included under 'S3: Incorrect algebraic manipulation' might not be merely due to a careless slip but rather could betray a more fundamental misunderstanding. However, it is not generally possible for a marker to ascertain the reason why a student made a particular error, and since it is to be hoped that undergraduates studying for a mathematics degree are able to perform basic algebraic operations and manipulations competently, these errors are classified under slips of action.

The next group (U) contains errors of understanding – errors which demonstrate a lack of understanding of the mathematics on mathematical concepts being used. These errors go right to the heart of what assessing mathematics is about, and this grouping contains the largest number of errors of all the categories. Examples range from situations where it is clear to the student themselves that there is a gap in their understanding, e.g. where they are unable to finish a solution or arrive at a result which they know to be incorrect, to situations where the student might be oblivious to a fundamental error that they have made, e.g. where they divide by zero or make an argument which is not logically sound. It is of course possible that some of these errors might be merely slips,

Table 1. Taxonomy of Mathematical Errors.

| Code | Error | Examples |
|---|---|---|
| S1 | Copying error | Incorrect copying of the question, incorrect copying from one line to another, mistake copying information into a graph/diagram |
| S2 | Careless errors on simple calculations | Errors in addition or multiplication, overlooking negative signs, cancellation errors |
| S3 | Incorrect algebraic manipulation | Incorrect roots of equation, incorrect expansion of a bracket, incorrect handling of powers, incorrect addition/multiplication of fractions, incorrect partial fractions, incorrect manipulation of equality |
| U1 | Confusing different mathematical structures | Stating that a matrix is number, confusing a set with an element of a set, confusing definite, indefinite and/or improper integrals |
| U2 | Incorrect argument | Claiming an implication which is not true, incorrectly assuming additivity/commutativity |
| U3 | Lack of consideration of potential indeterminate forms | Division by zero, division by infinity, checking for zero determinants, checking a function is differentiable, logarithm not defined on non-positive numbers |
| U4 | Proposed solution is not viable | Area found is negative, probability found is not between 0 and 1, contradictions within solution |
| U5 | Definition/method/theorem not recalled correctly | |
| U6 | Partial solution given | Correct workings but unfinished solution, did not prove both implications of a bi-conditional statement |
| U7 | Incorrect assumptions | |
| U8 | Misinterpretation of results | |
| CM1 | Applying an inappropriate formula/method/theorem | Using irrelevant knowledge, uses the formula/method/theorem which is not relevant or not valid in the situation |
| CM2 | Correct solution, but a simpler/quicker approach could be used | Integration could be made simpler by using another method |
| UM1 | Does not consider all factors | Not checking all axioms are satisfied, stating conclusions with insufficient evidence |
| UM2 | Error in use of an appropriate definition/method/theorem | Algorithm is incorrectly followed, incorrect numbers applied |
| P1 | Result proved in specific/restricted cases | Formal proof replaced with specific examples, not proving all cases |
| P2 | Circular argument | Using the conclusion of the statement in order to prove the conclusion |
| P3 | Incorrectly proving backwards | Incorrectly starting with the conclusion, manipulating it and arriving at the statement |
| C1 | Undefined variables or objects | |
| C2 | Notational issues | Incorrect use of quantifiers, sets of numbers, summations or implications, omitting limit/sum/product/differentiation/integration symbols, ambiguously written fractions, no use of brackets |
| C3 | Graph/diagram presented poorly | Axes not labelled, important coordinates not labelled, use of ruler required |
| C4 | Solution difficult to read/follow | Bad prose, not communicating in coherent logical flow, insufficient workings, unclear justifications, unclear what is being done |

for instance a student may be aware of the danger of dividing by zero but may have simply missed the possibility of this happening in a given situation. However, from experience these kinds of errors are more often made by students who do not appreciate the need for care and precision when making mathematical statements, hence their inclusion as more serious errors of understanding rather than slips of action.

The following two categories (CM, UM) both relate to methods used, with the difference between the two groups being due to the distinction between the two types of rule-based mistakes: misapplication of a good rule and application of a bad rule. The first of these (CM) are errors in choice of method, even though the method itself may have been applied ‘correctly’, corresponding to the misapplication of a good rule. Here for instance an irrelevant statistical test may have been applied, or a theorem may have been used where the assumptions are not all satisfied. The second group (UM) contains errors in the use of a method, for instance, a student may have missed out an important step in the method.

The next category (P) contains errors specifically related to proof, for example using a circular argument or claiming general results from a few examples. The final category (C) relates to the student’s communication of their mathematical solutions, and covers mechanical aspects such as correct use of notation or labelling, as well as more qualitative judgements on clarity of expression or the amount of workings that have been included.

Table 2 shows the alignment of the proposed taxonomy with the model of human errors summarised in figure 2. We see that the taxonomy exhausts the main types of error indicated above. Table 2 shows that each error is typically related quite closely to the other errors within its own category in the taxonomy. Although there is overlap between groups of errors, this overlap has been minimized.

## 4. Conclusion

In this paper we have given a taxonomy of mathematical errors that has been informed by the literature on the different types of human error. This resource has the potential to decrease marking time by enabling tutors to quickly flag up general errors using the short codes for each error (e.g. S1), although it is likely that any feedback provided using the taxonomy would still need to be supplemented with some written comments to provide richer feedback.

If this taxonomy was embedded at programme level and the feedback provided was recorded, then students could be provided with an overall picture summarising the frequency of the types of errors they make, with advice on how to move towards eliminating these errors where possible. It might also be possible to design a resource to help students engage with higher-level reflection upon the types of errors they make, and how they may reduce the probability of these errors occurring. Another possible use of this taxonomy is to inform question setters of the errors they may be likely to see when designing or marking a problem. The notion of informed question design is reminiscent of work by Quinney (2008).

## 5. Acknowledgements

We thank Cardiff University’s Centre for Education Innovation for funding the work of the first listed author as part of a Cardiff University Student Education Innovation Project. We are grateful to the two referees for careful reading of our manuscript.

Table 2. Alignment of the taxonomy with types of error.

| Code | Slip of Action | Memory Lapse | Knowledge-based Error | Misapplication of Good Rule | Application of Bad Rule |
|---|---|---|---|---|---|
| S1 | X | | | | |
| S2 | X | | | | |
| S3 | X | | | | |
| U1 | | X | X | | |
| U2 | | X | X | | |
| U3 | X | X | X | | |
| U4 | X | X | X | | |
| U5 | | X | X | | |
| U6 | | X | X | | |
| U7 | | X | X | | |
| U8 | X | X | X | | |
| CM1 | | | X | X | |
| CM2 | | | X | X | |
| UM1 | X | | X | | X |
| UM2 | X | X | X | | X |
| P1 | | X | X | X | |
| P2 | X | | | | X |
| P3 | | | X | | X |
| C1 | X | X | | | |
| C2 | X | X | X | | |
| C3 | X | X | X | | |
| C4 | X | | X | | |

## 6. References


Ashlock, R., 2010. *Error patterns in computation*. 10th ed. Columbus, OH: Pearson.

Bailey, K.D., 1994. *Typologies and Taxonomies: An Introduction to Classification Techniques*. Thousand Oaks, CA: Sage Publications.

Ben-Zeev, T., 1998. Rational errors and the mathematical mind. *Review of General Psychology*, 2(4), pp.366-383. https://doi.org/10.1037%2F1089-2680.2.4.366

Foster, D., 2007. Making meaning in algebra: Examining students' understandings and misconceptions. In: A.H. Schoenfeld, ed. *Assessing mathematical proficiency*. Cambridge: Cambridge University Press. pp.163-176.

Idris, N., 2011. Error patterns in addition and subtraction for fractions among form two students. *Journal of Mathematics Education*, 4(2), pp.35-54.

Kieran, C., Booker, G., Filloy, E., Vergnaud, G. and Wheeler, D., 1990. Cognitive processes involved in learning school algebra. In: P. Nesher and J. Kilpatrick, eds. *Mathematics and Cognition*. Cambridge: Cambridge University Press. Ch. 5.

MacGregor, M. and Stacey, K., 1997. Students understanding of algebraic notation. *Educational Studies in Mathematics*, 33(1), pp.1-19. https://doi.org/10.1023/A:1002970913563

Matz, M., 1982. Towards a process model for school algebra error. In: D. Sleeman and J.S. Brown, eds. 1982. *Intelligent tutoring systems*. New York: Academic Press. pp.25-50.

Quinney, D., 2008. So just what is conceptual understanding of mathematics? *MSOR Connections*, 8(3), pp.2-7.

Radatz, H., 1979. Error analysis in mathematics education. *Journal for Research in Mathematics Education*, 10(3), pp.163-172. https://doi.org/10.2307/748804

Radatz, H., 1980. Students' errors in the mathematical learning process: a survey. *For the Learning of Mathematics*, 1(1), pp.16-20.

Rasmussen, J., 1986. *Information Processing and Human-Machine Interaction: An Approach to Cognitive Engineering*. New York: Elsevier Science.

Reason, J., 1990. *Human Error*. Cambridge: Cambridge University Press.

Senders, J. and Moray, N., 1991. *Human Error: Causes, Prediction and Reduction*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Simpson, G.G., 1961. *Principles of Animal Taxonomy*. New York: Columbia University Press.

RESEARCH ARTICLE

# The impact of peer assessment on mathematics students' understanding of marking criteria and their ability to self-regulate learning

Chris Brignell, School of Mathematical Sciences, University of Nottingham, Nottingham, UK. Email: chris.brignell@nottingham.ac.uk.
Tom Wicks, School of Mathematical Sciences, University of Nottingham, Nottingham, UK. Email: tom.wicks@nottingham.ac.uk.
Carmen Tomas, Educational Excellence Team, University of Nottingham, Nottingham, UK. Email: carmen.tomas@nottingham.ac.uk.
Jonathan Halls, Educational Excellence Team, University of Nottingham, Nottingham, UK. Email: jonathan.halls2@nottingham.ac.uk.

## Abstract

At the University of Nottingham peer-assessment was piloted with the objective of assisting students to gain greater understanding of marking criteria so that students may improve their comprehension of, and solutions to, future mathematical tasks. The study resulted in improvement in all four factors of observation, emulation, self-control and self-regulation thus providing evidence of a positive impact on student learning.

The pilot involved a large first-year mathematics class who completed a formative piece of coursework prior to a problem class. At the problem class students were trained in the use of marking criteria before anonymously marking peer work. The pilot was evaluated using questionnaires (97 responses) at the beginning and end of the problem class. The questionnaires elicited students' understanding of criteria before and after the task and students' self-efficacy in relation to assessment self-control and self-regulation.

The analysis of students' descriptions of the criteria of assessment show that their understanding of the requirements for the task were expanded. After the class, explanation of the method and notation (consistent and correct) were much more present in students' descriptions. Furthermore, 67 per cent of students stated they had specific ideas on how to improve their solutions to problems in the future. Students' self-perceived abilities to self-assess and improve were positively impacted. The pilot gives strong evidence for the use of peer-assessment to develop students' competencies as assessors, both in terms of their understanding of marking criteria and more broadly their ability to self-assess and regulate their learning.

**Keywords:** peer-assessment, assessment criteria, formative assessment, rubric-based scoring, analytic rubrics.

## 1. Introduction

### *1.1. Assessment context – NSS and marking criteria*

In the UK it is established that assessment related National Student Survey (NSS) questions perform consistently lower than the other areas of satisfaction or even the overall satisfaction. This study pays attention to a particular element of the satisfaction with assessment: 'assessment criteria have been made clear to me in advance'. The significance of this question is primarily about validity of assessments. For an assessment to be valid, the expected or required performance should be understood by all stakeholders, of which students are a primary one (Messick, 1994). Whilst the NSS questions have adopted a political significance, particularly with the introduction of regulatory subject level Teaching Excellence Framework (TEF) ratings, this paper explores how marking criteria can be communicated clearly to students in advance of mathematics assessments.

The very nature of marking criteria is contested in the literature, and no less in practice, but can criteria be accurately communicated to ensure validity of the assessment? Mathematics is often marked in a holistic manner, where an overall judgement is made, but analytic rubric marking, where several judgements are made on identified individual criteria, is sometimes proposed with the intention of increasing openness and objectivity (Swan and Burkhardt, 2012).

Sadler (2009) expresses the view that criteria are intrinsically vague and cannot be defined clearly. As a consequence, he argues in favour of holistic marking and urges practitioners to engage students in the practice of ‘evaluative experiences’. This particular line of argument has seen the development of a novel form of peer-assessment within mathematics known as ‘comparative judgement’ (Jones and Alcock, 2014; Jones and Sirl, 2017).

In contrast, several reviews have indicated rubrics are beneficial instruments for instruction by clarifying goals for all users, both markers and students (Jönsson and Svingby, 2007; Reddy and Andrade, 2010; Brookhart, 2018). Also, in agreement with Sadler’s early discussion of evaluative experiences, student engagement in evaluative judgement has grown conceptually (Boud et al., 2018). There is also empirical evidence that engaging students in peer, self and co-assessment can have a positive impact on students’ self-regulation, motivation and self-efficacy, their own confidence in self-perceived abilities (Winstone et al., 2017; Evans, 2013; Boud et al., 2018).

In a recent article Dawson (2017) identified 14 elements that practitioners need to define when designing rubrics and marking criteria. These elements span the objective of the assessment (what knowledge or skill is being tested) and the scoring strategy (how are marks arrived at) but also how the criteria are articulated to students and other markers required to implement the criteria. In practice, many of these optional decisions are left to the discretion of the practitioners.

### *1.2. Making criteria clear and self-regulation*

The importance of the ways in which marking criteria are used with students has been stressed with the development of the concept of evaluative judgement (Boud et al., 2018). Evaluative judgement provides a conceptual framework for practitioners that brings together multiple known formative practices (e.g. rubrics, peer and self-assessment, use of exemplars). In the absence of the ‘evaluative judgement’ umbrella these practices are understood as separate methods. Evaluative judgement provides a coherent framework for practitioners to actively and explicitly promote students’ ability to judge their own work and that of others. Similar concepts exist and predate evaluative judgement in the literature (e.g. evaluative competence by Sadler, 1989; assessment literacy by Price et al., 2012).

Within this evaluative judgement framework, Panadero and Broadbent (2018) make connections with self-regulated learning. Four levels of self-regulation exist (observation, emulation, self-control and self-regulation). Each level is incremental, although not in a linear fashion. In the observation level students observe an expert performing the task. In HE mathematics this might be during a lecture or problem class. Emulation is students performing the task themselves in the presence of the example. For mathematics students this might be a formative homework task attempted using the lecture notes as guidance. However, our aim is for students to reach self-control and self-regulation where students can attempt similar, and then unseen, problems in the absence of experts or model answers. Panadero and Broadbent (2018) propose that rubrics and peer-assessment tasks can assist students in achieving this aim. This framework for instruction including the use of rubrics derives from a pre-existing evidence base of the positive impact of the formative use of rubrics for learning (Panadero and Jönsson, 2013).

### *1.3. Scoring systems in mathematics*

Swan and Burkhardt (2012) note that while all assessment involves judgement, scoring systems in mathematics tend to reward answers, which is quick and objective, rather than mathematical reasoning, which is arguably more important but harder to judge. The main scoring systems currently in use are summarised below.

- Point-based scoring. A common and traditional scoring system where numerical marks are awarded for method, accuracy or explanation at each step of the solution. While easy to implement, marks are task-specific rather than an absolute measure of mathematical ability.
- Criterion-based scoring. The whole response is assigned a level based on pre-defined descriptors. The descriptors enable the student to be measured against absolute standards but converting levels to numerical scores is somewhat subjective.
- Rubric-based scoring. This retains the holistic element of criterion-based scoring but levels are awarded for different elements of performance, e.g. formulating a model or interpreting an answer, to pinpoint areas of strength and weakness.
- Comparative judgement. Responses are ranked by making relative judgements rather than judgements against criteria, which may be easier for inexperienced markers. However, scores are norm-referenced and the basis for judgements can be unclear.

Newton (1996) shows that point-based scoring has high reliability. However, our experience is that students don't benefit from points-based scoring. In the 2018 NSS, only 69% of mathematics students at our institution agreed that marking criteria were made clear, and only 64% agreed that they had received helpful comments on their work. Similarly, in a focus group in 2017/18, four out of seven of our mathematics students reported the current feedback did not help them understand where marks had been discounted.

Swan and Burkhardt (2012) suggest that criterion-based scoring is more useful for formative work because of its ability to feedforward to unseen tasks. Rubric-based scoring gives a more detailed judgement that communicates which facets of an answer are valued. See Mertler (2001) for examples of criterion-based and rubric-based scoring methods.

### *1.4. Objective of the present study*

This study, in seeking to enhance transparency of assessment criteria to mathematics students, trialled the use of analytic rubric-based scoring and peer assessment. In the context of mathematics, we wanted to evaluate the impact of these alternative instructional approaches on helping mathematics students to become self-regulated learners. The study aimed to provide insights into students' understanding of expectations of the present task (self-control) and their ability to plan future actions (self-regulation). Both aspects are crucial to student autonomy and evaluative judgement.

## 2. Developing and evaluating a peer-assessment activity

### *2.1. Preparation and class format*

Approximately 250 first year mathematics undergraduates studying a compulsory first year module were asked to complete three questions for homework. A copy of the questions is in appendix 1. The questions related to linear systems of equations and the use of row and column operations to invert matrices and find determinants. Solutions were submitted in advance of the class and anonymised via the use of student identification numbers.

At the beginning of the class a module lecturer explained the aims of the activity, to help students understand the marking criteria so they can self-evaluate the work. Students were provided with a copy of the model solutions and the marking criteria, as shown in table 1. An example mock script was projected on the screen and the lecturer demonstrated the application of the criteria and anticipated common errors were discussed.

Students marked an anonymous piece of coursework, rating the script for accuracy and clarity using the level descriptors in table 1. They also provided written feedback to justify their decision, and discussed in small groups of two to four students their perceptions and decisions on scoring.

The class was concluded with a class discussion where around five students were invited to present their scoring and feedback. Marked scripts were then returned to staff for checking prior to being returned to students via personal tutors.

Table 1: Original marking criteria rubric

| Mark | Accuracy | Clarity |
|---|---|---|
| 0 | No genuine attempt made at answering the question. | No genuine attempt is made to explain the method or use correct notation. |
| 1 | The solution contains multiple errors. | There is little explanation or correct use of notation. |
| 2 | The correct method is applied but with one or two minor errors (e.g., incorrect addition at an intermediate step). | Most steps are explained and notation is mostly correct. |
| 3 | The method is correctly applied with no minor errors. | Clear explanation of method. Consistent and correct use of notation throughout. |

## 2.2. Evaluation

Students completed a questionnaire at the beginning and end of the class in order to capture students’ understanding of criteria in the assessment before the class and after. A copy of the questionnaire is in appendix 2. In particular, students were asked, ‘In your own words, what makes a problem solution excellent?’ both pre-class and post-class. In order to capture the impact on planning actions, in the post-class questionnaire they were also asked, ‘Have you had any specific ideas on how to improve your solutions to problems in the future? If YES, please describe these briefly.’

One part of evaluative judgement and students’ autonomy is linked to confidence in their own abilities. A Likert scale question posed before and after the class aimed to capture their confidence in performing well in types of assessment and their ability to self-assess.

Table 2: Subdivision of marking criteria rubric

| | Accuracy | | Clarity | |
|---|---|---|---|---|
| Mark | Method | Errors | Explanation | Notation |
| 0 | No genuine attempt made at answering the question | No genuine attempt made at answering the question | No genuine attempt is made to explain the method | No genuine attempt is made to use correct notation |
| 1 | An incorrect method is applied | The solution contains multiple errors | There is little explanation | There is little correct use of notation |
| 2 | The correct method is applied | One or two minor errors | Most steps are explained | Notation is mostly correct |
| 3 | The method is correctly applied | No minor errors | Clear explanation of method | Consistent and correct use of notation throughout |

## 2.3. Data analysis

Students' responses to the questionnaire were originally coded according to the marking criteria rubric, shown in table 1. However, it was decided that the areas of Accuracy and Clarity could be subdivided into Method and Errors, and Explanations and Notation, respectively. This subdivision is shown in table 2. It is maintained that this refined rubric is more suited to the current analysis, as the amalgamation of areas could mask differences in student responses between pre-class and post-class. For example, students discussing Method pre-class and Method and Errors post-class would be viewed as scoring the same if their responses are coded only by the area of Accuracy.

Investigation of students' responses indicated that some discussed *legibility*. For example, they suggested that an excellent answer should be 'legible', 'neatly presented' or 'clearly written'. Therefore, students' responses were also analysed for the area of legibility. In the initial whole class discussions, legibility of answers (in terms of neatness of handwriting) had not been discussed.

Consequently, students' responses were coded for five categories: Method, Errors, Explanation, Notation and Legibility. Students' responses were marked for presence (1) or absence (0) of each category. To establish the extent of inter-rater reliability, 16% of the data were coded by a second researcher. Cohen Kappa indicated a very high level of agreement ($K = .953$, $p < .001$). McNemar's test was used to test for significant differences from pre- to post-class.

Students' self-reported ratings in their confidence at assessing their own work and writing good solutions, both pre-class and post-class, on a Likert scale from 1 (not confident at all) to 5 (very confident) were analysed using a related samples Wilcoxon signed ranks test.

## 2.4. Sample and Ethics

In total 97 students responded to the pre- and post-class questionnaire, but not all questions were answered by all students. Where appropriate, the number of responses to specific questions is provided in the analysis below. Prior to data collection, students were informed the questionnaire was optional, that data would be anonymised and separate to any other assessment activities, and asked to provide informed consent. The ethical procedures applied in this study were approved by a University of Nottingham ethics committee and we report no conflicts of interest.

# 3. Results

## *3.1.* Students' awareness of task requirements: observation level

To evaluate students' awareness of marking criteria, data is taken from students' pre- and post-class explanations of 'what makes a problem solution excellent?'. Table 3 shows the percentage of responses which related to the five coding categories (see table 2). It shows improvement in all five categories from pre- to post-class. That is, all areas of the subdivided rubric (Method, Errors, Explanation and Notation) were discussed more post-class than pre-class. A similar increase was also observed for responses discussing legibility. Analysis using McNemar's test indicates that there was a significant difference from pre- to post-class in the areas of Method, Explanation and Notation.

Table 3: Differences in rubric areas mentioned in students' responses (n=80)

| Area | Pre-class percentage of participants | Post-class percentage of participants | Percentage point change | *p.* |
|---|---|---|---|---|
| Method | 32.5% | 52.5% | 20.0 | .001 |
| Errors | 42.5% | 55.5% | 10.0 | .096 |
| Explanation | 75.0% | 87.5% | 12.5 | .041 |
| Notation | 2.5% | 22.5% | 20.0 | <.001 |
| Legibility | 17.5% | 26.3% | 8.8 | .118 |

## *3.2.* Student awareness of their own abilities: self-control

Students were asked to rate their confidence at assessing their own work and writing good solutions, both pre-class and post-class, on a Likert scale from 1 (not confident at all) to 5 (very confident). In both categories the median rating rose from 3 pre-class to 4 post-class, with 47% of respondents reporting an increased confidence in assessing their own work and 61% reporting an increased confidence in writing good solutions. A Wilcoxon signed-rank test showed both these improvements to be significant ($p < .001$).

## *3.3.* What to include in future assessments: self-regulation

When asked 'have you had any specific ideas on how to improve your solutions to problems in the future?', 67.3% of students stated that they did have ($n$ = 95). Table 4 shows the rubric areas that related to students' explanation about what to include in future work. The table shows no student discussed the rubric areas of Method or Errors. The most popular areas discussed were Explanation and Notation. These results do not map to the significant improvements found in the previous section (see table 3). Refinement of the questionnaire to tailor it to the types of question being discussed may improve the reliability of the responses.

Table 4: What to include in future work, coded by rubric area (n=63)

| Area | Percentage of participants |
|---|---|
| Method | 0.0% |
| Errors | 0.0% |
| Explanation | 84.1% |
| Notation | 31.7% |
| Legibility | 17.5% |

## 4. Discussion

The ubiquity of point-based scoring in mathematics for summative assessment is likely to prevail for some time to come. Its convenience, speed and high reliability are all good reasons for its dominance in a landscape dominated by traditional closed book exams (Iannone and Simpson, 2011). However, for formative assessment, the peer assessment activity described in this article has been found to be highly effective in enhancing students' understanding of both the current task's requirements and their ability to plan next steps to improve solutions to future unseen problems.

This contrasts to standard feedback methods, such as annotations on student scripts or general feedback to the class, which may highlight errors with the solution to the present task but might not enhance the feed-forward to future solutions. Indeed, this cohort has been exposed to these standard feedback methods previously, so the fact this task gave students new insights demonstrates that peer-assessment using absolute performance descriptors provided students with fresh understanding of marking criteria that was not gained from standard feedback and point-based scoring implemented previously. The dissatisfaction in the quality and quantity of feedback to mathematics students to support student learning is well-documented (Bidgood and Cox, 2002), but this study shows further research into the use of rubric-based scoring for formative assessment is worth pursuing.

Other forms of peer-assessment, such as comparative judgement (Jones & Alcock, 2014), are available. However, rubric based scoring with explicitly stated criteria has been shown here to enhance aspects of observation (students understanding what is required) and of self-regulation (planning actions). By contrast, comparative judgement requires students to make judgements in the absence of absolute performance criteria. This could be an advantage to inexperienced markers but could also lead to judgements based on tangential criteria. For example, this study shows students became distracted by the legibility of the writing as a proxy for the level of explanation.

This small-scale study shows a positive impact on student learning. Further research is needed to investigate the integration of peer-assessment with other forms of evaluative judgement, such as self-assessment, over a longer period of time in a mathematics learning context. The present study is a pilot of a much larger project in mathematics and another university-wide initiative to see the development of longer-term approaches to developing students' evaluative judgement in multiple subject areas.

## 5. References


Bidgood, P. & Cox, B., 2002. Student Assessment in MSOR. *MSOR Connections*, 2(4), pp.9-13.

Boud, D., Ajjawi, R., Dawson, P. & Tai, J., eds., 2018. *Developing evaluative judgement in higher education: Assessment for knowing and producing quality work.* Abingdon, Oxon: Routledge.

Brookhart, S.M., 2018. Appropriate criteria: Key to effective rubrics. *Frontiers in Education*, 3, 22. https://doi.org/10.3389/feduc.2018.00022

Dawson, P., 2017. Assessment rubrics: towards clearer and more replicable design, research and practice. *Assessment & Evaluation in Higher Education*, 42, pp.347-360. https://doi.org/10.1080/02602938.2015.1111294

Evans, C., 2013. Making sense of assessment feedback in Higher Education. *Review of Educational Research*, 83, pp.70-120. https://doi.org/10.3102/0034654312474350

Iannone, P. & Simpson, A., 2011. The summative assessment diet: How we assess in mathematics degrees. *Teaching Mathematics and its Applications*, 30, pp.186-196. https://doi.org/10.1093/teamat/hrr017

Jones, I. & Alcock, L., 2014. Peer-assessment without assessment criteria. *Studies in Higher Education*, 39, pp.1774-1787. https://doi.org/10.1080/03075079.2013.821974

Jones, I. & Sirl, D., 2017. Peer assessment of mathematical understanding using comparative judgement. *Nordic Studies in Mathematics Education*, 22, pp.147-164.

Jönsson, A. & Svingby, G., 2007. The use of scoring rubrics: reliability, validity and educational consequences. *Educational Research Review*, 22, pp.130-144. https://doi.org/10.1016/j.edurev.2007.05.002

Mertler, C.A., 2001. Designing scoring rubrics for your classroom. *Practical Assessment, Research and Evaluation*, 7, pp.1-10. Available at: https://pareonline.net/getvn.asp?v=7&n=25 [Accessed 4 September 2019].

Messick, S., 1994. The interplay of evidence and consequences in the validation of performance assessments. *Educational Researcher*, 23, pp.13-23. https://doi.org/10.3102/0013189X023002013

Newton, P.E., 1996. The reliability of marking of GCSE scripts: mathematics and English. *British Educational Research Journal*, 22, pp.405-420. https://doi.org/10.1080/0141192960220403

Panadero, E. & Broadbent, J., 2018. Developing evaluative judgement: a self-regulated learning perspective. In D. Boud, R. Ajjawi, P. Dawson & J.Tai, eds, *Developing evaluative judgement in higher education: Assessment for knowing and producing quality work*. Abingdon, Oxon: Routledge.

Panadero, E. & Jönsson, A., 2013. The use of scoring rubrics for formative assessment purposes revisited: A review. *Educational Research Review*, 9, pp.129-144. https://doi.org/10.1016/j.edurev.2013.01.002

Price, M., Rust, C., O'Donovan, B., Handley, K. & Bryant, R., 2012. *Assessment literacy: The foundation for improving student learning.* Oxford Centre, for Staff and Learning Development.

Reddy, Y.M. & Andrade, H., 2010. A review of rubric use in higher education. *Assessment and Evaluation in Higher Education*, 35, pp.435-488. https://doi.org/10.1080/02602930902862859

Sadler, D.R., 2009. Indeterminacy in the use of preset criteria for assessment and grading. Assessment & Evaluation in Higher Education, 34, pp.159-179. https://doi.org/10.1080/02602930801956059

Swan M. & Burkhardt H., 2012. Designing assessment of performance in mathematics. Educational designer, 2, pp.1-41. Available at: https://www.educationaldesigner.org/ed/volume2/issue5/article19/ [Accessed 4 September 2019].

Winstone, N.E., Nash, R.A., Parker, M. & Rowntree, J., 2017. Supporting learners' agentic engagement with feedback: A systematic review and a taxonomy of recipience processes. *Educational Psychologist*, 52, pp.17-37. https://doi.org/10.1080/00461520.2016.1207538

# 6. Appendices

*Appendix 1 – Homework questions*

**1** Write the system of linear, simultaneous equations

$$\begin{aligned} x + 2z &= 10 \\ 2x + 3y + z &= 5 \\ y + z &= 3 \end{aligned}$$

in matrix form. Use the Gauss-Jordan (and no other) method to find the inverse of the matrix and hence find the solution to the system.

**2** By performing suitable row and column operations show that

$$\begin{vmatrix} x & y & z \\ y & x & x \\ z & z & y \end{vmatrix} = (x-y)(y-z)\alpha(x,y,z)$$

where $\alpha(x,y,z)$ is a linear term in $x, y, z$ which you should determine.

**3** By performing suitable row and column operations show that

$$\begin{vmatrix} x+2 & 3 & 3 \\ 3 & x+4 & 5 \\ 3 & 5 & x+4 \end{vmatrix} = 0$$

has solutions $x = 0, 1, \beta$ where the value of the constant $\beta$ is to be calculated.

## *Appendix 2 – Questionnaire*

**BEFORE TAKING PART IN THE WORKSHOP TODAY**

Please circle your answers.

**1** Please rate how nervous you feel about the coursework

Very nervous 1 2 3 4 5 Not nervous at all

**2** Completing the coursework seems…

Very difficult 1 2 3 4 5 Very easy

**3** Please rate how confident you feel about assessing your own coursework

Not confident at all 1 2 3 4 5 Very confident

**4** Please rate how confident you feel about how to go about writing good solutions to problems

Not confident at all 1 2 3 4 5 Very confident

**5** In your own words, what makes a problem solution excellent?

*Please complete the rest of the survey at the end of the workshop*

**AFTER COMPLETING THE WORKSHOP**

**6** Please rate how nervous you feel about the coursework

Very nervous 1 2 3 4 5 Not nervous at all

**7** Completing the coursework seems…

Very difficult 1 2 3 4 5 Very easy

**8** Please rate how confident you feel about assessing your own coursework

Not confident at all 1 2 3 4 5 Very confident

**9** Please rate how confident you feel about how to go about writing good solutions to problems

Not confident at all 1 2 3 4 5 Very confident

**10** Have you had any specific ideas on how to improve your solutions to problems in the future?

YES NO

If YES, please describe these briefly:

## CASE STUDY

# A statistical learning exercise based on a modified Rock-Paper-Scissors game

Paola Bortot, University of Bologna, Bologna, Italy. Paola.Bortot@unibo.it.
Stuart Coles, Smartodds Limited, London, England. Stuart.Coles1111@gmail.com.

## Abstract

The standard version of the game Rock-Paper-Scissors is interesting in terms of game theory, but less so in terms of Statistics. However, we show that with a small rule change it can be made into an interactive exercise for degree-level students of Statistics that leads to a Bayesian change-point model, for which the Gibbs sampler provides an intuitive method of inference. First, students play the game to generate the data. Second, they are encouraged to formulate a model that reflects their experience from having played the game. And third, they participate in the development of a suitable MCMC algorithm to fit the model.

**Keywords:** Bayesian Statistics, change-point analysis, Gibbs sampler, teaching.

## 1. Introduction

The processes of data collection, model building and inference are the main themes of any statistical analysis. For teachers of Statistics, however, there are few opportunities to involve students in the whole operation. This is especially true for analyses requiring advanced statistical techniques — the physical and time constraints of standard teaching environments are simply not conducive to this aspect of statistical learning. The aim of this article is to suggest a teaching exercise which illustrates the entire sequence of a statistical analysis from data collection to inference, with model and inference developed from a knowledge of the data-generating mechanism that is also part of the exercise. We have run the exercise ourselves with degree-level students following a course in Computational Statistics, though it could work equally well as a practical lesson in a Bayesian inference course. In either case, some basic knowledge of Bayesian Statistics is required, as is an understanding of the Gibbs sampler and Markov chain Monte Carlo (MCMC) in general. Our objective is to provide an interactive platform through which students can see and exploit the links between Bayesian theory, model building and simulation-based inference.

The starting point for our developments is the well-known game of Rock-Paper-Scissors. Each of two players, labeled A and B respectively, simultaneously selects by hand caricature one of the three elements Rock, Paper and Scissors, which we abbreviate to $\{R, P, S\}$. If both players select the same element the game is a draw; otherwise, using obvious notation, $R > S$, $S > P$ and $P > R$. Several rounds are usually played, and the overall winner is the player with the most rounds won. The possible outcomes of the game are labeled A, X and B, corresponding to a win for A, a draw, and a win for B, respectively. Clearly, if each player chooses between $\{R, P, S\}$ at random, the A/X/B probabilities are identically 1/3. In general we write $\theta = (\pi_A, \pi_X, \pi_B)$ to denote the vector of A/X/B probabilities, and use $\theta_0$ to denote the special vector $(1/3,\ 1/3,\ 1/3)$.

From a game-theoretic point of view, Rock-Paper-Scissors is a simple zero-sum game whose Nash equilibrium solution corresponds to each player playing the elements of $\{R, P, S\}$, each with probability 1/3 (see van den Nouweland, 2007, for example). There have also been various statistical studies of Rock-Paper-Scissors. For example, Wang, Xu and Zhou (2014) compare player behaviour in laboratory conditions, with expected behaviour under optimal Nash Equilibrium rules. At a more general level, Walker and Walker (2004) provide a strategy handbook for players. They describe, for example, empirical evidence suggesting that players generally tend to choose $S$ on only around 30% of occasions. Knowing this, and assuming everything else equal, opponents have a slight advantage in selecting $P$ more often than random selection would imply. However, strategy issues like this are more interesting for their psychological or game-theoretical implications than for their statistical relevance.

## 2. A Modified Version of Rock-Paper-Scissors

Though the standard version of Rock-Paper-Scissors is of limited interest from a statistical point of view, a modification of the rules can lead to versions of the game that are more stimulating. Specifically, we suggest a novel variation which limits the play options for one of the players. In this way the other player has an advantage, but only once they correctly identify the limitations imposed on their opponent. The idea is then to challenge students to play this version of the game and develop a statistical model that will allow an observer to learn about the way the game has been played from the collected data.

Students are placed in groups of three, comprising two players and an observer. The role of the observer is to record the sequence of match outcomes, A/X/B, though not the actual play choices of either player. Once all rounds have been played, these are the data that form the basis of the analysis.

Before the game starts the two players are randomly given one of two instructions, I1 and I2. They are additionally told that one set of instructions corresponds to complete freedom in gameplay, but that the other imposes restrictions. In practice, we write the instructions on cards and ask each player to randomly select one without replacement. The instructions are:

I1: You are free to choose from $\{R, P, S\}$ in every round;

I2: You may only choose from $\{P, S\}$ in every round;

and without loss of generality we can assume these are assigned to players A and B respectively. Therefore, Player A is playing to standard rules, while Player B is prohibited from playing $R$. As such, player B is at a disadvantage, but only once player A has deduced the limitation imposed on player B. The results themselves are analysed from the point of view of the observer, who sees only the sequence of A/X/B match outcomes, and is unaware of which player has which instruction, and indeed what the limitation is in instruction I2.

Though players are not constrained to play randomly from the options available to them, our experience is that they tend to do so, at least approximately. Under random selection from the available sets, the A/X/B probabilities are still $\theta = \theta_0$. However, it is likely that Player A will eventually realise after a number of rounds that their opponent never plays $R$, and deduce that they have an improved strategy by selecting just from $\{R, S\}$. The logic is that since player B never plays $R$, it is wasteful for player A to ever play $P$. If both players then select randomly from their reduced sets, it is easy to check that $\theta = (1/2,\ 1/4,\ 1/4)$. In real play, where players may choose not to make random plays, $\theta$ may differ slightly from this theoretical value, just as the standard version may have probabilities that differ from $\theta_0$. In practice we have found that playing the game for 100 rounds provides a reasonable chance for player A to identify the limitations of player B and to change strategy accordingly.

Once the game is played, students are asked to develop a statistical model based only on the A/X/B data with the objectives:

1. To assess whether there has been a change of strategy during the 100 rounds, and if so to identify where it occurred;
2. To estimate $\theta = (\pi_A, \pi_X, \pi_B)$ for each round, accounting for the fact that there may have been a change of strategy for one of the players at some point.

## 3. Model Building

Getting students to play the game themselves serves two purposes. First, to obtain the data; second, to provide students with an experience of the data-generating process which, in turn, assists with appropriate model building. In practice what we have found is that some student pairs have made no change to strategy within the 100 rounds, and others have attempted several strategy changes. In most pairs, however, player A realises their advantage within the allocated 100 rounds and changes their play accordingly.

The discussion of these various playing strategies is an integral part of the exercise. To simplify the model development, we ask students to make three assumptions when model-building:

1. Player B maintains the same strategy at all rounds;
2. Player A also maintains a single strategy, except possibly at one point where they realise their potential advantage and change strategy for the remaining rounds;
3. In all rounds, both players make random choices from either all of, or a subset of, the options available to them.

These are reasonable assumptions from both a game-theoretic and statistical point of view, but they may be inconsistent with some players' actual strategy. This point itself can generate interesting discussion, but the bottom line is that simplifying assumptions of this type are necessary to construct a model which is both feasible and meets the stated objectives.

Step-by-step, students can be led to the natural model that these assumptions imply: a change-point model with at most one single unknown change-point corresponding to the round in which player A exploits their advantage and no longer plays $R$. In greater detail:

1. The game consists of $n$ rounds, each of which is a multinomial trial:
$$Y_i \mid \theta_i \sim \text{Multinomial}(1, \theta_i), \ \ i = 1, \dots, n,$$
where the levels of $Y_i$ are A/X/B with probabilities given by the vector $\theta_i$;
2. There is an unknown change-point $k$ such that for $i \le k$, $\theta_i = \theta^{(1)}$, while for $i > k$ , $\theta_i = \theta^{(2)}$;
3. There is the possibility that $k > n$, corresponding to the situation where no change of strategy occurs within the $n$ observed rounds;
4. In the early rounds the vector of A/X/B probabilities is likely to be close to $\theta_0$, regardless of the strategies assigned to the players;
5. There is likely to be a change in the pattern of A/X/B results as one of the players discovers their superior strategy;
6. A priori we have no information about $\theta^{(2)}$;
7. Any change in the pattern of results is likely to occur within a reasonable number of rounds, but unlikely to occur within the first few rounds.

In terms of inference there is a strong argument to be made for the use of a Bayesian rather than a classical model (Killick, 2011, for example). The arguments are two-fold, and are worth elaborating with the students. The first argument is technical: Bayesian methods are better suited than classical methods for change-point problems, since they naturally admit marginalising over the uncertainty in the change-point. The second argument is practical: we have different knowledge about the A/X/B probabilities both previous to and after any possible change-point, and this is much more naturally expressed via a Bayesian model. Previous to the change-point, for reasons discussed above, we anticipate $\theta_i$ to be close to $\theta_0$. On the other hand, exchangeability in the players implies that $\theta_0$ remains a reasonable 'best guess' for $\theta_i$ even after the change-point, though there is no reason to believe that the actual probabilities will be close to this value. Lack of information on $\theta^{(2)}$ implies that any element of the valid space for the A/X/B probabilities,

$$\Delta_2 = \{(z_1, z_2, z_3): z_1 \geq 0,\ z_2 \geq 0,\ z_3 \geq 0,\ z_1 + z_2 + z_3 = 1\},$$

is equally plausible for $\theta^{(2)}$ prior to observing the data.

The next point we try to emphasise to students is the interplay between model structure and inference. What we are aiming for is an understanding that while a fundamental aspect of Bayesian inference is the inclusion of prior knowledge through prior distribution specification, such knowledge is generally limited to summaries of centrality and variability. More precise details about the shape of the prior distribution can be selected on grounds of computational convenience, which usually implies exploiting conditional conjugacy. For the multinomial change-point model this means choosing Dirichlet prior distributions for $\theta^{(1)}$ and $\theta^{(2)}$, most conveniently parametrised as

$$\theta \sim \text{Dirichlet}(\phi, d),$$

where $\phi \in \Delta_2$ is the mean and $d > 0$ is a dispersion parameter. Full definitions and properties are given in the Appendix. For the change-point model we then set

$$\theta^{(1)} \sim \text{Dirichlet}(\theta_0, d_1),\ \ \theta^{(2)} \sim \text{Dirichlet}(\theta_0, d_2),$$

independently.

The extent to which students can come up with these choices themselves depends on whether they have studied conjugacy both in general, and specifically in the context of multinomial models. Nonetheless, the arguments in favour of these choices are easily understood:

1. The support of the Dirichlet distribution, $\Delta_2$, coincides with the domain of $\theta^{(1)}$ and $\theta^{(2)}$;
2. The parameter choices ensure that $\theta_0$ is the prior mean for both $\theta^{(1)}$ and $\theta^{(2)}$;
3. The scale parameters $d_1$and $d_2$ enable flexibility in the prior distributions for the $\theta$ parameters. Setting $d_2 = 3$ gives a uniform prior on $\Delta_2$ for $\theta^{(2)}$, but specifying a considerably larger value for $d_1$ leads to a greater concentration of the prior distribution of $\theta^{(1)}$ around $\theta_0$.

The remaining parameter is the change-point, $k$, whose theoretical domain is the entire set of positive integers, as the model assumes that a change will occur, even if this might happen after the allotted $n$ rounds. However, the Gibbs sampler is considerably simplified (see Section 4 below) if the prior distribution for $k$ is bounded above at some pre-specified value $k_{max}$, which might be chosen to be much greater than $n$. Apart from this restriction, any choice can be made that is consistent with the prior knowledge that Player A is unlikely to learn their optimal strategy in the first few rounds, but is also unlikely to need very many rounds to learn it. To account for these aspects, we suggest a truncated Negative Binomial model for the prior probability function of the change-point:

$$h(k) \propto g(k; m, v),\ \ k = 1, \ldots, k_{max},$$

for some value of $k_{max} \geq n$, where $g(.; m, v)$ is the probability function of the Negative Binomial distribution parametrised in terms of mean $m$ and variance $v$. This choice affords considerable flexibility in prior elicitation for the change-point through the specification of $m$, $v$ and $k_{max}$. However, it is necessary that $m \leq v$ to satisfy the validity requirements of the Negative Binomial distribution. Furthermore, to avoid a monotonically decreasing prior distribution with mode at 1, the additional constraint $v \leq m^2 + m$ should also be respected.

## 4. Model Inference

Assuming students have some background knowledge of Gibbs sampling, they can formulate, at least in outline, the following steps, all of which are implicitly conditional on the observed data:

1. Choose arbitrary initial values for $k$, $\theta^{(1)}$ and $\theta^{(2)}$. Then iterate over the following steps:
2. Given $k$, simulate from
$$\theta^{(1)} \sim \text{Dirichlet}(\theta_0 + c_1, d_1),$$
where $c_1 = (c_1^{(A)}, c_1^{(X)}, c_1^{(B)})$ is the vector counts of the outcomes A, X and B, respectively, among $y_1, \ldots, y_k$.
3. Similarly, again given $k$, simulate from
$$\theta^{(2)} \sim \text{Dirichlet}(\theta_0 + c_2, d_2),$$
where $c_2 = (c_2^{(A)}, c_2^{(X)}, c_2^{(B)})$ is the vector counts of the outcomes A, X and B, respectively, among $y_{k+1}, \ldots, y_n$, with the convention that $c_2 = (0,0,0)$ if this set is empty;
4. Since $k$ is discrete and bounded, given $\theta^{(1)}$ and $\theta^{(2)}$, simulate from the full conditional probability function which, up to proportionality, is given by
$$f(k) \propto h(k) \prod_{i=1}^{k} f(y_i \mid \theta^{(1)}) \prod_{i=k+1}^{k_{max}} f(y_i \mid \theta^{(2)}).$$

In this expression $h$ is the prior change-point probability function and $f(y_i \mid \theta^{(1)})$ and $f(y_i \mid \theta^{(2)})$ are, respectively, the multinomial probability functions before and after round $k$, subject to the convention that $f(y_i \mid \theta^{(2)}) = 1$ whenever $i > k_{max}$. In summary, the multinomial-Dirichlet conjugacy has been exploited to enable simple updates of the multinomial probability vectors given the current value of the change-point, while the change-point itself is updated via an enumeration of the full conditional probabilities, which is feasible because its support is discrete and bounded.

We assume that students are familiar with the Gibbs sampler and issues about mixing and convergence of MCMC series. The neat structure of the above model leads to a Gibbs sampler that behaves well in both these respects. For protocol we assume a small burn-in period, discarding the first few simulations of the simulated Markov chain, but even this is not strictly necessary.

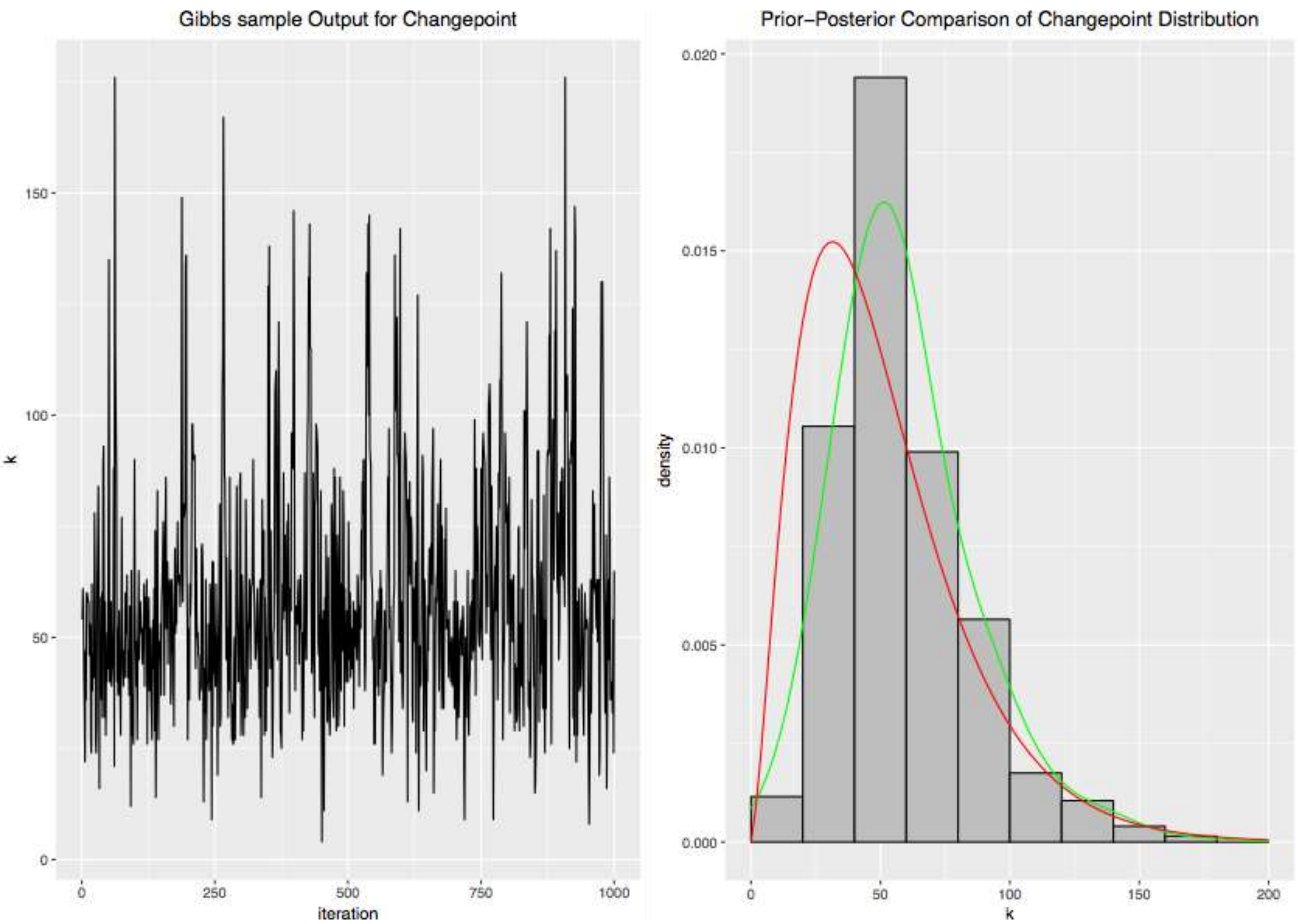



Figure 1. *Left*: Gibbs sample output of change-point parameter. *Right*: Comparison of prior (red) and posterior (green or histogram) distributions of change-point parameter.

## 5. Analysis of Data

Though students have generated their own data, it is easier for practical purposes to demonstrate the inference on simulated data. Using the statistical language R (R Core Team, 2017), functions for both simulating the data and fitting the model via the Gibbs sampler described above – either to genuine or simulated data – are available as supplementary material at the journal website. The only package required outside of the base R language is ggplot2 (Wickham, 2016), which we use to enable improved graphics.

As an illustration, to simulate $n = 100$ rounds with the default settings as described above, with Player A switching to the reduced set of plays $\{R, S\}$ after the 50th round, we write

```
> rps_data<-rps_tournament_changepoint(n_games=100, changepoint=50)
```

The output comprises the sequence of A/B/X results:

```
> head(rps_data$simulated_data)
```

[1] “B” “B” “X” “X” “X” “B”

The Gibbs sampler is then run on the simulated object as follows:

```
> rps_gs_out<-rps_gs(rps_data$simulated_data, d1=100, d2=3, m=50, v =1000)
```

For this example we have set $d_1 = 100$, giving a prior choice on $\theta^{(1)}$ that is strongly concentrated on $\theta_0$, and $d_2 = 3$, giving a uniform prior for $\theta^{(2)}$ on $\Delta_2$. The arguments for these choices have been discussed above. The prior for *k* has mean close to 50 but with a large variance. This choice of mean is slightly unfair, since in practice the true value would be unknown, but the consequences are mitigated by also having a large variance *v*. In any case, the Gibbs sampler is quick to run, so the sensitivity of results to these choices can be examined in real-time during class if that seems appropriate. Obviously, the simulated object can be replaced with actual data once collected to effect an analysis on the students' own data.

As with any Gibbs sampler, the output from these functions can be studied graphically to assess the performance of the sampler and to obtain summary inferences. For example, figure 1 shows the Gibbs sample output and a comparison between the prior and posterior distributions of the change-point after 100 rounds. The Gibbs sampler tracer provides visual evidence of the satisfactory mixing and convergence of the chain. The comparison of prior and posterior distributions of the change-point show the extent to which the data have transformed prior beliefs: having observed the data, the change-point is more likely to have occurred after 50 rounds than the prior assumed, and has a greater concentration than the prior. Nonetheless, the prior and posterior are reasonably similar, but this is hardly surprising given the limited amount of data with which the inference is being made. Note that both prior and posterior distributions are discrete having support on the positive integers, but the histogram and smoothed curves are shown on a continuous scale for ease of interpretation.

Similar graphical analysis can be made on the multinomial probabilities $\theta^{(1)}$ and $\theta^{(2)}$, but more interesting in practice is the posterior for $\theta_i$ in the original model specification

$$Y_i \mid \theta_i \sim \text{Multinomial}(1, \theta_i), \; i = 1, \dots, n.$$

In other words, having observed the data, what can be said about the A/B/X probabilities for each round?

Since the change-point is unknown, the appropriate choice for $\theta_i$ between $\theta^{(1)}$ and $\theta^{(2)}$ is also unknown, but this is exactly the sort of situation in which the Gibbs sampler can be fully exploited, as the output for $(\theta^{(1)}, \theta^{(2)}, k)$ can be transformed to give a Gibbs sample for $\theta_i$, through

$$\theta_i = \theta^{(1)} \text{ if } i \leq k$$

$$\theta_i = \theta^{(2)} \text{ if } i > k.$$

Applying this mapping to the original Gibbs sampler output generates a Gibbs sample of $\theta_i$. Figure 2 summarises the result of this procedure for $i = 1, \dots, 100$. The three panels represent respectively the three components of $\theta_i$, namely the A/X/B probabilities respectively. In each case, for each round $i$, the summary is a box plot of the Gibbs sample of the relevant component of $\theta_i$, and therefore a graphical approximation to its posterior distribution. The central black curve is therefore a trace of the median of the distribution as a function of $i$; the red region is the inter-quartile range; the black stems extend to somewhere around 1.5 times the quartiles; and the yellow points indicate outlining points. Note that the posterior distributions here are conditional on data observed from all rounds, not just those up to round $i$; that's to say, the distributions in figure 2 provide a smooth of the data, not a filter.

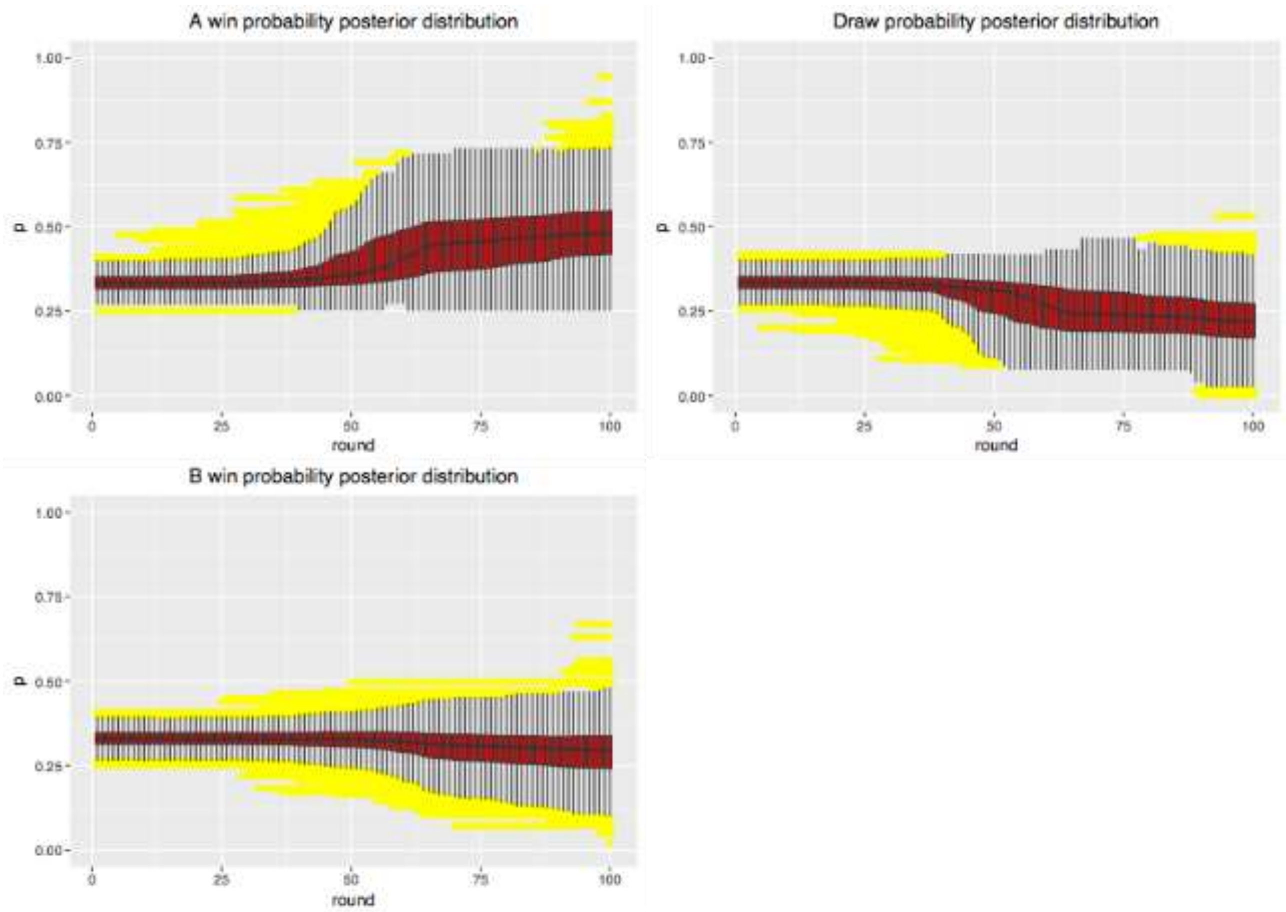



Figure 2. Gibbs sampler distributions of marginal posterior for game outcome probabilities as a function of round. Distributions are represented in standard boxplot form.

For the early rounds, the posterior mean for $\theta_i$ is approximately $\theta_0$, with a small posterior variance, in accordance with the strong information provided by the prior. After about 35 rounds, the possibility of a change-point starts to affect the posterior distributions on the $\theta_i$, to an increasing extent as more data become available. By the time we have the full set of data, the posterior mean has shifted much closer to the true value of $\theta = (1/2, 1/4, 1/4)$, albeit with larger posterior variances. Given what students have learnt about the process from playing the game, all aspects of the inference are entirely convincing.

## 6. Discussion

The exact way this exercise can be used will depend on the type and level of class in which it is introduced. The whole exercise, including data generation, model building and inference can usually be completed within two or three hours. Our own experience was with final-year undergraduate students, who had previously covered the basics of Bayesian Statistics, and were following a course on general computational methods, which included Bayesian techniques such as the Gibbs sampler. In that setting we used the exercise towards the end of the course as a way of reinforcing the links between inference and computation, emphasising the role of model construction for both aspects. In other teaching programmes which include separate modules based on case studies, this exercise could be used as one of the cases. The obvious limitation is that a knowledge of Bayesian inference and computational techniques is required to a level that is typically not studied until second-year undergraduate programmes.

The feedback we received from students, both informally and through student feedback questionnaires, was overwhelmingly positive. In our first attempt we were more vague about the instructions given to students, which led to inconsistencies for many pairs between the way they actually played the game and the single change-point model we had anticipated they would develop. By being more precise with the instructions, and also giving a stronger steer towards our intended model structure, we found the exercise to work much better, and students' satisfaction to be greater.

Possible tasks and extensions that can be suggested to students for further study include:

1. A study of the sensitivity of results to prior choices.
2. A more detailed analysis of the mixing and convergence properties of the Gibbs sampler.
3. A change of protocol so that both players are given the same instructions.
4. Running the Gibbs sampler on subsets of the data, using results from just the first $n^*$ rounds, for $n^* = 10,20,\dots,100$ . How does inference on $k$, $\theta^{(1)}$ and $\theta^{(2)}$ change as $n^*$ increases?

Finally, although application of a change-point model to the modified Rock-Paper-Scissors game is just an educational exercise, students can also be made aware that change-point models of the type developed here have many real-world applications, including the identification of irregularities in DNA sequences.

## 7. R code

A zipped version of the R Studio project is available alongside this article from the MSOR Connections journal website https://journals.gre.ac.uk/index.php/msor/. Unzipping the file and opening in Studio gives immediate access to the functions and a script we used to produce the figures.

## 8. Acknowledgements

We would like to thank Luigi Colombo and Paul Wikramaratna for giving us the initial idea of using the Rock-Paper-Scissors game as a teaching exercise and Philip Giles for pointing out the similarities of the Rock-Paper-Scissors change-point model with models used for DNA sequencing. We are also indebted to Professor Nicola Sartori and the students of the Statistica Computazionale 2 course in the Department of Statistics, University of Padova, for allowing us to carry out this teaching exercise. Finally, we are extremely grateful to the editor and two referees for insightful and helpful comments on a previous version of the paper.

## 9. Appendix: The Dirichlet distribution

The 3-dimensional Dirichlet distribution $D(\phi, d)$ where $\phi = (\phi_1, \phi_2, \phi_3) \in \Delta_2$ and $d > 0$ has probability density function

$$f(\theta) \propto \theta_1^{d\phi_1 - 1} \theta_2^{d\phi_2 - 1} \theta_3^{d\phi_3 - 1},$$

for $\theta \in \Delta_2$. Its expectation is $\phi$ and its variance decreases as $d$ increases, with a limiting variance of zero as $d \to \infty$. The choice $d = 3$ and $\phi = (1/3,1/3,1/3)$ results in a uniform distribution on $\Delta_2$. It is a convenient prior distribution for a random variable that comprises a probability vector both because it has the correct support and because it provides a conjugate family for the Multinomial distribution. Specifically, if $x$ has the Multinomial distribution

$$x \mid \theta \sim \text{Multinomial}(n, \theta),$$

where $x_1 + x_2 + x_3 = n$, and

$$\theta \sim D(\phi, d)$$

then

$$\theta \mid x \sim D(\phi + x/d, d).$$

## 10. References


Eckley, I.A., Fearnhead, P. and Killick, R., 2011. Analysis of changepoint models. In: D. Barber, A.T. Cemgil and S. Chiappa, eds. *Bayesian Time Series Models*, Cambridge: Cambridge University Press. pp.205-224.

R Core Team, 2017. *R: A language and environment for statistical computing*. Available at: https://www.r-project.org/ [Accessed 3 September 2019].

van den Nouweland, A., 2007. Rock-Paper-Scissors; A New and Elegant Proof. *Economics Bulletin*, 3(43), pp.1-6.

Walker, D. and Walker, G., 2004. *The Official Rock Paper Scissors Strategy Guide*. Touchstone.

Wang, Z., Xu, B. and Zhou, H.-J., 2014. Social cycling and conditional responses in the Rock-Paper-Scissors game. *Scientific Reports*, 4(5830). https://doi.org/10.1038/srep05830

Wickham, H., 2016. *ggplot2: Elegant Graphics for Data Analysis*. 2nd ed. New York: Springer-Verlag.