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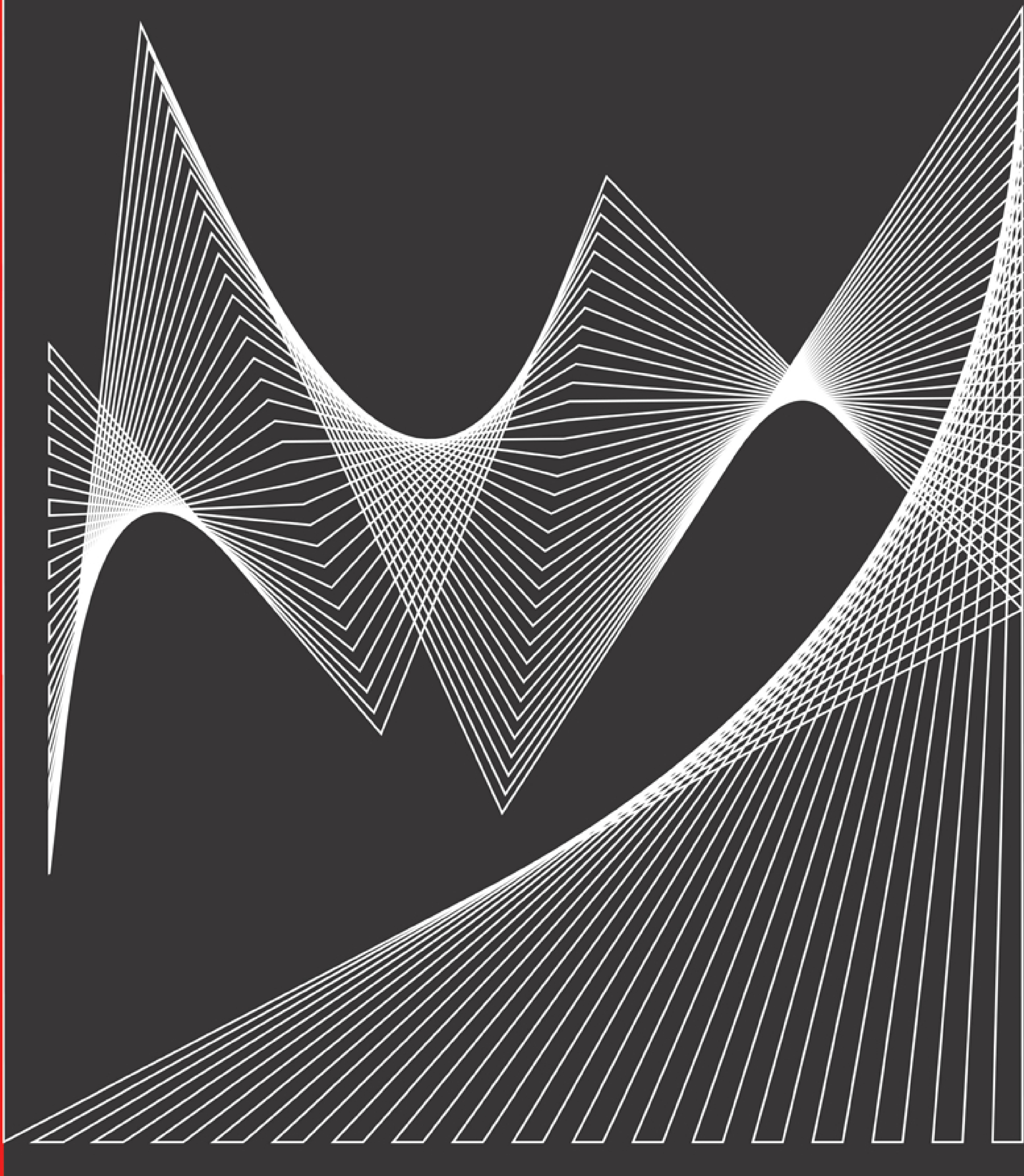

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EDITORIAL

Editorial

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It is only right that this latest issue of MSOR Connections starts with a sincere apology for the significant delay in its publication. I would like to take this opportunity to personally say sorry for any disruption and uncertainty caused, and I would like to thank the authors, reviewers, readers and my co-editors for their ongoing support and patience during this period.

This issue contains both national and international contributions on topics from mathematics support and both specialist and non-specialist mathematics learning and teaching. We begin with an article from Mac an Bhaird et al. who report on student awareness and perceptions of the Maths Support Centre website at Maynooth University. They also investigate how students interact with the website and discuss potential future developments. The next paper from Steckles et al. investigates how mathematics undergraduates engage with informal math-based activities prior to university study and how it might impact on subsequent decisions to study degree-level mathematics.

The next two case studies explore essential mathematical skills from different perspectives. The paper by Hodds reviews the impact of a programme at Coventry University to support the development of mathematical skills in nursing students, while the paper by Lishchynska et al. discusses an intervention to enhance the algebraic manipulation of higher education students (third-level) at Cork Institute of Technology.

The following case study is one of two papers in this issue originating from the US. The first paper by Spindler examines student perceptions toward statistical projects and how this relates to the outcome of such assignments. The penultimate paper by Andris et. al., also from the US, provides an example of how students were given the opportunity to engage with a “live” operational research project to optimize a local borough’s refuge collection route. Details of the approach taken and its outcome are presented.

The final article in the issue takes us back to Ireland. Mac an Bhaird et al. review the results of a survey aimed at determining how students utilise the online learning resources developed for the Maths Support Service at Maynooth University. Further discussion is also provided on the worth of providing such resources.

Finally, it is hoped that everyone is keeping safe and well during this difficult and unprecedented time as the impact of the COVID pandemic continues to reverberate across the globe.

RESEARCH ARTICLE

Student perspectives on their engagement with a mathematics support centre website

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Abstract

In this paper, we focus on survey results of students' use of a Mathematics Support Centre website. We discuss potential modifications to the advertisement of our online services based on student responses. We also consider the purpose of an MSC website and who should be responsible, in an institution, for the provision of topic specific online resources for students.

Keywords: Mathematics support, website, online, resources.

1. Introduction and background

The Mathematics Support Centre (MSC) was established by the Department of Mathematics and Statistics (the Department) in Maynooth University (MU) in 2007. Its remit, similar to that of most MSCs (Lawson et al., 2012), was to tackle retention issues, especially in first year service mathematics courses. The majority of MSC attendees take service mathematics and there is also a small class of pre-degree Certificate students. However, it is not just 'at-risk' first years who avail of MSC services. In MU (Mac an Bhaird et al., 2009) and elsewhere (Grove et al., 2019), students with stronger mathematical backgrounds also attend MSCs to improve their grades.

In 2007, appropriate advertisement was identified as crucial to raise awareness of our MSC amongst undergraduates. One approach advised by colleagues, both in person and through publications (Lawson et al., 2003; Mac an Bhaird and Lawson, 2012), was the establishment of an online presence. Initially, based on online reviews of MSCs in other institutions, we had a webpage with information on opening hours, venue, and a brief description of what students should expect in the MSC. As the MSC became firmly established, the webpage became a website in 2009, which contained similar information but presented in a more coherent fashion. The main MSC services at MU were (and still are) drop-in and weekly workshops. All workshops were held in the MSC and were optional. They focus on specific material requested by students from their modules; the material could not be due on a subsequent module assignment and was subject to lecturer approval. Each first year workshop had a dedicated page on MU's virtual learning environment (VLE) where workshop notes were placed along with links to related online materials. For other on-demand workshops for non-first years, notes were placed on a dedicated MSC webpage for one week only due to Department instructions. We also added a page for online resources, which similar to other MSC websites, linked to locally developed resources and online repositories, e.g. www.mathcentre.ac.uk. Over a subsequent four-year period, the website saw mostly ad hoc changes, including moving servers due to changing internal university policies. While the impact of institutional procedures and policies on MSC websites appears relatively common, the fact that we retained control of editing and content is less common (Mac an Bhaird et al., in preparation).

Anecdotal feedback during this period identified that students were rarely accessing resources on our website, as they did not see how they related to their modules. We decided to change both the layout and extent of online resources in summer 2013. A tutor gathered online materials relevant to the main 1st and 2nd year modules in the Department, single and multivariable Calculus, Linear Algebra, Data Analysis and Statistics. The selected material, usually in video or text format, was reviewed to ensure academic accuracy and placed on a topic webpage. In keeping with the MSC's ethos as a supplement to, not a replacement for, Departmental course material, there is a statement to that effect on each topic webpage. We did not focus on materials for 3rd and 4th year modules as they tend to be more advanced in nature, with content often changing depending on the lecturer. Our current website (http://supportcentre.maths.nuim.ie) has changed little since 2013, the only significant changes of note have been the addition of our Twitter and Facebook feeds to the main page.

Key to the success of MLS is that best practice is based on informed research. In MU, research priority was given to the potential impact of our drop-in service (27 hours per week) on the retention of students (Mac an Bhaird et al., 2009; Berry et al., 2015). This is a common area of MLS research (Matthews et al., 2013). We had never evaluated our website and in early 2018, the possibility arose that control of the structure and layout of the MSC website might go to staff outside the Department. A review of literature in MLS and related fields identified that, to the best of our knowledge, very little work had been carried out in this area. Indeed, recent reports recommended that steps be taken to address this, for example

"Further investigation be undertaken to explore how MLS providers can enhance the online resources and services available to students, and increase student awareness of and improve student accessibility to these ICT enabled Supports." (O'Sullivan et al., 2014, p.83)

We decided to investigate student use of our website and, to provide a broader context, we also conducted a survey of MLS practitioners in Ireland and the UK on the extent of their online MLS presence. The results of that survey are available in Mac an Bhaird et al. (in preparation). In this paper, we briefly describe our methodology and subsequently focus on the results of the student survey. We close with a discussion based on the results and consider potential next steps.

2. Methodology

In March 2018, the authors developed a survey hosted on Google forms. The focus, comprising of 14 survey items, was to try to establish whether MU students used the MSC website, why they did or did not engage, whether or not they used online resources on the MSC website and the reasons why. We added 13 extra questions on background and on student use of online resources in general. These are reported on in Mac an Bhaird et al. (2020a). The anonymous survey was trialled by MSC tutors. The structure was subsequently altered and some of the questions were reworded.

Ethical approval was received for the final survey and, in April 2018, the survey was placed on the VLE page for all Department students who also received an email inviting them to participate. MSC attendees were also made aware of the survey and invited to participate. There were 99 respondents in total, and the results were downloaded to SPSS (Version 24) and analysed. Any responses to open questions were coded separately by the authors, who subsequently met and finalised categories of response.

3. Results

All respondents (n=99) were asked if they had ever used the MSC website (Q1), with 68 selecting that they had, 15 that they had not, and 16 that they did not know the MSC had a website. The majority of those who answered no (11 of 15) and all who did not know about the MSC website were Certificate, 1st or 2nd year students.

3.1. Students who have not used or did not know about the MSC website (n=31)

Respondents were asked, in an open response question (Q2), why they had not used the website. The 16 students (from Q1) who did not know about the website repeated this fact. Twelve of the remaining 15 respondents gave relevant comments. The most common response (n=7) was that students did not need it, with some adding extra details such as 'have my notes', 'I usually just get help in the MSC', 'I prefer to get hands on assistance'. Four further comments referred to a lack of continuous visibility or advertisement of the website, e.g., 'It was only mentioned to us during our first week; I'd actually forgotten there was a site'. The remaining response raised broader orientation issues:

"As a mature student I rely on traditional study methods and am very backward when it comes to technology. I don't understand its potential. I just don't know or understand how it might be helpful to me. I'm very surprised that the university didn't alert people like me beforehand about the extent of the use of technology in student life today. If it had I would have tried to get into it a bit." (1st Year student)

In another open response question (Q3), respondents were asked what would encourage them to start using the website. Twenty (of 31) students gave relevant responses. Nine comments related to how they thought the website should be advertised, including advice on how to communicate the information to students 'A link/ advertisement, as well as promotion by MSC staff' and suggestions on information to include in the advertisement 'helpful notes on topics we are dealing with such as at the moment linear algebra'. Eight said 'if they had known about it', with five of these specifying that 'Now that I know it exists, I will use it'. The remaining three comments were 'more time', 'an online chat with a support', and 'not being able to make it into college'.

3.2 Students who have used the MSC website (n=68)

Respondents who have used the MSC website were asked to select what they used the MSC Website for (Q4), with fixed options 'To find out MSC opening hours' (63), 'To find out when a particular tutor is on' (44), 'To find out what topics each tutor is comfortable with' (31), 'To find MSC contact details' (6). There were 151 responses, including 7 open response answers all of which related to accessing resources, e.g. 'List of additional resources/websites on maths', 'notes from workshops'. Of the 7 students who gave an open response, only one had not selected any of the fixed options. This indicates that 67 of the 68 students were using the MSC website for information purposes. Third and 4th years were the most common groups to use the MSC website to find out when a particular tutor was on duty (34 of 44) and to find what topics a tutor is comfortable with (25 of 31).

Thirty (of 68) students indicated (Q5) that they used online resources on the MSC website, and 38 that they did not.

3.2.1 Students who have used the MSC website but not the MSC online resources (n=38)

Respondents who have used the MSC website were asked why they did not use online resources on the MSC website (Q6), with fixed options 'I did not think of them' (n=20); 'I did not know they were there' (n=14); 'I find it too difficult to find what I am looking for' (n=8); 'I do not think that the online resources that are there are helpful' (n=6). There were 48 responses, including three open responses answers 'I usually have a specific question and that's why I come to the msc. I prefer one on one' (2nd Science), 'No relevant information for modules (4th Science)', and 'I try to stick to the lecturer's notes' (3rd Arts).

These respondents were asked, in an open response question (Q7), what would encourage them to use the online resources. Thirty-four (of 38) responses were relevant to the evaluation of the MSC website. Seventeen comments referred to the resource content of the website. Eleven of these were general resource requests, e.g. ‘more videos’, and the other 6 were more topic specific. Four of these 6 respondents were final year, e.g., ‘If they contained lots of worked examples and notes on courses similar to ours.’ (4th Science), ‘Specific module content, almost like tutorials.’ (4th Science).

Ten of the 34 comments were related to advertisement, focussing on what methods we should use to communicate about the website to students and what detail to include, e.g. ‘More emphasis/encouragement from lecturers and tutors in the MSC.’ and ‘emails or reminders about topics being on them when covered in lectures’. Two further comments could also be considered as advertisement because they referred to the lack of resources or material that are in fact already on the MSC website, e.g. *“Maybe online videos like Khan Academy cause they are very handy to follow worked solutions cause just reading a solution you can get lost if it’s something you struggle with to begin with…”.* There were two comments on the interface (a more modern layout of the website), two on accessing resources if they needed help, and one final comment referred to the fact that the student would start using the resources now that they were aware of them due to the survey.

3.2.2 Students who have used the MSC website and the MSC online resources (n=30)

Respondents who have used the MSC website and MSC online resources were given fixed options on how often they used the online resources (Q8), ‘at least once a day’ (1), at least once a week (7), ‘at least once a month’ (14), and ‘at least once a semester’ (8). Asked (Q9) about the ease of finding suitable resources, the fixed options selected were ‘very easy’ (4), ‘easy’ (16), ‘neither easy or nor difficult’ (9), and ‘difficult’ (1). When asked to rate these online resources (Q10), 11 and 16 respondents selected very good and good respectively. The remaining three selected neutral. Asked how likely they were to use the resources and links on the MSC website instead of those from another source or website (Q11), respondents selected ‘More Likely’ (19), ‘Neither likely nor unlikely’ (9), ‘Less Likely’ (2). Question 12 was optional and asked for lists of topics or resources respondents would like added to the website, there were three responses. One listed a third-year module which doesn’t currently have online resources; the other two mentioned resources that they found had worked for them, e.g. quizzes and videos *“I used a link once which was a quiz. When you got an answer wrong it showed you how to do it correctly, more of them maybe?”*

Finally, in this section of the survey, there was an open question (Q13) asking for suggestions on how we might improve the MSC website, 27 (of 30) responses were relevant. Nine comments related to the structure or user interface of the website. Six of these nine were suggestions on modernising the website, e.g. ‘Update it. More modern and user friendly’. Three of the nine, all from 3rd Science and two of whom used the website at least once a week, were happy with the current design, e.g. ‘It’s very easy to navigate, I'm happy with it’.

Another nine (of 27) relevant comments related specifically to the content of the MSC website, these were from 3rd years and one 4th year student. Six were general suggestions for more content and more topic specific content, the other three comments specifically requested that MSC workshop notes be left on the website for longer, e.g. *“Workshop notes should be left online and not removed following the workshops.”*

Seven respondents stated that they had no suggestions, and five of these had indicated in Q9 that it was easy or very easy to find resources on the website. The remaining two comments, both from 1st year Science students, stated that they were happy with the website. It was not clear if they were referring to the structure or content, or both.

3.3. Communications about the MSC website (n=99)

All survey respondents (n=99) were given (Q14) fixed options on the best way for the MSC to communicate about its website and online resources. They selected: Email (65); Message on the module VLE class forum (58); Announcement in lectures (62); Facebook and Twitter (27). No students selected ‘I do not want to receive these communications’. The 214 responses included two who selected the ‘other’ open option. One respondent, who had also selected the email and VLE forum options, commented ‘Email with a link, because students are actually that lazy’. The second student, who had also selected all the fixed options, added ‘Maybe posters in the MSC walls and door as you walk in, leaflets with specifics about MSC’.

4. Discussion, conclusion and future work

This was a short local survey of limited scope, with a small number of respondents, all of whom used the MSC drop-in. However, it does provide an interesting ‘first look’ at students’ engagement with an MSC website. We are now reconsidering our advertisement of the website, particularly with early year students. To this end, we are automatically including a website link in all communications. Tutors have been instructed to remind students of the website and how to use it, and the website is open and displayed on our screens during drop-in sessions. All but one respondent indicated that they used the website to find practical MSC information, which was its original purpose, and the majority of early year users were happy with the resources provided. A small number of website users and non-users stated a preference for face-to-face support, which is consistent with findings in our staff survey (Mac an Bhaird et al., in preparation). Indeed, when we first expanded the provision of topic specific resources for 1st and 2nd years, it had been in an effort to provide extra reliable online support for ‘at-risk’ students who were not getting as much one-to-one attention in our drop-in due to overcrowding.

Some responses referred to the lack of institutional information and training received on the use of technology, and the lack of guidance on how to use the website. Online web searches for topic specific questions can lead to unmoderated sites where information is often wrong or misleading (Kiili et al., 2008). Analysis of our extra questions, to be reported on elsewhere, indicate that 23 of 59 respondents always think about the reliability of online resources before they use them, 33 sometimes and three never. Goldman et al. (2012) found that when sourcing information on the internet, ‘better students’ were more readily able to identify reliable sites than ‘poorer’ students. Therefore, it could be argued that MSCs have an important role to play in the provision of appropriate online resources, especially as Henderson et al. (2015, p.1577) state that, *“…students’ uses of digital technologies (and perceptions of ‘what works’ best) are clearly being shaped by the university contexts within which students are situated…”*

However, some of the early year student responses indicated that they wanted a more precise breakdown of resources to match the weekly demands of their courses, and students of more advanced modules identified the lack of resources for their modules as an issue. These responses have caused us to reassess the MSC’s role in providing this level of online support for students, in addition to reviewing where the divide lies between lecturer and MSC responsibility. We recommend that practitioners considering the expansion of resource provision on their MSC website contemplate this issue.

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RESEARCH ARTICLE

Pre-university informal engagement with mathematical activities and the decision to study mathematics at university

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Abstract

A survey was created to investigate the experiences of mathematics undergraduates with informal mathematical activity prior to starting university, and links these with the decision to study mathematics. A questionnaire was completed by a small sample of first-year undergraduates at two UK universities. Generally, incoming undergraduates are shown to have a high level of enjoyment of mathematics and engagement with informal mathematical activity. Popular activities included mathematical puzzles and games, and online videos about maths. Students were often engaged with family or via social media, playing computer, tablet or phone games, watching TV game shows with mathematical aspects and participating in organised competitions. Only around half engaged via talks or workshops organised through school and watching more structured documentaries or videos of lectures. Few participated in organised clubs. It seems there was greater engagement with 'fun' aspects of mathematics than with activities which demonstrate mathematics linked to career choice. The link to goals of outreach and similar initiatives is discussed, with further research indicated.

Keywords: outreach, university, engagement, informal learning.

1. Introduction

The study of effective outreach practice in STEM is relatively new and developing, but is still under-studied in mathematics in particular. We sought to understand informal engagement with mathematical activity experienced by mathematics undergraduates prior to arriving at university, and to examine any links with the decision to study mathematics at university. A significant feature of this research is that it draws on the experience of a successful freelance maths communicator with many years' experience (KS) to establish potential experiences and activities, and is completed in collaboration with authors who have experience of outreach and public engagement projects (PR) and a formal outreach role in a university mathematics department (AU). A survey was designed, with the aim to gather data on students' recollections of informal mathematical activities and the decision to study mathematics at university.

Informal mathematical activity in universities often takes the form of public engagement and outreach. Public engagement is *"the myriad of ways in which the activity and benefits of higher education and research can be shared with the public"* and is *"by definition a two-way process, involving interaction and listening, with the goal of generating mutual benefit"* (NCCPE, 2018). University outreach activities are undertaken in order to promote public awareness and understanding of subjects and make informal contributions to education (Varner, 2014), usually with a secondary goal of creating and maintaining relationships with schools and colleges and encouraging young people to continue to higher education in the subject (University of Sheffield, 2019). A related concept is widening participation, which directs outreach and marketing activities to particular under-represented demographics. In addition to university activity, there is much amateur and freelance maths communication. In this article, we consider any activity outside of school and formal education which encourages participants to engage with mathematical ideas and techniques. These activities may cross over one or more of the above categories, and from the point of view of the recipient may be indistinguishable.

The remainder of the article is structured as follows: a discussion of the types of informal mathematics activity, based on our experience, is followed by a discussion of background literature. Details of method are followed by results and discussion.

2. Types of informal mathematics activity

Separate from formal school curricula, we consider informal engagement with mathematics through a huge variety of activities, ranging from organised extra-curricular activities participated in at school through to measurements and calculations performed as part of activities around the home. Based on our experience, we listed relevant activities. This was particularly informed by the professional practice of KS via a review of freelance outreach/public engagement projects and other activity she has been involved with. This list of activities was then sorted into broad categories: organised; independent; online/digital; and, other media.

Organised activities are tangential to formal schooling. This might include visits to a school from STEM professionals for talks and lectures, organised workshops such as Ri Masterclasses, maths/STEM clubs and school trips to STEM institutions.

Independent activities might include talks and lectures visited with family or other non-school organisations, mathematical or logic-based toys or board games, reading maths books in their own time, or mathematical activities at home, such as measuring/calculating in order to perform DIY or cooking tasks. We argue that these are distinct from ‘organised activities’ because either they take place informally in the home, or if they are arranged, they are not for a specific group of students.

The category of online/digital activities includes online courses, YouTube videos, mathematical apps/games, and maths/logic puzzles shared on social media. This is distinct from ‘independent activities’, as they take place online and are largely self-directed by the student.

The final category concerns ‘other media’, and includes mathematical television documentaries or lectures, radio shows or segments, and TV game shows with a mathematical basis. We have this as separate because mass media is fundamentally different to social media and other digital activities.

The categories are not totally distinct. It might not be very different, say, for a parent to share with their child a mathematical board game or a mathematical app, or to watch a video on YouTube or one on iPlayer, but we have these as separate activities. For this reason, the categories are mainly a useful division to break up the survey questions and analysis.

3. Background

Our list of reasons students give for studying mathematics at university is drawn from a study by Robinson, Thomlinson and Challis (2010). They surveyed 223 students on arrival at mathematics departments in *“a handful of diverse institutions”* (p. 8), and do not claim the sample as representative. They found that around three quarters of students chose maths as a degree subject due to their interest in, enjoyment of or ability in mathematics, with around a third saying their choice was due to maths having a good reputation as a subject with high earning potential that could lead to good jobs. Of those who had an interest, enjoyment or ability, they note that these categories range from *“being the ‘only subject I was good at’ through to a student talking about their ‘passion’ for maths”* (p. 11).

Robinson, Thomlinson and Challis do not report asking students about their experience of pre-university extracurricular engagement with maths. The Smith Inquiry ‘Making Mathematics Count’ reports that outside of the school and college system, *“the UK has a tradition of independent small-scale voluntary initiatives to support particular aspects of the teaching and learning of mathematics”* and recognised the need to evaluate these (Smith, 2004).

Archer et al. (2013) highlight *“participation in science-related activities outside of school”* and *“parental attitudes to science”* (p. 17) as some important factors relating to aspirations in science among 10-14 year olds. Cooper (2011) warns of the impact of formal school settings on *“our perspectives of mathematics”*, with *“most students”* experiencing *“isolated instruction that prepares them to perform well on standardized tests”*, but not opportunity to apply mathematics in real situations or integrate this in other areas such as the other STEM disciplines (p. 48). She explains (p. 51)

“It is important for young learners to realize that mathematics is more than counting and number facts, or recognition of geometric shapes, and the application of mathematical procedures. They also need to see the mathematics that makes up the world they live in, such as the growth patterns in nature, the steepness of a slide, why something is pleasing to the eye (symmetry, proportion), and other important connections that informal learning environments might allow learners to explore.”

Stirling et al. (2009) suggest that outreach work to show that *“mathematics is more interesting or applicable”* or that *“it leads to more appealing and varied careers”* can help encourage more school students to continue with the subject (p. 5). Johnson and Mulligan (2016) recommend outreach can address *“a common misconception that there are limited career opportunities within the field of mathematics”* by relating learning to *“real life situations and potential career paths”*, with particularly high potential impact for girls (p. 31). They also recommend designing engaging activities using puzzles, games and challenges.

In terms of research into outreach and informal learning, there is relatively little available. UPMAP (Hofkins, 2017), took place in 2008-2011 and surveyed 23,000 UK school students at different ages, asking them about their intentions to study maths post-16 and their current experience. The questionnaire included a section on engagement in enrichment activities - including maths clubs, masterclasses and competitions. A number of research papers have resulted from this study, but most have a main focus on other factors and have not analysed the contribution of such activities beyond statistical correlation.

Stirling et al. (2009) offer sample feedback sheets for asking about the effectiveness of a maths outreach activity at the point of delivery. This type of evaluation is common, but cannot effectively determine whether the activity has resulted in an increased likelihood of further education as it rarely includes any long-term follow up of participants (Rodd, Reiss and Mujtaba, 2011). Cooper offers some insights, but suggests the need for further research into how mathematical thinking can be enhanced through informal learning experiences. Denson, et al. (2015) suggest there is value in informal learning environments, making a link to learning *“outside the constraints of standards-based testing and state-wide curriculums”* (p. 14), and again suggest more research is needed.

We did not find a study of the range of informal experiences with mathematics among students who eventually chose to study mathematics at university, which our project attempted.

4. Method

The survey was conducted as a questionnaire, emailed to first year mathematics students at Sheffield Hallam University and Sheffield University, and is available in the appendix.

It included four main questions about types of informal experience of mathematics prior to attending university. These questions were written based around the discussion given in section 2 above, with one question for each of the four categories of activity giving checkboxes to indicate which activities the respondents recalled participating in, with the options ‘Yes’, ‘No’, ‘I think so’ and ‘I don’t know’. Each question also contained a free text field to allow respondents to expand on or clarify their answer, including the prompt *“For example, what do you particularly remember about the activities you took part in? What did you most enjoy?”* A fifth free-text question asks for activities not already covered.

A final question explored the reasons for studying mathematics at university, giving students a list of reasons based on an analysis of free-text responses given by Robinson, Thomlinson and Challis (2010, p. 11) and discussed earlier in this article.

There is much more that could have been asked, but the decision was made to balance thoroughness against the length of the survey. The idea was to obtain coverage of the topic for the surveyed audience, in hopes of highlighting particular areas that might form the basis of further investigations.

Ethics approval for the questionnaire in the appendix was granted by the Sheffield Hallam University research ethics committee in March 2019. The survey was promoted to first year undergraduate students in class and by email over two weeks in March and April 2019 and data collected over a five week period.

5. Results

The questionnaire was completed by 33 students, 23 from Sheffield Hallam University and 10 from the University of Sheffield, all of whom consented to participate.

5.1. Organised activities

Responses to question 1a about organised activities are given in table 1. The most popular response was to competitions and challenges, with 70% of respondents saying ‘Yes’ or ‘I think so’. Otherwise, around half of participants reported engaging in a school trip to a maths lecture or workshop, and around half reported a talk or activity by an external person at their school. Around a third of respondents reported attending a careers event that involved meeting mathematicians, scientists or engineers, and around a third reported a school trip to a relevant museum, centre or exhibit.

One free-text response reported attending a *“‘getting girls into STEM’ style day”* and said *“I found the talks from female mathematicians motivating to go into maths”*, and also taking part in UKMT challenges and that these *“built on my confidence in my maths ability at different ages”*. Another commented on UKMT challenges, saying *“I loved all the UKMT stuff, because it was maths that made you think, rather than memorise.”*

Table 1. Responses to ‘Please indicate which of the following organised activities you took part in before starting university’.

Activity	Yes	No	I think so	I don't know
Someone visited my school and gave a maths talk or ran an activity.	13 (39%)	16 (48%)	3 (9%)	1 (3%)
School trip to maths lecture or workshop.	16 (48%)	15 (45%)	2 (6%)	0

School trip to science/engineering/maths museum, STEM centre or exhibit.	9 (27%)	23 (70%)	1 (3%)	0
School trip to industrial site/factory/business to learn about how they apply maths/science.	4 (12%)	29 (88%)	0	0
I attended a careers event involving meeting mathematicians/scientists/engineers (in or out of school).	12 (36%)	21 (64%)	0	0
I participated in Maths Club/STEM Club activities.	6 (18%)	26 (79%)	1 (3%)	0
I participated in STEM activities through Scouts/Guides or similar.	2 (6%)	29 (88%)	2 (6%)	0
I attended CodeClub/Coderdojo/Hackday events or any other events to learn coding.	1 (3%)	32 (97%)	0	0
I participated in STEP prep sessions.	3 (9%)	30 (91%)	0	0
I visited Sheffield Hallam for the PopMaths quiz.	2 (6%)	31 (94%)	0	0
I participated in another maths competition/Olympiad/quiz/UKMT challenge - through school or individually.	21 (64%)	10 (30%)	2 (6%)	0

5.2. Independent activities

Responses to question 2a about independent activities are given in table 2. The most popular responses are puzzles (85%), board games (76%), sharing mathematical puzzles/ideas with friends and family (67%) and toys (58%). 42% had visited a relevant museum, centre or exhibit not with school, and around a third had attended a maths lecture or workshop not with school, participated in an escape room or read popular maths books.

One free-text response referred to the Rubik's cube and speed solving. Three referred to parents and grandparents, including one who enjoyed *"testing my parents"*. One had read biographies of mathematicians and another had read a mathematics study guide.

Table 2. Responses to ‘Please indicate which of the following other activities you took part in before starting university’.

Activity	Yes	No	I think so	I don't know
Attended maths lecture or workshop (not with school).	11 (33%)	21 (64%)	0	1 (3%)
Visited science/engineering/maths museum, STEM centre or exhibit (not with school).	14 (42%)	18 (55%)	0	1 (3%)
I participated in an Escape Room.	12 (36%)	21 (64%)	0	0
I attended a science/maths themed birthday party.	1 (3%)	32 (97%)	0	0
I read popular maths books (covering maths topics, biographies of mathematicians/scientists).	11 (34%)	21 (66%)	0	0
I solved Sudoku and other logic or maths puzzles.	28 (85%)	5 (15%)	0	0
I played maths/logic based board games.	24 (73%)	8 (24%)	1 (3%)	0
I played with maths/logic based toys.	18 (55%)	14 (42%)	1 (3%)	0
I did practical building/measuring/crafting/carpentry/ engineering tasks around the house/with parents.	16 (48%)	17 (52%)	0	0
I shared mathematical puzzles/ideas with friends/family.	21 (64%)	11 (33%)	1 (3%)	0

5.3. *Online/digital activities*

Responses to question 3a about online/digital activities are given in table 3. Here, participation was generally high, with around 70-75% reporting having watched online videos about maths, accessed mathematical content via social media and played maths/logic based computer games or phone/tablet apps. Around half had watched online lectures/talks about mathematics but fewer than a quarter had studied online courses.

Table 3. Responses to ‘Please indicate which of the following online/digital activities you engaged with before starting university’.

Activity	Yes	No	I think so	I don't know
I watched online videos about maths (Numberphile, Standupmaths, etc.).	18 (56%)	8 (25%)	6 (19%)	0
I watched online lectures/talks about maths (e.g. on YouTube, Ri Channel, a university website, etc.).	15 (45%)	17 (52%)	1 (3%)	0
I studied online courses such as Khan Academy or a MOOC.	6 (19%)	24 (77%)	1 (3%)	0
Mathematical puzzles/ideas shared on social media.	21 (66%)	10 (31%)	1 (3%)	0
I played maths/logic based computer games or phone/tablet apps.	23 (70%)	10 (30%)	0	0

Free-text responses do not add significantly to the details here, simply naming specific games and YouTube channels/presenters.

5.4. Other media

Responses to question 4a about other media activities are given in table 4. Here, around 70% reported having watched TV game shows or quizzes with a mathematical aspect, and around half had watched TV documentaries about mathematics, with lower proportions for the other options.

Table 4. Responses to ‘Please indicate which of the following other media activities you engaged with before starting university’.

Activity	**Yes**	**No**	**I think so**	**I don't know**
I watched TV documentaries about maths.	14 (42%)	17 (52%)	2 (6%)	0
I watched TV game shows/quiz shows with a mathematical aspect (Dara O Briain’s School of Hard Sums, Countdown, Golden Balls etc.).	22 (67%)	10 (30%)	1 (3%)	0
I watched Royal Institution Christmas Lectures on TV (or attended a live recording).	6 (18%)	27 (82%)	0	0
I heard maths puzzles on the radio (e.g. Radio 4 Puzzle For Today, or otherwise).	6 (19%)	25 (78%)	1 (3%)	0

One free-text response clarifies that they watched quiz shows rather than game shows.

5.5. Other activities

A free-text question 5 asked for other activities not covered in the questions. Two affirmative responses were received. One referred to UKMT challenges, which *was* included as an option in an earlier question to which this student had answered ‘Yes’. The other said *“Tried to do a step exam”*. This respondent had answered ‘No’ to participation in STEP prep sessions, which are different but related.

5.6. Decision to study mathematics at university

Responses to question 6a about the decision to study mathematics at university are given in table 5. Considering a response of ‘5’, ‘6’ or ‘7’ as indicating that this reason was important, the most important reasons were wanting to learn more maths (97%), enjoying maths (94%), being good at maths (84%), finding maths a challenge (72%) and career prospects (72%).

There were two free-text responses giving other reasons. One said *“To improve my ability to communicate maths to others”*, and the other said *“To put off getting a proper job for a while! I just want to mess around with maths for a career, if that's possible.”*

Table 5. Responses to ‘Please indicate how important the following were in your decision to study mathematics at university’.

Reason	1 - least important	2	3	4	5	6	7 - most important
I enjoy maths/maths is my favourite subject	1 (3%)	1 (3%)	0	0	7 (22%)	6 (19%)	17 (53%)
I wanted to learn more maths	0	0	1 (3%)	0	9 (28%)	12 (38%)	10 (31%)
Maths is a challenge	0	3 (9%)	2 (6%)	4 (13%)	8 (25%)	11 (34%)	4 (13%)
I am good at maths/best at maths compared with other subjects	1 (3%)	1 (3%)	1 (3%)	2 (6%)	6 (19%)	9 (28%)	12 (38%)
Good job prospects/Wide range of jobs open to maths graduates	2 (6%)	1 (3%)	3 (9%)	3 (9%)	5 (16%)	9 (28%)	9 (28%)
Maths is a prestigious/valued degree	2 (6%)	2 (6%)	2 (6%)	7 (23%)	5 (16%)	11 (35%)	2 (6%)
Maths fits into my specific career plans	6 (20%)	1 (3%)	4 (13%)	8 (27%)	3 (10%)	6 (20%)	2 (7%)
Earning potential	5 (16%)	2 (6%)	5 (16%)	5 (16%)	6 (19%)	6 (19%)	2 (6%)

5.7. Links between experiences and the reason to study

The small sample size precludes meaningful statistical analysis, and also many students rated several options as ‘7 - most important’, meaning we don’t have distinct groups with different motivations. However, it may be interesting to explore two cases in greater detail, chosen for apparently different motivations. While not necessarily representative of particular groups within the cohort, these two cases illustrate combinations of attitudes we saw in the responses which may be of interest – many students were positive about their attitude to mathematics, but some were less likely to be motivated by its impact on their career prospects, like Student A. We are planning a further study which will investigate the connections between these attitudes more closely.

Student A

Student A is a first year student at Sheffield Hallam University. In terms of reasons for studying mathematics, they rated ‘I enjoy maths/maths is my favourite subject’ and ‘I wanted to learn more maths’ as ‘7 - most important’, and all other reasons less important. They rated ‘Good job prospects/Wide range of jobs open to maths graduates’, ‘Maths fits into my specific career plans’ and ‘Earning potential’ as ‘1 - least important’. It seems reasonable, therefore, to classify student A as motivated by enjoyment of the subject and not at all by future career prospects.

In terms of organised activities, student A engaged with none of the activities in question 1a, nor did they engage with informally-organised talks or visits in question 2a. It is clear, then, that this student’s enjoyment of the subject did not arise from organised extracurricular activities. They answered ‘Yes’, however, to reading popular maths books and playing with mathematical games, puzzles and toys, and to engaging mathematically with family members. In question 2b, this student reported teaching themselves to solve the Rubik’s cube, playing Sudoku, Chess, Othello and other games with their dad and grandad, and doing woodworking projects with their dad. This student also reported engaging with all the online activities in question 3, as well as some engagement with TV documentaries and the Ri Christmas Lectures. In question 3b, they report watching YouTube videos by Numberphile, Matt Parker, Grant Sanderson, Sal Khan, Michael Stevens and others, and playing the Euclidea mobile game.

Generally, then, the picture presented by student A is of someone inspired to an enjoyment of the discipline on the basis of high engagement with individual and informal family activities, rather than organised outreach and public engagement events.

Student B

Student B is a first year student at Sheffield Hallam University. They rated ‘Good job prospects/Wide range of jobs open to maths graduates’ as their sole ‘7 - most important’ response to question 6a. They answered ‘5’ to the reason ‘I enjoy maths/maths is my favourite subject’ - still a positive response, but note that 72% of respondents gave a more positive response to this question. (In fact, only two students gave outright negative answers to the enjoyment question and neither appears to be an illustrative case for reasons omitted for space.) It may be fair, then, to characterise student B as more motivated by career goals.

Student B was highly-engaged in organised activities, reporting experience of mathematical events at school, trips to events and locations with school and outside of school, involvement in STEM careers events and involvement in STEM clubs outside of school. They also report involvement with mathematical puzzles, games and toys, and engaging in all online activities in question 3a.

6. Discussion

There is little research on engagement in mathematics by school-age students outside of the formal curriculum. We are interested in engagement in such activities by students who eventually chose to study mathematics at university, and the links between engagement in informal activity and motivations to study mathematics further. A survey of a small group of students at two UK universities gives some insight into such links.

Students have many different reasons for choosing to study mathematics at university. Certainly many are motivated by an interest in or enjoyment of mathematics, while the motivation offered by career prospects varies in our sample. Advice for universities on offering informal engagement activities varies, but often includes a link to applications and career prospects (e.g. Stirling et al., 2009). It is also suggested that there is a role for informal engagement in exploring the true nature of mathematics (beyond standardised testing) and developing mathematical thinking (Cooper, 2011).

The most popular activities among responses were individual ones, playing with mathematical puzzles and games, and watching online videos about maths, all with at least three-quarters participation. Around two-thirds engaged in sharing mathematical ideas with family or via social media, playing online games, watching TV game shows with mathematical aspects and participating in organised competitions. Around half engaged via talks or workshops arranged through school, and watching documentaries and videos of lectures online. Few participated in organised clubs.

It is likely that some of these results reflect societal trends, for example for young people to watch online videos, or around the level of engagement with regular out-of-school clubs. Practicalities also play a role, with free, individual activities more frequently engaged with than paid-for organised events. That said, it is perhaps surprising to note that the most frequently engaged with activities are those which may demonstrate the fun in mathematics (puzzles, games, short videos on YouTube), with lower engagement for activities that may demonstrate mathematics as a career choice.

There may be a link between activities engaged with and motivation to study, though more work is needed to investigate how widespread these results are and to explore causal relationships. It seems reasonable to highlight that student B has a more career-focused interest in mathematics and that their informal experience of mathematics was more organised and less personal or family-oriented than student A. It may be that playing with mathematical games, puzzles and toys with enthused family and watching YouTube videos about mathematics supports an enjoyment of mathematics, while organised activities have greater focus on applicability and career links.

Further research to explore the links between the goals of different outreach/public engagement methods and student motivations for study would be potentially useful. Activities have myriad goals, including informal learning, furthering engagement in mathematics or broadening public awareness of mathematics. For example, if a project aims to encourage uptake of university mathematics, it would be useful to know whether a focus on applications and career prospects is a reliable approach, or whether fun mathematics is more engaging to the target audience. This is a subtle question; it may be that those who are engaged by fun mathematics are already predisposed to study mathematics further, while those who are uncertain about further study may be more likely to be motivated by applications or career links.

A key limitation of this study is the small, self-selecting sample. It would have been good to ask students to rank their reasons for studying mathematics or at least give a single 'most important' reason, because a lot of students were generally positive across many different reasons. We did not examine the level of engagement in these activities, in order to keep survey completion manageable - for example someone who had engaged in an activity once would give the same response as someone who did so regularly. Given the link to gender revealed by Johnson and Mulligan (2016), perhaps it would have been fruitful to ask for this information.

On a more fundamental level, there are goals of outreach and public engagement that cannot be examined via this sample population - for example, an activity may aim to engage those who aren't going to study mathematics at university in taking maths further via A-level or non-mathematics STEM university study, or to engage the general public to see the value of mathematics. Students who considered studying mathematics but chose not to, are not included. It may have been useful to consider whether students had access to opportunities they did not take, or whether some of the activities we listed were not available to them, though recall of childhood activities may not be the method to obtain such information.

Overall, it is pleasing to note that incoming mathematics undergraduates have a high level of enjoyment of mathematics and of engaging with informal mathematical activity, particularly so on an individual basis where presumably they are able to make choices about how to spend their time on a phone or tablet or around the home. The level of engagement in more organised informal mathematical activities is lower, and the link between the goals of such activities and students' motivations warrant further study.

7. Appendix

Pre-university informal engagement with mathematics - Questionnaire Questions

- (Consent to participate question)

- At which university do you study mathematics? *

  - Sheffield Hallam University
  - University of Sheffield
  - Other: [free text]

- 1a. Please indicate which of the following organised activities you took part in before starting university. [Yes/No/I think so/I don't know]

- Someone visited my school and gave a maths talk or ran an activity.
- School trip to maths lecture or workshop.
- School trip to science/engineering/maths museum, STEM centre or exhibit.
- School trip to industrial site/factory/business to learn about how they apply maths/science.
- I attended a careers event involving meeting mathematicians/scientists/engineers (in or out of school).
- I participated in Maths Club/STEM Club activities.
- I participated in STEM activities through Scouts/Guides or similar.
- I attended CodeClub/Coderdojo/Hackday events or any other events to learn coding.
- I participated in STEP prep sessions.
- I visited Sheffield Hallam for the PopMaths quiz.
- I participated in another maths competition/Olympiad/quiz/UKMT challenge - through school or individually.

- 1b. Please use this box to expand on or clarify your answer if you want to. For example, what do you particularly remember about the activities you took part in? What did you most enjoy? [free text]

- 2a. Please indicate which of the following other activities you took part in before starting university. [Yes/No/I think so/I don't know]

    - Attended maths lecture or workshop (not with school).
    - Visited science/engineering/maths museum, STEM centre or exhibit (not with school).
    - I participated in an Escape Room.
    - I attended a science/maths themed birthday party.
    - I read popular maths books (covering maths topics, biographies of mathematicians/scientists).
    - I solved Sudoku and other logic or maths puzzles.
    - I played maths/logic based board games.
    - I played with maths/logic based toys.
    - I did practical building/measuring/crafting/carpentry/engineering tasks around the house/with parents.
    - I shared mathematical puzzles/ideas with friends/family.

- 2b. Please use this box to expand on or clarify your answer if you want to. For example, what do you particularly remember about the activities you took part in? What did you most enjoy? [free text]

- 3a. Please indicate which of the following online/digital activities you engaged with before starting university. [Yes/No/I think so/I don't know]

    - I watched online videos about maths (Numberphile, Standupmaths, etc.).
    - I watched online lectures/talks about maths (e.g. on YouTube, Ri Channel, a university website, etc.).
    - I studied online courses such as from Khan Academy or a MOOC.
    - Mathematical puzzles/ideas shared on social media.
    - I played maths/logic based computer games or phone/tablet apps.

- 3b. Please use this box to expand on or clarify your answer if you want to. For example, what do you particularly remember about the activities you took part in? What did you most enjoy? [free text]

- 4a. Please indicate which of the following other media activities you engaged with before starting university. [Yes/No/I think so/I don't know]

    - I watched TV documentaries about maths.
    - I watched TV game shows/quiz shows with a mathematical aspect (Dara O Briain's School of Hard Sums, Countdown, Golden Balls etc.).
    - I watched Royal Institution Christmas Lectures on TV (or attended a live recording).
    - I heard maths puzzles on the radio (e.g. Radio 4 Puzzle For Today, or otherwise).

- 4b. Please use this box to expand on or clarify your answer if you want to. For example, what do you particularly remember about the activities you took part in? What did you most enjoy? [free text]

- 5. Are there any other extracurricular maths or related activities not covered in the questions above which you took part in? [free text]

- 6a. Please indicate how important the following were in your decision to study mathematics at university. [Rated from 1 (least important) to 7 (most important)]

    - I enjoy maths/maths is my favourite subject
    - I wanted to learn more maths
    - Maths is a challenge
    - I am good at maths/best at maths compared with other subjects
    - Good job prospects/Wide range of jobs open to maths graduates
    - Maths is a prestigious/valued degree
    - Maths fits into my specific career plans
    - Earning potential

- 6b. If you have a reason for studying maths at university not covered by the list above, please indicate this here. [free text]

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CASE STUDY

Supporting nursing students' mathematical understanding

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Abstract

A key component of any nursing course is the ability to confidently, and competently, use basic mathematical skills. Indeed, without such skills they would be unable to work safely and successfully in the profession (McMullan, Jones, and Lea, 2012; Choudhary and Malthus, 2017). Furthermore, many nursing students come on to their courses having not studied any form of formal mathematics for years and are very likely to have some form of maths anxiety (McMullan, Jones, and Lea, 2012). **sigma** Maths Support at Coventry University have developed a programme, in partnership with the Faculty of Health and Life Sciences (HLS), to support these students with their mathematical skills using a variety of small interventions. The interventions require little time and effort to prepare and have proven to be very successful. This article will discuss the methods used, including diagnostic testing, the use of 'Numbas', and self-explanation training (as described in Hodds, 2017), alongside the outcomes of the programme.

Keywords: Mathematics support, Nursing, Self-explanation, Numbas, Coventry University.

1. Introduction

When students make their decision to study for a degree that is traditionally considered non-mathematical, they often do not consider that there may indeed still be mathematics contained within it. Moreover, if a student has a fear of mathematics then that will influence their choice of degree once they leave school (Dowker, Sarkar, and Looi, 2016), particularly since studies have shown that attitudes towards maths deteriorates as students reach adolescence (e.g. Wigfield and Meece, 1988, Ma and Kisnor, 1997). To try to help these students, many universities in the UK now have a form of mathematics support provision. Initially, these provisions were set up to help improve the declining mathematical skills of undergraduate students on courses with high mathematical content, such as engineering (Lawson, Croft, and Halpin, 2003), but more recently these provisions have expanded to offer more support to non-mathematicians (Grove, Croft, and Lawson, 2019). Non-mathematicians are more susceptible to maths anxiety however and, therefore, just seeking out the support and stepping through the door can be a daunting task.

One such group of students are nurses. These students have very basic mathematical content on their course, such as percentages, fractions, simple addition and subtraction, and using formulas. It is important that these students have good mathematical abilities in order to have a successful career and practice safely (Choudhary and Malthus, 2017); a lower mathematical ability has also been shown to be the main factor in predicting drug calculation ability (McMullan, Jones, and Lea, 2012). Of course, safe drug calculation is important for nurses as without those skills they pose a risk to patient safety (Renmarker and Carlson, 2019). However, nursing courses also often require their students to achieve 100% in any final year mathematics exam putting more stress on students who are already usually anxious about mathematics.

The demographics of nursing students may also prove to be a hindrance to their mathematical understanding. Nursing students are often older than typical undergraduate students are and therefore may not have had formal mathematics training for many years. As stated previously, it has been shown that as students get older their attitude towards mathematics deteriorates (e.g. Wigfield and Meece, 1988, Ma and Kisnor, 1997) so, for these students in particular, this can be a serious problem. Furthermore, Zakaria and Nordin (2008) showed that there can be a negative correlation between maths anxiety and motivation to study mathematics. Therefore, the combination of not studying any form of mathematics for many years and generally being older students, leads to low motivation to study mathematics and high anxiety towards it.

To help students on all courses at Coventry University, the **sigma** Mathematics Support Centre offers support to students through drop in sessions for over 50 hours a week in term time, alongside many other forms of support. Data on student entry to the support centre is collected via a card reader which students tap their ID card on to when they first arrive. Figure 1 below shows the data for student entries by faculty for the academic years from 2015 to 2018.

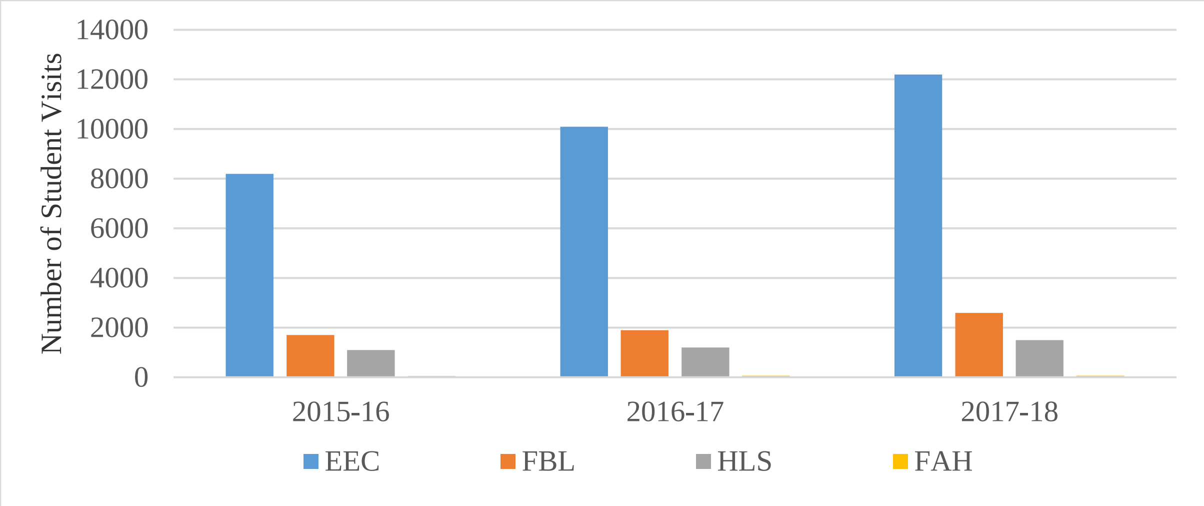

Figure 1: Visits to Coventry University Mathematics Support Centre by Faculty and Academic Year

The data shows that the majority of student visits come from the Faculty of Engineering, Environment and Computing (EEC), which has engineering, computing and mathematics courses. These courses have high mathematical content whereas the other faculties (Faculty of Business and Law, FBL; Health and Life Sciences, HLS; Faculty of the Arts and Humanities, FAH) have less mathematical content in their courses. Nursing is based in HLS where numbers year on year had increased for visits from students in that faculty. However, for 2017-18 visits from nursing students had declined significantly. Furthermore, students who were visiting the centre were mainly visiting in the week of, or the week before, their final exams. Obviously, this was not enough time to get the support they needed and pass rates for their final numeracy exams were only at 75%.

Data from a survey given to first year students revealed some of the reasons why non-mathematics students in general were not coming to seek the support they needed. Comments included *"I should be able to do this – it's not even GCSE level maths"*, *"I'm worried my lecturer will find out that I am struggling"*, and *"I hate maths, I'm not very good at it."* For those who work within mathematics support, this is probably unsurprising but there is a clear trend in these comments that link back to maths anxiety and motivation discussed previously. However, a further comment suggested that when they do come for support, they gain confidence: *"I have just had a… session with a tutor and honestly it was amazing. She was so kind and patient with me. I feel confident now to continue on the biomedical science course knowing that I can come here as often as I like for maths support… I was thinking I would fail just because of the maths side of the course. I am not even that bad at maths, I just need a little extra help to understand what I am doing. Thank you so much and please continue the… support sessions."* It is clear that if students access the support on offer it can be beneficial to them. It therefore seemed reasonable to improve nursing student awareness of the support on offer and increase the support offered to them. Hence this provided the motivation for producing some interventions for student nurses at Coventry University to see if their mathematical understanding could be improved.

2. Interventions provided to improve understanding

In the 2017-18 academic year there were only three main methods of maths support available to nursing students. These were formal teaching and office hours from nursing lecturers within the faculty, an intervention session on the basics of nursing mathematics from a member of **sigma** staff, and general maths support within the **sigma** Mathematics Support Centre. Not all nursing courses received the intervention session due to time available from **sigma** staff, so some nursing courses only had two methods of maths support available. For 2018-19, four further methods of support were made available.

The first method of support was a bespoke diagnostic test designed specifically for nursing students. The test contains questions on order of operations, percentages, basic calculations, fractions, conversions of units and simple drug calculations using given formulae. The test was given in week one of the course and the results showed students their areas of strength and weakness. For the topic areas that needed improving, the results provided links to worksheets available from the **sigma** support website that students could work through in their own time.

The second intervention was to provide students with a self-explanation training booklet, as described in Hodds (2017). The self-explanation training method has been shown to be effective at supporting the understanding of both mathematics students (e.g. Hodds, Alcock, and Inglis, 2014) and non-specialist mathematics students (Hodds, 2017). The training booklet is designed to help nursing students understand how to deconstruct questions and unpick the mathematics, allowing them to answer the question using the methods taught within lectures. Indeed, one of the reasons why some nursing students fail is not because they cannot do the maths, but instead they cannot understand what maths to use given a particular situation (Hodds, 2017).

The third intervention was to provide students with online interactive questions using the platform ‘Numbas’. These online questions are designed so that students can attempt exam style questions many times but with different numbers and variables for each attempt. Since the questions are online, the students can attempt them anytime, allowing for a continuous method of support. Furthermore, these questions were embedded within the university’s online learning environment (CU Moodle) so the lecturers could see how well students were progressing through each topic. This allowed the lecturers to invite students who were struggling to consider getting further support to improve their understanding.

Finally, to encourage students to visit the **sigma** Support Centre for support, members of the nursing team provided up to 5 hours per week of support time within the centre. Not only does this help students to find the centre, it helps them to realise the centre is a place where they can get support without judgement, breaking down the anxiety barrier. As the member of the nursing team is a familiar face, students are more likely to come and use the centre whilst also seeing that permanent members of the **sigma** team are available for friendly help and support also.

For clarity, the table below compares the previous support available to nursing students in 2017-18 compared to the support available in 2018-19.

Table 1: A comparison of the support available to nursing students in 2017-18 to 2018-19

Support during academic year 2017-18	**Support during academic year 2018-19**
Formal teaching from faculty staff	Formal teaching from faculty staff
Intervention session from **sigma** staff	Intervention session from **sigma** staff
General support in the **sigma** Support Centre	General support in the **sigma** Support Centre
	Bespoke diagnostic test in week one
	Self-explanation training booklet
	Interactive 'Numbas' questions online
	Formal support from Nursing lecturers in **sigma** Support Centre

3. Results

Firstly, the pass rate in the end of year maths exam for three nursing courses (where the full range of support during the academic year was offered) were considered: Non-Medical Prescribing (NMP), a final year masters course, Community Nursing Prescribing (CNP), an undergraduate course, and Adult Nursing first year undergraduates. For NMP, the pass rate was 80.3% and for CNP, the pass rate was 97.3%, both up from 75% in 2017-18. For CNP, only two students failed in the entire cohort of 73 students. For Adult Nursing, the pass rate was 92.6% but there was no comparison as there previously was no exam in the first year. Although direct causation cannot be suggested, it does appear that the interventions had at least some positive effect on understanding as all other factors to do with the courses (such as timetabled sessions, exam timings, types of exam questions, etc.) remained consistent.

Secondly, the diagnostic test scores for the first year Adult Nurses were correlated with their final exam scores to determine whether there was any relationship. Indeed, a Pearson's Correlation revealed that the diagnostic test score was a good predictor of the final exam result, $p < 0.001$, $r = 0.666$, and is shown in the graph below, coloured by how many visits to the **sigma** Maths Support Centre each student made.

The result suggests that the better the student did on the diagnostic test, the better they did on the final exam. However, since it is not a perfect correlation, it shows that students were able to improve if they came to the centre for support. Moreover, the more times they visited for support, the more they generally improved. Indeed, those students who visited the centre more than once had an increase of 23.1% on their score from the diagnostic test to the final exam, compared to those who did not visit or just visited once, who had an increase by 14.5% on average. However, this difference in improvement did not quite reach significance, $t(65) = 1.829$, $p = 0.072$. Nevertheless, the result indicates that the new diagnostic test was a good predictor of exam performance but by visiting the centre, students' understanding would generally improve, resulting in an improved performance in the end of year exam.

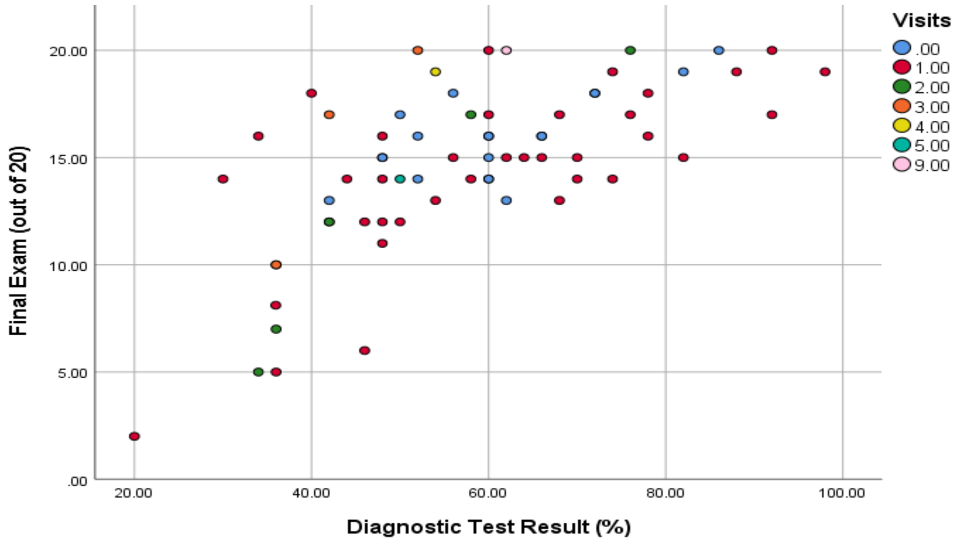

Figure 2: Correlation between diagnostic test score (%) against final exam score (out of 20) for first year Adult Nursing undergraduates by how many visits made to the **sigma** Maths Support Centre

Finally, the number of nursing student visits to the sigma Mathematics Support Centre was considered to see if more nurses were seeking the support they needed. Indeed, the number of visits from nursing students increased significantly from 60 visits in 2017-18 to 340 in 2018-19. Having a member of staff provide support hours in the centre really increased awareness of the service to the students and helped to reduce the worry about crossing the threshold of the centre to obtain support. The graph below shows the change in the number of nursing visits to the sigma Mathematics Support Centre for the academic years from 2012-13 to 2018-19.

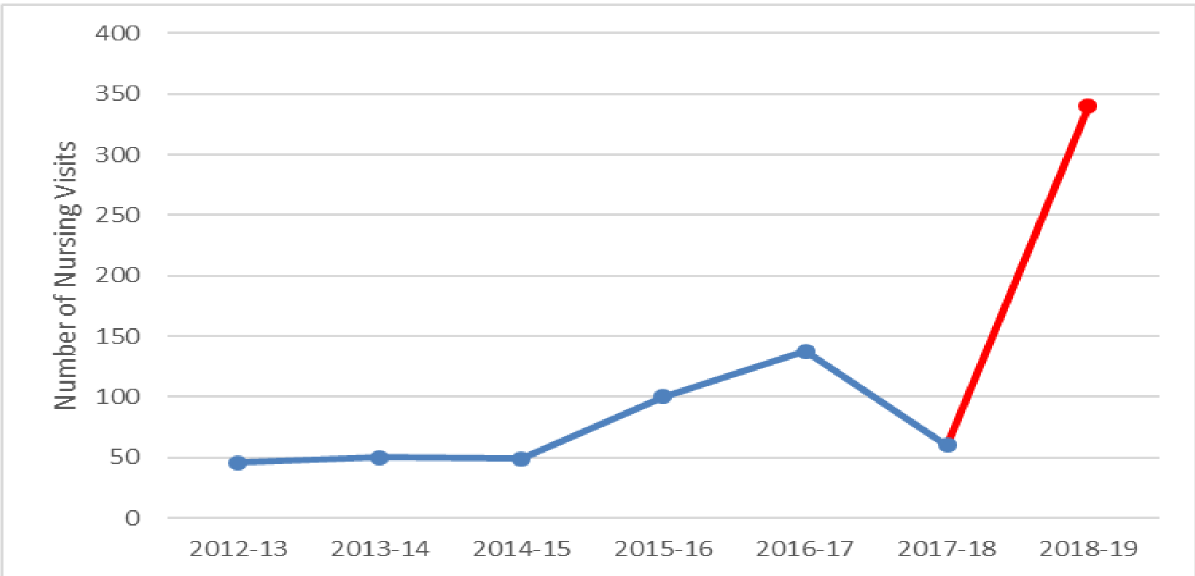

Figure 3: Nursing student visits to the **sigma** Maths Support Centre by academic year

4. Discussion

The interventions described have proven to be successful in improving nursing students' mathematical understanding at Coventry University. These interventions have had at least some influence in helping 289 nursing students pass exams with mathematical content in 2018-19. Clearly, this is a positive outcome and will therefore have had some positive financial impact on the university, but exactly how much is a difficult question to answer.

One measurement we can consider is how many nursing students from various vulnerable groups we are helping to progress. As part of universities' impact statements to the Office for Students, mathematics support centres and provisions in the UK now have to consider exactly how they are supporting students from several key groups, historically shown to struggle, to pass their degrees. Two of the key areas are students from minority ethnic backgrounds and mature students. The initial work described above is a good case study for the sigma Mathematics Support Centre as 34.2% of the nursing students in 2018-19 were from minority ethnic backgrounds and 53.8% of the nursing students were mature students. More work needs to be done to complete the statistics for this but it will provide a useful evaluation tool going forward.

Work has already begun to increase the support offered to nursing students in 2019-20. For example, the Adult Nursing course will now have online work each week to complete using 'Numbas' in the second year and collaborative workshops with both sigma staff and faculty staff in the final year. Furthermore, in September 2019, the first cohort of nurses at the Scarborough satellite campus completed the diagnostic test as it was made available online for the first time. Offering support to students not based on the main campus in Coventry is a big gap that sigma are looking to fill and online remote support seems to be the way to do this. Indeed, work is currently taking place to train staff to be able to provide one-to-one mathematics support sessions using an online meeting space called 'Big Blue Button' whilst writing on screens using styluses and tablets.

To conclude, this article has shown that it is possible to improve the mathematical understanding of non-mathematicians, and nursing students in particular, by adding a few simple interventions. Each intervention was rarely time consuming and, once completed, can be used over again and improved upon, reducing the time needed to produce support materials in the future. Furthermore, getting faculty staff to offer time and support within your own support provision can really help to break down any barriers or stigmas attached to seeking much needed support. As a maths support practitioner, you may need to approach faculty staff and offer a quid-pro-quo in order to get them to support you but doing so can be extremely effective. Since non-mathematicians are often reluctant to ask for help, as they are either anxious or they see it as a weakness, faculty staff can be a useful link between your provision and a student. Providing a supportive environment with a friendly face students recognise can therefore be the first step on the road to helping these anxious students see maths support as a useful tool in their degree journeys. Indeed, in 2019-20, there have been 390 nursing student visits (up to November 2019) to the sigma Mathematics Support Centre which is already 30 more than the whole of last academic year. Finally, it is important to stop and reflect to ensure what has been implemented is having a positive impact. The work of maths support provisions in the UK is being further scrutinised by universities, so detailed evaluations showing exactly how the provision is increasing student engagement and retention can only help to support the impact the service is making.

5. Acknowledgements

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CASE STUDY

Rearranging equations: (concepts – misconceptions) × peer discussion

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Abstract

Transposition of formulae (also known as rearranging equations and changing the subject) is a skill vital for professionals in many fields of science and engineering. It is however a topic with which many students, and particularly students of weaker algebraic competency, struggle and often do not master sufficiently. This paper proposes an intervention strategy for improved teaching and learning of transposition of formulae at third level. The intervention aims to address three key issues thought to inhibit students' understanding of the topic: (1) a lack of conceptual understanding of equations and equality, (2) prior misconceptions and (3) a fast paced learning environment that does not account for diversity in knowledge and aptitude. The strategy consists of three hour-long lesson plans that emphasise conceptual understanding while also dispelling the relevant misconceptions, using a peer discussion teaching model as a vehicle for consolidating and propagating the right concepts. To account for diversity in prior knowledge and aptitude an online tutorial on the topic of transposition has been developed using an online e-assessment platform that allows students to practice at their own pace and receive instant feedback as they progress.

Keywords: Transposition of formulae, rearranging equations, changing the subject, algebra.

1. Introduction

Transposition of formulae (also known as rearranging equations and changing the subject) is a fundamental skill for quantitative disciplines; not only does it form a key step in more advanced problem solving but it also facilitates understanding of the relationships between quantities of interest. From a learner's prospective, it is a rather demanding task as it represents a culmination of many algebra skills and concepts coming into play at once. As a result, it is a topic with which many students, and particularly students of weaker algebraic competency, struggle and often do not master sufficiently. Despite the topic being taught using a variety of styles and approaches both at school and often again at third level (higher education in the Republic of Ireland), students' difficulties persist (O'Brien & Ní Ríordáin, 2017; Stephens, et al., 2013; Pendergast M. & Treacy P., 2015; Chow, 2011). In our own experience, after giving a diagnostic test on transposition to over 350 first year students at Cork Institute of Technology, 75% of respondents were not able to correctly rearrange the following benchmark equation to isolate g:

$$T = \frac{2v}{g} + 5.$$

The problem is not limited to the topic of transposition itself and in fact perpetuates far beyond a class on rearranging equations. The difficulties that students encounter in a wide variety of further mathematical problems, e.g. separating variables in differential equations, arise from a lack of proficiency in rearranging equations. Moreover, mathematical proficiency is a major contributory factor to the success of a student in any science or engineering course. In many applied disciplines the topic of transposition is so important and integral to the understanding of applied concepts that students' struggles with transposition significantly impact on their progress in the applied subject. At Cork Institute of Technology, lecturers from applied disciplines, such as engineering, physics and business studies, reported that difficulties with the topic of transposition inhibited the teaching and assessment of applied concepts. Interestingly, even some online resources in Physics and Chemistry have dedicated sections to rearranging equations thus confirming the importance of the transposition skills in these disciplines but also highlighting students' difficulties with it in an applied context (Konig, 2015; Scott, 2012; Southall, 2016; Isaac-Physics, 2019). These resources also emphasise the importance of a conceptual learning of the method versus memorising some shortcuts like 'a formula triangle'.

The origins of students' difficulties with transposition, discussed in detail in (Lishchynska M. et al, 2019), are complex and multifaceted. Two major technical contributors to the issue, identified in (Lishchynska M. et al, 2019), are the lack of understanding the concept of balancing equations, as well as strong misconceptions of an algebraic and conceptual nature. It seems that the issue persisting does not so much depend on a particular style or method of teaching but more on the fact that mathematical fluency in the topic of transposition can only be acquired if one has formed a strong conceptual understanding of the principles while also managing not to form erroneous beliefs (misconceptions). The third issue identified in (Lishchynska M. et al, 2019) is the limited time available to teach the topic to a large number of students that vary in prior knowledge. Since the topic is taught at second level, transposition is viewed as a prerequisite in first year Mathematics modules and the three-hour lecturing time allocated is for the purpose of revision and consolidation. However, for many students the gaps in their knowledge are too large to overcome in this short time frame. In response to the technical findings and with the aim to rectify the problem, this paper concentrates on cultivating the key concepts and dispelling students' misconceptions pertinent to rearranging equations. We also seek to address the time constraints and variation in prior knowledge. The work aims to develop an approach to teaching transposition that will address the above deficiencies effectively and will ultimately help the students to master the skill.

2. Building concepts

To gain a better insight into the students' difficulties, and more importantly the basis for those, an error analysis was performed on students' work (Lishchynska M. et al, 2019). One of the two major deficiencies identified was a lack of understanding of the equality sign and how equivalent equations are generated. This agrees with the findings in studies by Bush and Karp (2013), Stephens et al. (2013) and Byrd et al. (2015) where the lack of understanding of the equality sign and equivalent equations was at the core of students' problems with solving equations. The lack of conceptual understanding plays a major role in rearranging equations as without the deep understanding of what equations are, how they are formed and what the equality sign means, other algebraic skills become unproductive in the context of transposition. Conversely, when a student has developed a solid understanding of the main concepts and principles, they should be able to apply them and rearrange any formula or equation, not just a particular set of easy or typical ones.

To account for the above, teaching with a strong emphasis on conceptual understanding of balancing equations needs to precede the delivery of the method itself and practical work on rearranging equations. Moreover, to foster a greater understanding of the fundamental principles and concepts, students need to be confronted with a variety of concept questions relevant to transposition. A good concept question should satisfy the following criteria (Crouch, 2007): focus on a single important concept, ideally corresponding to a common student difficulty; require thought, not just plugging numbers into equations; provide plausible incorrect answers; be unambiguously worded; be neither too easy nor too difficult. In relation to transposition of formulae, the thought process behind forming concept questions applied in this work is schematically presented in Figure 1 and explained in more detail in (Lishchynska M. et al, 2019). With these criteria and approach in mind, a bank of concept questions on equations and transposition has been developed, with several examples given in the Appendix. Such questions can be used as a basis for peer discussion and teacher-students discussions in class as well as in diagnostic tests.

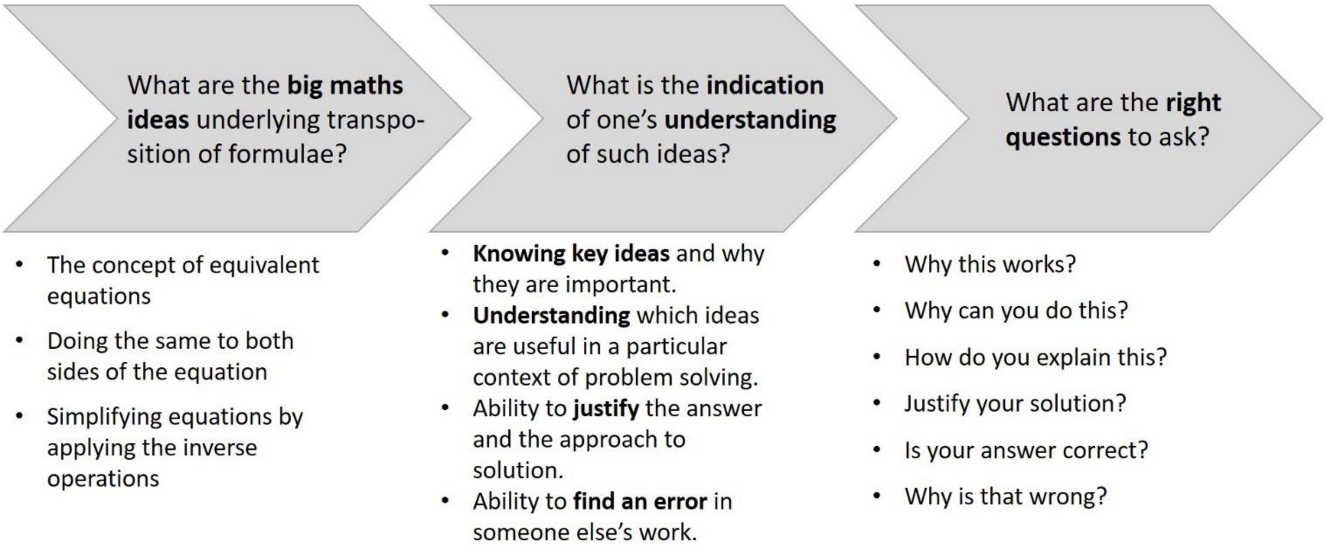


Figure 1. The approach to creating concept questions.

It is worthwhile acknowledging that in the Mathematics Education research there exists a decades long 'the chicken or the egg' type debate about what needs to come first, conceptual understanding or procedural fluency (Rittle-Johnson, et al., 2001; Haapasalo, 2003; Star, 2000). With regard to rearranging equations, we believe, a successful intervention must firstly focus on building up a solid understanding of the concepts of equality and equations. And this must come before the procedural fluency because *"to be able to apply the procedure for solving equations 'accurately, efficiently, and flexibly', students really need to know what they are doing to a far greater extent than when applying Pythagoras's theorem or multiplying fractions"* (Barton, 2018). In the case of rearranging equations, conceptual learning supports and generates procedural learning but procedural learning on its own does not support conceptual learning. Once students have a solid conceptual understanding of the principles of balancing equations, they can then be presented with a procedure, examples and practice questions.

However well intended and planned, focusing solely on cultivating a conceptual understanding may not be sufficient to affect students' skills in transposition. Students' erroneous beliefs about equations need to be dealt with as well.

3. Misconceptions cannot be ignored!

Another major contributor to the poor transposition skills pinpointed by the error analysis was a whole myriad of misconceptions that students hold about equations and manipulating them. In addition to this, there are also misconceptions related to the algebraic procedures involved in transposition of formulae. As it happens, students arrive at third level with some cognitive baggage and that likely includes erroneous beliefs and incomplete knowledge of some topics and concepts. Students' misconceptions are a known phenomenon with mathematics teachers and have been the focus of multiple studies. There is a general agreement in the literature that students' misconceptions form barriers to further learning and need to be addressed and dispelled in class (Lucariello, et al., 2014; Bush & Karp, 2013; Barbieri & Booth, 2016). Students can only benefit from developing good conceptual understanding while simultaneously challenging misconceptions (Stephens, et al., 2013). The research into the topic suggests that, while necessary, focusing an intervention solely on cognition or motivation may not lead to improvement in algebra skills whereas a combined intervention approach that includes error reflection may be beneficial (Barbieri & Booth, 2016).

Misconceptions most relevant to transposition may be roughly grouped in two classes: conceptual and algebraic. Conceptual misconceptions manifest in the lack of respect for the equality and hence mishandling the equations, incorrect use of inverse operations and, very commonly, illegal ‘moving’ entities across an equation in a random fashion. Algebraic misconceptions affecting transposition are mostly, but not limited to, misunderstanding and misuse of the distributive law, incorrect simplification of expressions (illegal ‘cancelling’) and the very common, intuitive but wrong assumption about distributivity of exponents (squaring, taking roots etc.). A review of algebraic misconceptions given in (Chow, 2011) highlights the latter group of misconceptions as over-generalisation of rules and/or operations. A schematic classification of misconceptions relevant to transposition and examples of typically encountered errors are given in Figure 2.

In our experience, the errors due to the above misconceptions are so common and persistent that they simply cannot be ignored and need to be resolved effectively. While there are plenty of resources of ‘spot the mistake’ style exercises, an active intervention needs to take place in class for a teaching strategy to be effective. The misconceptions of conceptual nature can be addressed through deliberate teaching with an emphasis on the right concepts of balancing equations discussed in the previous section. The issue of illegal ‘moving’ of entities across an equation is linked to that and can be addressed *after* the key concepts have been presented and practiced by the students. Algebraic misconceptions relate to rearranging somewhat harder, more convoluted formulae, and therefore can be addressed at a later stage but before considering discipline relevant equations/formulae. The idea here is to reinforce the necessary laws of algebra and principles of distributivity and simplification approaches, then challenge the students with concept questions testing these laws and discuss the common misconceptions.

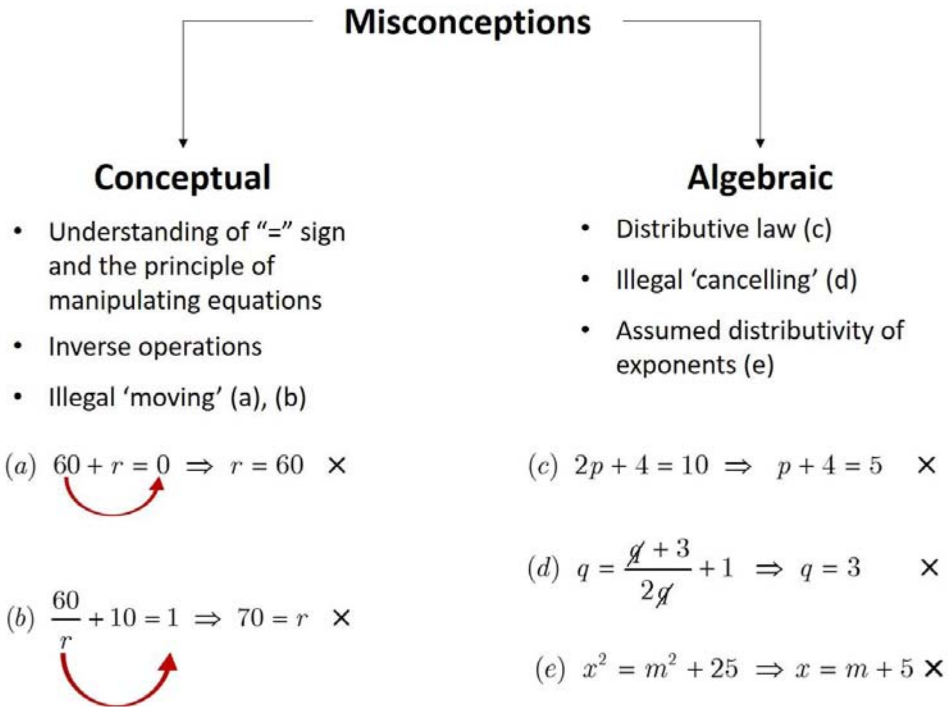


Figure 2. Summary of misconceptions and typical errors relevant to transposition.

Given the limited time lecturers at third level can dedicate to teaching the topic, e.g. a lecturer at Cork Institute of Technology has at most three hours of lectures and a tutorial, it is vital to plan the classes carefully and use this time effectively. Traditional teaching approaches, even with an increased emphasis on concept building and addressing misconceptions, may still be insufficient to induce a tangible change in students' learning and acquiring a solid skill of transposition. To be effective, an intervention needs to include a radical aspect in comparison with existing methods. *"Memory is the residue of thought"* and therefore *"students remember what they think about …"* (Barton, 2018). If one aims for the right ideas to be firmly planted in students' memories and discourage the wrong ones from forming, one needs to stimulate students to think, question and remember. Therefore, a peer discussion teaching model was adopted to support the taught element of the topic.

4. Peer discussion: a vehicle for propagating right concepts

Eric Mazur was teaching introductory physics to undergraduates and used a traditional method: the lecturer speaks, the students listen and take notes. One day Mazur decided to test the students' understanding of a particular concept not by doing traditional problems, but by asking them a set of basic *conceptual* questions, he discovered that the students had great difficulty with the conceptual questions. As a result, Mazur had to completely re-think his approach to teaching and proposed a peer instruction (also known as peer discussion) model to be used in conjunction with lectures (Mazur, 1997). Peer instruction is designed to engage students during class through activities that require each student to apply the core concepts being presented, and then to explain and discuss those concepts with their fellow students. After a particular topic is presented by a lecturer, a concept question (a question probing understanding of a particular concept) is posed. Students are given one or two minutes to formulate individual answers and vote, using clicker devices or mobile phones, for an answer from a choice displayed on a screen. Students then discuss their answers with others sitting around them; the instructor urges students to try to convince each other of the correctness of their own answer by explaining the underlying reasoning. After a few minutes, the instructor calls an end to the discussion, polls students for their answers again which may have changed based on the discussion.

Above anything else peer instruction makes each student *think* and think twice: once individually and then again when trying to explain their point of view to the fellow students. Mazur's results show a significant increase in the percentage of students answering the concept question correctly. Peer instruction has been shown to be effective across disciplines at second and third level (Cummings & Roberts, 2008; Smith, et al., 2011) and combining peer discussion with instructor explanation increases student learning from in-class concept questions (Smith, et al., 2009). The logic underlying the success of peer discussion based on concept questions is that it continuously engages students' minds and provides feedback to both students, and the lecturer, about the level of understanding. It also allows students to construct their own knowledge of the topic. Other benefits include the (usually positively received) social aspect of the peer discussion and the students' greater openness to accept information from their peers who know and are convincing in their explanations. There is also the use of a different, and perhaps simpler, language with explanations.

Importantly, as a basis for peer discussion, concept questions can be designed to uncover and highlight students' misconceptions in the material thus bringing the students' attention to the issues and allowing firstly the students themselves, and if necessary the lecturer, to address those. The authors in (Giuliodori, 2006) have found that 56.8% of the students with incorrect individual answers switched to a correct answer after peer discussion. Equally significant was the fact that only 6.5% of students changed their originally correct response to an incorrect one. Quoting (Mazur, 1996) it is *"easier to change the mind of someone who is wrong than it is to change the mind of someone who has selected the right answer for the right reason",* thus peer discussion tends to propagate the right ideas not the misconceptions.

Given the thought stimulating potential of the peer discussion teaching model, it may prove to be an effective part of an intervention strategy for teaching transposition. We want our students to think hard about the concepts and principles underpinning the process of rearranging equations and we want these thoughts to make a lasting imprint in their memories. Peer discussion may act as a vehicle for propagating and consolidating the right concepts.

5. Intervention strategy and lesson plan

This section brings together the ideas from the previous three sections in an intervention strategy. Considering the above, a successful intervention teaching model comprises three main components: (1) building of a solid conceptual basis, (2) resolving/dissolving relevant misconceptions and (3) a peer discussion acting as a vehicle for propagating and consolidating the right concepts (Figure 3). It is important to emphasise here that one needs to be cautious when dealing with misconceptions in terms of presenting the right ideas first, through explicit instruction, and only then moving onto clearly labelled incorrect examples once the students are familiar with the concepts.

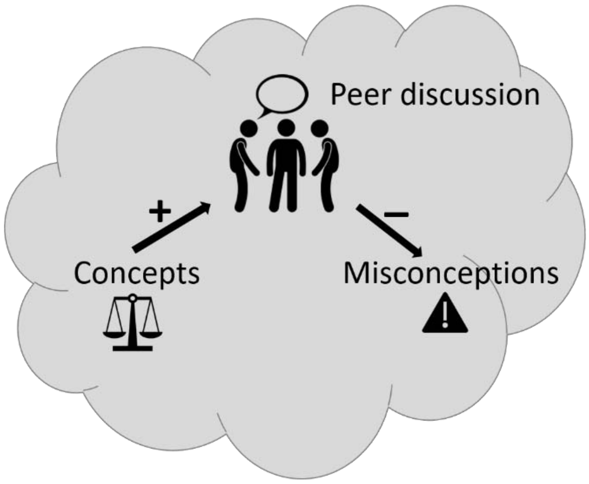


Figure 3. Schematic view of the intervention model.

Going back to the dilemma of the right order between procedural and conceptual knowledge acquisition, O'Connor and Norton (2016) investigated the mathematical difficulties encountered with quadratic equations by analysing student error patterns and the relationships between them. The study found that students' lack of procedural skills in prerequisite algebra hindered their conceptual understanding of quadratic equations. Similarly, our own error analysis found that many of our students did not have the pre-requisite algebra skills to apply to transposition problems. Star (2000) gives an interesting dissection of the 'which comes first' debate and concludes that *"understanding in mathematics is the synthesis of knowing and doing, not the accomplishment of one in the absence of the other"*. Indeed, conceptual errors and procedural errors are often intertwined. Rittle-Johnson et al. (2001) put forward the idea of an iterative model of the development of conceptual and procedural knowledge which we think may be especially relevant to the process of learning to rearrange equations. This factor is also taken into account in the intervention strategy.

In summary, the proposed intervention strategy flows as follows (Figure 4):

- Develop concept of an equation and balancing equations (supported by peer discussion);
- Address misconceptions about handling equations (supported by peer discussion);
- Teach, through explicit instruction, manipulating equations and basic principles of transposition;
- Address relevant algebraic misconceptions (supported by peer discussion);

- Teach and practise transposition of harder (more convoluted) formulae.

Concept questions similar to the examples given in the Appendix can be used as a basis for peer discussions.

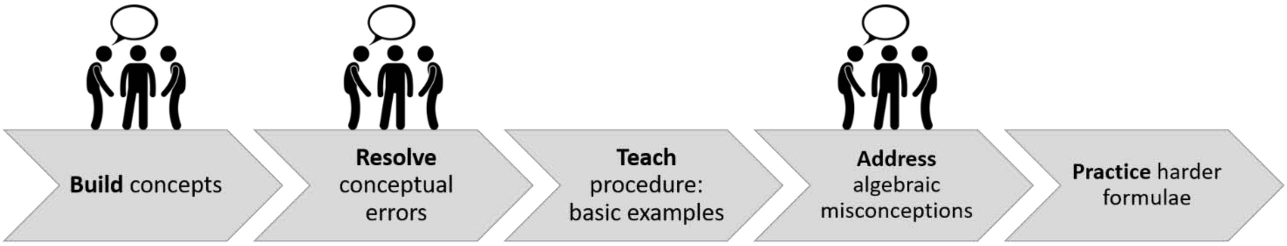


Figure 4. Teaching sequence in the proposed intervention strategy.

Specifically, the proposed intervention comprises a three-lecture plan. Lecture 1 focuses on the main concepts and fundamentals of balancing equations: motivation for transposing formulae; the concept of an equality and balancing scales; inverse operations; common conceptual misconceptions. The objectives of Lecture 2 are to review and reinforce algebra pre-requisites of transposition (simplifying expressions, distributive law, non-linearity of exponents), to discuss the relevant misconceptions and to present the general approach to rearranging equations. This class also addresses the difficulty of 'where to start' which is a common stumbling stone with weaker students. Lecture 3 is dedicated to dealing with harder (more convoluted) formulae involving the use of the distributive law, a subject appearing on both sides of the equation and fractions. Purposeful practice using students' discipline specific formulae here aims to help students master the skill.

6. Time constraints and variation in prior knowledge

One of the non-technical factors contributing to the students' difficulties with transposition identified in (Lishchynska M. et al, 2019) is the difference in prior knowledge. The students have been exposed to the topic at school and this prior exposure made the topic particularly challenging to teach; some students already know how to transpose equations and are bored in class distracting others whereas some think they know it (when they do not) and are not paying attention. There is also the issue of fast delivery pace at third level which does not suit some learners. All this does not create an optimum learning environment. It is hoped that the interactive nature of peer discussion will engage students who already understand the topic allowing them to consolidate their own knowledge through explanation to their peers. The quiz element of the peer discussion will also provide instant feedback to those students who think they know the material when in fact they do not.

To deal with the diversity of skill in class as well as encouraging all students to practise more, an online tutorial using the open source maths e-assessment platform NUMBAS (NUMBAS, 2015) has been developed at Cork Institute of Technology (NUMBAS-CIT-Transposition, 2019). This is an optional tool to be used as a supplement to lectures and standard tutorials. NUMBAS allows a lecturer to create a question framework from which an unlimited number of random variations of a question can be generated, allowing a student to practice a question of a certain type or difficulty as often as they feel necessary. As well as instantly informing the student whether their answer is correct, NUMBAS also provides options for 'show steps' and 'advice' where either partial solutions or full solutions can be displayed thus providing instant detailed feedback to a student when required. In line with the *"learning for mastery"* strategy (Bloom, 1968), where the emphasis is on students achieving a level of mastery in prerequisite knowledge before moving forward to learn subsequent information, the NUMBAS online tutorials are intended to help the students to spend as much time as individually necessary on tasks of various difficulty, and hence master the skill at an individual pace.

7. Conclusion and future work

Students' lack of proficiency in rearranging equations is an issue perpetuating far beyond a maths class and often hinders progress in a science or engineering course. This paper proposes an intervention strategy for improved teaching and learning of transposition of formulae. The intervention is based on an increased emphasis on conceptual understanding while also dispelling the relevant misconceptions. This is to be supported by a peer discussion teaching model as a vehicle for consolidating and propagating the right concepts. Differences in prior learning and a fast-paced learning environment are countered through the use of interactive online NUMBAS tutorials.

The intervention is currently being implemented at Cork Institute of Technology. A full evaluation study is underway. Future work will focus on quantitative analysis of the outcomes of the implemented intervention. The 'before' and 'after' diagnostic tests will aim to quantify the impact of the intervention on student attainment in the topic of transposition in a first-year service mathematics course. A qualitative study intended to gauge the effect on students' experience and attitudes when learning transposition will also be undertaken.

8. Acknowledgements

The authors would like to acknowledge the useful discussions with the Team Transposition of the Mathematics department at Cork Institute of Technology. The authors also recognise the support of the Teaching and Learning Unit at Cork Institute of Technology.

Appendix

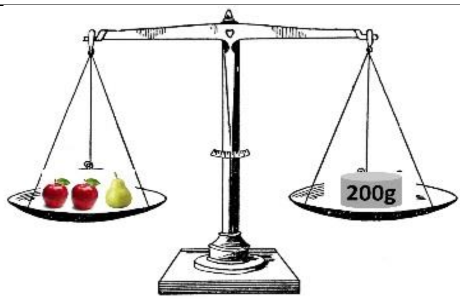
Examples of concept questions on equations and transposition.

Q1. Which of the following are equations? Choose all the options that apply.

□ x^2	□ $x^2 - 5x + 6 = 0$
□ $x^2 = 2x$	□ $x = 2b$
□ $x + 5$	□ $T = 2\pi\sqrt{\frac{l}{g}}$

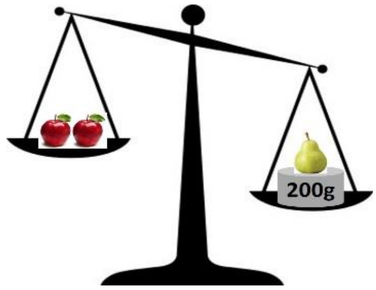
Q2.

$$2a + p = 200$$



What happened here?!!

$$2a = 200 + p$$



Can you explain why the scales are no longer balanced?

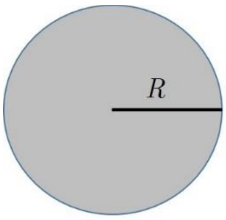
Q3. If $3m = 5$, which of the following are true? Explain why. Choose all the options that apply.

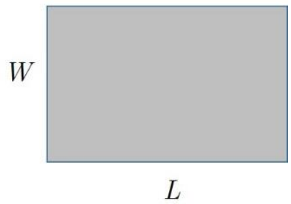
☐ $6m = 10$ ☐ $9m^2 = 25$

☐ $3m - 2 = 3$ ☐ $\sqrt{3m} = \sqrt{5}$

☐ $3m^2 = 25$ ☐ $m = \frac{5}{3}$

Q4. Two plots of land, of circular and rectangular shapes, have equal areas.





Is the following statement true or false? Justify your answer.

$$L = \frac{\pi R^2}{W}$$

Q5. Apples and pears are used to balance the scales shown below.

If a is the weight of an apple and p is the weight of a pear, which of the following statements are correct? Explain why.

(a) $10a = 2a + 6p$ (b) $3p = 4a$ (c) $5a = 3p$ (d) $a + 3p = 5a$

Q6. In an exam, students had to re-arrange (transpose) the following formula to express r which represents the thickness of an engineering part:

$$p = \frac{r^2 + q^2}{L}$$

Below are two solutions by two students. One of these solutions is incorrect.

a) Identify the incorrect solution.
b) Circle the part of the incorrect solution where the mistake is made.
c) Use the box below to explain briefly why it is wrong.

$p = \frac{r^2 + q^2}{L}$ $pL = r^2 + q^2$ $pL - q^2 = r^2$ $r = \sqrt{pL - q^2}$	$p = \frac{r^2 + q^2}{L}$ $pL = r^2 + q^2$ $\sqrt{pL} = r + q$ $r = \sqrt{pL} - q$	

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CASE STUDY

An analysis of student reflections of semester projects in introductory statistics courses

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Abstract

Projects in an introductory statistics course have been implemented for many years as an authentic assessment and learning activity. This article examines student perceptions toward semester projects and the relationship of those perceptions to outcomes. Student semester project data including personal reflections and assessment scores from 254 undergraduate students in six semesters of introductory statistics were analysed using qualitative and quantitative methods, relying heavily on content analysis. The projects comprehensively assessed introductory statistics content, and student reflections were based on three prompts. High levels of learning and interest were reported by the students. Workload issues were not expressed by students to an appreciable degree. Finally, this study illustrates the value of using qualitative methods with case studies to inform teaching and learning.

Keywords: Statistics, Teaching, Project, Qualitative Method

1. Introduction and Literature Review

Projects and other authentic assessments, defined by directly examining student performance on worthy intellectual tasks (Wiggins, 1990), have been proposed as important tools to not only monitor but improve performance, paraphrased from Wiggins (1990). There are many examples and case studies of authentic tasks available. For a few recent examples, Shekar (2014) describes a re-design of an engineering curriculum around project-based learning, Marshall (2019) discusses how statistical education guidelines were used to change a first year probability and statistics module into a project based learning approach, and Spindler (2019) describes modelling projects in differential equations and a rubric for assessing them. For many years the statistical education community has expressed concern about the focus on learning formulas and techniques in introductory statistics courses, instead of on statistical thinking and the statistical process of collecting data to answer questions about the 'real' world (e.g., Garfield, 1994; Cobb and Moore, 1997; Cobb 2007; American Statistical Association, 2016). In addition, in any field student assessment test anxiety can be a concern and performance assessments may lower that anxiety (Onwuegbuzie, 2000).

Students' perceptions about assessment are correlated with what and how they learn. In a comprehensive review of the literature of students' perceptions about assessment, Struyven, Dochy, and Janssens (2005) reveal that the type of assessment influenced the type of learning that students engaged in (i.e. shallow versus surface learning). Thus, it is important to understand students' perceptions of their work. Using case studies with interviews, Sambell, McDowell, and Brown (1997) studied students' perceptions of alternative assessments in a wide variety of subject areas. They reported that students viewed alternative assessments to tests as more authentic compared to the artificial nature of traditional assessment, as a more accurate measure of learning compared to traditional assessment, as measuring complex skills (compared to memorization), and to be fairer (closely related to impressions of validity) than traditional tests. Similarly, Onwuegbuzie (2000) also reported that students rated authentic statistics assessments as promoting higher-level thinking compared to tests. Early (2007) used a phenomenological approach with student interviews to better understand student experiences in introductory statistics courses. Likewise, using qualitative methods, Neumann, Hood, and Neumann (2013) studied students' perceptions of using real-life data in introductory statistics course instruction and identified authenticity as being positively associated with student learning experiences. Finally, Moreira and Pinto (2014) created an end-of-course evaluation that students completed at the end of a 'learning project', which they describe as *"a dialogical teaching approach which can remove emotional inhibitions of students who are fearful of calculations involved in statistics and encourage them to interact among themselves and learn via a team approach"* (Moreira and Pinto, 2014, p. 178). The data from the end-of-course evaluation included both quantitative and qualitative information, and they concluded that the learning projects help students connect statistical concepts and practical applications.

The present study builds on the work of these findings on students' perceptions of alternative assessments by studying students' reflections on a semester project for 254 students in an introductory statistics course and examining different aspects of their reflections. In addition, this study provides a case study of how qualitative methods may inform instructors on their teaching and on student thinking. The specific intent of this retrospective study is to explore students' perspectives of an introductory statistics semester project and the impact of the semester projects on student learning for students at primarily 4 year public universities. These projects were authentic assessments since students were analysing real data and whose goal was to reach conclusions based on students applying appropriate statistical analyses. Student project reflections were studied both through content analysis and quantitative methods on prior known data. Prior known data included final exam and project scores, project types, and the semester when the projects were completed.

The primary goal was to explore and identify undergraduate student impressions of the project and to explore relationships between these impressions and outcomes. The central questions explored are as follows. What did students feel or express about the projects and the value (or not) of the projects? What themes or categories of student sentiment emerge? How are those categories of student expressions related to each other and the known prior outcomes of the course, especially project scores? Philosophically, the approach here is a practical one and could be considered *interpretive description* which is *"to discover association, relationships, and patterns within the phenomena that has been described"* (Thorne, 2016, p. 56).

2. Method

2.1 Participants

The participants of this retrospective study were undergraduate students in an introductory statistics course where a semester project assessment was used. The students for the first five semesters of this study were at a primarily undergraduate traditional mid-western regional public university consisting of approximately 10,000 students with a strong liberal arts focus but also with substantial professional programs such as nursing and business. The sixth and last semester students were similar, attending a primarily undergraduate eastern regional public university consisting of approximately 6,000 students again with liberal arts as an important focus along with some professional programs. Each section typically consisted of 30-35 students. The students were a diverse set of majors, some requiring statistics and some meeting just general education requirements. Since this is a retrospective study, students were not randomly assigned to the introductory statistics sections. However, all introductory statistics sections were listed in the course catalogue in the same way and so the section using projects were not identified as such.

2.2 Course Background

All of the students in this study were taught by one instructor (the author), and I had taught introductory statistics for many semesters. Prior to the Spring of 2014, over a few semesters I had shifted the course from a standard lecture-based course to a flipped classroom model. Students would read or watch videos before class. Then the students worked in groups on learning activities and problems of varying types in class, coupled with class discussions and mini-lectures.

For assessment, during that transition time period a major semester project was instituted to be due at the end of the semester. To ensure that students received feedback on their understanding of the content and to ensure accountability, midterm exams were still part of the course, but their weight on the final grade was reduced. The students still took a mandatory, department-wide common final exam, but its weight on the final grade was small compared to the semester project. The semester projects were instituted prior to Spring of 2014, but those data were not included in this study since important changes during those semesters were made to the instructional approach and the assessment of the projects as I learned and adapted. By the Spring of 2014, the course instruction and the project were stable.

In the semester project, I encouraged students to work in pairs but they could choose to work alone. They could choose between utilizing authentic data provided to them, or creating a survey of their own and gathering the data, which were called personal projects. In the former, I provided National Survey of Student Engagement (NSSE) data for the institution. In addition, I obtained real data about food insecurity in the community gathered in prior years by a community research team independent of the course. Thus, students had a choice of three project types: personal, NSSE, or food insecurity. For the initial segment of the project, students chose survey questions from the surveys used to gather the data or created their own if doing a personal project. In addition, they created research questions based on those survey questions. Note the process was not necessarily sequential this way, as creating the research questions and choosing/creating survey questions were dependent on each other. This was typically a productive learning process for the students to understand the difference between research questions and survey questions. I emphasized the importance of this part of the project by receiving two submittals to review to ensure the students would have solid research and survey questions to work with. In addition, students were required to submit a basic timeline with dates for finishing project milestones, which included a written introduction, a written methods section, data acquisition (if a personal project), descriptive statistics analysis, inferential statistics analysis, and a written conclusion with reflection.

The heart of the project was the analysis. In the final project paper submitted at the end of the semester, students were required to answer their research questions using both exploratory (descriptive) and inferential statistics from the data. Once students learned enough procedures, either descriptive or inferential, in class they were guided by activities to help them decide what procedure made sense with what type of research question. Although working through the procedures was important, a more critical part of their work was interpreting and evaluating their results, justifying if a procedure used made sense and was consistent with other results. Throughout the semester, in class students would learn, practice, and elaborate on those elements. When students had learned enough descriptive statistics content in the course, I required students to submit draft work on the descriptive statistics portion of their analysis to help with time management and to benefit from some feedback. This helped students think about time management and planning during the semester. In addition, as the deadline for the project approached, I provided time in the classroom for students to work together and to gain feedback on their progress.

A detailed rubric with specific guidelines was used to guide the students and for assessment. Suskie (2004) gives an overview of assessment tools and in particular rubrics. Menendez-Valera and Gregori-Giralt (2015) cites the literature on the many potential benefits of using rubrics, which include setting learning standards, allowing students to make academic decisions, giving students tools to assess their own progress, making grading more transparent, providing students feedback on their work, and enhancing communication between faculty and students focused on learning. Similarly, Chance (1997) stresses the importance of providing students ‘guidelines of what is expected’.

Transparency is an essential part of using rubrics (Jonsson, 2014), and so activities on interpretation and evaluation in class helped students understand the semester project rubric. Thus, a rubric provides important support for learning beyond grading.

2.3 Data Collection

The project rubric included a short component for the students to reflect on the project experience. The reflection prompts given to the students were:

- Did you encounter any problems?
- Did other questions arise? Identify interesting follow-up investigations.
- Reflect on the project experience as a whole from a personal point of view. For example, did you learn from it? Did you enjoy working with real data? Was it meaningful?

The purpose of these reflections was for students to think about what they gained from the project experience (or not). The data analysed in this study are the reflections based on those prompts, semester project scores, final exam scores, and type of project over six semesters from Spring, 2014 to Fall 2016, for each student. At the time, I had no thought about using their reflections for analysis, so the study is retrospective. (Since then, I did receive university IRB approval to analyse the data.) Data from a few students that did not write a reflection and students that did not take the final exam were eliminated before analysis. This resulted in reflection data for 254 students. Students that worked as partners could write separate reflections or write one reflection together if they both agreed to the content of the reflection.

2.4 Data Analysis

I used both qualitative and quantitative analysis techniques to analyse the data from these 254 students. Partners that wrote the same reflection were counted twice since the implication is that both felt the same way as reported in that reflection. The text of the reflections were analysed qualitatively using an inductive coding approach as described in the grounded theory method (Babbie, 2014, p. 405). However, it should be noted that theoretical saturation was not used because this study analysed fixed, historical data. Thus, the qualitative aspect of this study was not a true grounded theory approach but primarily used coding ideas from grounded theory. Of course, because most of the information comes from a content analysis of the reflection data, categories or themes were not known ahead of time. However, data that were available prior to the analysis were project scores, final exam scores, project type, and semester. Although the original student words and sentiments summarized in the quotes and content analysis are the most meaningful, I also used quantitative methods to analyse both emergent categories and prior data, and also to analyse relationships between these.

The reflections were coded in an iterative process by beginning with open coding and then continuing with focused (or selective) coding (Glaser, 1978). Thus, first an initial reading and review of the reflections indicated some possible categories or themes that appeared regularly in the reflections. Tentative categories were created based on that initial review. Then, a more thorough reading of the reflections was done with coding and categorizing. This round of focused coding caused a refinement of categories, including re-naming and combining categories, as well as creating sub-categories to parse out finer variations of the categories. This process was simplified by using an electronic spreadsheet, placing all reflections in one column and categories (or sub-categories) in new columns as they emerged from the data. Because the coding was done over multiple sessions, it was important to return to previously coded reflections to understand the meaning of those categories before going forward again.

3. Results

3.1 Categories from the content analysis

Several major categories emerged from the content analysis: Learning, Interest, Time Management/Organization, and Challenge. In addition, a few other less common categories included Partners and Confidence. Table 1 summarizes sub-categories that describe each of these categories.

Table 1. Categories resulting from coding student reflections

Category	**Description through sub-categories**
Learning (L)	Overall learning, Application, Comprehensive Coverage, Concepts, Particular topics (in the student project).
Interest (I)	General Interest, Enjoyed applying, Future interests, Fulfilling, Liked Options, Satisfied/Proud.
Time Management /Organization (T)	Good Time Management, Learned about Time Management, Poor Time Management, Put More Time into.
Challenge (C)	Workload (Much work/Time-consuming), Other challenges.

The Learning category indicated whether a student expressed learning gains and, if possible to identify, in what way. The sub-categories that emerged were general overall learning, applying statistics to real life, comprehensively covering the course, reviewing concepts, and learning about the topic of the student's particular project. Table 2 in the Appendix contains multiple examples of such quotes. Here is one:

"I think it is an excellent educational tool that allows students to not only understand elementary statistics, but also to practically utilize those skills."

In the Interest category, students expressed their interest in the project or other similar positive emotions, such as fulfilment or satisfaction. The sub-categories here included general statements of interest, enjoying applying statistics to real problems, creating future interest in statistical problems, fulfilling work, liking the ability to choose between options for the project, and being proud of the work. Here is one example from Table 3 in the Appendix:

"This project was very interesting overall!"

The Time Management category indicated a comment about time management and organization. Some students explicitly reported having good time management (no problems with time management or managing their time well). Some students reported having some issues with time management. A subset of these latter students reported learning from the experience (expressing positive reactions to it), while some were negative about it, for example blaming the instructor. Many students were silent about time management. Thus, the subcategories include statements indicating time was managed well, learning about managing time, poorly managing time, and wishing more time as put into it (either out of class or in class). Table 4 in the Appendix contains examples, such as

"I did not realize how big this project actually was until I started it and realized I was not going to have enough time to finish it by the deadline. I definitely should have followed the schedule provided for me, so I would have finished the project on time."

Finally, in the Challenge category students expressed the project as challenging in some way with a general sub-category and a workload (hard work or time consuming) sub-category. Table 5 in the Appendix provides examples, including:

"It was a difficult and time consuming project."

3.2 Quantitative analysis of categories

Over three-quarters of the students, 78.3% (199 out of 254), stated some kind of learning benefit, which makes sense since one of the reflection prompts suggested commenting about learning. The largest number of comments were on the great overall learning experience or learning how to apply statistics, with the fewest commenting about learning or reviewing concepts. This also makes sense since the project was focused on analysis, applications, interpretation, and evaluation of results while exams were used to test statistical procedural knowledge and some understanding of concepts. Approximately a third of the students reported learning in more than one way (two or more learning subcategories). Also, over two-thirds of the students, 67.7% (172 out of 254), stated some kind of interest characteristic, with most students stating they enjoyed or were interested in the project and many stating it was interesting to apply the statistics. Almost a quarter stated two or more interest subcategories.

Most of the students did not make a comment about time management. Of the 254 students, only 47 (18.5%) mentioned time management in any form. To break it down further, 15 (5.9%) of all students expressed they had good time management (managed their time well or had no problems with time management), while 32 (12.6%) of all students stated they had issues with time management. Of those 32 students, 16 expressed a negative statement about time management. The remaining of the 32 students expressed learning about time management from the project experience. Finally, 76 (30%) of all students stated it was challenging in some way (though not necessarily in a bad way), while 31 (12.2%) of all students specifically stated it was hard work or time consuming.

3.3 Assessment Score Analysis

Students did reasonably well on projects, especially compared to the common final exam. The median project score was 84% (mean 78%) yet the median final exam score was 71% (mean 67%). In addition, the variability of the project scores (standard deviation 18 percentage points) was lower than for final exams (standard deviation 21 percentage points). The relationship between project scores and final exam scores was also examined, illustrated in the scatterplot in Figure 1. Pearson's linear correlation coefficient for this relationship is 0.39. There is not a clear linear relationship, but it appears reasonable to conclude that the relationship is positive. When the scatterplot is broken down by project type, the resulting three scatterplots are not appreciably different from Figure 1.

Figure 1. Scatterplot of final exam scores versus project scores for all 254 students.

3.4 Relationships with project scores

Project scores did appear to be related to students' expression of learning, since the median score when no learning was reported was 75 (mean 74.1), and the median score when learning was expressed was 86 (mean 83.8), also illustrated in Figure 2. A Mann-Whitney test yielded a p-value of less than 0.001.

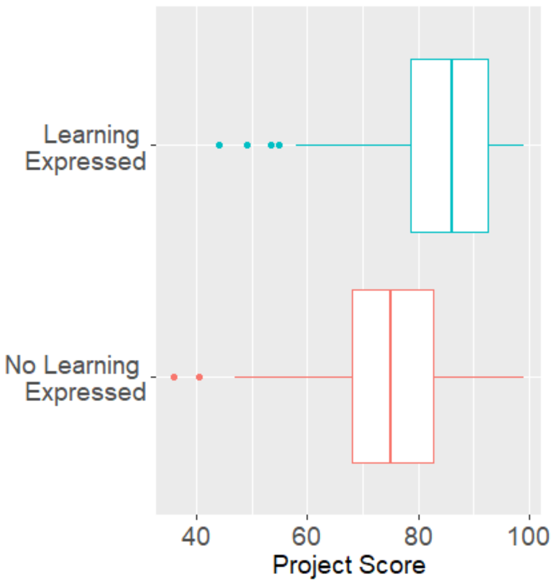

Figure 2. Project Scores by whether learning was reported. Out of the 254 students, 199 expressed learning from the project in at least one way.

The 15 students in the sub-category who explicitly said they had good time management appeared to do appreciably better on the project. The median score when good time management was reported was 90.5 (mean 86.0) and for all other students the median score was 84.0 (mean 81.5). However, the small sample and a borderline p-value of 0.05 with the Wilcox-Mann test implies a questionable significance. The relationship between time management and project score is supported somewhat by combining students that expressed they had good time management with students that had problems with time management but were positive about learning time management. There were 31 students that expressed either trait. The median score for students that expressed good time management or were positive about learning time management was 90.0 (mean 85.2) and for all other students the median score was 83.5 (mean 81.2), with a Wilcox-Mann test p-value of 0.02.

Project score did not appear to be affected by students' reporting the work being challenging in the Challenge category. However, within that category, some students (31) specifically said the project was hard work or time consuming. It was found that there may be some relationship between the project score and if the student specifically expressed the project being hard work or time consuming (Wilcox-Mann p-value of 0.05). The median score when hard work was reported was 88.0 (mean 85.0) and for all other students the median score was 83.3 (mean 81.3).

Project scores did appear to be affected by whether students had partners (Wilcox-Mann p-value less than 0.001). The median score of students with a partner was 86.0 (mean 84.7) and for students without a partner, the median score was 81.3 (mean 77.7). In addition, the variability of scores was appreciably lower for students with partners (standard deviation 9) versus for students without partners (standard deviation 15). Figure 3 illuminates the results about partners further, also showing the left skew of project scores for students without partners.

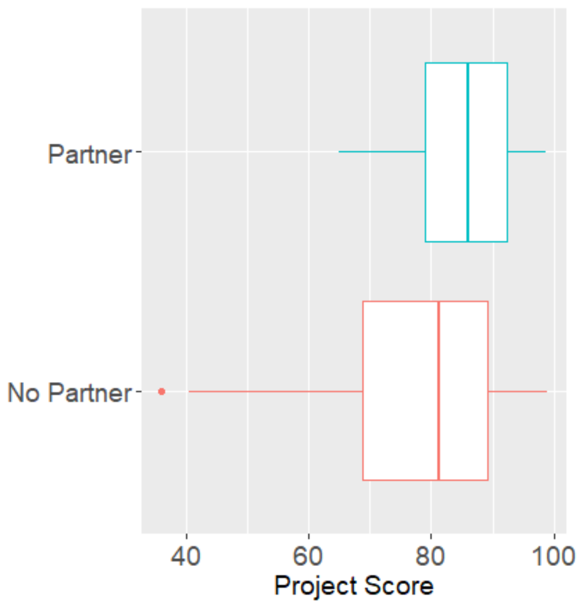

Figure 3: Project Scores by whether a student had a partner. Out of the 254 students, 107 did not have a partner. (Note that an odd number of students with a partner in the data occurred because one student with a partner did not write a reflection and so his/her data was dropped.)

3.5 Relationships with expressions of learning

The possible dependence of students' expression of learning on other factors is examined next, starting with time management. Recall some students reported good time management (having no problems with time management or managing their time well), some students reported having some issues with time management (a subset of these latter students reported learning from the experience, while some were negative about it), and some students were silent about time management. Again, note the low percentage, 18.5%, that expressed any sentiment about time management. A comparison of the learning expressed between students that reported a problem with time management and were negative about it (n=16) compared to all other students indicated that a negative attitude about time management may be associated with a lower expression of learning, but the numbers are too small to make any conclusions, confirmed using a chi-square test. In addition, unlike project scores, there was little difference in expressed learning between students that were positive about time management (good time management or learning from it) and the rest of the students.

Similarly to project scores, a student expression of learning does appear to be related to having a partner (Chi-squared test p-value of 0.003). Of students with a partner, 85% expressed learning, while of students without a partner, 69.2% expressed learning. Finally, there did not seem to be an appreciable difference in scores, learning, interest, or challenges across semesters or across the three types of projects.

4. Discussion

The analysis above documents and quantifies student expression and student outcomes on an authentic assessment. Students overall viewed the semester statistics project as a comprehensive, useful, and interesting learning experience, as illustrated by the quotes in Table 2 in the Appendix on comprehensive coverage and learning applications. In addition, a student's expression of learning was highly related to the project score. There could be a number of reasons for this relationship, but it seems reasonable to conjecture that students who think they are learning much from the project, aware of its importance in their learning, will try to do good work. This observation is related to the concept of self-regulated learning (SRL). Self-regulated is 'the degree that they [students] are meta-cognitively, motivationally, and behaviourally active participants in their own learning process' (Zimmerman, 1989). In this case, a reasonable conjecture is that if motivation related to learning is high, then outcomes will improve.

It is possible that the third reflection prompt may have implicitly biased students to state that they learned from the experience. In part of that prompt, students were asked: 'Reflect on the project experience as a whole from a personal point of view. For example, did you learn from it? ….' Thus, perhaps more diligent or ambitious students would make a point of mentioning learning and also those same students would do well on the project. However, the extent of some of the comments, stating not only whether they learned but what they learned (with a third of the students stating more than one subcategory), and the intensity of the student comments on learning, appear to indicate that there is more to those reports of learning than to meet a reflection component of the rubric or to please the instructor. For example, students genuinely expressed a range of areas of what they learned, including how to apply the content, comprehensive coverage of the material, deeper understanding of statistical concepts, and learning particular topics (such as about food insecurity), as illustrated in Table 2 in the Appendix. The reflection also had a low impact on their project score and saying something specifically about learning was not required.

Students also reported a high degree of interest in the project, as illustrated in Table 3 in the Appendix and in the content analysis. One of the prompts was *"Did you enjoy working with real data?"* which again may have biased student reflections. But, again, the depth and variety of the comments (with a quarter of the students expressing more than one category) seem to reveal genuine interest rather than to gain points on a low impact part of the project. It would be interesting to apply more formal attitude tools, such as described by Gal and Ginsburg (1994), to study the role of interest and beliefs about learning in determining outcomes in the context of semester statistics projects.

A positive result was how well students overall performed on projects compared to final exams, and more consistently as shown by the lower project score variability. There could be many factors related to these results, including that projects don't measure the same outcomes and learning characteristics as final exams. This is partly true in that the projects assessment is more inclusive.

The final exams focused mostly on procedures and some understandings of statistical and probability concepts, while the projects included application, interpretation, and evaluation, along with the procedures and some analysis. It is also possible that students approached a project more seriously than a final exam since the project was worth appreciably more than the final exam as a percentage of the final grade. However, the median score of the final exams for five sections over the several years prior to the use of the projects (with the same instructor) was a 74 (mean 73). This is still 10 points lower than the median project score. In other words, students did better on projects even when compared to students that had only a final exam as the last major course assessment (worth about 25% of the grade).

An expectation I had entering into this study was that a high proportion of students would say the semester project was a lot of work and time consuming. But the data did not show that. This conclusion is reinforced in that a relatively small percentage mentioned time management, good or bad. Although this seems counterintuitive, Kember (2014) examined case studies revealing that students' impression of workload is not equivalent to the time actually spent on tasks. He found that some factors that influence and correlate with a lower impression of workload include concentrating on concepts and understanding, active engagement of students ['projects seem to be particularly effective', (Kember, 2014, p. 182)], positive student-student-teacher relationships, and class coherence. All of these were generally present in this course.

On the other hand, students that reported the project to be hard work or time consuming tended to perform somewhat better on the projects than students that did not, although the difference was not large and the sample size of 31 is relatively small. One conjecture is that the expression of hard work may be an indication of persistence and how seriously the student took the project. Also, time management is important for a potentially time-consuming project, and the results show that. The data on time management, though not definitive and from a small sample, indicate that a student's approach to time management plays a role in success as measured by project scores. This is also supported by the results indicating that poor time management may be related to lower expressions of learning.

Having a partner did appear to improve student project scores, where students with partners scored better on projects and tended to express learning. These results seem to reflect the idea that teamwork and cooperative learning supports student learning and outcomes (Springer, Stanne, and Donovan, 1999; Gaudet, et al, 2010; Slavin, 2014). In addition, not having a partner appeared to produce higher variability in student scores. The primary source of this higher variability was a left skew, where 25% of non-partnered students had a project score below 69%. In their meta-analysis study, Springer, Stanne, and Donovan (1999) describe the positive motivational effects of group learning in science, mathematics, engineering, and technology. A semester project is a complex and intensive undertaking, and so it may be that some of the non-partnered students did not have the persistence and mutual support that is available to partnered students to finish the project completely and strongly. It would be interesting to explore this further to determine if this subset of students has particular characteristics, for example, under-represented groups, traditionally less college-prepared students (such as first generation students), or other characteristics impeding their reception to group work.

Finally, the type of semester project did not appear to be associated with outcomes and other factors. Thus, students creating their own survey and gathering their own data did not appear to influence their outcomes differently when compared to being provided authentic data that they could analyse instead. Thus, it may be that simply having interesting and authentic data regardless of the source is the important factor.

Because this study was done retrospectively and the data is mostly qualitative in nature, the conclusions made here are necessarily exploratory and tentative. In addition, the participants were not chosen randomly. However, one strength of this study is that several important factors were kept constant: one experienced instructor throughout, a consistent course curriculum and pedagogy in the classroom, the same assessment tool of the projects through a rubric, and a mostly consistent department-wide final exam. Except one semester, all students were attending the same university, and both universities were similar in purpose and types of students they served. The courses were always taught Tuesday and Thursdays during the day. The lack of differences in project and final exam scores across semesters is likely a consequence of the consistency in those factors. Another strength is that there is a significant number of participants. The student reflection questions were open-ended which can be both a strength and a weakness. As discussed above, there may have been implicit bias in these questions. Also, students expressed what is most important to them, yet every student did not express thoughts on every category. So, students could have felt a certain way on a characteristic but just didn't write it. The assumption was made that a student didn't feel strongly about that particular characteristic if the student did not report on it.

5. Conclusion

To conclude, this retrospective exploratory study utilized qualitative and quantitative methods to analyse student reflections by 254 students on semester statistics projects. It appears that many students working on projects believe they make important gains in learning on how to use real data within the big picture of statistical analysis and also many appear interested in such projects. Students generally performed better on a project than on a final exam. In addition, student expression of learning gains appeared to be strongly associated with improved project scores. Perhaps surprisingly, a low percentage of students reported problems with having enough time to complete the semester project. This study also supports the notion that cooperative learning, at least through partners, can be beneficial to student learning. Finally, these results suggest the value of future qualitative studies and case studies examining ways for students to be given assessments and activities where students feel they are having significant learning experiences and are actively engaged in authentic activities.

6. Acknowledgements

I am grateful for the advice and manuscript review generously given by Dr. Elizabeth Onasch at SUNY-Plattsburgh. In addition, I want to thank Rosa Canales for assistance in reviewing and analysing the student reflections.

7. Appendix

Table 2. Student examples of the Learning Category

Description through sub-categories	Quotes
L: Overall learning	*"Doing this also taught me to figure out when and how to use each test in different scenarios without a book telling me to use a specific test or graph to test and show my data."* *"I think it is an excellent educational tool that allows students to not only understand elementary statistics, but also to practically utilize those skills."*
L: Application	*"It was a good experience working with a project that had real data and was local."*

	"This was the most hands-on learning I can recall participating on and I feel that is how I am better suited to learn."
L: Comprehensive Coverage	*"This project served as a good review for inferential stats because we had to use many different types of tests. We had to do quite a bit of reviewing."* *"As a whole, I think this project is a great way to end the semester because it tied in everything we learned throughout the entirety of this course. I remember learning about qualitative and quantitative variables the first day of class."*
L: Concepts	*"It helped us understand the concepts within the class much more thoroughly."* *"Not only do I recognize the concepts I learned in class, but I also understand what those concepts measure and how they apply to answering a problem and forming a conclusion."*
L: Particular topics (in the student project)	*"This project taught both of us a lot about food insecurity. We now we realize that this is a growing problem and is more wide-spread than just large urban populations."* *"We thought it was super interesting finding out the different answers that we got and getting an idea of what the population of active individuals is doing with their workouts."*

Table 3. Student examples of the Interest Category

Description through sub-categories	Quotes
I: General Interest	*"This project was very interesting overall!"* *"The project as a whole was very engaging."* *"Overall, the project was a very interesting topic, especially when applied to our area right here."* *"Overall we enjoyed the experience since we were able to work at a pace that was more flexible toward our different schedules."* *"It was cool to put all of the information we had been learning over the semester into application and see what could happen."*
I: Enjoyed applying	*"So this has been my favorite final project in a college class since it has real world application and really is a*

	good representation of all that we have learned throughout the semester." *"It was enjoyable to research something that students deal with on a day to day basis and to use what we have learned to conclude the things that we did."*
I: Future interests	*"This project hit right on the spot for what I want to do for a career making it very interesting to me and made me want to learn more and do more research on the topic as to how I can help now and later on."*
I: Fulfilling	*"I enjoyed this challenging but fulfilling project!"* *"I think that I definitely grew from this project because it pushed me to really work hard and to look at connections from data, which I normally would not do if I hadn't been assigned to do so in class. As a nursing student, I don't really get to spend too much time making calculations and analyzing data too often, so I enjoyed this challenging but fulfilling project!"* *"Thank you for making me do this project and forcing me to realize some pretty awesome things."*
I: Like Options	*"We are glad that we had the option of doing the food insecurities project because it was really eye opening for us to see that the city of Eau Claire isn't as perfect as we thought."* *"We also appreciated the fact that we got to choose our own subject of research and also chose our options of how we would like to analyze our data."*
I: Satisfied/Proud	*"We are extremely happy with this project and our results."* *"I was proud of myself"* *"We are proud of our work."*

Table 4. Student examples of the Time Management Category

Description through sub-categories	Quotes
T: Good Time Management	*"We knew it was going to be a process and take quite a bit of time, so we met at least once most weeks throughout the semester. Because we managed our time well, we were able to finish on time with no struggles or rushing."*

	"This project wasn't too much of a workload as long as we stayed on top of it, which we did a pretty good job on."
T: Learned about Time Management or admitted to poor Time Management	*"I should have made sure that I managed my time better when trying to complete these end of the semester projects and papers. I should have made myself a schedule that I would work on a couple pages a day to complete it. I was able to complete this project, but I could of spread out the work more equally throughout many days."* *"I did not realize how big this project actually was until I started it and realized I was not going to have enough time to finish it by the deadline. I definitely should have followed the schedule provided for me, so I would have finished the project on time."* *"If I could give one piece of advice someone who is just starting this assignment, it would be create a schedule and stick to it."* *"I truly wish I had stuck to my timeline more closely."*

Table 5. Student examples of the Challenge Category

Description through sub-categories	Quotes
C: Hard work/Time-consuming	*"I really enjoyed this project, it was just a lot of work. Which is not necessarily bad if students keep on the pace you set for them."* *"It was a difficult and time consuming project."* *"This project has truly pushed both of us to new limits. This was a very challenging project."*
C: Other challenges	*"This project was not easy by any means, and took a lot of time and dedication and critical thinking."* *"Minitab was very difficult to use at times."* *"Personal things in my life also made problems arise. It was very hard to focus on school in general for a while and I put things on the back burner so that I could overcome the obstacles, which I am still working on."*

Quotes about confidence:

"We feel confident in our research and conclusions."

“I will definitely take away a lot of confidence and understanding of what we have covered all semester.”

Quotes about partners:

“Doing it on my own also made it more difficult because I didn’t discuss questions and conclusions I made with other people, preventing me from organizing my project and thoughts as well as I could of.”

“I enjoyed working with [name removed], and am very grateful that we each pulled our own weight. This project would not have been completed as thoroughly and on time, if we didn’t work together as well as we did.”

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CASE STUDY

Math students help their community develop balanced refuse collection routes

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Abstract

In fall of 2017, the Superintendent of Public Works for Kutztown Borough approached Kutztown University's Department of Mathematics seeking help in "re-balancing" refuse collection routes in the Borough of Kutztown. Historically (for many decades), refuse was collected two days a week on the south side (Mondays and Thursdays) of Main Street and two days a week on the north side (Tuesdays and Fridays) of Main Street. Wednesdays were used for recycling collection. Over the years, new housing development was primarily on the north side of Main Street. As a result of this development, refuse collection time had become "unbalanced"; requiring more time for the north side collection. During the spring semester 2018, several math majors in their last semester at Kutztown University developed a new refuse collection strategy. This strategy balanced collection times over the four collection days and just as importantly, minimized the modifications to the existing routes. Additionally, a minimum number of residents were impacted while accounting for future housing development. Their strategy has been successfully used in the Borough of Kutztown since August 2018.

Keywords: waste collection vehicle routing problem, operational research, undergraduate student projects.

1. Introduction

Kutztown is a borough in Berks County, Pennsylvania, USA, with a population of about 5000 inhabitants. For many decades, refuse collection was performed on Mondays and Thursdays for residents living on the south side of Main Street. Residents on the north side of Main Street had their refuse collected on Tuesdays and Fridays. Recycling is collected on Wednesdays for the entire borough. Over the years, new housing development has been primarily on the north side of Main Street. This has resulted in it taking longer to collect refuse on the north side of Main Street.

The Superintendent of Public Works for the Borough of Kutztown approached the Kutztown University (KU) Department of Mathematics toward the end of the 2017 fall semester. He was seeking KU's help in "re-balancing" Kutztown Borough's refuse collection routes. A primary goal of the re-balancing was to have each refuse collection day be approximately equal in terms of time needed to complete the refuse collection. A second major goal was to modify the existing routes as little as possible. A third goal was to impact as few residents as possible. It is important to note that the Public Works Department of the Borough of Kutztown felt that the existing refuse collection routes, subject to the constraint that south side collections were not mixed with north side collections, had been continually improved and fine-tuned over the years and were reasonably optimized.

In the next section we will briefly review the literature of vehicle routing for refuse collection. This will be followed by the solution approach and recommendation made by the KU math students.

Subsequently, we will discuss the implementation process of their recommendation and the educational benefits of this project for the students.

2. Approaches in the literature for refuse collection vehicle routing problem

Although the vehicle routing problem literature is quite extensive, the literature on vehicle routing applied specifically to refuse collection is quite limited. A small number of specific examples include Benjamin and Beasley (2010) who discuss a meta-heuristic for waste collection using multiple vehicles that takes into account customer time windows and driver rest periods. Buhrkal et al. (2012) use a large neighbourhood search to solve the waste collection vehicle routing problem with customer time windows. Malakahmad et al. (2014) use a Tabu search approach that optimizes waste collection routes to reduce route lengths and collection time. Finally, a paper by Wy et al. (2012) discusses a garbage collection problem dealing with construction sites and shopping districts that involves the use of huge containers.

3. A strategy for re-balancing refuse collection

3.1. Background

Based on the Public Works request, the professor of the two undergraduate operations research courses (OR) (MAT361 and MAT362) approached three students who had taken these courses as juniors and were already registered for their final semester at KU (Spring 2018). Although all three students had a full semester of courses, they were willing to register for two credits of independent study during which time they would study the refuse re-balancing problem and hopefully develop a solution strategy. Their professor, having several decades of industrial experience, was not optimistic that the students would be able to develop a practical solution approach to the problem in the limited time period of one semester - a little more than three months. He certainly did not expect that, if the students developed a practical solution strategy to re-balance refuse collection, that it would necessarily be implemented by Kutztown Borough. Remember that the current refuse collection routes had basically been used for several decades and the need for re-balancing the routes had been recognized for years, but no alternatives had been implemented to solve the problem. What chance did three undergraduate students with no former knowledge of the problem have to solve this problem in three months? However, the OR professor felt that, even if the students did not solve the problem, exposing the three students to a real-world problem provided them the opportunity to interact with real-world “clients” - both management and workers. This interaction would be helpful in further developing the students’ modelling and interpersonal skills.

3.2. Student solution approach

The OR professor stressed to his students that they needed to get familiar and thoroughly understand the real-world problem before they could expect to solve it. To that end, the students were interested in doing a “ride along” during refuse collection in order to get first-hand knowledge of the process. Unfortunately, for several reasons, this was not possible. The students needed to try to understand and solve the refuse re-balancing problem by studying a street map of Kutztown Borough (see appendix) and asking questions of the Public Works Superintendent. Typically, the refuse collection crew were not directly available for questions due to time constraints. The key aspect of the problem that the students focused on was the need to re-balance the collection days. Simply using Main Street as a dividing line was no longer a viable alternative, but how were they to better balance the collection routes with minimal disruption to the existing routes? Additionally, the students soon learned that even trying to obtain collection time data would be time consuming and not a straightforward task.

Given all the constraints of the problem, the students decided on the following strategy. They would develop a number of scenarios in which, during the south side refuse collection route, when the truck was at Main Street, it would cross over to the north side and do a limited amount of refuse collection and then return to the cross over location and continue with the south side collection as usual. Although not a very sophisticated approach, it appeared to be the best strategy for balancing the collection times with minimal disruption. The problem was to determine the best crossover location and which north side streets to collect during the crossover. After considering several options, the students determined that the best crossovers would be at either the west end of the borough or the east end of the borough. The reason for these two choices was that by including one of the ends of the north side collection with the south side collection, disruption to the north side route would be minimized. In contrast, if the collection truck, during the south side collection, crossed Main Street in the middle of the borough, there would be a "gap" in the north side collection route.

Since new housing development was taking place at the northeast section of the borough, it made sense to include the refuse collection for the northeast end of the borough with the south side collection route. After discussions with the Public Works Superintendent, it was decided that the recommendation would be that during the south side collection route, when the truck was close to the intersection of Main Street and Elm Street, the truck would cross to the north side of Main Street and collect refuse at the east end of the borough. This would include the new housing development on Seem Drive (see map of Kutztown Borough in the appendix). The collective opinion of the group was that this modified collection strategy would, on average, reduce the north side collection time by about 30 minutes and add 30 minutes to the south side route. Thus, considerably improving the balance between collection times of the new "south side" route and the new "north side" route while minimizing disruptions to the existing routes.

4. Implementing the recommended strategy for re-balancing refuse collection

The students' recommendation (discussed in the previous section) was accepted by the Public Works Superintendent of the Borough of Kutztown at the end of April 2018. However, the Public Works Superintendent wanted the students to present their re-balancing strategy to the refuse collection crew for their reaction and, hopefully, approval. At the beginning of May 2018 (right before the end of the spring 2018 semester), the students, their OR professor, and the Public Works Superintendent met with the head of the refuse collection crew who had been on the job for several decades. After the students explained their proposed strategy for re-balancing refuse collection, the crew head expressed concern as to what degree the proposed strategy would actually re-balance the collection routes. After additional discussion in which the students addressed his concerns, the refuse collection crew head felt that the proposed strategy was worth a try. Next, in early June, this recommendation was presented and accepted by the Public Works Committee. Later in June, this recommendation was accepted by Kutztown Borough Council. The new routes were implemented in August 2018 and are being used currently by the Borough of Kutztown.

In an email communication (dated November 15, 2018) between the Borough of Kutztown and KU, the Public Works Superintendent stated that "*Overall I am happy so far with changing the route….Depending on how the last areas of town that can be developed are developed we may have to revisit this again at some point down the road. For now this has given me what I was looking for. A more balanced route and less involvement by the public works guys to help the refuse guys.*"

5. Educational benefits and final thoughts

In their two undergraduate OR courses, the students had worked on group projects in which they solved complex (scaled-down) real-world problems involving strategic planning, both single-period and multi-period production planning and inventory problems, distribution and location problems and multi-period personnel requirements planning problems. However, in all these cases the professor was the “client” that the students needed to interact with when modelling and solving the problem. The refuse collection re-balancing project afforded the students the unique opportunity to see what was required to get their solution approach actually implemented in the real-world by interacting with real-world clients—both workers and managers.

The OR professor was very pleased with how his students appropriately confronted, solved and were successful at getting their strategy implemented especially given the short time that the students had to work on the problem. This project was certainly a very positive aspect of interview discussions that the students had for industrial OR positions.

6. Appendix

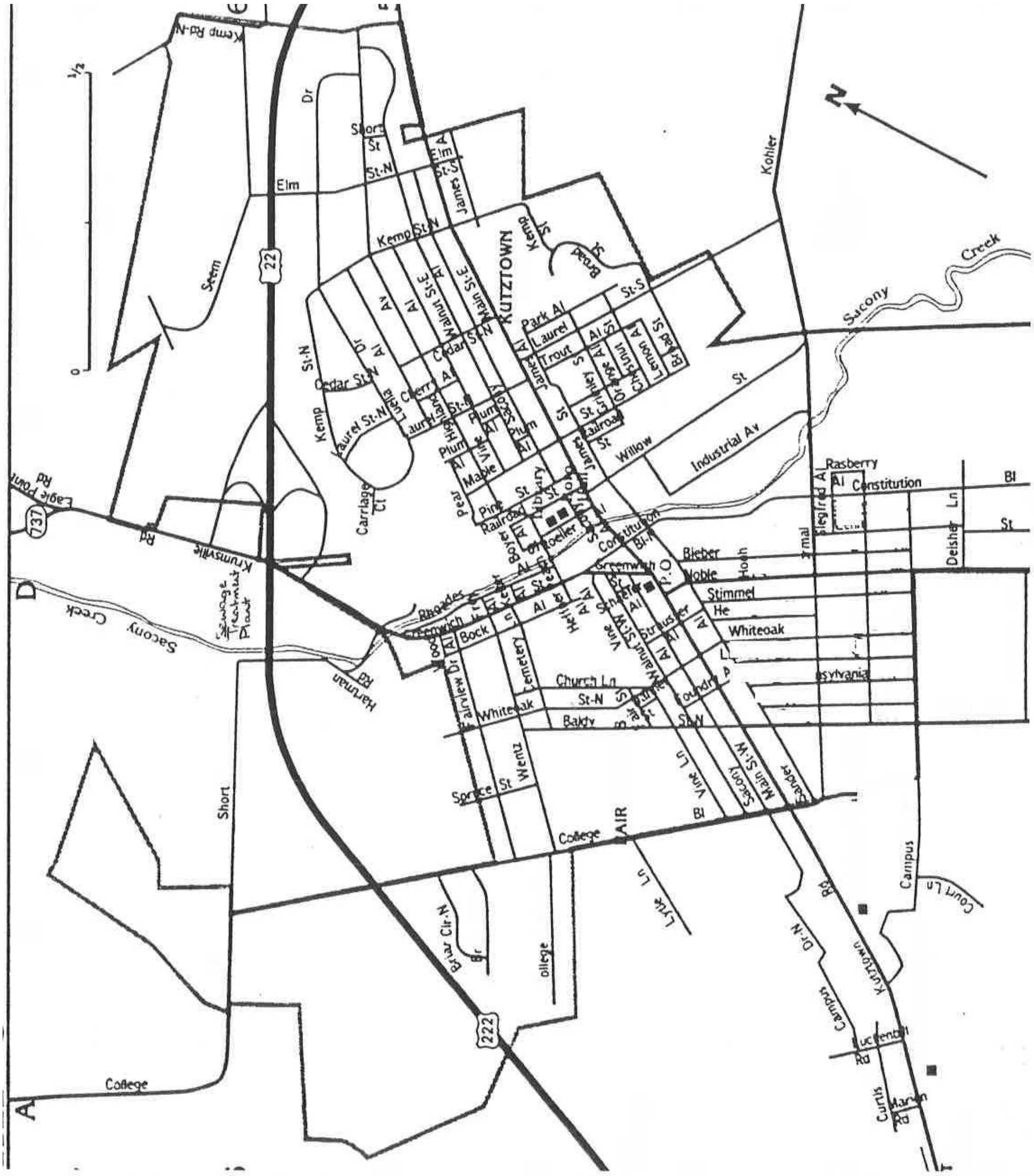

Figure: Map of Kutztown Borough

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RESEARCH ARTICLE

Mathematics support centre attendees and their use of online resources

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Abstract

In this paper we consider survey results which focus on student use of online resources, where all respondents were attendees of a Mathematics Support Centre. We see that while most respondents are engaging with online resources, some are not using resources in an appropriate fashion. We discuss how these preliminary findings might inform discussions on the merits, or otherwise, of institutions providing training for students on their use of online materials. We also identify several areas of potential further research.

Keywords: Online resources, mathematics support, usage, reliability.

1. Introduction

In Mac an Bhaird et al. (2020), we present an evaluation of the Mathematics Support Centre (MSC) website (http://supportcentre.maths.nuim.ie/) at Maynooth University (MU). The evaluation is based on findings from a survey issued to MU undergraduates in 2018. Prior to the development of the survey, a literature review identified that, to the best of our knowledge, no research on MSC websites had been undertaken. Thus, in order to give the website evaluation some local context, we decided to place additional questions at the start of our survey on student use of online supports in general. The responses to these questions are the main focus of this paper. To provide some broader international context to our website evaluation, we also sent a survey on the online presence of MSCs across Ireland and the UK to institutional co-ordinators of Mathematics Learning Support (MLS), and we will report on those findings elsewhere (Mac an Bhaird et al., in preparation).

In this paper, we provide a brief overview of the rationale for and methodology of our study. These are presented in more detail in Mac an Bhaird et al. (2020). The main focus of this paper is on the results, and a discussion on how this small study may link to literature and further research regarding why certain groups of students use online resources in particular ways. We close with some consideration of how our findings, albeit from a small sample, highlight a need for institutions to promote and advertise both online resources and appropriate student engagement with these resources.

2. Background

The MSC was established in MU in 2007 and, from the very beginning, a visible online presence was identified as playing a key role in student awareness of and engagement with the MSC, especially in terms of advertisement of services (Lawson et al., 2003). Initially our website was a basic webpage with operational information such as location, opening hours etc. It went through several iterations and now has a series of webpages featuring operational information and links to a large number of mostly external online resources. The online resources are organised according to subject area, e.g. Differential and Integral Calculus, Ordinary Differential Equations, Introduction to

Statistics etc. These subject areas are the normal source of queries with which 1st and 2nd year students attend the MSC drop-in.

The MSC normally provides 27 hours of drop-in each week to MU undergraduates. It is based in the Department of Mathematics and Statistics (the Department), and the majority of MSC attendees are students taught by the Department, i.e. degree and pre-degree (Certificate) undergraduates. The majority of students in our Department take ‘service mathematics’ courses, for example mathematics as part of a Science, Arts or Finance degree. Also, for all first year Science and Certificate students, mathematics is compulsory. Generally, students attend the MSC to work on their assignments, the results of which contribute to their continuous assessment grade. Students in the Department receive at least one assignment each week, and tutors at the MSC assist students if or when they get stuck but will not do the assignment for them.

Furthermore, the MSC also provides weekly workshops for the main first year classes, and workshop notes are placed on dedicated Moodle pages along with links to related online resources. Moodle is MU’s Virtual Learning Environment. Until 2019 there were also ‘on-demand’ workshops for other year groups, and the notes were hosted on the MSC website for one week.

In early 2018 we were made aware that the MSC might lose control of the layout and content of the MSC website. In order to challenge this potential change, we decided to try to evaluate the impact of our website. As outlined previously, a review of MLS literature identified no research in this area. Indeed, a recent paper, which gives a literature review of the evolution of MLS, identifies no research in this area (Lawson et al., 2019). Thus, we decided that it was important to conduct this research project, to inform our and potentially other practices in the provision of MSC websites and MLS online presences.

3. Methodology

In early 2018, we developed and tested a survey which was hosted on google forms. The survey had two parts. The first section, which is reported on in this paper, mainly focussed on students’ use of online resources. The second part related to the MSC website and is reported on separately (Mac an Bhaird et al., 2020).

Ethical approval was sought and granted for the survey and during April 2018, the survey was placed on Moodle. Via the Moodle messaging facility, all students registered with the Department had access to the survey and were invited to participate. The survey was also advertised in the MSC. In total, 99 undergraduates completed the survey, with responses from all academic years and all respondents were MSC users. The results of the survey were coded and analysed using the software package SPSS.

Responses to all questions reported on in this paper were crosschecked with other survey questions and issues of importance to the research questions were noted.

4. Results

All respondents were asked ‘Do you use online resources to assist with your MU coursework?’, 59 indicated that they did and 40 that they did not.

4.1. Students Who Use Online Resources (n=59)

The 59 respondents were asked to select how often they used online resources to assist with their MU coursework, with fixed options ‘At least once a day’ (10), ‘At least once a week’ (25), ‘At least once a month’ (16), and ‘At least once a semester’ (8).

Respondents were then asked to indicate which option best described how they used online resources. They were given five fixed options, and an option to give an open response which no one did. Students could select more than one answer. Forty selected ‘I use online resources to help understand material from my lectures’, 38 that ‘I use online resources to help me understand how to approach a homework problem’, and 30 that ‘I use online resources to help study for exams’. Fourteen respondents selected either ‘I use online resources to find exact worked solutions to a homework problem’ (10) or ‘I use online resources to replace any lecture material that I've missed’ (6). Of these 14 respondents, 13 also selected at least one of the other fixed options, and nine were Certificate or First Year students.

We asked respondents, in an open question, to briefly describe how they find the online resources that they use. There were 79 responses, of which 76 contained relevant information. Coded responses fell into the following categories: 29 MU or MSC specific guidance such as ‘MSC Workshops’ (11), ‘Staff Recommendations’ (9), ‘Other Student Recommendations’ (6) and ‘Moodle’ (3); 32 ‘Search Engine’ use, e.g. Google (21), and ‘YouTube’ (11); other responses highlighted specific websites such as ‘Khan Academy’ (10) and ‘Paul’s Online Notes’ (2); and three mentioned ‘Online Calculator’.

Respondents were then asked, in an open question, if they used a search engine to source online resources, to briefly describe how they choose which of the resources to use. There were 34 responses, mostly from ‘Search Engine’ respondents to the last question but also including some from respondents who had selected ‘Moodle’, ‘Staff Recommendations’ or ‘MSC Workshops’. Coding of responses gave two broad categories. Fourteen referred to using familiar websites or resources they have used in the past ‘I tend to look at YouTube views or websites/users I have seen before.’ with three of these (in response to a later question) stating that they always considered the reliability of online resources.

Twenty referred to new searches, in other words random or arbitrary searches and not previously used resources or specific websites. Fifteen of these 20 reported some level of consideration about the suitability of what they found ‘Whether the title of the link closely resembles what I'm looking for’. Six of these 15 always thought about the reliability of online resources. The remaining five (of 20) gave little or no justification ‘Just pick the first link that comes up’, though three of these indicated that they always thought about the reliability of online resources.

Respondents were asked to select what type of resources they found most helpful and, in an open response, to briefly explain why they found this type of resource helpful. The majority (30) selected ‘either video or text, depends what I find’, 20 selected ‘Videos’, and 9 selected ‘Text’. No respondents selected the ‘Other’ option. Of these 59 respondents, 48 provided the explanation requested. Forty-two of these had selected ‘Videos’ or ‘either video or text…’ while 6 had selected ‘Text’, and responses were spread across the following five categories of codes. Twenty-six liked the step by step nature and high-level of detail provided in online examples ‘Goes through examples explaining every step’. Nine made reference to the ability to go at a pace that they are comfortable with ‘It’s like being at a lecture that I can pause at any moment and replay’. Four preferred to use resources that appeal to their learning style ‘I'm a visual learner’. Two referenced the engaging nature of videos for their choice of resource ‘If you find someone that has a good personality and enthusiasm then video is much more enjoyable than reading pages of size 11 black text on an a4 piece of paper’. The final 6 gave very general responses, such as ‘everything helps’ or ‘both are equally helpful’.

Finally, in this section of the survey, we asked respondents if they thought about the reliability of online resources before they used them. They were given fixed options ‘Always’ (23), ‘Sometimes’ (33) and ‘Never’ (3). When we considered the various year groups, responses from undergraduates were spread across each of the fixed options, though for each year group the majority of respondents did not pick ‘Always’. All four Certificate students who answered this question selected ‘Always’.

4.2. Students Who Do Not Use Online Resources (n=40)

These 40 respondents were evenly spread across the various year groups and 37 of them reported that they attended the MSC at least once a day or at least once a week. They were asked why they did not use online resources, and of the responses given, 38 were relevant. Seventeen stated a lack of awareness of online supports. Eleven referred to a preference for physical one-to-one support 'I find that I can understand more easily from the tutors' explanations'. Nine of these 11 respondents were 3rd year or higher and they also selected elsewhere that they attended the MSC at least once a day. Three, two final years and one 1st year, cited a lack of suitable online material for their course 'Most of the time they don't seem fully relevant to my course material……which is different to the online resources available'. Three other respondents, two 3rd year and one 2nd year, said that they did not need online resources. Two felt that online resources are unreliable 'Don't trust the content majority of the time…', while the final two suggested that online resources do not help them 'Sometimes they don't help me understand it enough'.

The 40 respondents were also asked what would encourage them to start using online resources and 39 gave relevant responses. Seventeen referred to advertising and providing information about the available resources 'If people mentioned them and said that they were useful. Sometimes I wouldn't even think to look'. Eleven said they would use online resources is they were tailored to match the content of their lectures and assignments 'If they were personally adapted to our modules and courses'. Seven of these 11 respondents were 3rd year or higher. Three respondents suggested availability issues, and two said they would use online resources if they could not access the MSC 'If the maths support centre was no longer available for help'. There were also 5 other miscellaneous responses such as 'more time', 'I will now research the online resources' and 'Unsure exactly what I could personally further require'.

5. Discussion, conclusion and future work

The survey reported on in this paper and in Mac an Bhaird et al. (2020) is of narrow scope, and there are several papers which look at student use of (online) resources in a broader (non-MSC specific) context, for example Ní Shé et al. (2017) and Van de Sande et al. (2014). However, the responses to our survey do suggest preliminary discussion points on MSC attendees' use of online resources, which may be of use to those considering the expansion of their provision of online mathematical resources or, indeed, to any researchers who want to investigate this subject area in further detail.

Some of the data seems to paint a positive picture. In our Department, students have at least one graded assignment each week, and the majority (59) of the 99 respondents indicated that they were using online resources to assist with their coursework, and 35 of these reported using online resources at least once a day or once a week. Respondent approaches to finding online materials, for example via search engines, do not appear unusual when compared to those reported in other studies (Conole et al., 2006, Ní Shé et al., 2017).

When asked why they were using online resources, the majority of respondents selected options which mentioned understanding lecture material, understanding how to approach assignment questions or when studying for exams. This appears encouraging, for example Anastasakis et al. (2017), who considered links between student goals and their choice of educational resource, also reported high incidences of students expressing a desire to improve their understanding and to prepare for exams.

There is also some evidence that respondents' use of online resources '…(and perceptions of 'what works' best) are clearly being shaped by the university contexts within which students are situated as much as they are being driven by individual 'choice' and agency,…' (Henderson et al., 2015, p.1577). Twenty-nine of 76 responses mentioned MU guidance when referring to how they find the resources that they used. Furthermore, the reasons given by respondents for the helpfulness of the resource types that they used were very positive and are similar to those reported elsewhere (Henderson et al., 2015, Ní Shé et al., 2017).

Bray and Tangney (2017, p.265), state '…although use of technology in the classroom is increasing, its implementation in the mathematics classroom, and indeed in related research, still lags behind its perceived potential to enhance the learning experience'. As we carefully consider what future role we may take in the provision of online resources for students, there are several causes for concern arising from this study that require, at a minimum, further investigation.

Fourteen respondents indicated that they were not always using online resources appropriately, i.e. they used them to replace missed lecture material or looked up exact solutions to assignment problems. Nine of these respondents were 1st year or Certificate students, for whom mathematics is compulsory. This is worrying and suggests several avenues for research which lie outside the scope of this survey data. For example, are there links with existing research, such as that of Baynard reported in Hilton (2002), who noted that some engineering students viewed their first year mathematics course as an obstacle to overcome in order to study the topics they are interested in. Furthermore, while we did not ask for respondents' mathematical background, in a previous study (Berry et al., 2009) at MU we found that 1st year students who were at-risk of failing or dropping out were more likely to attend the MSC and stay for longer than their peers who were not at-risk. There is a danger, as reported in Chi et al. (1989), that 'poor' students are not capable of extracting the information they need from worked solutions to apply it to their problems. Institutions can play a key role in addressing these concerns. Also, as reported in Mac an Bhaird et al. (in preparation), there are other potential downsides for students who chose online resources instead of face-to-face support, as suggested by the following comment from an institutional co-ordinator of MLS:

"Some students may see ICT support as a sufficient alternative to visiting the MSSC and speaking to a tutor face-to-face. Without doing so, they may not correctly self-diagnose whatever issues they are facing and this may lead them to concentrating on material that won't help them overcome these issues."

Thirty-two respondents mentioned the use of search engines when finding resources, of these 20 used 'new searches' and only nine of these always considered the reliability of the resources that they used. Indeed, overall, just 23 of 59 respondents always considered the reliability of the resources that they used. Again, the institution has the opportunity to play an important role, as reported in our survey of MLS practitioners (Mac an Bhaird et al., in preparation) and highlighted by the following comment:

"I suspect that students just use google to search for materials rather than looking at the collated sets of materials collected on a site. I think where it can be helpful to have collections of resources is for lecturers who can link to these from their VLE module pages to ensure consistency and quality of additional resources provided to students that way. I think students will go to specific resources on a particular topic that are linked from the VLE page." [Institutional Co-ordinator of MLS].

If we consider those who reported that they did not use online resources, the most common reason given was lack of awareness, something that could be resolved within the institution, for example, via the lecturer or via an MSC if they provide links to online resources. Once the advertisement issue is resolved however, there is no guarantee that students will avail themselves of online resources. Respondents, mainly those towards the end of their degree, indicated a preference for 1-1 support. A similar point was raised by institutional co-ordinators of MLS in Mac an Bhaird et al. (in preparation), for example:

"…*it's just that our students value our face-to-face provision so much they find that much more useful, engaging, and suitable* [than online supports]."

When asked what would encourage them to use online resources, again the majority referred to appropriate advertisement. The second most common response was if the resources were more tailored to respondents' courses, with the majority of these comments coming from higher year students. These findings are consistent with what we found in Mac an Bhaird et al. (2020), when respondents were asked what would encourage them to use the MSC website and MSC resources.

We believe that our limited findings bolster one of the recommendations from Mac an Bhaird et al. (2020), that institutions should consider the provision of training to students on appropriate use of online resources as part of their education. There appears to be a need, amongst our student population at least, to be guided on how to source and evaluate online resources, as well as how to use them in conjunction with more traditional forms for learning. There also needs to better and continuous advertisement of the online resources that are available to students.

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