MSOR Connections

Articles, case studies and opinion pieces relating to innovative learning, teaching, assessment and support in Mathematics, Statistics and Operational Research in higher education.

Volume 20 No. 1.

CETL-MSOR Conference Special Issue 1: Responding to the COVID-19 pandemic

Contents

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This journal is published with the support of the **sigma** network and the Greenwich Maths Centre.

EDITORIAL

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Welcome to the first of two special issues of *MSOR Connections* containing papers from presentations at the CETL-MSOR 2021 conference held at Coventry University on 2nd-3rd September 2021. The conference was special for many reasons, including that it was the first CETL-MSOR conference for two years, the first to take place since the COVID-19 pandemic began, and the first hybrid version of the conference. As pandemic effects continue to be felt, this first special issue focuses on matters arising from it in maths, stats and operational research.

The first paper is written specially for the issue by one of us, Mark Hodds, who acted as chair of the CETL-MSOR 2021 internal organising committee and guest editor of this edition of *MSOR Connection*s. The article describes the processes of running a hybrid conference for the first time, offering advice, ideas, and support to anyone who plans to do something similar in the future.

The remainder of the issue contains a wide and interesting variety of papers based on excellent presentations at CETL-MSOR 2021 which discuss many issues surrounding pandemic learning.

First there are three papers that discuss the transition from school and college to university and the support that is then available. Price et al. describe the effects of the pandemic on A level learning showing that many topics were not covered adequately and there was an emphasis on pure knowledge at the expense of applied (statistics and mechanics) knowledge. Furthermore, they show there is evidence of an association between lockdown maths experience and mathematical confidence, suggesting that mathematics and statistics support services at universities maybe more crucial than ever. Indeed, mathematics and statistics support services had to quickly adjust to the working from home environment caused by the pandemic, and the paper by Smith discusses student preferences for online maths and stats support with anxiety factored in. Smith suggests that students have a preference for online over in-person support in their study, highlighting however that different centres will have different needs and facilities for their own students. Support should therefore be given according to the scenario as, for example, maths support is easier to do in-person than online when compared to statistics support. This is also shown in the paper by Gilbert et al. who reflect on how the views of mathematics and statistics support practitioners have changed from the start of the pandemic to the middle of 2021 when many measures were starting to be relaxed. The paper shows that initially 54% of practitioners in the study said online support was worse than in-person support but this had been replaced by acceptance in 2021 with 100% saying they would continue with online support after the pandemic. Flexibility and accessibility of online support were identified as key advantages for continuing with online support although this had waned when compared to the initial stages of the pandemic.

The next papers focus on the online teaching of mathematics with Russell discussing the use of "games" in remote active learning. To improve attendance in online synchronous activities, Russell encouraged students to take part in "games" and this resulted in 70% of students finding the sessions useful, as well as an increase in attendance. Alarfaj et al. discuss a survey conducted with lecturers in Calculus and Linear Algebra revealing some mixed views regarding the importance of offering on-campus classes once restrictions were lifted slightly. Many of the lecturers in the study preferred and were enthusiastic about retaining what had been created and provided during the pandemic, suggesting online learning is perhaps here to stay.

The following two papers focus on collaborative online learning. Jones et al. discuss the use of iPads at Middlesex University, something that has been provided to students since before the pandemic. The paper shows that iPads can facilitate communication online, allow easier collaboration between staff and students, enable quicker feedback that is more relevant, and encourage equality of access so long as all students have fair and equal access to them. Carey et al. discuss some changes that were brought in around the pandemic, resulting in better collaborative teaching and learning that will remain once the “new normal” begins. These included induction seminars, different question types, new mark schemes and changed feedback procedures, all of which were shown to be benficial to students’ learning.

The penultimate paper by Marshall et al. looks at statistics anxiety and the challenges faced when working online. The authors show that students are less likely to actively engage with material, ask for help, or work with peers in statistics scenarios when compared to in-person. Those with compulsory statistics modules felt it was less likely they could learn remotely but were more likely to ask for help when compared to students who did not have compulsory statistics modules. Statistics anxiety was however shown to be generally lower than in-person settings. Finally, Casey et al. present the development of a community of practice during the pandemic around staff-student partnerships. The paper gives two project case studies showing that shared experiences and communities of learning can be developed outside of traditional methods.

Producing an issue of *MSOR Connections* is always a big collective effort involving a host of authors and peer reviewers, and we are delighted to have had so many submissions – more than typical for an issue of *MSOR Connections* – due to the numbers of authors keen to write papers. We would like to thank all of the authors for their contribution to this special issue as well as all of the reviewers we invited to review these papers. It is brilliant to see such a vibrant community hard at work and willing to share its emerging practice. We would also acknowledge delays to production caused by ongoing pandemic effects and the UCU industrial action over ‘Four Fights’ and ‘Action for USS’, and we look forward to a day when our community can put the pandemic and the issues underlying the dispute behind us.

We will follow this issue with the second CETL-MSOR 2021 special issue on innovations in teaching, learning, assessment and support more broadly, and are hopeful of publishing this and a regular issue of *MSOR Connections* before the end of the academic year.

Don’t forget that you can submit a paper to *MSOR Connections* at any time or become a reviewer by visiting the journal website: http://journals.gre.ac.uk. The journal is always keen for writers and reviewers so please do consider making a contribution in the future.

WORKSHOP REPORT

CETL-MSOR 2021 – How did we do it?

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Abstract

In 2021 the Continuing Excellence in Teaching and Learning in Maths, Stats and Operational Research (CETL-MSOR) was hosted as a hybrid event for the first time. The event was attended by delegates from around the world both in-person and virtually through the use of Microsoft Teams live events. Presenters were also able to present from their homes or present with others in different locations, including at the conference venue in Coventry. This article is written by the chair of the organising committee and provides an insight into how the conference was organised and run behind the scenes, giving advice and feedback for future hybrid conference organisers to learn from.

Keywords: Hybrid teaching and learning, Hybrid conferences, CETL-MSOR, Online learning, Conference organisation.

1. Introduction

CETL-MSOR 2021 was the first hybrid version of the conference with delegates able to attend either virtually or in person. Hosted at Coventry University, presenters and delegates from around the world were able to collaborate and discuss issues and research regarding mathematics, statistics, and operational research under the theme of "*celebrating our past, embracing our future*". Although the main reason for hosting the conference as a hybrid event was the continuation of the Covid-19 pandemic, teaching and learning in the virtual world was increasing when the conference was awarded to Coventry in early 2020, so having an online option for attendance and presentation seemed natural. This article provides readers with an insight into how the conference organisers were able to provide a hybrid experience. It discusses the set-up, challenges, and feedback from delegates so that future conference organisers can learn and improve upon what was offered at CETL-MSOR 2021.

2. The set-up

Having decided to go with a hybrid conference, the choice of online platform to host sessions was vital in order to make it a success. The platform needed to allow for multiple presenters to present whilst being in different locations but also allow for delegates to view the presentations at the same time. It also needed to be a secure system that could not be shared to non-paying delegates. Having considered several systems, it was decided to use Microsoft Teams utilising their live events feature for the majority of presentations. There were several reasons for this choice. Firstly, it was free for delegates to join and therefore they would not need to subsidise the system. Indeed, the university had already paid for a licence for Microsoft Teams for use for teaching prior to the pandemic. Secondly, the host organisation was using Microsoft Teams throughout the pandemic so the staff involved in running the conference were confident and competent in using it. Thirdly, the live events feature meant the presentations were secure and could not be shared. The email addresses of delegates were added to the presentation stream ensuring that only they could get in to view the presentations. Even if the link was shared and clicked on by a non-paying "delegate" they would not be able to access the stream. Finally, having discussed options with several regular delegates, we found many other institutions were using Microsoft Teams also and were therefore used to the system. Trial sessions were used to ensure that delegates and presenters could access the events before the conference began and any issues found could be resolved.

Figure 1: Professor Ian Dunn gives his opening address in-person and live on Microsoft Teams simultaneously.

The downside of Microsoft Teams live events is that there is less interaction with participants watching the stream. Only text-based questions could be asked by online participants and these had to be moderated by our student proctors who were directing the streaming of the event. This did mean a solution had to be found where in-person delegates and online delegates could interact. In workshops, where interaction is key, Microsoft Teams meetings were used. These events allow anyone to present and speak although they are not secure. This meant that if the link was shared, anyone could access these sessions even if they were not a delegate of the conference. Outside of this, we used Big Blue Button to stimulate interactions between delegates. Big Blue Button is a separate conferencing platform designed for educational purposes. Coventry University purchased licences for Big Blue Button to deliver their online degree programs. The platform was also used by Coventry University's mathematics and statistics support service throughout the pandemic to provide online support and is still being used at the start of the 2021-22 academic year. Big Blue Button allows the creation of breakout rooms and, more importantly, anyone can control who goes in and out of them. Microsoft Teams has a breakout room feature but, at the time of writing, does not allow attendees to choose who goes in and out of each room. The breakout rooms were important to have if delegates felt they wanted a private chat with other delegates.

Links to each session were provided on the conference website so that online delegates could easily jump between sessions. The links corresponded to the physical rooms that presentations were taking place in and were colour coded to match the conference timetable. Within the physical rooms, a webcam and microphone and speakers, which were inbuilt into the design of the room, were provided. Student proctors, as mentioned previously, provided the directorship of the streams within the rooms, and were paid for their time throughout the conference. The proctors were able to control who was presenting, which webcam was streaming, and were able to mute microphones if necessary.

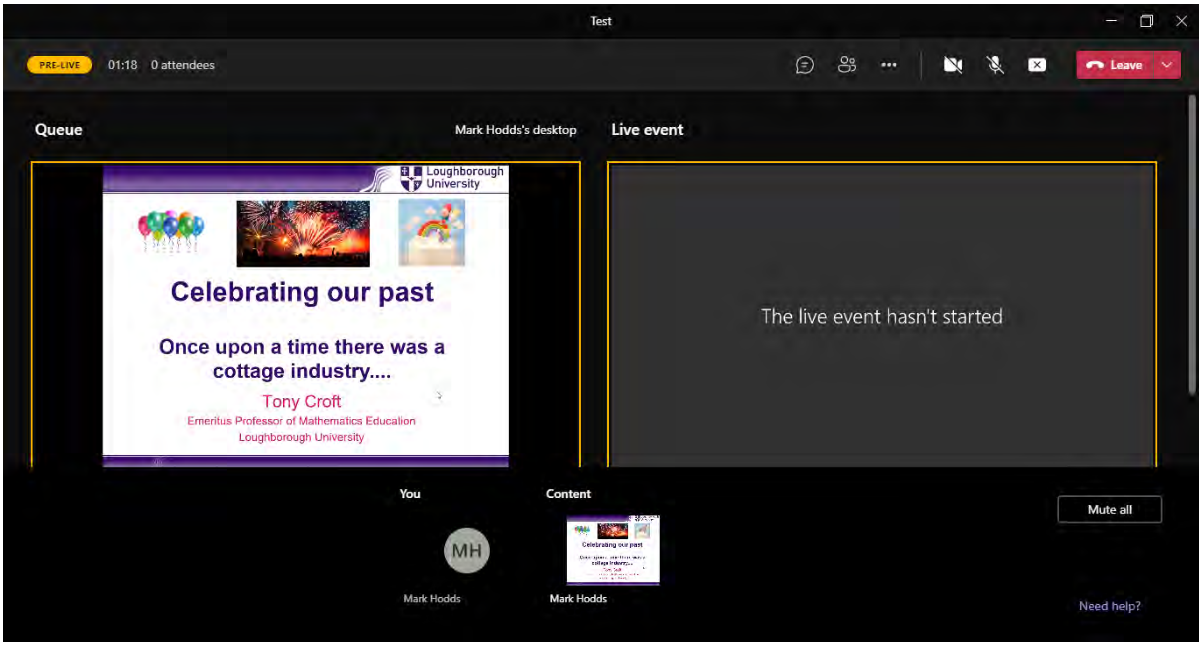

Figure 2: An example of the director screen in Microsoft Teams, controlled by the student proctors during the conference

We learnt quickly that the room microphone, for example, needed to be muted to prevent feedback if a presenter was presenting online. At this point it is worth expressing our gratitude to the proctors for the excellent job they did. They received one hour of training with only one external presenter to work with so to learn as quickly as they did and engage with the conference in the positive manner that they did was fantastic and really helped the organisers.

The in-person experience at the conference was effectively unchanged to that of previous CETL-MSOR conferences. Dedicated rooms were provided for the different parallel sessions with a main room for the keynote sessions. In each room however, large screens and speakers ensured that virtual presenters were seen and heard clearly. The rooms used were brand new and had not been used for teaching at the time of the conference due to the pandemic.

3. What did we learn?

The most important thing we learnt was it is possible to hold a successful hybrid event. We had presentations where a presenter was in the room in Coventry and their co-presenter was in Australia whilst someone in Ireland was watching, for example. In a scenario like this, a presenter in Australia was timetabled to be in the early sessions due to the time difference. We showed that collaboration around the world at the same time is possible and it can be done to a high standard of quality.

We also learnt that you cannot predict every eventuality. There are events that will happen on the day that no-one could forsee. However, delegates are understanding and as technology improves, as we learn and as we improve, the experience will improve. The way to deal with unforeseen events is to think about the situation and quickly provide the best solution given the circumstances. One example of this was a presenter who was unable to log in to their presentation session on Microsoft Teams despite being able to log in during the trial session and throughout the conference. Quickly we had to find a solution and that was to stream the session in Big Blue Button and share the webpage through the live event stream.

The workload and preparation of a hybrid event is much larger than a purely in-person or purely online event as you would expect. Research, testing, training, and trials of the systems need to be considered in advance. Furthermore, there is a lot of administration to go through at each stage. Having our own website which we could edit ourselves really helped as we were not relying on others or support from IT to update things when we needed it. A dedicated conference website which is editable by the organising committee is a must.

Something that required more consideration was how to get delegates to interact more. Indeed, having dedicated time to encourage in-person delegates to interact with online delegates would have improved the conference experience for many, particularly those online. There is however a balance to be considered as having a dedicated time for interactions reduces the number of presentations that can be delivered.

Finally, a list of instructions for chairs of each session needs to be provided. We did provide this set of instructions and a quick two-minute training session/introduction was given to some of the chairs who arrived early at the conference. Going forward, more emphasis would be needed on reminding chairs to repeat the questions of delegates in the room as often the microphone did not pick up those questions. We also found the student proctors provided more of the warnings regarding timings of presentations than the chairs did. This was because from the producer role of the live event, the proctors were able to text chat directly to the presenters and therefore this reduced the interruptions to the flow of the presentations.

4. Feedback from delegates

A survey was conducted to obtain feedback from delegates from the conference. Although the survey was sent to all delegates, only 26 delegates responded and all of whom attended virtually. It is perhaps that those who attended in-person felt the experience was similar to previous CETL-MSOR conferences and did not feel the need to comment, although this is unknown. The results of the general feedback questions are provided below:

Table 1: Survey results from delegates on general feedback questions

	Strongly Agree	Agree	Neither Agree or Disagree	Disagree	Strongly Disagree
CETL-MSOR 2021 was a successful event	21	3	2	0	0
CETL-MSOR 2021 was better than past versions of the conference for being a hybrid event	5	6	14	1	0
CETL-MSOR 2021 was well organised	21	5	0	0	0
CETL-MSOR 2021 utilised technology in the best possible way	12	10	4	0	0

I would not have been able to attend CETL-MSOR 2021 if it had been in-person only	18	3	1	4	0
Being able to view the recordings of talks at CETL-MSOR 2021 is useful	18	6	2	0	0
Knowing there were online and in-person delegates at CETL-MSOR 2021 made the conference experience richer	6	5	12	3	0
Microsoft Teams Live Events was the right choice to host online sessions at CETL-MSOR 2021	4	10	7	4	1
Big Blue Button was the right choice to host the discussions with other delegates at CETL-MSOR 2021	1	1	18	5	1

This feedback suggests that delegates thought the conference was well ran and provided the best experience given the constraints of the technology used. Despite this, delegates were mixed on the use of Microsoft Teams and Big Blue Button however some of this may be down to personal preference. The majority of delegates thought being able to see the recordings of the talks after was useful with several commenting on how at previous CETL-MSOR conferences you only get to see one talk in a parallel session when you may have wanted to see several. The recordings allow for delegates to see as many talks as they want after the event but watch live the presentation they most wanted to see.

We also asked about the experience as an online delegate specifically and the feedback is provided in Table 2 below.

Table 2: Survey results from delegates on being an online delegate

	Very Easy	Fairly Easy	Neither Easy or Difficult	Fairly Difficult	Very Difficult
How easy did you find it to ask questions and get answers?	8	11	6	0	1
How easy did you find it to listen and follow the presentations and talks?	17	8	0	1	0
Once connected to Microsoft Teams, how easy did you find it to use the technology and the conference website to navigate between sessions?	14	6	2	3	1

Here we can see the majority of delegates found the online experience easy to follow. However, when asked whether delegates felt they were part of the conference community when compared to attending in-person, 76% of respondents said they felt less a part of the community than in-person. This is perhaps due to in-person delegates not utilising Big Blue Button to connect with online delegates and there not being a scheduled time for interactions as discussed previously.

Finally, we asked presenters to provide feedback. Of the 26 respondants to the questionnaire, 15 were presenters. Their responses to our questions are provided below in Table 3.

Table 3: Survey results from presenters on their experiences

	Very Easy	Fairly Easy	Neither Easy or Difficult	Fairly Difficult	Very Difficult
How easy was it to interact with the in-person audience?	2	4	5	3	1
How easy was it to interact with the online audience?	2	2	4	6	1
How easy did you find it to present with your colleagues? (9/15 presenters had other presenters)	3	4	0	2	0

We can see that from the presentation side, generally it was a smooth experience but the lack of interactions certainly unnerved many people. One presenter commented that they did not like not knowing how many people were watching and that it felt like they may have been presenting to no-one. However, some presenters said they were used to this as it was similar to how it felt during the pandemic when teaching to students who did not put their cameras on. The student proctors were able to see how many people were in the virtual room but the presenters were not so perhaps we could have put a text comment to let presenters know how many people were watching. Going forward, although Microsoft Teams has many useful advantages incluing the needed functionality of only allowing delegates to attend, an alternative solution maybe needed where the presenter can see how many people are in the virtual room and be able to interact with them more.

5. Conclusions

Putting on the first hybrid version of CETL-MSOR was no easy feat but the outcome shows that it was possible. Right up to the night before, different scenarios and issues with the technology were being considered. As mentioned however, you cannot predict everything that may happen. Having some ideas regarding back-ups is helpful but remaining calm is the most important thing to obtain a logical conclusion. Hybrid conferences are going to be more and more common as many institutions will no longer be able to afford to send staff to conferences due to the pinch of the pandemic, environmental concerns, and Brexit. The technology will improve and we will improve as we learn to work with online systems in better and more efficient ways.

My advice to future hybrid conference organisers is to get as many people on board to help before and during the conference as possible. Utilise expertise to make sure you have a smooth online experience whilst providing the traditional in-person experience at the same time. Find a system that allows protection for paying delegates but allows for greater interactions. If the conference is free and hybrid, Microsoft Teams meetings will do a good job but the live events feature could certainly be better for paying delegates.

I hope this short article has provided an insight into how we hosted CETL-MSOR 2021. The hybrid world will be with us for years to come so future conferences need to be prepared to hold them as hybrid events. I would be very happy to discuss and share further ideas and thoughts with anyone planning to host a hybrid conference in the future.

6. Acknowledgements

Thanks to the conference supporters and sponsors for making the event happen, to the internal and external organising committees for all their support, and to **sigma** Coventry staff and student proctors for all their help on the two days and in the lead up to the conference.

7. Links

The conference website is available at http://sigma.coventry.ac.uk/cetlmsor2021 and those who were delegates are able to access the recordings of the sessions through the links on that page.

RESEARCH ARTICLE

The Impact of COVID-19 on Year 13 A level Mathematics Students: findings of a small-scale survey

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Abstract

In England, students who were awarded A levels in the summer of 2021 experienced disruption in their education during Years 12 and 13 (i.e., throughout the whole of their A level studies) due to the COVID-19 pandemic. National examinations were cancelled in 2021 (as in 2020) and students were given Teacher-Assessed Grades (TAGs). In determining these grades, teachers were instructed to judge students' achievements in relation to the material that had been covered during Years 12 and 13 rather than against the complete A level syllabus. This small-scale study investigated potential impacts of the unusual experience of Year 13 A level mathematics students on factors including topics covered, confidence in mathematics, post-A level intentions and readiness for higher education, with a view to considering challenges around the transition to higher education. Data were gathered using student and teacher questionnaires. In total, 174 students and 27 teachers from 19 schools/colleges across the country (including one college in Wales) completed the questionnaires. This paper gives a high-level overview of the main messages emerging from the questionnaire responses. It is intended to undertake a more in-depth analysis of the data at a later date and report the findings from the more detailed analysis in subsequent papers.

Keywords: Impact of COVID-19, A level mathematics, syllabus coverage, mathematics confidence.

1. Introduction

The cohort of students in England that began studying for A levels (aged 16+) in September 2019 experienced two years of study vastly different to previous cohorts. The 2019-20 academic year began as usual, but concerns around COVID-19 were increasingly reported in the news. By March 2020, England had gone into 'lockdown' (ONS, 2021). This included the closure of schools and colleges to all but the children of key-workers and, in a very short space of time, education moved primarily online. From mid-June 2020 this cohort of students was permitted to return for some face-to-face teaching. From this time, the A level experience for the majority of these students was a mixture of online learning and in-class teaching in bubbles. Recent evidence suggests a decline in both the quality and quantity of learning during the period March 2020 to March 2021 (Howard et al, 2021).

It was quickly recognised that the disruption caused by the pandemic meant that, in some institutions at least, it would not be possible to cover the complete A level syllabus in the same depth as usual and, indeed, that some topics may not be covered at all. Guidance issued to teachers by the Joint Council for Qualifications in March 2021 openly acknowledged this, advising teachers that the process of determining grades for their students should:

"*Look at the specification that has been taught to consider:*

- *what content has been taught?*
- *what content has not been taught to this cohort because of the impact of the pandemic?*
- *has the content that has been taught been covered deeply or superficially?*

The evidence used to make judgements must only include the appropriate assessment of content that has been taught" (JCQ, 2021a, p. 20)

While there were good reasons to assess students only on topics taught, the possibility of gaps in content learned, alongside the general disruption to education, poses potential problems for students transitioning to higher education as well as for the higher education providers themselves. With a view to exploring the issue of transition to university following the disruption caused by the pandemic, this study focused on three main themes: content covered in year 13 A level mathematics programmes during the academic year 2020-21, confidence in mathematics and post-18 intentions.

2. Methodology and Respondent Demographics

The group of interest for this study is Year 13 A level maths students in England from the academic year 2020-21. The researchers developed two questionnaires to investigate a range of issues related to learning in mathematics and the potential impact of the Covid-19 pandemic on the learning experience: one for students and one for teachers. The full questionnaires can be found in the Appendix. In this paper, we will focus on issues relating to the coverage of the A level syllabus, the impact on students' mathematical confidence and on their intentions following their A level studies, and readiness for higher education study. Additional and deeper analyses will be presented in subsequent publications.

It was expected that students would answer their questionnaire based on their own individual experiences but that teachers would reflect on the experiences of their classes as a whole. As such, in addition to providing triangulation, it was anticipated that some of the potential bias that might result from the self-selecting nature of the student respondents could be offset by the more holistic assessment of the teachers.

The study began late in the academic year in April 2021. Recognising the challenge of securing student responses to the questionnaire at this time, it was determined to distribute the questionnaires as widely as possible. Participants were primarily sought through mailings to relevant teacher networks, such as the MEI (https://mei.org.uk/) and the Royal Society's ACME Contact Groups for A Level Mathematics and Post-16 Pathways (https://royalsociety.org/about-us/committees/). In addition, colleagues within Higher Education networks, such as the **sigma** Network, were asked to share the invitation to participate with their teacher contacts. These mailings provided a brief description of the study and asked interested teachers to contact one of the researchers. The two questionnaires were made available using JISC Online Surveys (https://www.onlinesurveys.ac.uk). Once a teacher had indicated their willingness to complete the teacher survey and to distribute the questionnaire to their students, they were sent the appropriate links. The survey links were unique to each institution, thus enabling us to distinguish responses across the institutions. As such, although the initial step of asking teachers to contact the researchers may have hindered uptake of the survey, it was a crucial step in the process. Teachers and students were provided with a detailed information sheet and gave informed consent to participate in the study. All participants, including the student respondents, were aged 18 years or over at the time of completing the surveys, and ethics approval for the study was obtained from the Coventry University Research Ethics Committee.

A total of 18 schools and colleges participated in the research from across England and 1 college in Wales. Although the focus is on England, we decided to include the latter to maximise data usage and because lockdown restrictions were similarly imposed in Wales in March 2020 (Senedd Research, 2021). In summer 2021 there were 90,047 A level mathematics entries in England (JCQ, 2021b, p.12).

Our sample is therefore small relative to this population and also clearly self-selecting. We therefore make no claim that the sample is representative of the overall population.

Not all schools provided both student and teacher data and we obtained responses from 174 students across 17 institutions and 27 teachers across 16 institutions. The mean number of student responses per school was 10, with a minimum of just one response and a maximum of 21. The demographics of the student respondents are summarised in Table 1. It can be seen that a little over half of the respondents were male (54.6%) and the respondents were predominantly White (74.7%). Since we did not ask schools for a breakdown of their cohorts, it is unclear whether this breakdown of gender and ethnicity is representative of the participating schools.

The respondents were overwhelmingly high achievers in GCSE mathematics with 69.0% obtaining a grade 8 or 9 (A*). According to the MEI, "Most schools and colleges require at least a grade 6 for entry to an A level Mathematics course" (MEI, 2021). However, the respondents had almost all achieved grade 7 (A) or higher (90.8%).

Table 1: Demographics of student respondents (n = 174).

Characteristic	Count	Percent
Gender:		
Female	76	43.7
Male	95	54.6
Other/not stated	3	1.7
Ethnicity:		
White	130	74.7
Asian	27	15.5
Other/not stated	17	9.8
Attainment in GCSE Maths		
Grade 8, 9 or A*	120	69.0
Grade 7 or A	38	21.8
Grade 6 or B	12	6.9
Did not study GCSE maths	3	1.7
Not stated	1	0.6

3. Coverage of Syllabus

In order to ascertain how well the A level mathematics syllabus had been covered, we asked both students and teachers to comment on the coverage of twelve advanced mathematics topics, primarily focusing on pure maths. The majority of the A level Mathematics syllabus is pure mathematics (approximately two thirds) with the remaining content being mechanics and statistics - see DfE (2016) for full details. In the teacher questionnaire, they were asked to respond relative to their usual coverage of these topics (Q10, Appendix). Teachers were also asked more generally if they had covered the A level maths syllabus as completely as usual (Q9, Appendix), and both questionnaires included space for free text comments for further elaboration.

Some 63.0% of teachers (n=17) stated that some parts of the syllabus that are usually covered had not been covered since the pandemic started, with a further 6 teachers (22.2%) indicating that the syllabus had been covered but some parts had not been covered as thoroughly as usual. Only 4 teachers from 3 schools stated that the syllabus had been covered as thoroughly as usual. A Director of Mathematics from one of the sixth form colleges responding to the survey provided the following insight:

> "*We focused on covering pure content as thoroughly as possible. This meant that we ran out of time to cover all of the Year 13 Applied topics. We factored this into our final TAG* [Teacher Assessment Grades] *assessment papers which were heavily skewed towards Pure."*

This statement supports a larger-scale survey by the Joint Mathematical Council for the UK (2020) which found that, "More than half of A level mathematics teachers changed the Scheme of Work this academic year in order to accommodate students' learning gaps and requirements for blended learning".

With regard to the coverage of the twelve advanced mathematics topics, Table 2 shows the percentage of teacher respondents who indicated that they had covered (taught) these topics as completely as usual and the percentage of student respondents who said that they had definitely covered (been taught) the topic.

Table 2: Percentage of teachers indicating "covered as thoroughly as usual" and percentage of students indicating "definitely covered" on 12 advanced pure mathematics topics.

Topic	Teachers % (n=27)	Students % (n=174)
Proof by contradiction	63.0	65.5
Partial fractions	92.6	82.2
Parametric equations	70.4	82.2
Arithmetic sequences and series	77.8	83.3
Geometric sequences and series	77.8	87.4
Secant, cosecant, cotangent	88.9	69.5
Formulae sin(A+B), cos(A+B), tan(A+B)	85.2	92.0
Points of inflection	77.8	78.7
Chain rule for differentiation	92.6	93.1
Parametric differentiation	81.5	73.0
Integration by substitution	74.1	81.0
Integration using partial fractions	74.1	75.3

The results in Table 2 show that, from the teacher perspective, there has been a reduction in the thoroughness of the coverage of these topics during the pandemic compared to usual practice. In particular, there are seven topics where fewer than 80% of teachers felt that they had covered the topic as thoroughly as usual, with proof by contradiction standing out as a particular weak spot (63.0%).

Since the students could not compare with previous years, they were simply asked to indicate if they felt that they had covered the topics. Even with this lower threshold, there were still five topics where fewer than 80% of students indicated that they had definitely covered the topic and only two where more than 90% were certain they had. Although the teachers' question and the students' question were not measuring precisely the same quantity, it is interesting to note that the topic that fewest students were sure had definitely been covered was proof by contradiction (65.5%), similar to the teachers' response.

Students were given the opportunity to comment on topics they felt they did or did not cover during their A Level Maths studies. The full list of comments is shown in the Appendix. Almost all these comments relate to topics not covered. There are a small number of comments that simply list topics with no further context. However, it seems reasonable to infer that these students meant these as topics not covered, since students would be unlikely to limit their response to just those topics if these were those that were covered. Figure 1 shows a word cloud of these responses for 25 students who made a comment. The size of each word reflects its frequency, with larger words occurring more frequently.

Figure 1: Word cloud from comments about topics the students did or did not cover during their A Level Maths studies.

Although based on a small number of comments, the word cloud helps to corroborate the previous statement from the Director of Mathematics at one of the sixth form colleges, which suggested that emphasis was placed on pure topics at the expense of applied topics. From Figure 1, the words Mechanics, Vectors, Numerical Methods and Stats stand out, with terms such as Normal, Distribution and Applied also visible in the background.

4. Mathematics Confidence

We asked both students and teachers to comment on confidence in mathematics, with an emphasis on changes over the past year (Q12 student and Q11 teacher respectively, Appendix). Students were asked to reflect on their own confidence levels and also how they had found the experience of learning mathematics since the first COVID-19 lockdown. Teachers were asked to reflect on the confidence levels of their students, thus taking a view across the whole cohort. Table 3 shows changes in confidence level against the experience of learning mathematics during the pandemic for the students (Q14, Appendix), and Table 4 shows the correlation between them, broken down by school, assessed using Spearman’s correlation coefficient. Table 5 summarises the teacher perspective on the students’ confidence in mathematics as compared to previous cohorts.

Table 3: Student experience of learning mathematics against changes in confidence since the first COVID-19 lockdown in March 2020 (n=173)*. Percentages are shown across rows.

Q14: Since the first COVID-19 lockdown, I have found learning in mathematics:	**Q12: Over the past year, my confidence in my mathematical abilities has:**			
	Reduced	**Stayed the same**	**Increased**	**Total**
Easier	2 (7.4%)	4 (14.8%)	21 (77.8%)	27 (100%)
About the same	5 (9.4%)	25 (47.2%)	23 (43.4%)	53 (100%)
More difficult	62 (66.7%)	16 (17.2%)	15 (16.1%)	93 (100%)
Total	69 (39.9%)	45 (26.0%)	59 (34.1%)	173 (100%)

**1 student with missing data*

Table 4: Spearman's correlation coefficients between confidence in mathematical abilities (Q12) and experience of learning mathematics since the COVID-19 lockdown (Q14), broken down by school.

School*	**Spearman's correlation**	**N**
1	0.189	9
2	-0.066	7
3	-0.158	17
4	-0.408	5
5	-0.452	8
6	-0.504	21
7	-0.559	5
8	-0.621	8
9	-0.650	11
10	-0.667	18
11	-0.668	17
12	-0.717	9
13	-0.739	12
14	-0.794	11
15	-0.800	13

** Note: two schools did not contribute enough data to undertake this analysis*

Table 5: Teacher perspective on students' confidence in A level mathematics compared to previous maths cohorts (n=27)

Confidence is…	Count	Percent
Lower	19	70.4
About the same	6	22.2
Higher	0	0.0
Not sure	2	7.4

Since the first COVID-19 lockdown in March 2020, 93 students (53.8%) found learning in mathematics more difficult and 69 students (39.9%) stated that their confidence in their mathematical abilities had reduced (Table 3). There is strong evidence of an association between the experience of learning mathematics during the pandemic and mathematics confidence; $\chi^2 = 74.6$, df=4, $p < 0.001$ (chi-squared test). From Table 3, we can see that 66.7% of those who found learning in mathematics more difficult since the first COVID-19 lockdown stated that their confidence in their mathematical abilities had reduced (62 out of 93). This contrasts with just 9.4% and 7.4% for those who had found learning in mathematics about the same (5 out of 53) or easier (2 out of 27) respectively. After digging a little deeper, we can see a moderate to strong negative association between maths confidence and learning experience in mathematics across nearly all of the schools separately (Table 4). In other words, a decrease in maths confidence correlates with finding learning in mathematics more difficult since the COVID-19 lockdown. From the teacher perspective, 70.4% of teachers (n=19) felt that their students' confidence in mathematics was lower compared to previous cohorts (Table 5).

There is a sense from this data, particularly from the teachers who have reflected on their full cohorts, that mathematics confidence has been negatively impacted during the pandemic. There are, of course, a number of considerations to bear in mind when interpreting this data, not least the fact that the student respondents were predominantly high achievers at GCSE and their responses may not be representative of the whole cohort. We also need to be mindful of the fact that mathematics confidence and learning experience may naturally change with the transition from Year 12 to Year 13, regardless of the pandemic conditions. However, if a sizeable number of these higher achieving students have experienced struggles in their learning with accompanying reduced confidence, it seems plausible to hypothesise that the cohort-level picture may look bleaker.

When asked about A level grade expectations (Q15, Appendix), 66 students (37.9%) felt that their A level grade in mathematics would be worse than before the pandemic, and 53 of these felt that their confidence in maths had reduced since the first lockdown. For those with lowered grade expectations, we asked them to comment on why they felt this way. From the free text comments, words/themes such as motivation, mental health, struggle and difficult stood out by their repetition. Comments included the following:

> "*Just had difficulty with motivation, so I was not on top of the work"*

> *"Overall my mental health worsened as a result of increased isolation and decrease in support during lockdown"*

> *"I struggle to teach myself maths and I struggle to learn maths through a video call. I need a staff member with me to help me and consolidate things"*

When asked to comment on reasons for changes in student confidence, the teachers used words such as ‘lack’ and ‘less’ (Figure 2). Their comments indicated a number of areas where things were lacking for the current cohort; some examples are given below.

> “*They lack the help they normally get in the classroom”*
>
> *“Lack of time with teachers and other students”*
>
> *“I think the main problem is the lack of reinforcement over time”*
>
> *“… lack of resilience …”*

Taking the student and teacher responses and comments together, it seems clear that this cohort have found their A level mathematics experience to be challenging.

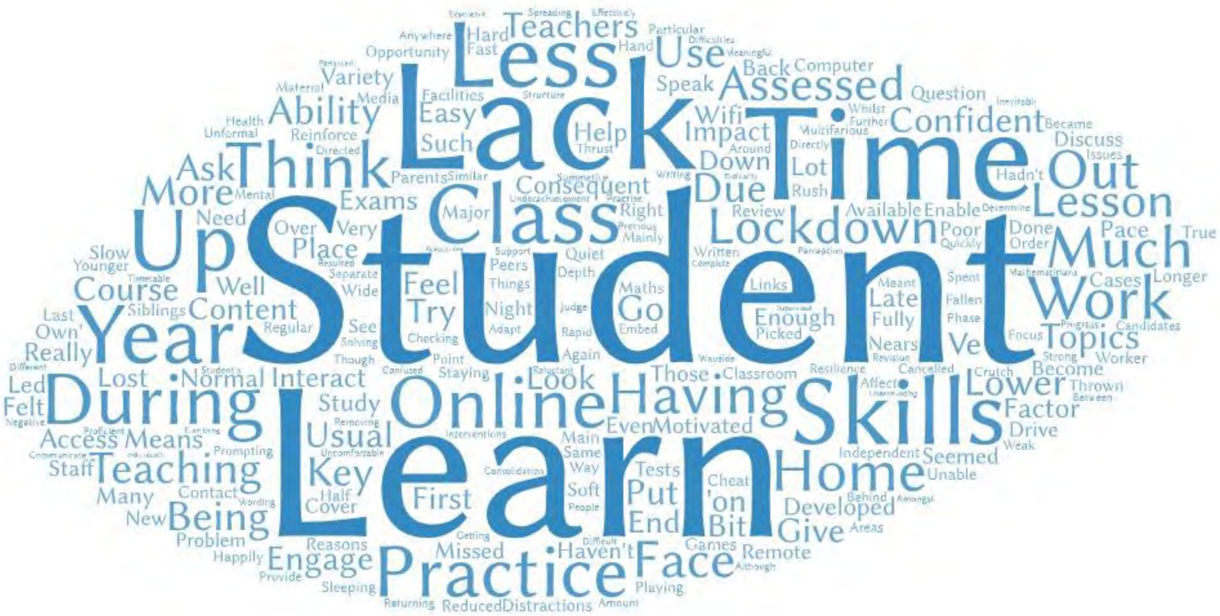

Figure 2: Word cloud of teacher comments to explore changes in students’ confidence in mathematics.

5. Post-18 Intentions and Readiness for Higher Education

Of the 174 student respondents, 135 (77.6%) indicated that they are considering studying for a degree after completing their Year 13 studies, with the majority specifying a degree subject that requires mathematics. When asked to specify their degree choice, frequent words (count) included mathematics (20), engineering (18), science (18) and economics (13).

From Table 6, it is clear that teachers feel uncertain about students being prepared for the transition to higher education, with only 4 teachers (14.8%) from 3 schools feeling confident that their students are prepared to cope with the demands of a higher education course that requires mathematics. In contrast, 14 teachers (51.9%) feel, to some degree, that their students are not as well prepared as previous cohorts.

Table 6: Teacher responses – are your Year 13 A Level students as well prepared for the mathematical demands of a higher education course as your pre-pandemic cohorts? (n=27).

Teacher View	Count	Percent
Definitely	4	14.8
Possibly	7	25.9
Not sure	2	7.4
Possibly not	8	29.6
Definitely not	6	22.2

6. Conclusions

This study was conducted to gain insights into the experiences of Year 13 A level mathematics students who studied under the unusual conditions imposed by the COVID-19 pandemic during the academic year 2020-21. While we do not claim that the respondents in this study form a representative sample of the Year 13 cohort or their teachers, this high-level analysis has indicated a sense of struggle and reduced confidence in mathematics. The teachers feel that the students are less prepared than previous cohorts for the mathematical demands of a higher education course, and responses from both students and teachers indicate potential gaps in mathematics knowledge, particularly in applied topics such as mechanics and statistics. This paints a troubling picture for those who have recently entered higher education, as well as for the institutions who must try to bridge the gaps in knowledge and boost confidence levels.

In the light of these findings, the role of mathematics and statistics support is likely to be more crucial than ever. Students may have to 'fill in the gaps' in their knowledge of A level topics themselves, outside of scheduled teaching. As such, there may be opportunities for mathematics and statistics support providers to assist these students through the provision of focused workshops on topics known to have been covered less thoroughly over the past year. There will be value in individual support providers, and the mathematics and statistics support community as a whole, reflecting on their experiences at the end of the first semester/term of academic year 2021-22, comparing with previous years and identifying lessons to be learned to take into future years.

7. Appendix

7.1. Student Questionnaire

Note: Questions 1-3 are consent questions.

4. What is your gender?

Male
Female
Other
Prefer not to say

5. What grade did you achieve in GCSE Maths?

A*	A	B
C	D or lower	9
8	7	6
5	4	3 or lower
Did not study GCSE maths	Prefer not to say	Other

6. Which option best describes your ethnic background?

White
Asian/Asian British
Black/African/Caribbean/Black British
Mixed/Multiple Ethnic Group
Prefer not to say
Other

7. What A level subjects are you studying? Please state below (free text).

8. Are you also studying for a BTEC qualification?

Yes
No

9. Since the first COVID-19 lockdown in March 2020, have you attended school or college during the lockdown periods (e.g., because your parent/carer is a key worker, or for other reasons)?

Yes, I attended school or college as usual during lockdown periods
Yes, sometimes
No
Not sure

10. Please read the following statements. In each case, tick the box that shows how strongly you agree or disagree with the statement.
Measured on a 4-point scale: Strongly Disagree, Disagree, Agree, Strongly Agree. Statements adapted from Strand and Winston (2008).

I have a quiet place in which to do school/college work
If I get stuck, I can usually work things out
Family members/carers help me with homework
I am good at solving problems
I feel good about myself
I know how to be a good learner
Family members/carers reward me if I do well at school/college
I have had access to the technology I need to learn from home during the COVID-19 pandemic
I am good at most subjects at school/college
I am good at working with others
Family members/carers usually come to open evenings/reviews
Family members/carers often ask me how I'm doing at school/college
I enjoy working on my own

11. **Before the first COVID-19 lockdown** in March 2020, compared to my other subjects, I found learning in mathematics (tick one of the following options):

Easier
About the same
More difficult
Not sure

12. **Over the past year**, my confidence in my mathematical abilities has (tick one of the following options):

Reduced
Stayed about the same
Increased
Not sure

13. **Since the first COVID-19 lockdown** in March 2020, I have found learning in all of my subjects (tick one of the following options):

Easier than before lockdown
About the same as before lockdown
More difficult than before lockdown
Not sure

14. **Since the first COVID-19 lockdown** in March 2020, I have found learning in mathematics (tick one of the following options):

Easier than before lockdown
About the same as before lockdown
More difficult than before lockdown
Not sure

15. As a result of the COVID-19 pandemic, I think my A Level Mathematics grade will be (tick one of the following):

Worse than expected before the pandemic
About the same as expected before the pandemic
Better than expected before the pandemic
Not sure

15a. If you think your grade will be worse, please describe the reasons for this below (free text).

16. During your A Level studies in Mathematics (not including A Level Further Mathematics), for each of the following more advanced topics listed below, please indicate if you think you have covered them. Measured on a 5-point scale: Definitely covered, I think so, Don't know, I don't think so, Definitely not covered.

Proof by contradiction
Partial fractions
Parametric equations
Arithmetic sequences and series
Geometric sequences and series
Secant, cosecant and cotangent functions
Formulae for sin(A+B), cos(A+B), tan(A+B)
Points of inflection
Chain rule for differentiation
Parametric differentiation
Integration by substitution
Integration using partial fractions

16a. If you have any other comments about topics you feel you did or did not cover during your A Level Maths studies, please state them here (free text).

7.2. Teacher Questionnaire

Note: Questions 1-3 are consent questions.

4. Please state the name of your school or college.

5. Please state the postcode of your school or college.

6. If you know it, please state your school or college's URN.
7. Please state your job title. *Note: if you do not wish to provide this information please write 'prefer not to say' in the box.*

8. What impact has the pandemic had on how A level Mathematics has been taught in your school or college? *For example, the extent to which online and blended learning have been used, the use of technology in teaching and learning, changes to your teaching practices etc.* (Free text).

9. Since the pandemic started, have you covered the A level maths syllabus as completely as usual?

Yes, as thoroughly as usual
Yes, but some parts have not been covered as thoroughly as usual
No, some parts of the syllabus usually covered have not been covered
Other *(please specify)*

10. For each of the more advanced topics below, indicate how thoroughly you have covered them with your current Year 13 A Level Maths students. *Note: please do not include topics covered in A Level Further Mathematics.* Measured on a 5-point scale: Covered as thoroughly as usual, Covered but not as thoroughly as usual, Not covered (but on our syllabus), Not on our sullabus, Not sure.

Proof by contradiction
Partial fractions
Parametric equations
Arithmetic sequences and series
Geometric sequences and series
Secant, cosecant and cotangent functions
Formulae for sin(A+B), cos(A+B), tan(A+B)
Points of inflection
Chain rule for differentiation
Parametric differentiation
Integration by substitution
Integration using partial fractions

11. What change, if any, have you noticed in the level of confidence in mathematics of your current Year 13 A Level Mathematics students, compared to your previous Year 13 maths cohorts?

Confidence is lower
Confidence is higher
Confidence is about the same
Not sure

12. Do you think that your current Year 13 A Level Maths students are as well prepared for the mathematical demands of a higher education course (not necessarily in mathematics, but one with some mathematical content) as your pre-pandemic maths cohorts?

Definitely
Possibly
Not sure
Possibly not
Definitely not

13. Do you think your current Year 13 A Level Maths students have had sufficient opportunity to practise writing mathematical arguments in an appropriate manner? (*i.e., formally writing out solutions and not just giving answers?*)

Definitely
Possibly
Not sure
Possibly not
Definitely not

13a. If you answered definitely or possibly, how has this been achieved under pandemic conditions? (Free text).

14. Have your students encountered any particular difficulties when studying A Level Mathematics over the past year?

Yes
No
Not sure

14a. If your answered Yes above, can you please describe those difficulties? (Free text).

15. Are you aware of any ways your Year 13 maths students have revised their aspirations because of the pandemic?

Yes
No
Not sure

15a. If you answered Yes above, please outline how students' aspirations have changed (free text).

7.3. Comments Recored for Q 16a of Student Quesitonnaire.

Did not fully cover vectors.
Sequences and series, logarithms
Did not finish mechanics or statistics
We haven't finished stats and mechanics
Some of the year 13 mechanics topic was missed out.
My learning of these topics was not impacted by the pandemic because we finished the A Level course in Year 12 to move onto the further syllabus
We didn't learn moments from mechanics or numerical methods or vectors
We did no differential equations either
Proof was extremely hard to cover and as the exams focus on problem solving questions i found it hard to actually complete it due to lack of practice as i had less questions to work with than i would in class
Didn't really cover Rational proof
Didn't cover all of Mechanics A2 content
A2 Mechanics, Normal distribution
Vectors was not covered
We didn't do much in vectors or in applied maths
Didn't get to cover a lot of the year 2 applied maths content, and missed out the trapezium rule for integration
Numerical method
Stats year 2 Mechanics chapter 8 year 2 Vectors year 2
Numerical methods
Did not cover numerical methods and one or two other units I cannot remember.
We didn't cover Year 2 Vectors, The Normal Distribution or Year 2 Numerical Methods.
Vectors
Only assessed on half
There was topics I missed in lockdown which I then had to learn independently.
We did not do any stats or mechanics in late 1st year or at all in 2nd year
Some were covered over online which meant help from the teachers was more awkward and some had to be self taught.
I had a very good teacher and online resources on a classroom

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CASE STUDY

Student Preferences for Online Maths and Stats Support and difference on Mathematics Anxiety

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Abstract

During the COVID-19 pandemic, many Maths and Stats Support (MSS) centres moved to offer wholly online support. By examining student preference scores to different types and deliveries of MSS, insight is gained into what forms of academic support students want going forwards. Students' mathematics anxiety levels are evaluated in three distinct situations: in general, before online support, and before in-person support. Thematic analysis of student preferences identifies the key themes for students preferring different forms of support. Factors such as accessibility and support logistics are discussed as key features in determining students preferred support method.

Keywords: Mathematics and Statistics Support, in-person support, online support, mathematics anxiety.

1. Introduction

In response to the COVID-19 pandemic many universities moved to online modes of teaching, and this included their maths and stats support (MSS) centres. These are facilities offered to students (not necessarily of mathematics) in addition to the regular programmes of teaching through lectures, seminars, tutorials, and problem-solving classes (Lawson et al., 2003). Workshops, drop-in sessions, and dedicated one-to-one appointments are currently some of the most common types of support. (Lawson et al., 2020).

As universities re-open and continue their face-to-face teaching, MSS centres must determine if they will resume face-to-face teaching, or maintain online teaching, or move to a more blended approach'. In making this decision MSS centres must consider student preferences and specifically consider that many students who are on their final year of a three-year course, will have only ever experienced online support during their degrees.

Mathematics anxiety is a negative emotional reaction to mathematics that has been defined as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in … ordinary life and academic situations" (Richardson & Suinn, 1972). Students with high levels of this situational-specific anxiety often practice mathematics avoidance (Hembree, 1990), by choosing subjects they think will not include maths or statistics. But with many more courses, particularly in social sciences, now including mandatory statistics modules, maths and stats is harder to avoid. Mathematics anxiety has also been found to be an important predictor of mathematics performance and achievement (Ashcraft & Moore, 2009) and MSS centres have observed the impact of anxiety in students for several years (Marshall et al., 2017).

To understand student's preferences for maths and stats support and to understand how levels of mathematics anxiety differ between delivery of support, three research questions were defined:

- RQ1 - Do students prefer online maths and stats support or in-person maths and stats support?

- RQ2 - Do students prefer a particular type of maths and stats support?
- RQ3 - Do students report higher levels of mathematics anxiety immediately before an online session or an in-person session?

2. Method

2.1 Participants

Mathematics and statistics support services at the University of Lincoln is delivered through the Maths and Stats Help Centre (MASH), one of the Learning Development Services based in the University Library. Before the pandemic the service offered almost exclusively in-person support and during the pandemic the service offered exclusively online support. Students seeking direct one-to-one support can book a maths or stats appointment and they are allocated an appropriate advisor. The service is staffed by specialist part-time advisors. Much of the support offered at MASH is dissertation statistics support such as helping students determine appropriate statistical tests and running statistical tests with sample data. In addition, the service offers maths support helping students learn and practice mathematical techniques such as algebra, calculus, and drug-dosage calculations.

Students who attended an online one-to-one appointment with the University of Lincoln MASH service in the 2020/2021 academic year were invited to participate in this study. Three hundred and fifty students were identified as possible participants as they provided their contact details for further work and 114 (32.6%) participated. Participation was voluntary and the project was approved by the University of Lincoln Ethics Committee. Students were invited from all years of study; this with their gender data is presented in Table 1.

Table 1. Sample demographics.

Year of Study	Gender		
	Female	Male	Total
Level 1 Undergraduate	3	1	4
Level 2 Undergraduate	10	0	10
Level 3 Undergraduate	46	4	50
Postgraduate Research	11	5	16
Postgraduate Taught	27	7	34
Total	97	17	114

The gender divide of the overall sample was significantly skewed towards females (85.1%), which is consistent with gender splits of MASH service users in previous years. The majority of participants were enrolled in Psychology (42.2%) or Clinical Animal Behaviour (14.7%), which are the two subject areas with the highest percentage of female students: 81.2% and 81.7% respectively (HESA, 2021). The remaining students come from a wide range of courses from Journalism to Computer Science with 38 different courses represented, 32 of those however are represented by single students, therefore between subject analysis was unable to be conducted.

2.2. Student Engagement

During the pandemic, the MASH centre was forced to move entirely online. Most of the student engagement with the service was through One-to-One appointments using the Microsoft Teams video call feature, with almost all participants receiving at least one hour of dedicated one-to-one support. The University of Lincoln MASH is rare in this regard as we are one of the very few centres to offer predominantly statistical dissertation support rather than a more holistic maths and stats learning environment.

2.3 Data Collection

A mixed-methods cross-sectional design was used to measure students' preferences for different types and delivery techniques of their MSS. Students were asked to evaluate how likely they were to attend three types of mathematics and statistics support: One-to-One Appointments, Workshops, and Drop-in sessions; for online and in-person delivery; giving six preference scores for each student. Each question asked students to record their likelihood of attendance on a 7-point scale from Extremely Unlikely (1) to Extremely Likely (7). In addition to their numerical scores, students were asked to give reasons for their preferences for different deliveries and types of support. These reasons offer qualitative insight into preferences and allow for direct observation of the student voice.

Students were also asked their levels of mathematics anxiety in three situations: in general, immediately before an online session, and immediately before an in-person session. Mathematics anxiety in general was collected to use as a baseline to allow for identification if mathematics anxiety changes before MSS, where students were asked to rate their levels of mathematics anxiety without a specific situation. Mathematics anxiety was measured using a modified version of the Single-Item Math Anxiety Scale [SIMA] (Nunez-Pena et al. 2014) due to its good convergent validity, efficiency, and ease of administration. The modification was made by appending a situation to the scale's base question, e.g. "On a scale from 1 to 10, how math anxious are you, immediately before an online session?". Of the 114 students who accepted the invitation to participate, all students completed the likelihood and mathematics anxiety questions. A single scale was used for students seeking maths or stats support because many appointments covered both areas of support, e.g., support the statistics of running an ANOVA and supporting the maths of calculating and understanding the means and differences.

Mathematics anxiety scores were not collected before appointments to encourage students to seek out MASH for their support needs without any pressure or obligation to participate in this study.
Many MASH service users access the service only once, therefore it was not possible to gain a sufficient sample of student who have experienced both in-person and online support. Students who have not experienced a type or delivery of MSS were asked to imagine how they would feel in those situations. Further research is currently being conducted to explore how mathematics anxiety levels change immediately before and immediately after single MSS interactions.

2.4 Data Analysis

Descriptive statistics were used to present overall mean preference scores and overall mean mathematics anxiety scores. The 2 x 3 Repeated Measures ANOVA was used to test whether mean differences between types of support and delivery of support were statistically significant. Pairwise comparison testing allows for direct examination of the within factor effects without increasing our Type 1 error rate. The One-Way Repeated Measures ANOVA was used to test whether mean differences between mathematics anxiety scores were statistically significant. Pairwise comparison testing again allows for direct examination of specific differences between scenarios.

Thematic analysis was used to identify, analyse, and report patterns and themes within the student preference open questions. Thematic analysis is conducted in six phases: familiarizing yourself with your data, generating initial codes, searching for themes, reviewing themes, defining and naming themes, and producing the report (Braun and Clarke, 2006).

3. Results

First, the student preference scores are analysed (Section 3.1) by way of direct analysis of means and then by two-way repeated-measures ANOVA analysis with post hoc pairwise testing. Next, analysis of mathematics anxiety scores is presented (Section 3.2) by direct analysis of means and by repeated-measures ANOVA. Finally, a thematic analysis of student responses is presented (Section 3.3) to show the two primary themes for student preference justification.

3.1 Student Preferences

Figure 1 shows the mean preference scores for different types of maths and stats support, separated by delivery technique, with error bars representing 95% confidence intervals also shown. Across all three support types, there was a greater preference for online support rather than in-person support. The largest observed difference between online and in-person support was through appointment support.

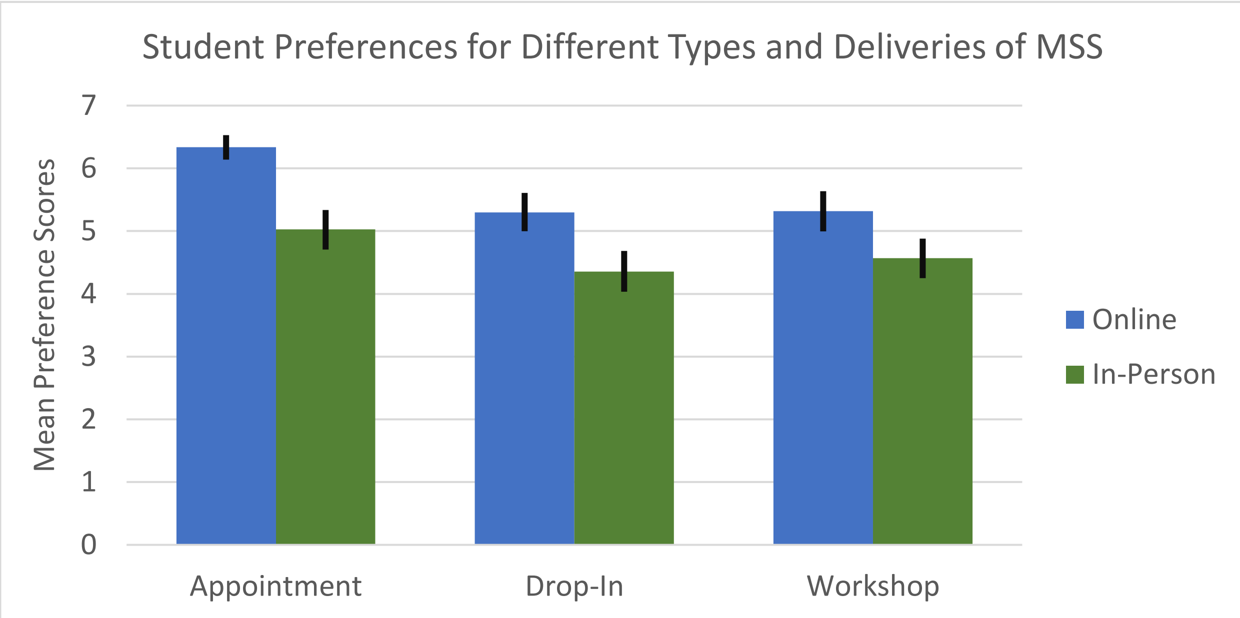

Figure 1. Student Preferences for Different Types and Deliveries of MSS.

To assess if observed differences are significant a 2 (Delivery) x 3 (Type) Repeated Measures ANOVA was conducted. There was a significant main effect for delivery of support on students' preferences ($F(1,112) = 47.04$, $p < .001$, $\eta^2 = .296$). Students reported a greater preference for online delivery of their maths and stats support (M=5.64, 95% CI [5.41, 5.86]) than in-person delivery of maths and stats support (M=4.65, 95% CI [4.38, 4.93]).

There was a significant main effect of type of support on students' preferences ($F(2,224) = 29.58$, $p < .001$, $\eta^2 = .209$). Students reported a greatest preference for One-to-One Appointments (M= 5.68, 95% CI [5.49, 5.87]), then for Drop-In support (M = 4.94, 95% CI [4.66, 5.22]) then Workshops (M = 4.82, 95% CI [4.55, 5.09]). Pairwise comparison showed that preference for One-to-One Appointments was significantly higher than both drop-in support ($p < .001$) and workshops ($p < .001$) however there was no statistical difference between drop-in support and workshops ($p > .05$).

There was also a significant interaction between delivery of support and type of support on students' preferences ($F(2,224) = 7.91$, $p < .001$, $\eta^2 = .066$). The greatest preference being for online appointments (M = 6.33, 95% CI [6.13, 6.52]) and the least preference for an in-person workshop (M = 4.35, 95% CI [4.03, 4.68]). This low value may be attributed to the risks of COVID-19 transmission, however, only one participant's responses indicated that COVID was a factor in their preferences.

There were no significant differences for gender in any of the 6 preferences scores, however, due to the comparatively small sample of male students and complete lack of non-binary students, this would benefit from further analysis in a larger study.

3.2 Maths Anxiety Scores

The mean score for the sample of 114 students was 5.53 (*SD* = 2.66) with a median score of 6. This is higher than the original SIMA test sample who had a mean score of 5.18 (SD = 2.43) and a median score of 5. Figure 2 shows the mean maths anxiety scores in three situations with error bars representing 95% confidence intervals also shown.

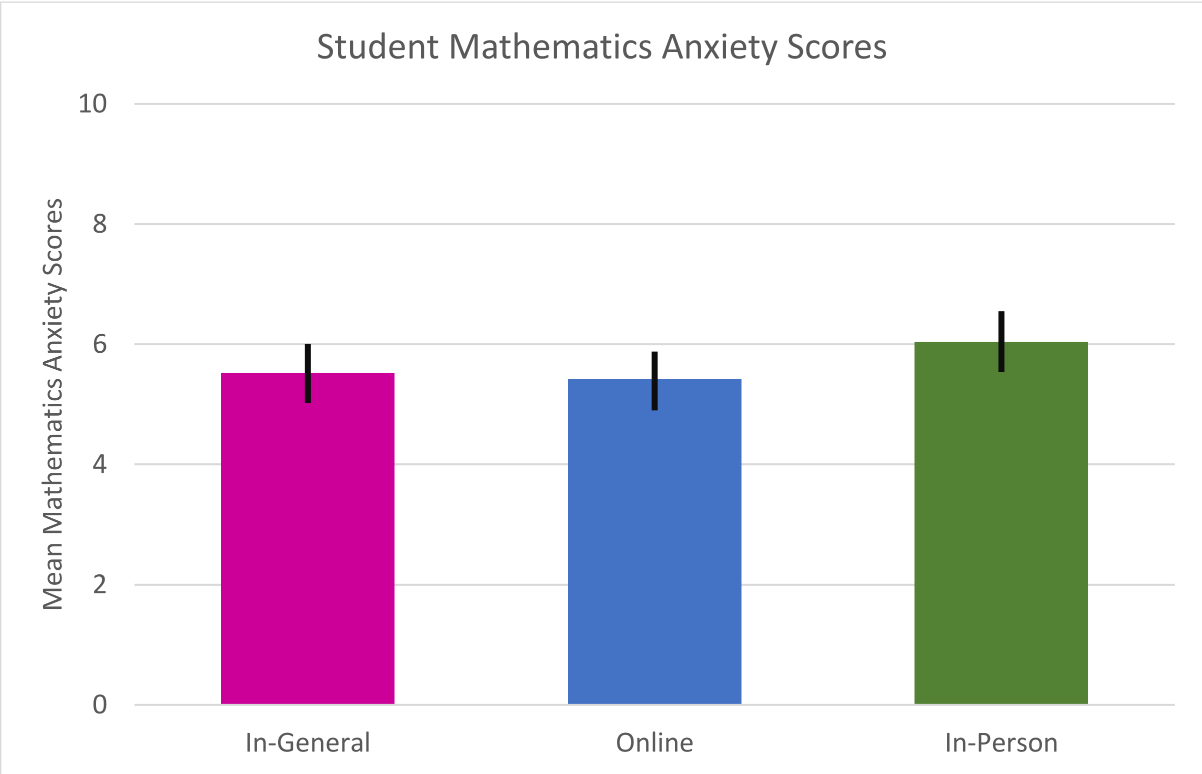

Figure 2. Student Mathematics Anxiety Scores.

To assess if observed differences are significant a One-Way Repeated Measures ANOVA was conducted. There was a significant main effect for situational mathematics anxiety ($F(2,228) = 10.43$, $p < .001$, $\eta^2 = .084$). Students reported experiencing the greatest levels of mathematics anxiety before an in-person session (M = 6.04, 95% CI [5.54, 6.55]); then comparable levels between mathematics anxiety in general (M = 5.51, 95% CI [5.02, 6.00]) and mathematics anxiety immediately before an online session (M = 5.39, 95% CI [4.90, 5.88]). Post hoc analysis showed that the differences between in-person and general (p=.004), and in-person and online ($p < .001$) were statistically significant; whilst the difference between general and online was not significant ($p = .455$).

3.3 Thematic Analysis

To gain a deeper insight into student motivations, participants were asked to explain their reasoning for their preference scores and 100 (87.7%) provided qualitative data. Two overarching themes, *Service Accessibility* and *Support Logistics* were identified from the students' responses. Quotations along with students' ages and genders are provided to illustrate specific sub-themes. The themes and sub-themes are summarised in Table 2.

Table 2: Themes and Sub-themes of the Thematic Analysis

Themes	Sub-themes
Service Accessibility	Travelling Time Commitment Personal Challenges / Disabilities
Support Logistics	Screen-Sharing Recordings Communication Problems

Service Accessibility

The *Service Accessibility* theme highlighted three sub-themes for student preferences: Travelling for Support, Time-Commitment and Personal Challenges including disabilities. A large number of students indicated that travelling for support was a major factor in determining which delivery of support they prefer. Many service users are off-campus students who expressed the added difficulties in travelling to campus for a single in-person session:

"…being online meant I didn't have to drive into the city…" [21F]
"I prefer online because I'm a commuting student… "- [25M]
"I'm not based on campus, online is my preference "- [32F]
"I live in Edinburgh so visiting Lincoln is a big trip"- [36M]

In addition to the travelling aspect, more students identified the time taken out of their day for support as another key factor in their preference choices.

"*Online appointments require less time out of the busy day*" [21F]
"*Online is more convenient if less time is available;*
don't have to walk into university" [21F]
"*I don't necessarily have the time to do a workshop*" [36M]

Direct accessibility was another reason given by students for a preference towards online maths and stats support delivery. Multiple students demonstrated how online support can lead to a more inclusive support centre.

"Shy/embarrassed in person. Online I can take notes easier"- [21F]

I get very embarrassed in person by my low ability in
maths due to having dyscalculia"-[22F]
"Online appointments are easily accessible;
Microsoft Teams has built-in subtitles"- [22F]
"...as someone who has a medical condition being able to
[have online support] in the comfort of my own home was great" – [21F]

Support Logistics

The *Support Logistics* theme highlighted three sub-themes around the practicalities of a maths and stats support session: Screen-Sharing, Recordings, and Communication Problems. A large proportion of maths and stats support in higher education is assisting students with statistical analysis for their dissertation. Through online support, students can share their screen/data which allows for more detailed support from advisers including giving examples using the students' data. This has been given by several students as their primary reason for preferring online delivery.

"I would be more likely to request an online appointment
due to being able to have all my data there on my PC" - [21M]
"Online is more convenient and is easy to follow the steps taken on SPSS
when the screen is shared" - [21F]
"An online appointment means the student can share
exactly what they've been working on"- [22F]
"I was able to work on my statistics whilst being showed
and told advice directly on my screen" –[24M]

In the recording sub-theme, students mentioned how they like the recording/playback feature of online video sessions and that group sessions can be replayed by students who have a timetable clash.

"I like to record via Microsoft Teams"- [29F]
"*Recording via Teams is great" – [29F]*
"[workshops] could have the possibility of being recorded so that people
with conflicting timetables/jobs/childcare can attend" – [22F]

Whilst many students prefer online maths and stats support delivery, particularly online appointments, this preference is not unanimous. There is a large proportion of students who dislike online MSS delivery due to the inherent communication problems of using video calls.

"It's easier to explain and show things in person as there are less barriers
and physical notes are easier for me than virtual ones"- [21F]
"In-person is better for me to explain myself and show what I mean,
I struggle doing this online"- [22F]
"I find it easier to communicate in person and there
were some connection issues at times."- [22F]

4. Discussion and Conclusion

This case study investigated what type and delivery of maths and stats support students prefer, reasons for their preferences, and how different deliveries of maths and stats support impact mathematics anxiety. We were looking for students' views on the advantages and disadvantages of different types of support and where those features impact the likelihood to attend support sessions.

Further, we looked for differences in mathematics anxiety levels considering different deliveries of MSS.

The results of the student preference questions indicate that for students in our sample there is a significant preference for online support over in-person support and a preference for tailored one-on-one sessions over group sessions, such as drop-ins and workshops. As each MSS centre caters for different types of students with different needs and expectations, this result will likely differ between centres.

Taking students' levels of mathematics anxiety in general as a baseline, students report no significant changes before receiving online support, however, they report a significant increase in their mathematics anxiety levels before in-person support. One reason for this, given by a student, is that they feel more nervous in person as they so don't want to be embarrassed. With students receiving the same support from the same people it is very concerning that purely being in-person raises their levels of mathematics anxiety, given its associations with maths avoidance and poorer maths performance. Mathematics anxiety scores before online support are similar to mathematics anxiety in general, therefore we can conclude that attending MSS does not itself raise anxiety scores, they are only raised when attending in-person MSS. To protect students who may be at risk from suffering the worst symptoms of mathematics anxiety, which can include physical pain, MSS centres must ensure they continue to offer some forms of online support.

The study also gave insight into how students determine which type of support they prefer with accessibility and support logistics being the two overriding themes. While there were many advantages to online support such as convenience, ease of use, and recording features, the inherent communication problems should not be ignored. These mixed preferences from students, with an overall preference towards online support, show that students have appreciated the move to online support during the COVID-19 pandemic and provide recommendations across academic and professional service support.

The delivery of support should be matched to the type of support required. Students needing advanced mathematics support, such as vector mathematics or linear algebra, are more likely to benefit from in-person support where problems can be solved together, and the advanced and often contradictory notation can be demonstrated more easily. In contrast students needing guidance on choosing appropriate statistical testing benefit from a more relaxed and open conversation around statistical research that can be conducted through online support.

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RESEARCH ARTICLE

Changes with time of Practitioners' opinions of online Mathematics and Statistics Support

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Abstract

As an established part of the infrastructure of many higher education institutions in the UK and across the world, Mathematics and Statistics Support (MSS), was forced to move to an online setting as a result of restrictions put in place due to the COVID-19 pandemic. With institutions offering limited online support prior to the pandemic, MSS practitioners were mostly unprepared to deliver online provisions. Survey data from May 2020 gives a first look at the immediate response of practitioners to online MSS. Interview data from January/February 2021 explores opinions after a period of reflection, and survey data from June 2021, over a year on from the initial sample, provides a direct comparison in opinion of online support a year later. This paper explores these three datasets, investigating the practitioner perspective and offers an overall reflection of how MSS practitioners' opinions of online support measures have changed from their "crisis-reaction" at the beginning of the pandemic to a more considered response as COVID-19 prevention measures are beginning to ease a year later.

Keywords: Mathematics and Statistics Support, online education, COVID-19, Practitioner perspective.

1. Introduction

Since its beginnings in the early 1990s, Mathematics and Statistics Support has become an established part of higher education infrastructure in many institutions across the UK and internationally (Lawson, Grove and Croft, 2020). However, prior to the COVID-19 pandemic, very little of this support was provided online (Cronin et al., 2016; Hodds, 2020); often a website presenting details about their in-person support being the only form on offer (Mac an Bhaird, Mulligan and O'Malley, 2020). Support practitioners were mostly hesitant and sometimes against using technology to deliver MSS online, as it was thought that support could not be offered as effectively as it could in person. The difficulty in writing formatted mathematical notation in an online setting, reported both prior to and during the pandemic (for example, Croft, 2000; Smith and Ferguson, 2004; Ní Fhloinn and Fitzmaurice, 2021), potentially being a key reason behind this. Regardless of opinion, when pandemic restrictions forced all levels of education online, MSS was not an exception and support practitioners were forced to move provisions online in order to continue supporting students.

This paper provides a three-stage reflection on how practitioners' opinions have changed over a year of pandemic restrictions, through the analysis of data sets collected at three sampling points. All data sets were collected by researchers based within **sigma**, Coventry University's mathematics and statistics support provision. The first stage is consideration of questionnaire data collected in May 2020. This offers a first look into the immediate response and MSS practitioners' initial opinions about online support methods, provided by institutions both in and outside of the UK, during the crisis-driven transition. The second dataset consists of interview data from January and February 2021. By this time, European practitioners had experienced a summer break and with that, a period of reflection on their institution's response, evaluating what had worked and what had not during the crisis reaction phase, with opinions not as influenced by the need for an immediate response. By the time the interviews took place, these practitioners had had a semester to implement a more planned online support provision based on their reflection. Finally, the third dataset, survey data from May and June 2021, provides a look into practitioners' opinions a full year after the initial data set, at a time where many pandemic restrictions were about to be lifted, influencing ideas of online MSS going forward. An overall reflection is provided, comparing how practitioner opinions have changed or remained the same between the sampling points, with concentration on identified advantages and disadvantages, as well as student engagement.

Phase one and three data sets were collected using JISC Online surveys. Participants in the surveys were mostly recruited via email using the **sigma** Network JISCmail list and were therefore mainly from the UK. Interview participants in phase 2 were selected from those that participated in the phase one survey and gave consent to be contacted for future research, to cover a range of different types of UK institutions and also an international perspective. A more detailed breakdown of locations is given in the following sections. The interviews were conducted using MS Teams where the recordings were uploaded privately to YouTube to automatically generate transcripts, and then removed. The transcripts were then carefully checked against the original recordings and edited if necessary. All data collection for this study received ethical approval from Coventry University Research Ethics Committee.

2. May 2020 – First look at the immediate response

The first dataset comes from a large survey conducted in May 2020, alongside the **sigma** Network's first online workshop, on the topic of online support, receiving responses from 72 individual institutions across the world; 53 from the UK and 19 from outside of the UK: 11 from Ireland and 8 from the rest of the world, namely Germany, Norway, Czech Republic, United States of America and Australia. A preliminary report was published the following month (Hodds, 2020), and some of the key findings were as follows:

- **Opinion: 54% of UK practitioners stated online support was worse than in-person support. This was also the consensus across locations outside of the UK.** At this stage many practitioners felt that in-person support had advantages that online support could just not offer.
- **Negatives: The most frequently identified negatives of online support were lack of non-verbal communication, technology, and lack of training.** It was found that students were less engaged and regularly not turning on their cameras, and as a result, practitioners could no longer utilise unspoken communication, such as body language and eye contact, to try to determine whether students were engaged and understood the content. Increased technology use led to an increase in technological problems; furthermore, it was also stated that some students and staff did not possess the equipment required to best utilise online support. Finally, at this stage practitioners felt that they were not pedagogically equipped to deliver support in an online setting.
- **Engagement: 74% of UK institutions, 82% of Irish institutions and 63% of institutions from the rest of the world saw a large decline in student engagement.** Some UK institutions were seeing average student numbers across the first two months of the UK lockdown, equivalent to what they would have seen in one week prior to the pandemic.
- **Positives: The most frequently identified benefits to online support were flexibility and timings.** Online support is not confined to a physical location, so commuting is no longer necessary. This means support can be offered at more times and navigated at a student's own pace, making it more accommodating to student schedules. This in turn increases accessibility to students who may not have previously been able to utilise support offered, such as mature

students with childcare commitments, students on other campuses, or those with anxiety about in-person attendance.

- **72% of UK practitioners, 80+% of Irish practitioners and 33% of practitioners from rest of the world stated they would continue with some form of online support post pandemic.** At this stage, over a quarter of MSS practitioners in the UK stated that they would not or were unsure whether they would choose to continue with online support once the pandemic was over.

The final question on the survey was open-ended, and asked participants whether they thought the nature of the approach should change now that MSS had moved online. In particular, whether it was best to replicate in-person techniques in an online setting, or adopt new approaches, and if so what approaches. Thematic analysis (Thomas, 2006) was conducted on the 74 received responses, identifying shared phrases or ideas to link answers together into underlying themes. This produced seven themes in total.

Three themes directly arose as a result of the question: answers stating replicate, not to replicate and cannot answer. The remaining four themes arose due to the open nature of the question and were identified from other issues raised by the participants: trying things out, blended approach, open to new approaches and looking for the best option. A more detailed explanation of methodology and thematic categories identified can be found in Gilbert, Hodds and Lawson (2021).

Overall, the most common characteristic across answers to this question was uncertainty. Not only was the largest theme ‘cannot answer the question’, containing 32.4% of responses, but uncertainty could be identified in the other six themes as well: for example, stating we should be replicating in-person methods online, but not knowing how to do so, in the ‘replicate’ theme, and just wanting the best for their students even if they did not know what that was yet, in the ‘looking for the best option’ theme.

3. January/February 2021 – After a period of reflection and further implementation

Twelve interviews, conducted in January/February 2021, produced the second data set (Gilbert, Hodds and Lawson, 2021). Practitioners completing the May 2020 survey had the opportunity to give their consent to being contacted in the future for follow up research. Of those, we selected twelve that came from a variety of countries, namely England (7), Ireland (3), the Czech Republic (1) and Australia (1), to be interviewed. The seven selected from the UK covered a variety of institution types for example Russell Group, post-92 universities and distance learning centres. These opinions were gathered at a time where the majority of practitioners had been delivering a more considered online MSS for the first semester of the 2020/21 academic year, as this delivery followed a summer break with little or no MSS provided, allowing time for reflection. Practitioners had the opportunity to consider what had worked well during the crisis-reaction phase, what had not worked and potential reasons for these difficulties. A question-by-question analysis produced these main findings:

- **All institutions were offering some form of online support.** Institutions were mainly offering online drop-in sessions and pre-booked appointments with several also offering online workshops.
- **Engagement: The majority of practitioners still reported a decrease in student engagement when compared to before the pandemic, however, they were starting to see an increase from when support first moved online.** This was mainly applicable to larger institutions.

- **Benefits: Accessibility to distance learners was the most frequently identified benefit of online MSS.** As with the benefits identified in the May 2020 data set, the flexibility of online support increasing accessibility for distance learners, referring to any student distant from the location of MSS provision (and at this point all students were effectively distance learners), was still identified as the largest benefit to online support. This was a particular benefit for universities that have multiple campuses where previously only students who studied on the campus where the support was located could utilise it.
- **100% of practitioners wanted to offer some form of online support post pandemic.** Every practitioner interviewed provided reasoning as to why some form on online support should remain in their institution post pandemic, a clear increase from the 72% in May 2020.

As with the open question data in the previous data set, thematic analysis was performed looking at the interview transcripts as a whole, and eight thematic groups were identified based on both how many practitioners mentioned the theme, and total number of mentions. Figure 1 is a bar graph depicting these eight groups. Each bar is segmented into 12 sections representing the 12 practitioners, and if all segments are filled, every practitioner mentioned that theme at least once during their interview. The bars are plotted in order of importance based on how many practitioners mentioned the theme (i.e., how many of the 12 segments are filled) and then by how many times the theme was mentioned overall.

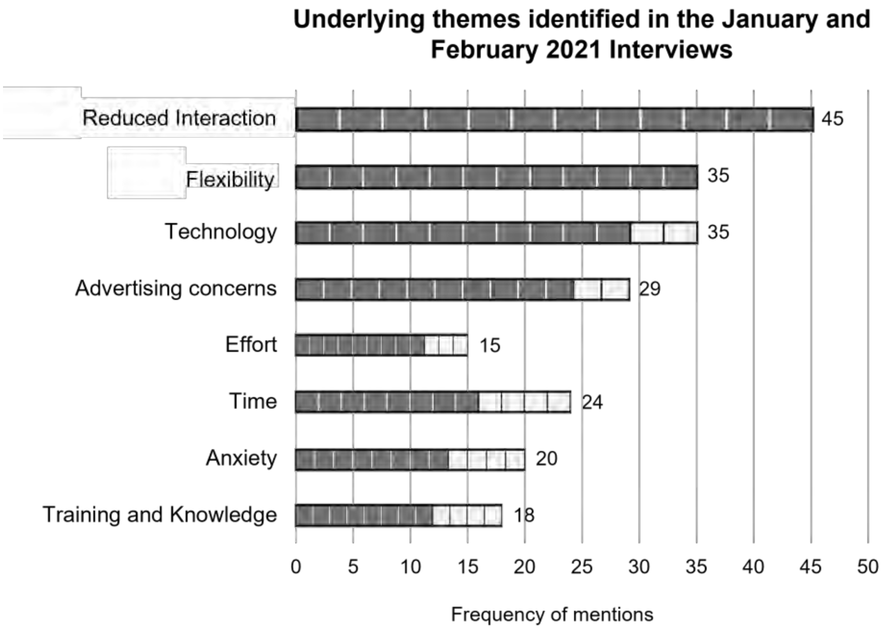

Figure 1. Bar graph depicting how many practitioners stated each of the underlying themes identified in the January and February 2021 interviews, and each theme's total number of mentions.

- **Reduced Interaction.** As with May 2020, lack of unspoken communication and the inability to use visual cues to gauge student understanding, due to students not having their cameras on, was identified as the biggest barrier in online MSS.

- **Flexibility.** Flexibility remained the largest benefit of online MSS in practitioner opinion. At this sampling time, forward thinking was beginning to be displayed. Many of the responses coded into this category talked about offering students the choice between in-person and online depending on which suited them best. A blended approach started to become more popular.
- **Technology.** In both data sets so far, we had responses regarding statistics support not being as affected when support transitioned online. This was particularly in relation to mathematics online support having to involve writing formatted mathematical notation online (ideally on a shared surface), and statistics online support not having to face this difficulty.
- **Advertising concerns.** Although this was the topic of a question in the interview, advertising concerns were mentioned so regularly and not just in response to that question, by 10 different practitioners, that it became a category in its own right. The three most prominent concerns were word of mouth, particularly peer recommendation, footfall, the loss of students spontaneously walking in, and email overload, now that all areas of university life were online.
- **Effort.** This relates to the effort from both students and staff alike. With support only online, students can no longer walk past the physical centre and decide in the moment to walk in. If they want support, they have to actively seek it out. Likewise, staff have to actively learn new methods and grasp how to use technology new to them in order to support students as effectively as they did in person.
- **Time.** In this category, opinion was divided. One side of the debate was that online support takes longer online. Technological issues such as a stable internet connection and issues such as not knowing if the student understands, can cause practitioners to regularly repeat themselves. On the other hand, some practitioners suggested that online support saves time, from not having to commute to a physical location, and so more can be spent on things such as preparation and ideas for the future.
- **Anxiety.** Anxiety also presented a split opinion. Some practitioners believed that as students can remain in their home and not have to turn their cameras on, they feel more anonymous and comfortable, empowering some anxious students to engage with MSS. However, an opposing argument was made that online support can cause its own anxiety. If you ask a question in a busy support centre, only the people around you are likely to hear, but in a conference software's chat facility, everyone in the meeting will see.
- **Training and Knowledge.** Finally, many practitioners felt that they do not possess the pedagogical skills necessary to support students online, which is a concern that has remained from the May 2020 data set.

4. June 2021 – A year of online support

Over a year after the initial data set, a June 2021 survey provides the final sample. At the time of the analysis here, there were 32 respondents from across 22 individual British Isles institutions. A similar survey was being conducted in Germany and an agreement was made with the German researchers that we would focus on UK and Ireland practitioner opinions and compare findings to those of the German study. The survey was designed based on the questions asked in both previous studies, so a comparison could be made.

The pattern in student engagement levels remained the same as identified in the previous data set. When comparing levels of engagement, during October 2020 – February 2021, to those before the pandemic, October 2019 – February 2020, practitioners were still seeing an overall decrease.

However, when compared to numbers during the initial phase of the pandemic, April 2020 – September 2020, numbers had increased.

Practitioners were asked about advantages and disadvantages in two open questions at the end of the survey. Flexibility and accessibility were again the most frequently identified advantages of online support. However, this was mentioned in only just over half (52.3%) of the 23 responses received for that question, a much lower percentage when compared to 100% of practitioners mentioning this benefit in the previous data set. This presents the question of whether flexibility and accessibility are becoming less of an advantage, or whether practitioners are just becoming desensitized to these benefits. The split opinion on time was also still present. There was a combined number of 40 responses across both the advantages (n=23) and disadvantages (n=17) questions; 20% of those were in reference to time. To break it down, 21.7% of advantage responses stated that online support saves time, and 17.6% of disadvantage responses stated online support is more time consuming.

Concerns about reduced interaction, particularly the lack of unspoken communication, were also still present in June. In May 2020, lack of non-visual communication was stated as the biggest issue with online support by a third of practitioners. Similarly in the second data set, reduced interaction was the largest underlying theme of the interviews, mentioned by 100% of the participants a total of 45 times. However, in June 2021, as shown in figure 2, when practitioners were asked whether they agreed with the statement that it is more difficult to determine how well students are engaged in online support, it resulted in more of a split opinion.

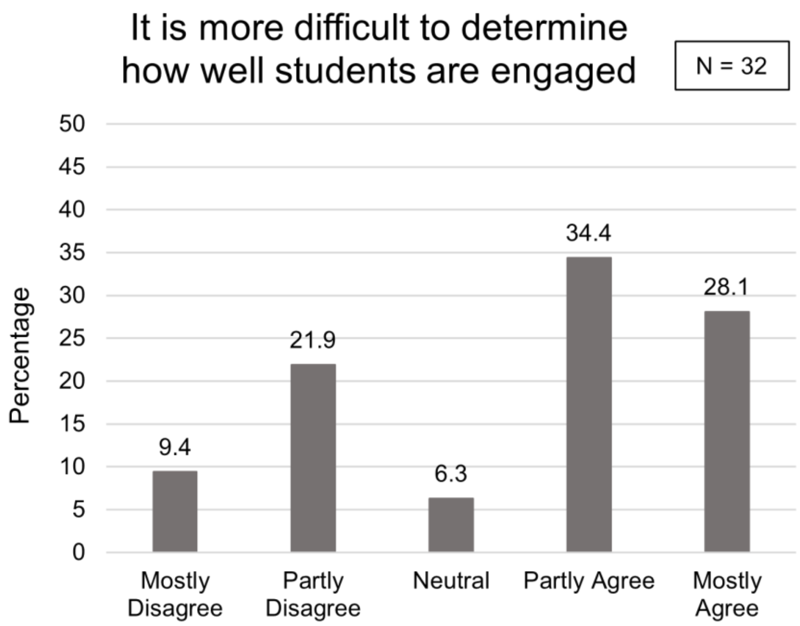

Figure 2. Bar graph showing the response distribution of practitioners when asked if it is more difficult to determine how well students are engaged during online mathematics and statistics support.

37.6% of 32 responses now either disagreed or were neutral, and 34.4% only partly agreed. Since student camera usage had not changed – when asked how often students have their cameras on, only 3.1% stated ‘always’ – something else has caused this change.

5. Discussion - The overall picture

The clearest change across the three data sets is that negativity and uncertainty regarding online support has changed to positivity. In May 2020, only 72% of practitioners stated that they would continue with some online support after the pandemic. This changed to 100% at both the second and third sampling times. When asked in the June 2021 survey how their attitude to online support had changed over the year, many respondents explained how hesitancy and even negativity towards online support before the pandemic, had been replaced by acceptance and positivity, using phrases such as *'greater understanding'* and *'enjoyment and confidence'*. The phrase 'in person is better' became much less frequent. This was foreshadowed in a case study in 2015 by Karal, Kokoc, Colak, & Yalcin. Two mathematics instructors, with no prior experience of distance teaching, taught their course using Adobe connect and a digital pen, and were observed and then interviewed. The authors stated that the instructors' negative attitude towards teaching mathematics online changed, as their experience of using the pen-based technology overcame their biases and resulted in them feeling as comfortable as they did teaching in person (Karal et al., 2015).

Flexibility and accessibility were most frequently identified by practitioners as the biggest advantages to online MSS at all three sampling points (however, in June 2021, when compared to the previous dataset, they were mentioned noticeably less). As the advantages of support being flexible and more accessible are well reported (as examples, see Bennett and Lockyer, 2004; Jaggers, 2014; Johns and Mills, 2021), it suggests that other benefits are potentially being recognized more as flexibility and accessibility are now just 'expected' properties online support has. Less time wasted, and convenience were jointly the second most mentioned benefits to online MSS in June 2021 (21.7% of 23 responses). Practitioners stated that less time is wasted particularly in regard to students missing sessions; they have not had to waste time traveling to a location and sit waiting just for the students not to arrive. Online support being more convenient also relates to this; accessing resources and sharing content can happen a lot faster in an online setting and can be stored in one location.

The hurdles that educators have faced with students not having their cameras on while learning in an online setting have been well documented (as examples, see Dennen, Aubteen Darabi and Smith, 2007; Roberts, Malone, Moore, Russell-Webster and Caulfield, 2020; Castellie and Sarvary 2021), as well as identified by practitioners in all three data sets in this study. Unspoken communication can no longer be utilised to try and gauge student understanding. However, in June 2021, there was more of a split opinion. As average student camera usage has not improved, something else has caused this divide in opinion. One explanation may be that over the year, practitioners have found ways to overcome this barrier, using other methods to gauge students' understanding successfully, therefore the lack of unspoken communication has become less of a concern. Or there may be other disadvantages that are becoming more prominent. The next two disadvantages of highest concern in June 2021, were online support being time consuming (17.6%), and needing a higher level of pro-activeness from students to access services (11.8%), which were both concerns previously stated by practitioners during the January/February interviews.

Engagement levels are increasing. Although student numbers at each sampling point have remained lower than before the pandemic, in January/February 2021 and June 2021, practitioners stated they had seen an increase compared to when support first moved online. However, as methods being offered have remained relatively the same over the year, with online prebooked appointments, drop-ins, and workshops remaining the most offered provisions in each data set, further exploration is needed into why student numbers are now increasing. As with practitioners, it may be that students are becoming accustomed to online learning generally, leading to increased engagement with online MSS; of course, there may also be other influences contributing to the growing numbers.

6. Conclusion

Overall, practitioners' opinions of online MSS have improved over the space of the year. Being forced into supplying MSS online by the pandemic, helped practitioners overcome their pre-existing biases and general belief that it was not possible to deliver MSS online successfully. Practitioners became more aware of the benefits online support has to offer, rather than focusing on the initial negatives in a time of crisis, and saw that online MSS is not only possible, but it can be delivered to a high standard. As time progressed, offering chances for reflection, uncertainty and negativity decreased and explanations for identified barriers were explored, offering opportunity for improvement. Practitioners have stated that student numbers are now increasing but are still less than before the pandemic, so will require further investigation into the student perspective of online support methods.

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CASE STUDY

Remote active learning

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Abstract

Following the necessary shift to online learning in mathematics due to the COVID-19 pandemic, it was widely reported across the sector that encouraging students to engage in online synchronous sessions was very challenging. In this case study, the design and delivery of synchronous sessions for a large Year 1 mathematics module will be discussed. These synchronous sessions utilised an active learning approach which sought to address the difficulties in online learning through a focus on playfulness and games. This approach created opportunities for peer learning and successfully encouraged student engagement and interaction.

Keywords: playful learning, active learning, online engagement, peer learning, polling software.

1. Background

As was the case across the sector, the institution in this case study had to respond to the challenges for mathematics teaching and learning presented by the COVID-19 pandemic. In line with most other institutions, the approach consisted of a mixture of asynchronous material and online synchronous sessions. Departmental guidelines for asynchronous content included expectations for complete lecture notes and/or high-quality video content recorded using tablets and annotation software.

Staff were provided with ideas for the online synchronous sessions, but module leaders were free to design the approach and activities themselves. The only clear expectation was that these sessions should take an active learning approach. It has been reported from the sector, at online workshops such as the Teaching and Learning Mathematics Online (TALMO) events, that engaging students in online environments can be very challenging. One specific issue which was highlighted was that students did not appreciate being moved into small-group breakout rooms against their will (Walkden, 2021). In addition to student engagement and motivation in general, another challenge which was identified was getting a sense of how students are coping with the material (Shaker, 2021; O'Malley et al., 2021). Feedback from Mathematics students at the institution indicated that they did not like online synchronous sessions which were unstructured or were run as drop-ins.

This case study will focus on the design of online synchronous activities for a Year 1, semester two module (115 students) which covers number theory and some initial ideas from group theory. This is a theoretical module, and mostly followed a traditional teaching approach pre-pandemic (a mixture of lectures and tutorials). The module is compulsory for all students on the BSc Mathematics and MMath programmes.

2. Development of the approach

Active learning is often characterised by students' cognitive investment and active participation in learning (Zepke and Leach, 2010). Intrinsic to active learning strategies is student engagement which is often encouraged through peer interactions (Kuh et al., 2006).

The synchronous session design was influenced by findings (both locally and from the wider sector) in semester one of the 2020/21 academic year. Based on experiences in semester one, colleagues flagged engagement as challenging and attendance as low at online synchronous sessions. The majority of online synchronous sessions in semester one attempted to simulate the tutorial environment, encouraging students to work on weekly exercise sheets in small-group breakout rooms. As stated previously, students felt that this approach was unstructured and did not encourage engagement.

After reflecting on experiences from semester one, the author identified specific aims for the online synchronous sessions in the Year 1 module. These were:

- encourage peer discussions
- determine whether students are engaging with the asynchronous resources for the module
- develop students' appreciation of the need for accuracy and clarity in the written presentation of mathematics
- develop students' enthusiasm for the campus and city region where they will be studying in-person in the future.

The final goal was defined in recognition of the fact that most mathematics students had not set foot on campus at all in the academic year due to UK COVID-19 restrictions. The author sought to build students' optimism about the prospect of being on campus in the near future.

It has been documented that a playful learning approach utilising games can foster creativity, encourage student enjoyment of learning, and provide a shared experience in a low-stress environment (Whitton, 2018). Highlighting the creative nature of the discipline can encourage learners to engage with the material as has been noted in several articles on recreational mathematics (e.g., Rowlett et al., 2019; Sumpter, 2015). The "safe space" concept associated with playful learning emphasises that it is acceptable and natural to make mistakes when learning new material, and that exploring or discussing mistakes can be very useful learning activities.

Inherent to the playful learning approach in group work is the encouragement of peer discussions and creating opportunities for students to learn from their peers. One method by which this can be achieved is peer instruction and participation (Crouch and Mazur, 2001). As students are tasked with convincing peers of their arguments, peer instruction has been observed to encourage the development of reasoning and communication skills.

When discussing problems with peers, differences in approach and method can lead to the identification of misconceptions or errors. From this perspective, engaging with peers in discussions has some overlap with learning from errors pedagogy (Tulis et al., 2016; Metcalfe, 2017). Under this pedagogy, errors are discussed openly and utilised as an important learning opportunity:

"Praise (for the correct answer only) curtails discussion and reinforces the teacher role as the authority who bestows rewards" (Tulis et al., 2016).

As encouraging discussion was a key aim of the endeavour, the author decided to adopt elements of learning from errors pedagogy in the approach for one of the games in the synchronous sessions. This was closely aligned with the approach of Groβe and Renkl (2007) primarily because their version does not rely on students making mistakes or errors themselves and subsequently having these picked up by peers (which could be disheartening for some students). Instead, the approach advocated by Groβe and Renkl tasks students with identifying mistakes or errors in worked examples. This approach pushes the learning from errors element further towards a game and a playful learning approach.

The approach adopted in the online synchronous sessions for the module therefore aimed to pursue a playful learning approach incorporating elements of peer learning and learning from errors. The peer instruction method of Crouch and Mazur was adapted to eliminate the initial (individual) vote which is usually included. One of the reasons for this omission is that in the purely online environment there is no way for the lecturer to ensure that any vote is not a collaborative effort. In the new approach, the initial vote will take place following peer discussions. It was hoped that these vote results would help to identify any widespread difficulties with the material. The combination of approaches aimed to provide an active learning environment where students were able to develop their confidence.

3. Structure of the online synchronous sessions

Every module in the Department was allocated a two-hour online synchronous session per week. The playful learning approach for the module in question consisted of three activities (labelled as "rounds") per week. At the beginning of the module, students were allocated to private channels on Microsoft Teams (approximately five students per channel). At the beginning of each round, the challenge was uploaded to the Team in PDF format. Students were then encouraged to move into their private channels to discuss the particular challenge. Student groups were given 20 minutes for each of these discussions. After 20 minutes, the whole class came together again in the main Teams call where anonymous polling was used to collect thoughts and opinions about the challenges. A summary of the three rounds is given in the table below.

Table 1 - breakdown of the three games ("rounds") in the synchronous sessions.

Round	**Focus**	**Format**
1	Revision of main theoretical ideas from asynchronous material for the week	5 or 6 multiple choice questions covering definitions and elementary examples from the weekly material.
2	Presentation of written mathematics related to the weekly material	4 sample answers to typical questions from the weekly material. Each sample answer contains an error. Students are challenged to identify these errors.
3	Consolidation of main ideas from asynchronous material for the week	Students are provided with coordinates in decimal degree format for an attraction in or near to the city. 8 of the digits are missing - students must solve clues relating to the weekly material in order to identify the mystery location. Students must also find out something interesting about the mystery location.

The University has a licence for Poll Everywhere and this was utilised in each of the three rounds to gather student views and encourage engagement. This software has many different formats for polls (including multiple choice, open text response, upvoting and clickable image). This range of polling offers the lecturer the opportunity to diversify the methods by which they invite students to engage. The anonymity feature also encourages student engagement.

3.1 Round 1

The focus of the initial round was on revision of the main ideas from the weekly asynchronous material. For this round, five or six multiple choice questions were designed which tested knowledge of definitions or simple examples from the asynchronous material. In line with recommendations in the pedagogic literature, questions in this round were not too complicated, and were designed with suitable distractors as options. The aim of this approach is to ensure that students cannot determine the correct answer by a process of elimination (King and Robinson, 2019). It is also noted that polling using electronic voting systems such as Poll Everywhere can catalyze active instructor-student feedback in real time. This was certainly a goal of the initial round of activity as the author aimed to determine what students had absorbed from studying the weekly asynchronous resources and highlight any potential issues with core concepts before progressing to subsequent rounds.

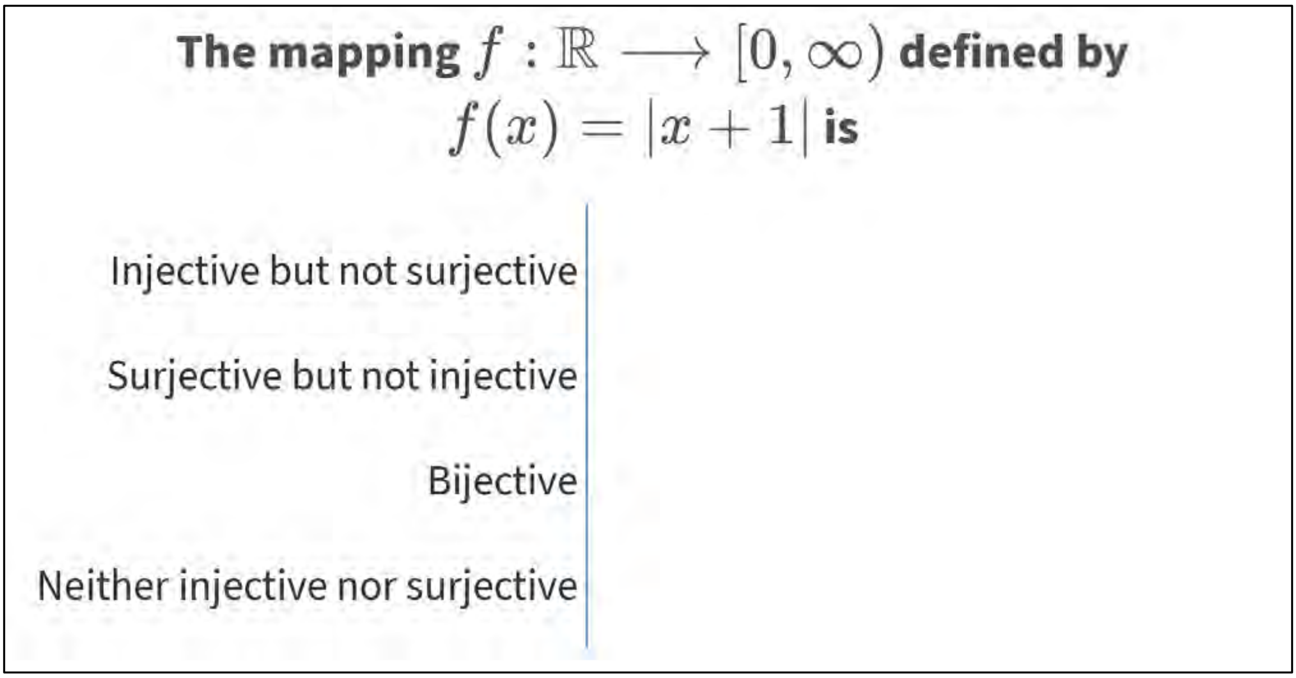

Figure 1. Example of a question in Round 1.

3.2 Round 2

The second round focused on the presentation of written mathematics and learning from errors. Over the years, the author has observed common errors in student submissions and this round was an opportunity to highlight these common errors by presenting sample solutions containing these errors to students for discussion.

As was the case in Round 1, a PDF document with 4 short sample solutions to questions from the recent material was uploaded to the Team. Students were advised that each sample solution contained an error and they were tasked with identifying these errors. Errors could be in terms of presentation (logical argument) or a more fundamental theoretical error in method. Again, students were encouraged to move into their private channels to discuss these sample solutions for 20 minutes (and most students did so). After the private channel discussions, the whole class came together again in the main call and a clickable image poll was conducted on Poll Everywhere. In this poll, the image of each solution was presented and students were asked to drop an anonymous pin on the image where they believed the first error occurs. This poll enabled the lecturer to observe where clusters of pins were grouped. As a follow-up poll, students were asked to elaborate on their pin choice in short text responses. Poll Everywhere then automatically runs a summary of these anonymous comments on the screen for students to view.

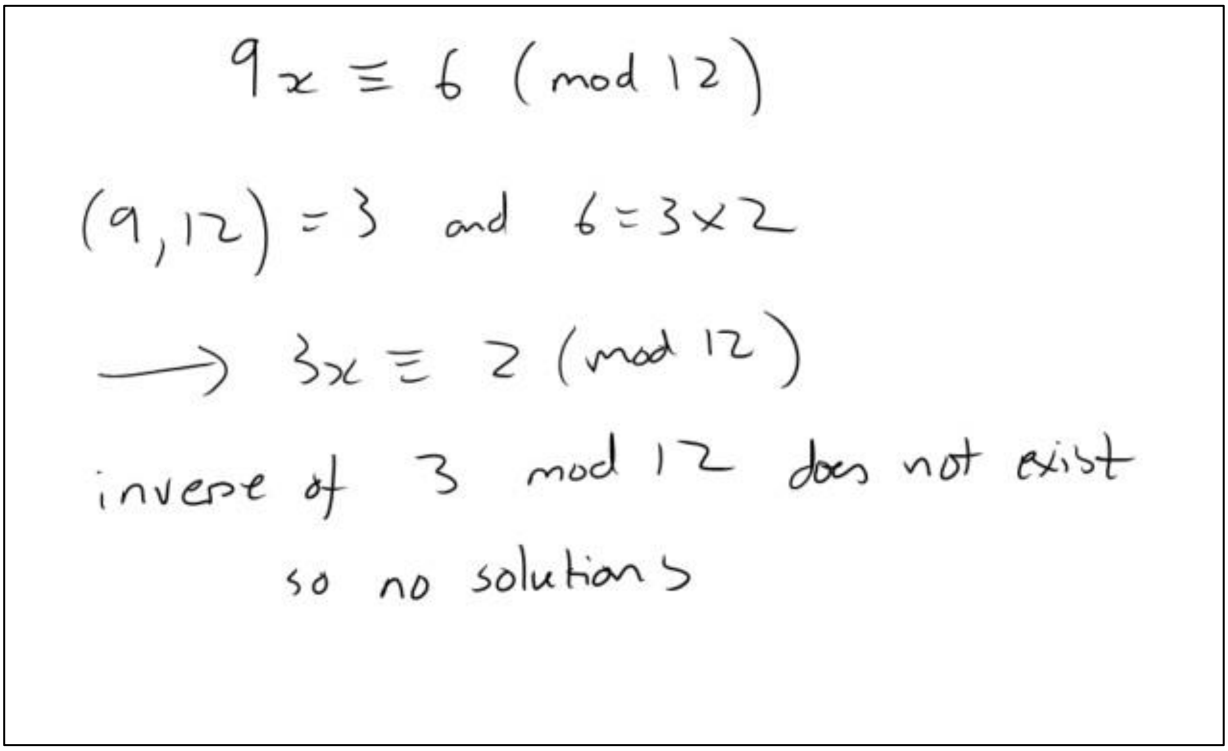

Figure 2. Sample answer with error from Round 2.

The example in Figure 2 is a particularly useful sample answer with error as there are several areas where students might identify an error (in method or in presentation). For example, some students identified the second line "6 = 3 x 2" as a point where an error takes place as the author did not explain why this is relevant in the progress of their solution. Perhaps this would not really be classified as an "error", but this did help to stimulate discussion around the incompleteness of the solution. Another subset of students identified the arrow in line 3 as an error (as many of them believed it would be more properly communicated with an implies sign). The majority of students identified the "12" in line 3 as the main error as the author of the solution did not divide the modulus (12) by the greatest common divisor (3). Each of these subsets of students make a valid point and thus this activity is very useful in discussing the range of errors which can be made, and the need for accuracy when presenting and communicating written mathematics.

3.3 Round 3

The third (and final) round challenged students to work in their groups to find a mystery location in or near to the campus location where they will be studying in-person in future. This challenge was specifically designed to consolidate knowledge from the asynchronous resources for the week in the guise of a game. Coordinates for the mystery location were provided in decimal degrees format. Eight of the digits in the decimal degree representation were missing (labelled as a, b, c, d, w, x, y, z). Students were provided with a PDF document consisting of a set of eight clues (one for each of the missing digits). The clues were not overly stretching and required students to demonstrate their grasp of fundamental topics / concepts from the recent material (see Figure 4). As with the previous challenges, students were encouraged to move into their private channels to work on this challenge, and most did so. Students worked on this challenge for 20 minutes.

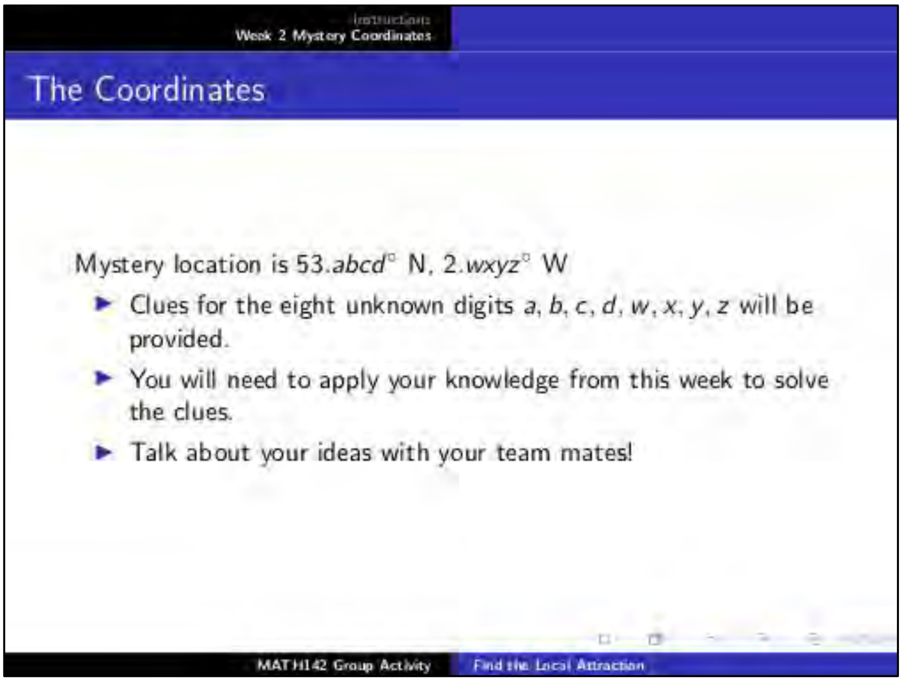

Figure 3. Example instructions for Round 3.

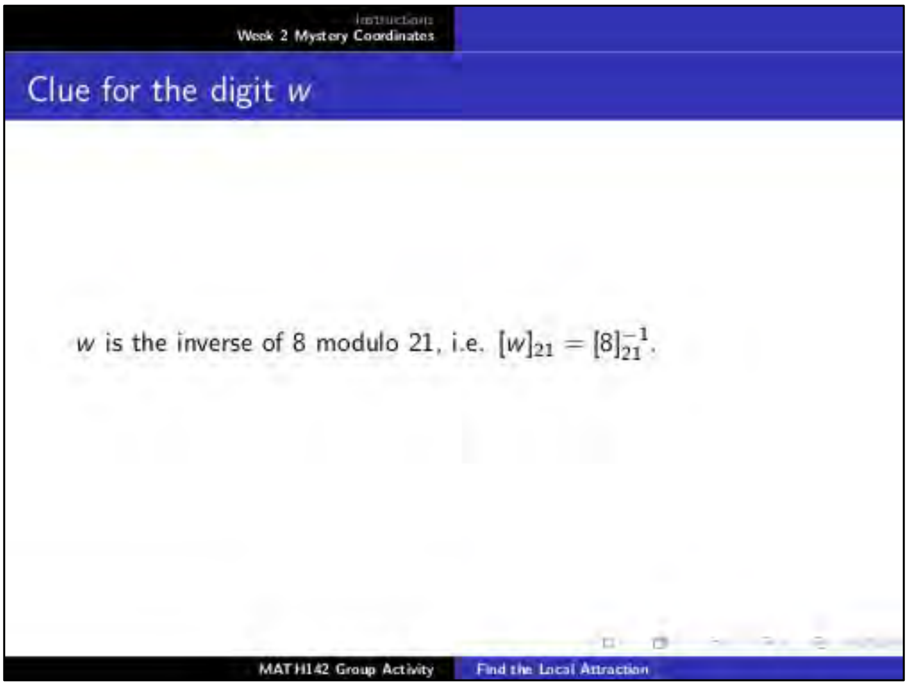

Figure 4. Example clue from Round 3.

When the class came together in the main call, a clickable image poll was run on Poll Everywhere. This time, the image was a map of the local region and students were encouraged to drop an anonymous pin on the image where they believed the mystery location was. Students were then asked to respond to a multiple-choice poll indicating which clue they found the most challenging. Once the correct answers were discussed, students were invited to fill in an open text poll with a fact that they had discovered about the mystery location.

4. Initial feedback and reflections

The online synchronous sessions for the module were well-attended with an average 75% attendance. For comparison, the average online synchronous session attendance across all core semester two Year 1 Mathematics modules was 50%. Students expressed their opinions on the module in a mid-semester survey (response rate 64%). The main results concerning the synchronous sessions are given in Figure 5 below.

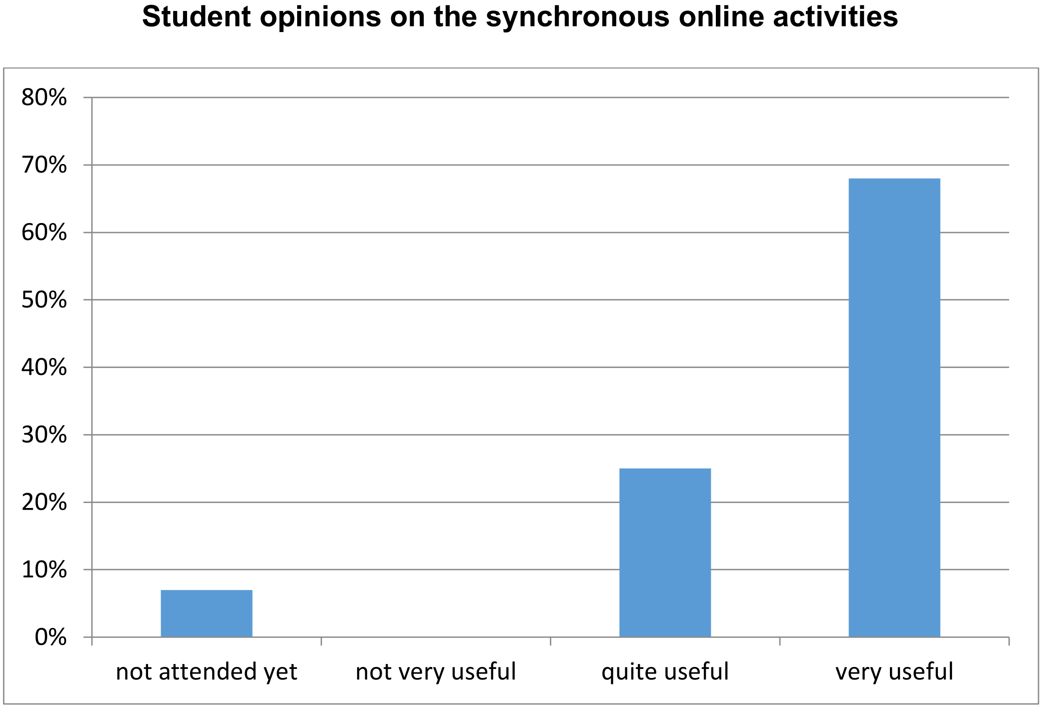

Figure 5. Responses to the survey statement "I have attended the live sessions and the activities are..."

As can be seen from the initial feedback, the synchronous sessions were well-received by students, with nearly 70% of respondents feeling that the activities were "very useful". It should also be noted that all students who attended live sessions and responded to the survey found the activities useful.

Especially encouraging is the fact that several (6) free-text student comments on the sessions described the activities as "engaging" and "interactive" – this was a key aim of the sessions and represents a significant achievement given the struggle across the sector to encourage this interaction in online mathematics sessions.

There are several components of the approach which the lecturer felt contributed to the success of the live session activities. These are given below.

- The challenges and games provided structure to the sessions. This is in contrast to the "any questions?" drop-in format which has been adopted by others to limited success.
- The focus of these sessions was on consolidation of the main ideas / concepts. The answers to clues were often stated clearly in lecture notes or very simple calculations which reinforce the fundamental methods. From this perspective, the challenges were not designed to be daunting for students and could instead be identified as "do-able". The activities were therefore able to act as a confidence-boosting springboard to higher order examples on additional question sheets.
- In contrast with some strategies employed in peer learning studies, students were given time to discuss the challenges with their peers in groups before any polling was conducted. This ensured that the poll results were the consensus view from group discussions and the lecturer was alerted to any widespread difficulties with the material.
- Students were constantly engaged in their work with peers and were not left lingering on one activity for too long. As an active learning strategy, the focus was rightly on what the students were doing and not on what the lecturer was doing.

Inevitably there are some trade-offs with this approach. In creating a safe space for peer discussions, the lecturer is not able to determine whether all students are engaging. While participation in polling and attendance at live sessions are positive indicators, the lecturer cannot be sure that all students in attendance are discussing the challenges with their peers. This would obviously be much easier to determine (and respond to) with in-person classes. There are many opportunities for individual support and feedback in the module via online office hours, exercise sheets etc. The lecturer viewed the synchronous sessions as confidence-building activities to encourage students to tackle the individual tasks and seek out individual support if required. The lecturer felt that the benefits of the approach outweighed the drawbacks, and that creating a safe space for students to develop their confidence in discussing the material was the most important feature of the approach.

5. Future plans – taking the initiative forward post-pandemic

As the sector moves towards a hybrid or blended future post-pandemic, many higher education practitioners are re-evaluating their approach to in-person student learning activities. This is certainly the case at the institution in this case study. As high-quality asynchronous resources have been created during the pandemic, there have been discussions on how best to utilise these and make effective use of in-person classes. For some practitioners (such as the author) this represents an opportunity to free up additional time for active learning and consolidation activities in place of lectures under a flipped classroom model.

The intention for the 2021/22 academic year is to evaluate the success when transferring the successful online model from 2020/21 to an in-person flipped classroom model. Explicitly, the playful learning activities (the three distinct "rounds") will be utilised in the large in-person classes. It is hoped that this will enable the lecturer to observe how students are coping with the activities and to step in when the class appears to be struggling. In the online format from 2020/21, the author deliberately stayed away from the private channel discussions and simply offered to come into the channels if the students made a request. This was intended to create student safe spaces (private channels) where the groups felt free to express their ideas and discuss topics openly. Twenty minutes appeared to be the suitable time for these rounds online, but in-person it should be easier to determine when each round should be brought to a close.

In the 2021/22, a study will be conducted which aims to capture any changes in student confidence when communicating mathematics and working with their peers in these activities. The findings will be reported soon after and it is hoped that data will be collected from several cohorts to determine a sustained impact of the activities.

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RESEARCH ARTICLE

Changes made to the teaching of linear algebra and calculus courses in the UK in response to the COVID-19 pandemic

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Abstract

In response to the COVID-19 pandemic, university mathematics departments in the UK adapted their teaching for 2020-21, with some courses being delivered digitally and others through a mixture of on-campus and digital delivery. A survey of linear algebra and calculus lecturers was carried out in the spring of 2021 to investigate what changes were made to courses, as well as lecturers' perceptions of institutional decision making and support. This survey found that a majority of the 41 participants were satisfied that the choice of delivery mode was correct, although views about the importance of offering on-campus classes were mixed. There was a significant increase in the use of video clips made by the lecturer, video-conferencing software, discussion forums, electronic submission of written work and on-screen marking tools. Most lecturers reported a reduction in the amount of time that students were expected to be taking part in live teaching activities and an increase in the amount of time they were expected to be working on asynchronous activities. While some were keen to return to their previous practice, others were enthusiastic about retaining features introduced in response to the COVID-19 pandemic.

Keywords: university mathematics teaching, COVID-19.

1. Introduction

1.1. Background

The COVID-19 pandemic forced university mathematics departments to adapt at speed to new constraints on the way they teach. On 18 March 2020, the UK government, the Scottish government, the Welsh government and the Northern Ireland Executive announced that state schools would close to most pupils. Around the same time, university departments across the UK began to pause or cancel on-campus classes and introduce emergency remote teaching. On 23 March 2020 governments across the UK announced stay-at-home measures to slow the spread of COVID-19 that prevented on-campus teaching.

Measures were put in place to complete the 2019-20 academic year and universities began to plan teaching for 2020-21 under the assumption that on-campus teaching would be limited or impossible. Throughout the summer of 2020 discussions took place within university mathematics departments and across the academic community, facilitated by initiatives such as the newly-formed Teaching and Learning Mathematics Online (TALMO) conference, about how mathematics courses could be taught digitally in 2020-21. This article aims to record what actually happened in university mathematics courses, specifically calculus and linear algebra courses, in 2020-21 and how any changes made might influence undergraduate mathematics teaching in the future.

1.2. Terminology

Many new terms were introduced during 2020-21 and the following will be used here to avoid ambiguity.

- *On-campus* teaching occurs with all participants in a university building; we use this term rather than *face-to-face* because digital teaching could also legitimately be described as face-to-face if cameras are used.
- *Digital* teaching uses platforms that can be accessed no matter where the participants are physically located; we use this term rather than *online* to avoid confusion with entirely online courses or distance-learning programmes.
- *Live* teaching is conducted in real-time, synchronously; *asynchronous* teaching is not live.

1.3. Research questions

The following research questions were considered.

(RQ1) What modes of delivery were used in 2020-21 for undergraduate mathematics teaching in the UK and did lecturers feel these were appropriate?
(RQ2) Did those lecturing undergraduate mathematics courses in the UK feel that their institutions were prepared for digital delivery in 2020-21?
(RQ3) How did teaching tools used change as a result of the COVID-19 pandemic?
(RQ4) How did expectations made of students change as a result of the COVID-19 pandemic?
(RQ5) Do lecturers intend to keep any changes to teaching made as a result of the COVID-19 pandemic in the longer term?

2. Method

2.1. Design and administration of survey

An online survey was designed with a mixture of open-response, multiple-choice and Likert scale questions. Survey items were inspired by the study of Drijvers et al. (2021). Participation was sought only from those teaching first courses in calculus or linear algebra as these are the only two subjects named explicitly as being common to mathematics degrees in the QAA's Subject Benchmark Statement for Mathematics, Statistics and Operational Research (2019). By focussing on calculus and linear algebra the responses can be compared directly, which would be difficult when including a broader range of subjects. This restriction does skew the results to courses which typically have larger classes and are taught in the earlier years.

The survey was administered electronically using JISC Online Surveys and responses were invited using the authors' professional networks, the Heads of Departments of Mathematical Sciences (HoDoMS) mailing list and a call in an email announcement by Teaching and Learning Mathematics Online (TALMO). The study was conducted according to the University of Edinburgh ethics procedures. The survey questions are given in the Appendix. The anonymised data, along with notes and code used during analysis, are available at https://osf.io/6pujb.

2.2. Analysis of responses

In Question 7 of the survey, lecturers were asked to indicate, from a list, which digital tools they had used before the COVID-19 pandemic and which they had used in the 2020-21 academic year. The data for each tool were subjected to the mid-*p* version of the McNemar test. This particular version of the McNemar test was chosen over the more common asymptotic McNemar test, following Fagerland et al. (2013), due to the modest sample size. To correct for the fact that multiple hypotheses were being tested simultaneously, and so rare occurrences were more likely to be observed in the results, the so-called multiple testing problem, the Holm-Bonferroni correction (Holm, 1979) was used to test for significance at the 5% level. The Holm-Bonferroni method was preferred to the more conservative Bonferroni correction (where an adjusted $\alpha = 0.05/15 \approx 0.003$ would have been used for each hypothesis test) to reduce the risk of not detecting a change (i.e., to reduce the risk of type II errors). However, for the collected data, the standard Bonferroni correction would have resulted in the same conclusion; that is, the same tools would have been identified as having seen a change in their use.

In Question 9, participants were invited to describe any other tools that had been used in 2020-21 and whether these tools had been used before the COVID-19 pandemic. On inspection of the responses to Question 9, some of the tools fell into the categories listed in Question 7. In each of these cases, the response given in Question 7 was checked for consistency with that given in Question 9; in one case, a response to Question 7 was modified accordingly. Notes were kept in the spreadsheet of survey results.

The open-response questions (5b, 8, 9, 11, 12 and 14-18) were analysed to identify common themes. Initially responses were coded according to the themes that they contained by one of the authors. All three authors then met together, looked through the themes and coding, and resolved disagreements by consensus.

3. Results

Between 22 March and 2 June 2021, 44 participants completed the survey. Question 2 was used to screen out three participants who had not taught linear algebra or calculus in 2020-21. The sample consists of 41 lecturers; 20 had been responsible for delivering a first course in linear algebra, 20 a first course in calculus, and one taught a course covering both subjects. Of these, 34 chose to identify their institution. There were participants from at least 21 distinct UK institutions; 20 in England and one in Scotland. Six of these institutions were members of the Russell Group. There were seven participants in the sample who did not identify themselves or their institution. For context, the Complete University Guide lists 72 departments in its UK Mathematics Subject League Table 2022, suggesting that the survey had a reasonably high response rate.

3.1. Modes of delivery (RQ1; survey Q3, Q4, Q5)

Figure 1. Summary of responses to Question 4: In 2020-21, what was the mode of delivery of your course?

Table 1: The number of courses delivered through on-campus and digital activities in different terms.

	A mixture of on campus and digital	Fully digital (with no on-campus activities)	Total
Before Christmas 2020	10	11	**21**
AAfter New Year 2021	1	9	**10**
Whole academic year 2020-21	6	4	**10**
Total	**17**	**24**	**41**

A small majority (24/41; 58.5%) reported no on-campus activities in 2020-21 (see Figure 1); the remainder reported teaching using a mixture of digital and on-campus activities. During October and November 2020 all areas of the UK introduced tiered systems designed to control the spread of COVID-19. On 19 December 2020 more restrictive measures were introduced in parts of England, in Scotland and in Wales. These measures constrained on-campus teaching and explain the marked shift in delivery mode after New Year 2021, with 9 of the 10 courses being fully digital (see Table 1). The responses to the question "Who made the decision about the mode of delivery?" are summarised in Figure 2. The two lecturers who responded "Other" both felt there was no decision to be made since government restrictions had forced them into digital delivery, and indeed these courses were both delivered after New Year 2021.

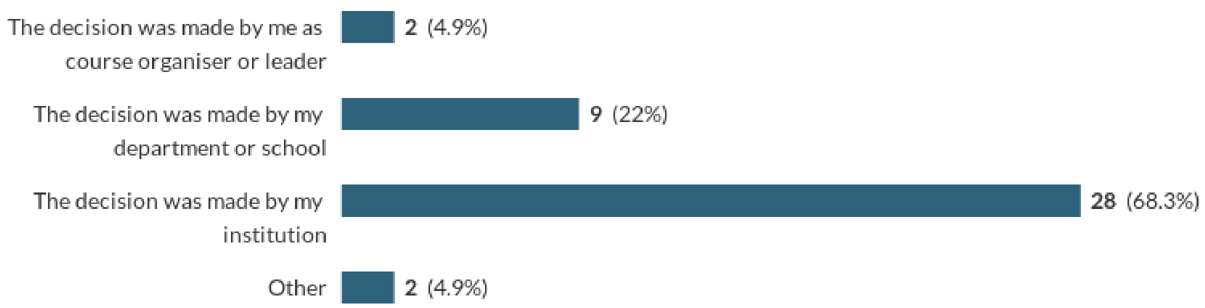

Figure 2. Responses to Question 5: Who made the decision about the mode of delivery?

Of those who expressed an opinion, there was near unanimity that the mode of delivery used had been the correct one given the circumstances. Some lecturers who had offered on-campus classes felt that these had been worthwhile, even under restrictions.

"I think it was correct, fully online would be worse, even the chance of having some in-situ examples classes were useful."

Several lecturers were concerned that students had lacked the opportunity for social interaction and that on-campus teaching activities would have been beneficial.

"The first year students would have benefitted more from more on-campus activity. The social cohesion of the students hasn't been great."

Others who had offered on-campus classes commented that these were poorly attended by students.

“Some element of in-person teaching was allowed, and reasonable to provide; however, it was hard to do, and attendance was very poor.”

One lecturer commented that, in their experience, while some teachers preferred to give on-campus sessions, the physical-distancing restrictions impinged on the operation of classes so much that offering fully digital classes was preferable for learning. However, they did concede that students may have missed out on the social aspect of on-campus teaching.

“Some of the problem class leaders preferred the on-campus versions, but the group work we usually do in those sessions made the safety very difficult. And on campus sessions were cut much shorter due to cleaning time. So educationally and from a health perspective, (though perhaps not socially), I think it was better online.”

Only two lecturers said explicitly that they felt the choice to have on-campus classes had been wrong. One cited the increased workload involved in delivering physically-distanced classes as the reason against offering on-campus activities; the other felt that having on-campus classes during a pandemic had put staff at risk.

Almost all lecturers who had taught fully digitally supported this decision, suggesting there was no other option (29/38; 76.3%). Three lecturers (3/38; 7.9%) raised concern about ensuring an equitable experience for students unable to travel to the campus. One felt students would have benefitted from face-to-face contact on campus while another suggested fully digital delivery provided a saving in workload.

3.2. Perceptions of institutional preparedness for digital delivery (RQ2; survey Q6)

A summary of the responses to Question 6 are shown in Figure 3. The vast majority (36/41; 87.8%) of lecturers agreed or strongly agreed with the statement “My institution as a whole has the technical infrastructure to deliver courses digitally”, with only three participants disagreeing; two did not know or were neutral. Confidence in technical infrastructure at a departmental level was similarly high; 35/41 (85.4%) agreed or strongly agreed that “My department/school has the technical infrastructure to deliver courses digitally”.

However, there was less confidence that institutions and departments had the expertise to provide excellent teaching digitally. The majority of those who expressed a view felt that their institution as a whole had the expertise to deliver excellent teaching digitally, but the most common response was “Neutral/Don’t know” (14/41; 34.1%). More participants (25/41; 61.0%) agreed or strongly agreed that their department had such expertise, but still a substantial number said they were neutral or did not know (11/41; 26.8%). There are many possible reasons for these results: it may be that a shared sense of what constitutes excellent digital teaching had not developed in UK mathematics departments or that communication within departments during the pandemic had been difficult.

The majority of participants (32/41; 78.0%) agreed or strongly agreed with the statement “My department/school supports me to deliver courses digitally”; seven did not know or were neutral and two disagreed or strongly disagreed. Inspection of Figure 3 suggests that participants felt more supported by their departments than by their institutions; this is perhaps unsurprising. Another possible explanation is that centralised support provided by universities may not have addressed the specific needs of mathematics lecturers, such as how to share writing containing mathematical notation. Similarly, participants had more confidence that students were supported by their department, rather than their institution as a whole, although most respondents agreed or strongly agreed that students had been supported to engage with digital activities.

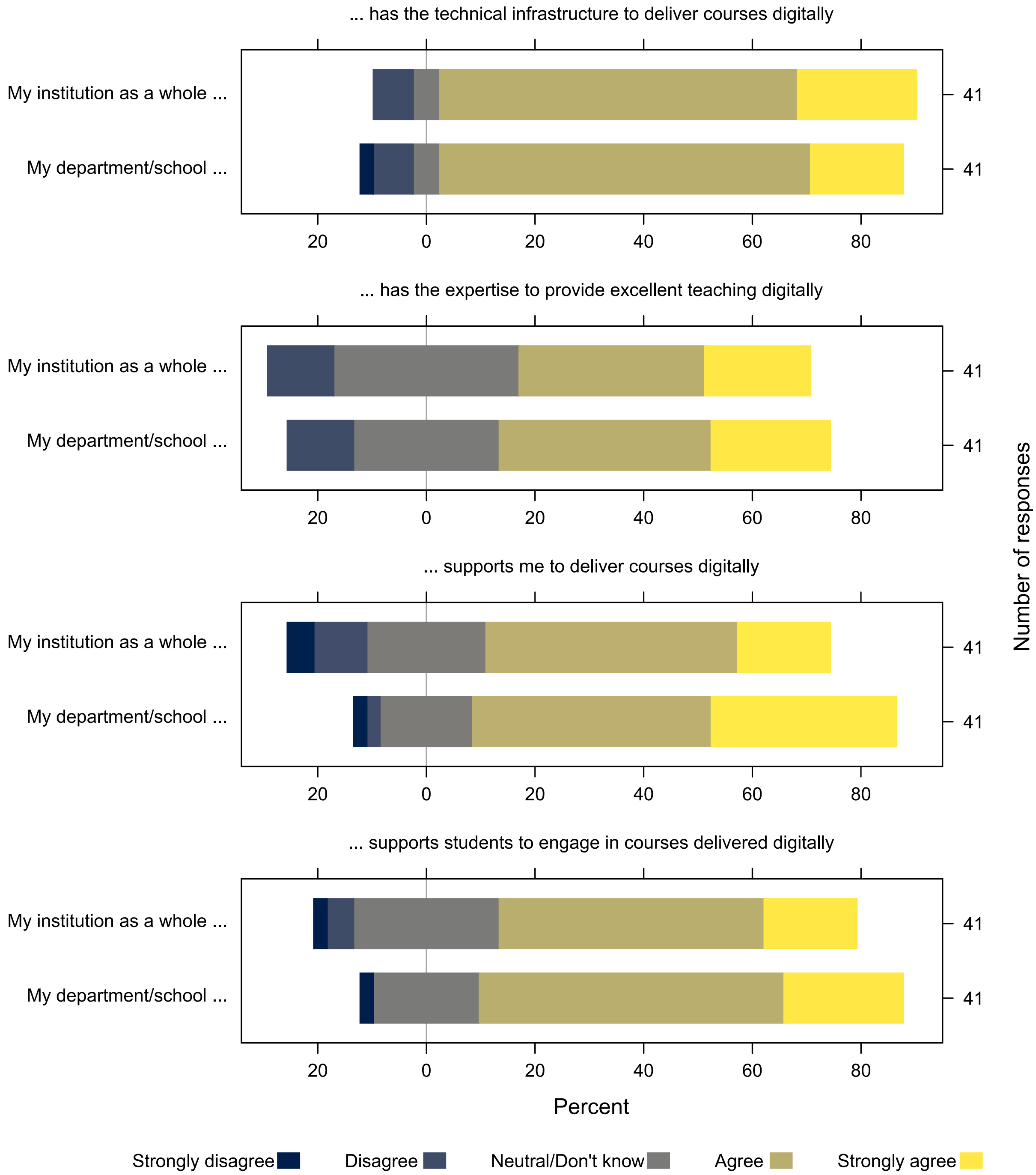

Figure 3. Summary of responses to Question 6: For each of the following statements, please choose the option which most closely aligns with your view.

3.3. Tools used in teaching (RQ3; survey Q7, Q8, Q9)

Responses to Question 7 of the survey are summarised in the alluvial plots in Figure 4. Each of these plots shows the proportion of participants using a given tool before COVID-19 and during 2020-21, and indicates those whose use changed in the interim. The results of the statistical analysis described in Section 2.2 are shown in Table 2.

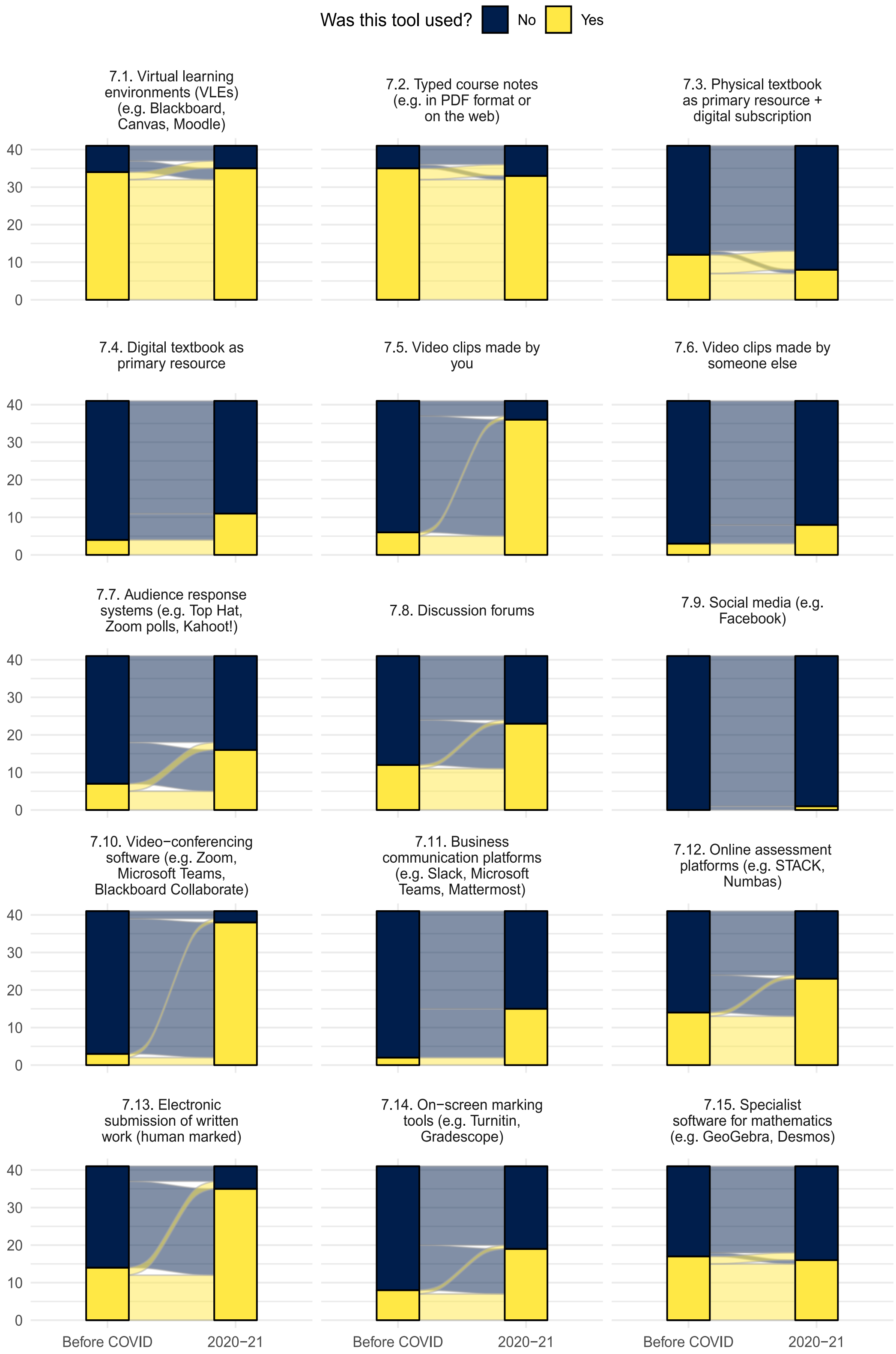

Figure 4. Summary of responses to Question 7: Thinking about how you would have delivered (or did deliver) the same course before the COVID pandemic, which digital tools would you have used (or did you use)? And which of these tools have you used to deliver the course in 2020-21?

Table 2: Results of the statistical analysis of responses to Question 7. Six tools saw a significant change in use in 2020-21 as compared to before the COVID-19 pandemic.

Tool	*p* value	Rank	Adjusted α	Significance
Video clips made by you	< 0.0001	1	0.003	sig
Video-conferencing software (e.g., Zoom, MS Teams, BB Collaborate)	< 0.0001	2	0.004	sig
Electronic submission of written work (human marked)	< 0.0001	3	0.004	sig
Business communication platforms (e.g., Slack, Microsoft Teams, Mattermost)	0.0001	4	0.004	sig
Discussion forums	0.0018	5	0.005	sig
On-screen marking tools (e.g., Turnitin, Gradescope)	0.0018	6	0.005	sig
Online assessment platforms (e.g., STACK, Numbas)	0.0063	7	0.006	non-sig
Digital textbook as primary resource	0.0078	8	0.006	non-sig
Audience response systems (e.g., Top Hat, Zoom polls, Kahoot!)	0.0129	9	0.007	non-sig
Video clips made by someone else	0.0312	10	0.008	non-sig
Physical textbook as primary resource + digital subscription	0.1250	11	0.010	non-sig
Typed course notes (e.g., in PDF format or on the web)	0.3750	12	0.013	non-sig
Social media (e.g., Facebook)	0.5000	13	0.017	non-sig
Specialist software for mathematics (e.g., GeoGebra, Desmos)	0.6250	14	0.025	non-sig
Virtual learning environments (VLEs) (e.g., Blackboard, Canvas, Moodle)	0.6875	15	0.050	non-sig

Six tools saw an increase in their use, significant at the 5% level. These were: (i) video clips made by the lecturer, (ii) video-conferencing software, (iii) business communication platforms, (iv) discussion forums, (v) electronic submission of written work (human marked) and (vi) on-screen marking tools. None of these results are surprising given the shift towards greater digital delivery, but these results do provide a record of changes that have occurred within this sample of mathematics departments across the UK.

Seven of the tools were used by more than half of respondents in their teaching in 2020-21: (i) virtual learning environments (VLEs), (ii) typed course notes, (iii) discussion forums*, (iv) video clips made by the lecturer*, (v) video conferencing software*, (vi) online assessment platforms, and (vii) electronic submission of work (human marked)*. Taken together, these appear to make up a common toolkit used to teach mathematics digitally. The four tools marked * also saw their use increase from pre-COVID times, according to the analysis in Table 2, suggesting that there was a major change in the tools used to teach first-year undergraduate mathematics.

In Question 8 participants were invited to describe any other tools that they used before the COVID-19 pandemic that they did not use in 2020-21. There were responses from 20 of the 41 participants. All respondents described tools or activities associated with traditional on-campus mathematics classrooms such as “Live in person lectures!”, “Chalk, markers and boards”, “A proper blackboard” and “Physical whiteboard and marker pens. Overhead projector”.

In Question 9 participants were asked to describe any other tools that they had used in 2020-21 and whether they had used them before the COVID-19 pandemic. There were two tools in the responses that were mentioned by more than one participant, namely: (i) writing tablets, mentioned by five participants; and (ii) online collaborative whiteboards (those named were Miro and OneNote), mentioned by two participants.

3.4. Expectations made of students (RQ4; survey Q10, Q11, Q12, Q13, Q14)

In Question 10 of the survey, participants were asked how much time students had been expected to participate in live teaching activities during 2020-21 compared with before the COVID-19 pandemic. In Question 11, they were asked to give reasons. Responses to Question 10 are summarised in Figure 5. A large majority of respondents (30/41; 73.2%) said that students had been expected to spend less time taking part in live activities than in previous years; only four (9.8%) said students were expected to spend more time, and seven just the same as before (17.1%).

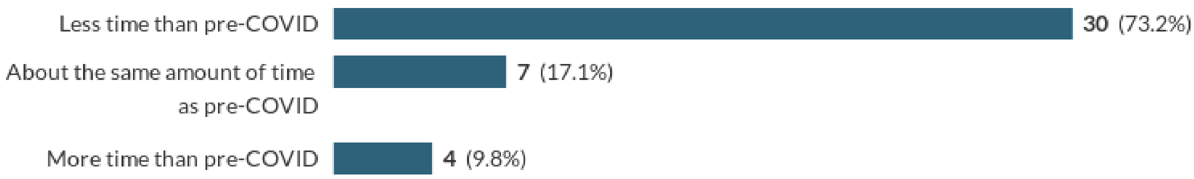

Figure 5. Responses to Question 10: In a typical week of your course in 2020-21, how much time did you intend for students to spend taking part in live activities compared with what you would have intended pre-COVID?

Continuity with previous years was mentioned by all five of those who gave reasons for expecting the same amount of time on live activities as before the pandemic. One lecturer suggested two arguments in favour of running sessions live, even if done so digitally: to cater for student preference and to avoid the workload associated with preparing video clips.

“[The amount of time is the same] Because I've continued to do essentially everything live. The students seem to prefer it like this, and saves me time as I don't need to spend lots of time preparing recordings.”

Of those who said that expected time participating in live sessions had increased, a common reason was the introduction of additional support sessions for students.

Among those who expected less live participation from students, the vast majority (27/29; 93.1%) referred to a reduction in the use of live lectures. Most (22/29; 75.8%) said that these had been replaced by pre-recorded video clips. The Teaching and Learning Mathematics Online (TALMO) meetings were mentioned by one participant as having influenced choices about how to run courses.

“We decided, given lots of advice (including TALMO), that a mix of live lectures and some pre-recorded videos would work best.”

Some of the participants (4/29; 13.8%) had concerns about the access to and the reliability of digital technology for giving live digital classes. There were concerns about students not having adequate software, hardware, a suitable working environment, or internet connection to participate in live digital classes. There were also concerns about reliability of software at the lecturer's end.

When asked, in Question 12, if the nature of live activities had changed compared to before the COVID-19 pandemic, all respondents suggested that changes had occurred. A sizeable number of participants suggested they had reduced the use of traditional lectures in place of other activities. Most common was replacing lectures with video clips and introducing office hours, in-class polling for more interactive sessions, Q&A sessions or review lectures.

Just under one third of respondents (13/41; 31.7%) said they felt they had less interaction with students during live classes. Reasons given included the apparent reluctance of students to appear on camera and speak out in digital classes, as well as difficulties seeing the reactions of the audience. More than a third of respondents (15/41; 36.6%) reported an increase in interaction with students during live activities, though sometimes through non-traditional means. Those who had designed live classes to encourage participation from students (for example through the use of a flipped classroom, in-class polling or Q&A sessions) seemed to report an increase in interaction.

"I did flipped classroom for the first time, encouraged by the Covid online situation. So the live sessions were entirely different to what I used to do before. Much much more interactive, with the lecturing of new material done in pre-recorded videos instead."

Several participants (5/41; 12.2%) mentioned that students seemed to prefer to make contributions using the text chat instead of speaking out during live classes. A few felt that this was a disadvantage.

"live seminars were less interactive due to students not switching on cameras and using time-consuming chat instead of speaking up"

Others saw the increased student participation as an advantage.

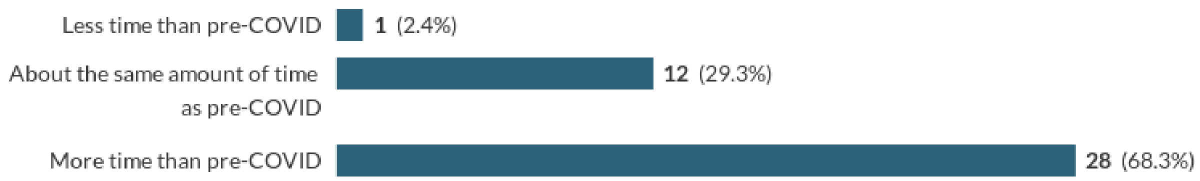

Figure 6. Responses to Question 13: In a typical week of your course in 2020-21, how much time did you intend for students to spend working on asynchronous activities compared with what you would have intended pre-COVID?

The responses to Question 13 are summarised in Figure 6. A large majority of respondents (28/41; 68.3%) said that students had been expected to spend more time taking part in asynchronous activities than in previous years; only four (2.4%) said students were expected to spend less time, and (29.3%) just the same as before. These results broadly mirror the changes in expectations of student participation in live activities discussed above.

In Question 14, participants were asked if students had acted as the lecturer had intended when designing the course, and on what evidence this judgement was based. More than a third (13/37; 35.1%) of the respondents said they thought students had broadly acted as intended, around a tenth

(4/37; 10.8%) suggested students had not acted as intended, around one fifth (8/39; 21.6%) said there had been mixed behaviour, and the same number (8/39; 21.6%) indicated that they did not know how students taking their course had behaved. Among the respondents who described how they had gauged student activity, most common was the use of some form of online activity tracking (14/37; 37.8%), for example the number of times a video clip had been viewed or reports from virtual learning environments. The next most common measure of how students had behaved was their engagement in live classes, which may have been digital or on campus (7/37; 18.9%). These respondents largely described a mixture of behaviour.

"I think the engaged students did work well and interacted with the material as intended. But there was a huge drop-off of engagement in the second semester […] there was reasonable engagement in the first semester (higher lecture attendance than in previous year for longer), but much lower in the second semester (much less than in previous years)."

A couple of respondents reported that students had not been keen to speak out in live digital classes. Several responses (5/37; 13.5%) mentioned that student attainment in coursework had been in line with expectations.

"The performance in assessments so far seem as good as in previous years, but we need to wait and see how the final exam goes."

Three of the responses (8.1%) suggested that lecturers felt some students had struggled with motivation and keeping pace with the course, as well as missing out on social interaction.

"There has been a bifurcation, with around half the students fully engaging as intended, but another half struggling to maintain work levels without the discipline of physical attendance and interaction with coursemates."

3.5. Looking to the future (RQ5; Q15, Q16, Q17, Q18)

In Question 15 of the survey, participants were asked to think about the digital tools they had used in 2020-21, and whether they intended to keep any changes to teaching made as a result of the COVID-19 pandemic. Only 4/40 lecturers suggested that they would like to return to the tools used before the COVID-19 pandemic while the rest expressed an intention to retain at least some of the changes that they had made for 2020-21, notably video clips (17/40; 42.5%). Five of these suggested that recorded lectures would be helpful resources for students.

"I may use the recorded video lectures as additional revision materials in the future. Students always appreciate having more help with their revision."

One suggested keeping the video recordings to help cater for students with varying prior attainment.

"Use of video material. There are parts of the course where students have varied background. Students who haven't seen the material at A-level can access the material at their own pace."

Of those who responded, 14 (35%) suggested that they would like to keep online assessment platforms. The reasons given included allowing students to practice with immediate feedback and preventing cheating on tests through question randomisation.

"I am planning to continue using STACK tests, since they worked very well, and are good both as a form of formative assessment and for providing immediate feedback. Finally, they make copying work from other students nearly impossible (although other forms of cheating may still be taking place)."

Nine lecturers (22.5%) said they would consider some form of flipped classroom, continue to use in-class polling or expressed a desire to have more interactive live sessions.

"I think polling is good because it gives the shy students the opportunity to still engage anonymously."

When asked to think about the mix of live and asynchronous activities (Question 16) half (18/36) suggested that they would like to go back to delivering courses largely as they had done before the pandemic, but with some indicating that they would like to retain video clips for use as an additional resource.

"I prefer to go back to the balance of classes that we had before the pandemic. Many students benefit from the face-to-face interaction. But I would like to make the recordings of lectures available for students that can't attend or need to watch twice."

Most of the remaining respondents (11/36; 30.6%) said that they would consider running their courses with a different mix of live and asynchronous activities than they would have traditionally. Reasons for this included giving students more time to learn at their own pace and having live classes focus on discussion or addressing students' difficulties.

"Yes. I think that the asynchronous delivery of more straightforward parts of the course was really helpful to allow students to go at their own pace. I would not return to the way that the course was delivered pre-COVID, but instead introduce some live delivery of the more tricky concepts."

In Question 17 participants were asked to suggest three things they had changed that they felt had helped students to learn and that they would recommend to colleagues for the longer term. Of the 35 responses, 12 (34.3%) mentioned online assessment, 6 (17.1%) suggested the importance of clear messaging about what is expected of students, and when. Five respondents suggested recording live sessions so that they could be accessed by "students that can't attend or need to watch twice". Four recommended using in-class polling to encourage active participation in live sessions.

Lastly, participants were asked for any recommendations of innovations that should be avoided (Question 18). Eighteen participants gave no response and the most common answer, given by 8/23 (34.8%), was that there was nothing they would recommend avoiding. Three participants recommended avoiding the use of breakout rooms in digital classes. All of these mentioned students' dislike of or lack of participation in discussions. However, this experience was not universal; other respondents elsewhere in the survey commented that they felt digital tutorials had been successful "[s]mall group synchronous maths tutorials worked reasonably well by electronic contact" and "[t]hey really enjoyed doing groupwork". Two responses recommended avoiding take-home assessments, with one participant commenting on the difficulty of ensuring academic integrity. However, the same response did suggest that open-book exams were new to the students; this may partly explain why this style of assessment had not worked well in this case.

4. Discussion and conclusions

4.1. Answers to research questions

(RQ1) What modes of delivery were used in 2020-21 for undergraduate mathematics teaching in the UK and did lecturers feel these were appropriate?

During 2020-21 more than half of courses in calculus and linear algebra were taught digitally with no on-campus activities; the remainder used a mixture of digital and on-campus delivery. On-campus activities were more common before New Year 2021 when COVID-19 restrictions were looser. Almost all respondents felt that their mode of delivery had been correct given the pandemic. Some of those who had offered on-campus teaching felt that it gave students valuable opportunities for social interaction, though it was reported that such activities had been poorly attended. Some institutions who had intended to have on-campus activities were forced to switch to digital delivery as a result of lockdowns and low student attendance. A small minority felt that offering on-campus classes at all during the pandemic had been the wrong decision. The responses suggest that fully-digital delivery was easier to implement than offering a mix of on-campus and digital teaching, and that doing so helped to make students' experiences more uniform.

(RQ2) Did those lecturing undergraduate mathematics courses in the UK feel that their institutions were prepared for digital delivery in 2020-21?

The vast majority of participants felt that both their institution as a whole and their department had the necessary technical infrastructure to deliver teaching digitally but only around half were confident that these also had the correct expertise to provide excellent teaching digitally. Perhaps more work is needed to develop a shared sense of what it means to provide excellent digital teaching in mathematics. The vast majority of participants felt supported to deliver courses digitally by their institutions as a whole and by their departments. Even more felt that their students had been supported to engage in digital activities, though it should be noted that the views of students were not sought in this research.

(RQ3) How did teaching tools used change as a result of the COVID-19 pandemic?

There was a change in the tools commonly used by lecturers to teach calculus and linear algebra courses. Seven tools were used by majorities of respondents in their teaching in 2020-21: (i) virtual learning environments (VLEs), (ii) typed course notes, (iii) discussion forums*, (iv) video clips made by the lecturer*, (v) video conferencing software*, (vi) online assessment platforms, and (vii) electronic submission of work (human marked)*. Four of these tools (marked *) saw increases in use in 2020-21 compared with before the COVID-19 pandemic, significant at the 5% level. A further two (on-screen marking tools and business communication platforms) saw increases in their use since before the COVID-19 pandemic, even though they had not been used by a majority of respondents during 2020-21.

These findings expand on work done to understand changes made during the period of emergency remote teaching (ERT) at the end of the 2019-20 academic year. For example, Ní Fhloinn & Fitzmaurice (2021) found large increases in the use of digital hardware and software for teaching. In particular, they report that video-conferencing software was used by almost all lecturers in their study. Unsurprisingly, this practice appears to have continued into 2020-21. Ní Fhloinn & Fitzmaurice found that only a very small number of lecturers had used pre-recorded video clips prior to ERT; our results similarly show that the use of video clips increased significantly during 2020-21. However, Ní Fhloinn

& Fitzmaurice found that delivering live digital sessions was the most common approach during the period of ERT; by contrast, we found that lecturers typically changed their approaches to introduce more asynchronous activity in place of some live digital classes for the 2020-21 academic year (see also (RQ4) below).

(RQ4) How did expectations made of students change as a result of the COVID-19 pandemic?

There was typically a decrease in the amount of time students were expected to participate in live teaching activities, with an increase in the amount of time students were expected to spend working on activities asynchronously. Almost all of those who expected less live participation from students had replaced live lectures with pre-recorded video clips to be studied asynchronously. Reasons given for this decision included concerns about the reliability of the technology needed to provide live digital classes and concerns that students may not have been able to access live sessions digitally.

Lecturers whose courses still had live classes (whether online or on campus) reported changes in the nature of those classes: more than a third said that they felt classes were more interactive than in the past, while around a third said they felt they had less interaction with students. It appears that lecturers who designed their classes with digital interaction in mind, and made full use of the tools available, saw more interaction than those who attempted to translate traditional lectures to a digital setting. Lecturers felt that students seemed reluctant to use video and contribute orally but were more willing to use text chat to interact during activities.

Lecturers gauged student participation using online activity tracking, their perception of student engagement during live classes, and student performance in coursework. Just over a third of lecturers felt that students had engaged with asynchronous material as intended and around a fifth said that they had seen a mixed response. Around a fifth of lecturers responded that they did not know how students had acted on their course. A few lecturers felt that, as well as missing out on social interaction, some students had struggled with motivation to study and keeping pace with the course.

(RQ5) Do lecturers intend to keep any changes to teaching made as a result of the COVID-19 pandemic in the longer term?

Around 90% of lecturers suggested they might keep some of the changes made in response to the pandemic; around 10% suggested they would like to return to their pre-pandemic teaching methods. More than a third of lecturers said they would keep using recorded video clips, either as an additional resource or to replace traditional on-campus lectures and free up class time for different activities. Just over a third said they would like to keep using online assessment platforms due to them providing students with opportunities to practice, the ability to give immediate feedback, and the ability to randomise questions. Around a fifth of lecturers said they would like to use some form of flipped classroom, in-class polling or expressed a desire to have more interactive classes than traditionally.

Half of the lecturers suggested they would like to return to their traditional mix of live and asynchronous activities in the longer term. Around a third said they would consider a different mix after their experiences in 2020-21, perhaps having students read or watch video clips asynchronously ahead of more interactive live on-campus classes involving polling.

When asked to choose three practices to recommend to colleagues for the future, over a third of lecturers again suggested keeping online assessment platforms. A sixth said that they would recommend clearly communicating expectations to students, perhaps using a to-do list or a week-by-week schedule.

Over a third of lecturers said explicitly that there was nothing they had tried that they would recommend colleagues avoid in the future. Two suggested avoiding take-home examinations in mathematics, with one mentioning concerns about cheating.

4.2. Concluding remarks

The intention of this research was to collect evidence of mathematics teaching practices in UK higher education during 2020-21 and to record the views of lecturers about their practice in the future. Lecturers responded from at least 21 distinct UK institutions, around 30% of all institutions offering BSc Mathematics. While this is a high response rate, it must be acknowledged that there was no response from most UK mathematics departments and so the results should be understood in that context.

The study only considered the responses of those who had taught calculus or linear algebra courses in the UK: one might wonder if the picture would be different for other courses and in other countries. No doubt further research will consider the wider picture.

7. Appendix

The survey questions are given below.

Page 2: Context and decision making in your institution

2. Which course or module were you responsible for delivering in 2020-21?
   - A first course in linear algebra
   - A first course in calculus
   - Other
3. When did your course take place?
   - Semester 1, 2020-21 (i.e. before Christmas 2020)
   - Semester 2, 2020-21 (i.e. after New Year 2021)
   - This course covers the full academic year 2020-21
   - Other

Mode of delivery

These next two questions are about the mode of the delivery of the course, by which we mean either fully digital (with no on-campus activities), fully on campus or a mixture of the two.

4. In 2020-21, what was the mode of delivery of your course?
   - Fully on campus
   - Fully digital (with no on-campus activities)
   - A mixture of on campus and digital

5a. Who made the decision about the mode of delivery?
   - The decision was made by me as course organiser or leader
   - The decision was made by my department or school
   - The decision was made by my institution
   - Other

5b. With the benefit of hindsight, do you think the choice of mode of delivery was the correct one? Why?

Page 3: Readiness and support for digital teaching

6. For each of the following statements, please choose the option which most closely aligns with your view.

   Options: Strongly disagree, Disagree, Neutral/Don't know, Agree, Strongly agree

6.1 My institution as a whole has the technical infrastructure to deliver courses digitally.
6.2 My department/school has the technical infrastructure to deliver courses digitally.
6.3 My institution as a whole has the expertise to provide excellent teaching digitally.
6.4 My department/school has the expertise to provide excellent teaching digitally.
6.5 My institution as a whole supports me to deliver courses digitally.
6.6 My department/school supports me to deliver courses digitally.
6.7 My institution as a whole supports students to engage in courses delivered digitally.
6.8 My department/school supports students to engage in courses delivered digitally.

Page 4: Tools

7. Thinking about how you would have delivered (or did deliver) the same course before the COVID pandemic, which digital tools would you have used (or did you use)? And which of these tools have you used to deliver the course in 2020-21?

   Options: Before COVID, 2020-21

7.1 Virtual learning environments (VLEs) (e.g. Blackboard, Canvas, Moodle)
7.2 Typed course notes (e.g. in PDF format or on the web)
7.3 Physical textbook as primary resource + digital subscription
7.4 Digital textbook as primary resource
7.5 Video clips made by you
7.6 Video clips made by someone else
7.7 Audience response systems (e.g. Top Hat, Zoom polls, Kahoot!)
7.8 Discussion forums
7.9 Social media (e.g. Facebook)
7.10 Video-conferencing software (e.g. Zoom, Microsoft Teams, Blackboard Collaborate)
7.11 Business communication platforms (e.g. Slack, Microsoft Teams, Mattermost)
7.12 Online assessment platforms (e.g. STACK, Numbas)
7.13 Electronic submission of written work (human marked)
7.14 On-screen marking tools (e.g. Turnitin, Gradescope)
7.15 Specialist software for mathematics (e.g. GeoGebra, Desmos)

8. Please describe any other tools that you would have used to deliver this course before the COVID pandemic but that you did not use in 2020-21.

9. Please describe any other tools that you used in 2020-21 and specify whether you had used them before the COVID pandemic.

Page 5: Live and asynchronous

Live activities

10. In a typical week of your course in 2020-21, how much time did you intend for students to spend taking part in live activities compared with what you would have intended pre-COVID?

   - o Less time than pre-COVID
   - o About the same amount of time as pre-COVID
   - o More time than pre-COVID

11. Why is this the case?

12. In 2020-21, did the nature of your live activities change compared to pre-COVID? For example, if you gave live lectures were these more or less interactive than in the past?

Asynchronous activities

13. In a typical week of your course in 2020-21, how much time did you intend for students to spend working on asynchronous activities compared with what you would have intended pre-COVID?
    - o Less time than pre-COVID
    - o About the same amount of time as pre-COVID
    - o More time than pre-COVID

14. Did students act as you had intended? On what evidence are you basing this judgement?

Page 6: Looking to the future

15. Thinking again about the digital tools you have used in your course delivery in 2020-21, are there any that you would want to keep for the longer term once the COVID pandemic is over? Why?

16. Thinking about the balance of live and asynchronous activities in your course in 2020-21, would you now consider a different live/asynchronous balance when delivering the same course once the COVID pandemic is over? Why?

17. Can you suggest three things which you changed that you believe were effective in helping students learn, and which you would recommend to colleagues for the longer term?

18. Is there anything which you changed that you would recommend colleagues avoid?

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CASE STUDY

Engaging with Maths Online - teaching mathematics collaboratively and inclusively through a pandemic and beyond

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Abstract

This case study details several concrete approaches to integrating the use of student-loaned iPads in the teaching of mathematics in Higher Education. Although there is a scarcity of rigorous studies into the efficacy of tablet devices for improved educational outcomes, previous case studies have argued that tablet devices, if used, should be integrated into the whole learning experience. The mathematics teaching team at Middlesex University have developed an inclusive digital pedagogy over the last five years that enabled us to effectively respond to the remote teaching imposed by the COVID-19 pandemic by loaning iPads to all students on specialist mathematics programmes. As we begin the return to campus, we continue to integrate these devices into our teaching to address the observed "digital divide" in Generation-Z students which is characterised not by access to smart devices but by the digital skills to use them as effective learning tools. This is particularly relevant at Middlesex University which is disproportionately affected by digital poverty amongst its student population. We discuss the use of virtual whiteboard apps, the necessity of handwritten mathematics, the rich integration of multimedia content, persistent collaborative "problem solving spaces", and how a common hardware platform allows for varied and equitable inclusive assessment. We also report the results of students' surveys of iPad use during the remote-only 2020-21 academic year.

Keywords: Inclusive assessment, digital poverty, technology enhanced learning, iPad, virtual whiteboard.

1. Introduction and background

For several years mathematics lecturers have been using technology enhanced learning to develop the student learning experience on the specialist mathematics programmes at Middlesex University. The university invested in a number of tablets and styluses for use by staff for teaching and feedback in 2017-18. This meant that when the UK went into lockdown in March 2020 due to the global pandemic the teaching team seamlessly moved to online teaching having previously prepared much of our resources and assessment electronically (see Figure 1 for an example of pre-pandemic lecture delivered from a staff iPad). Like many other HEIs, issues around the motivation and attendance at online classes soon began to appear. Speaking with students, it soon became clear that one significant reason was digital poverty, a problem that disproportionately affects students at Middlesex University. Around 58% of Middlesex Students are from the most deprived areas (2019-20 Index of Multiple Deprivation quintiles 1-2) and although this geographic data doesn't necessarily reflect students' individual circumstances, the university does have the seventh highest proportion of undergraduates who were eligible for free school meals at Key Stage 4 across the whole English Higher Education sector (Office for Students – Access and Participation Data Dashboard), and the second highest proportion across non-private universities (see Figure 2). We found that many students were unable to access the internet from their homes or didn't have the requisite hardware to access online content.

It became clear that the sector's default Bring Your Own Device (BYOD) approach to online teaching was not suitable for our students. As a result, the university decided to expand its investment in tablets and styluses to provide hardware for all specialist mathematics students from the start of the 2020-21 academic year. A total of 32 devices were distributed at a cost of approximately £400 per device including accessories. The devices are expected to have a 5-year life-span.

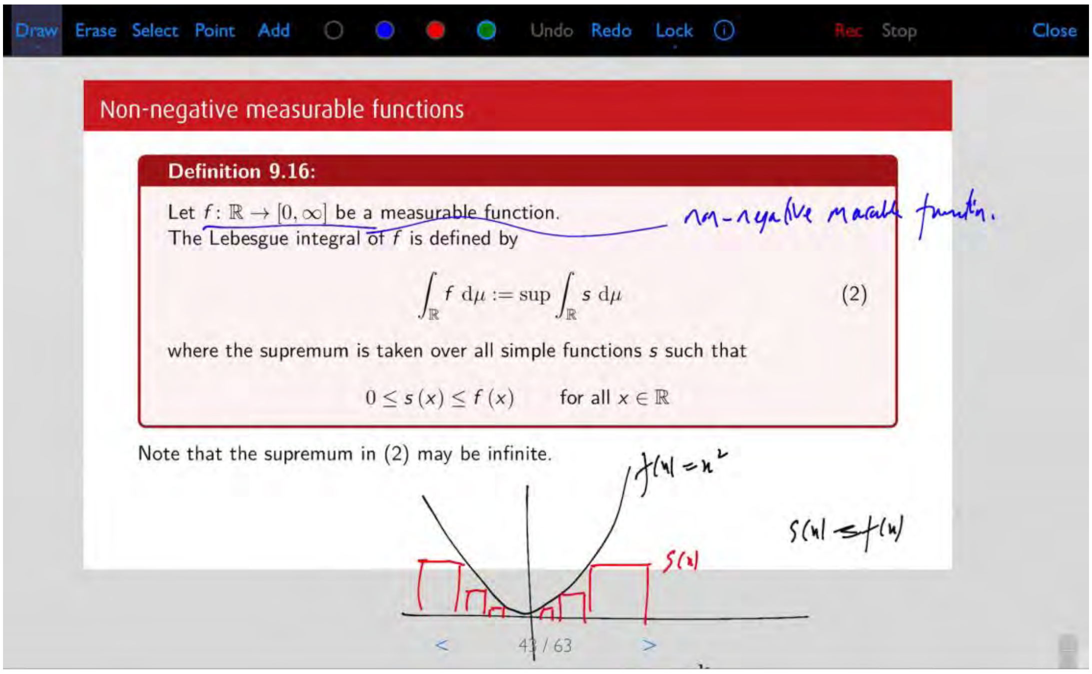


Figure 1. Screenshot of iPad app "Vittle Pro" used to record and mark-up slides during lectures.

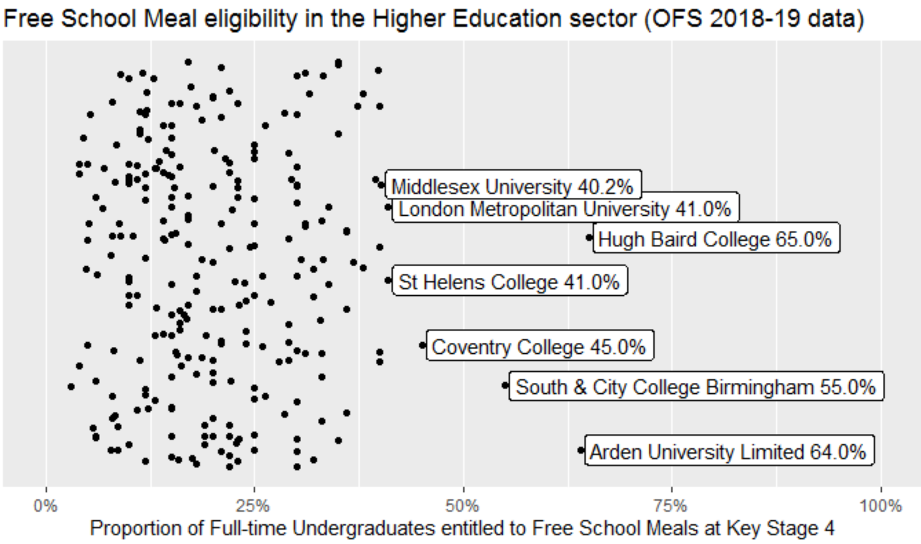


Figure 2. Proportion of full-time undergraduates eligible for Free School Meals at Key Stage 4 amongst 255 English HE providers. The vertical direction is jittered to prevent overplotting. Providers are labelled when proportion is greater than 40%. R code available at https://github.com/nicholassharples/OFSdata

In this paper we provide a case study for the approaches used in developing teaching, learning and assessment strategies based on the premise that students have a common hardware platform. Overwhelmingly we found that the use of the common hardware platform meant that we were able to leverage the wealth of software available that allowed us to develop a rich learning experience – in many cases improving on what was possible in a face-to-face setting, especially surrounding collaborative learning.

In the rest of this paper, we will describe and evaluate the novel pedagogic approaches to mathematics, enabled by the common hardware and software that the Middlesex University mathematics team have developed and will retain:

- **Courses are built around tablets to achieve true blended learning**
  Students use their tablets to work on collaborative, persistent whiteboards before, during, and after scheduled lecture times. All work can be completed, and feedback given, using the tablets to provide equality of access.

- **Collaborative "problem-solving spaces" create learning communities**
  Students contribute at times convenient for them, working with peers to build solutions to complex problems over an extended period.

- **Equitable access to technology allows for inclusive assessment**
  Students can choose their preferred submission format without disadvantaging others.

2. Discussion

Since the release of the iPad in 2010 the potential applications of these devices in education settings have been the subject of much qualitative work and case study. However, in a systematic literature review, Nguyen, Barton and Nguyen (2015) reported that studies were at an "early exploratory stage" and showed that "the iPad was found to enhance the learning experience but not necessarily lead to better learning outcomes" critically noting that "it is not clear how best to align and integrate it within the academic programmes". Haßler, Major and Hennessy (2016) describe a similar "scarcity of rigorous studies" in the primary and secondary education sectors. In a more recent systematic review, Svela et al. (2019) look specifically at the use of tablets in maths education but conclude that "almost eight years into this generation of tablet technology, the body of knowledge … is still limited".

Nevertheless, students are increasingly using digital devices to access education materials independent of universities. The Ipsos MORI (2018) report acknowledges the ubiquity of digital communication technology for Generation-Z students, but describes a significant digital divide in their use: students from lower incomes families may not be using the technology for learning and therefore are less likely to develop certain key skills as a result. Universities have an important role in addressing this gap: we can encourage the development of digitals skills by ensuring technology is integrated into courses. Elphick (2018) describes how embedding iPad use in HE courses can help improve student perceptions of their digital capabilities.

In one case study Mang and Wardley (2012) describe a trial of loaning iPads to 47 HE students across three summer courses to provide tablet-format lecture notes, electronic reading materials, electronic quizzes and a social platform to discuss lectures. The authors conclude their experiences with six recommendations including "Make the tablet an integral component of your class" to fully realise the potential benefits and prevent unstructured and distracting use.

Our case study therefore describes a variety of concrete ways to integrate iPads into mathematics teaching with a view to conduct a rigorous evaluation of learning experience and learning outcomes from this approach.

3. Virtual whiteboards and handwritten mathematics

Before the pandemic iPads were used by staff as virtual whiteboards (see Figure 1). Handwriting over prepared slides (digital ink) is a powerful way to distinguish mathematical content (such as typeset definitions) with pedagogic commentary (such as handwritten calculations). After lectures, marked-up slides are shared as PDF files as well as full-length audio/video capture of the whiteboard, which helps to improve the accessibility of the handwritten content.

When we loaned iPads and Apple Pencils to students a priority was to integrate digital ink into the learning environment: we designed lecture notes with spaces for annotations and encouraged working directly on the iPad rather than scanning or photographing work on paper. One approach was to use shared folders in Microsoft OneDrive that removed the need to submit work for feedback: once a student finished editing a document lecturers would be automatically informed.

Later we adopted the Miro platform (see Figure 3) as a more fully featured unbounded virtual whiteboard. This web application allows for slides and lecture notes to be easily imported, annotated and recorded but also allows a large number of multimedia resources to be directly embedded onto the whiteboard using HTML iFrames or third-party plugins. For example, YouTube videos, files through cloud storage services, or interactive graphing applications such as Demos or Geogebra can all be made available on a single whiteboard. Miro whiteboards themselves can be embedded in Virtual Learning Environments. We tried a variety of virtual whiteboard platforms, including Microsoft Whiteboard and Mural, but are continuing with Miro as they currently offer free educational licenses.

An immediate benefit with unbounded whiteboard space is the ability to present a non-linear discussion of mathematics (see Figure 3) so that dependencies and relationships between mathematical ideas are clear and spontaneous discussions can be appropriately placed. Common structures (such as definition/example/non-example) can be saved as templates to reduce preparation time. We also encouraged students to browse the virtual whiteboards using their own iPads rather than relying on projectors or screen sharing. Each student has acursor visible to all whiteboard users which indicates their current focusmaking it easy to judge engagement and respond to students' attention even during remote lectures. These cursors can be hidden or labelled with student names or pseudonyms.

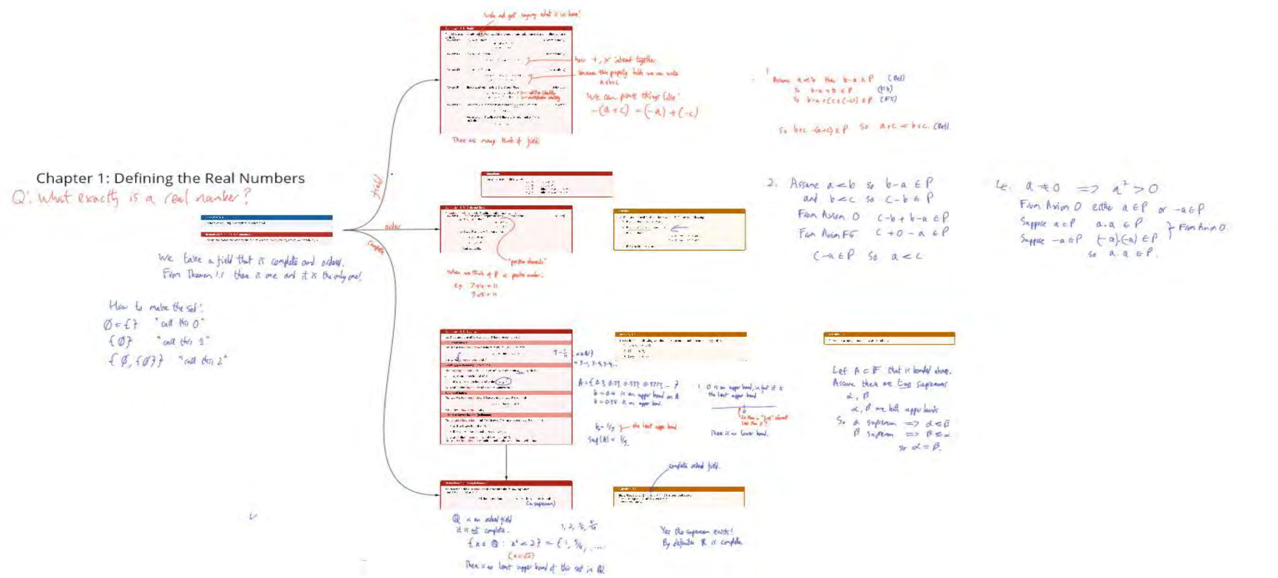


Figure 3. Non-linear virtual whiteboard with Miro.

4. Problem-solving spaces

Virtual whiteboard apps such as Miro also allow hosts and invitees to collaborate synchronously. Students can write to whiteboards, work in teams on problems or discuss material live in class. Importantly this can be done online so that students do not need to be in the same physical environment. A typical example of the output from a learning session is given in Figure 4. On the left is part of the whiteboard and the right is a closer look at a section where students have collaborated on a problem.

Influenced by approaches to teaching from other areas of the Design Engineering and Mathematics department, especially from colleagues from the product design and design engineering subject teams, the mathematics team have also developed Problem-Solving spaces for students in emulation of the “studio culture” of these other disciplines. Our approach is to use virtual whiteboards to build on lectures but give students their own space to interact with their peers outside the confines of timetabled sessions. These Problem-Solving spaces help develop a sense of a learning community: students contribute at times convenient for them, working with peers to build solutions to complex problems over an extended period, and can provide feedback and ask questions of their peers’ contributions.

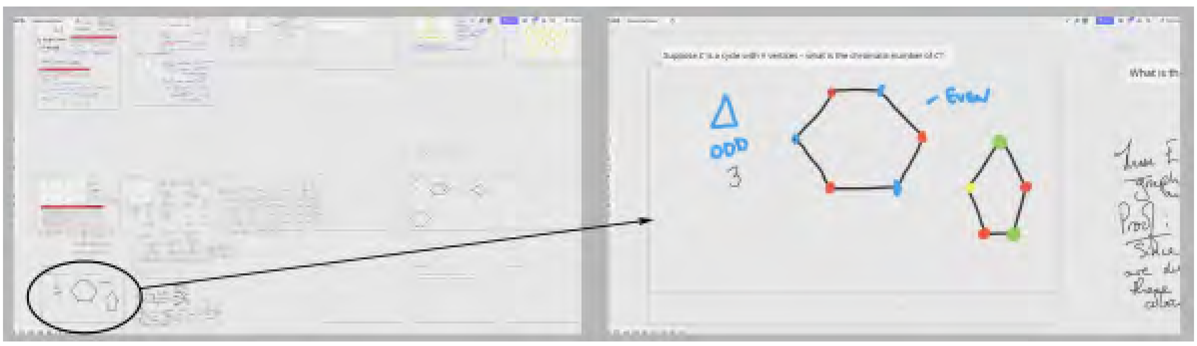


Figure 4. Student collaboration in a “problem solving space” using iPads in the Miro app.

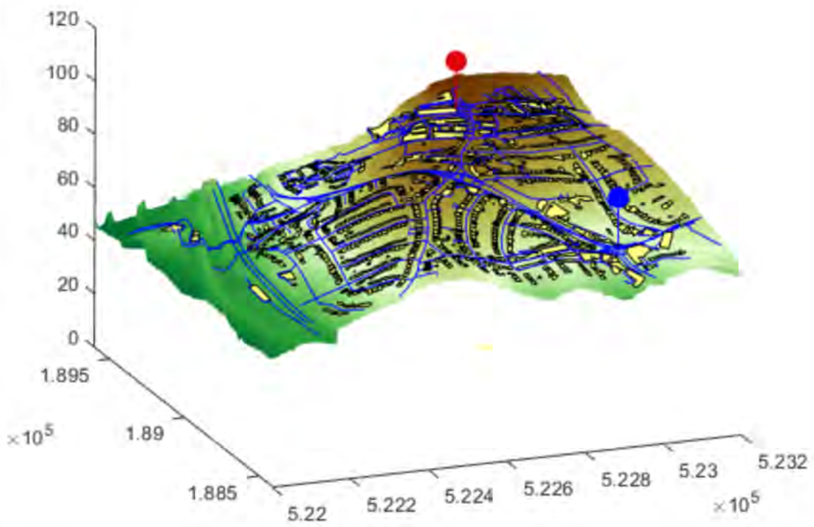
5. Inclusive assessment design

The mathematics team continue to embed the use of iPads within all learning activities and now are working more collaboratively with students by designing inclusive assessments that provide multiple equivalent options for demonstrating learning outcomes e.g., written work, video, audio, multimedia blogs. This aims to improve equality of opportunity, and be more inclusive, reducing the need for reasonable adjustment. Traditionally, assessment of mathematics is based on written coursework, timed assessment such as examinations and tests and project work. Often students are asked to communicate arguments or calculations and are assessed on the clarity of their exposition.

Although the programme assessment strategy had built in elements of choice, feedback from students indicated that digital poverty limited their engagement with it. Now that students have a common hardware platform and consistent access to specific software means that the mathematics team has been able to develop a more modern and inclusive approach to this type of assessment, removing barriers to learning (see Figure 5). Instead of asking students to write a report we now also give students the option to make a video commentary. Instead of asking students to show their working, we allow them to add an audio commentary to their work, or to record themselves working through the solution to a problem. While it is not unusual to assess students using audio and video submissions (these are used extensively in our Faculty of Arts and Creative Industries) this form of assessment is less common in the sciences. Our approach goes further by giving students the opportunity to choose the method they might employ to communicate their work whilst retaining the essence of the learning being demonstrated. The Faculty of Science and Technology at Middlesex have funded an evaluation into this radical form of assessment, which we expect to roll out further once successfully completed. It is hoped that the work will feed into the national conversation around authentic forms of assessment of mathematics and their integrity following the recent joint statement (London Mathematical Society, 2021) of mathematics learned societies concerning assessment adaptations necessitated by lockdown. The societies are concerned that open-book assessments don't provide the desired academic integrity and urge caution in the removal of traditional exams. We believe that technology-supported coursework provides greater opportunity for authentic real-world assessment, the connectivity afforded by this digital platform can improve academic integrity, and using technology to provide options to students is the best way to gauge their learning.

6. Evaluation

In order to evaluate the October 2020 roll-out of iPads for students we surveyed the recipients in February 2021 using a webform. The questions were a combination of number-of-star ratings and free-text responses, and an archived copy is available (Jones, Megeney and Sharples, 2021). All teaching provision had been done remotely during this period. At the time of this survey, during the 3rd national lockdown in the UK, students were using their iPads primarily to access live-streamed lectures and teaching resources, and to handwrite mathematics for feedback. Students were mainly submitting work asynchronously with some limited synchronous contributions during lectures and office hours. As detailed above we have since further integrated iPads into our teaching and the return to campus has enabled us to support and encourage students' use of iPads to a much greater degree. A follow up survey will be conducted at the end of the autumn term 2021 to continue this evaluation.



4. From first principles (i.e. working directly from Definition 3.3 and without using other theorems) prove that f is differentiable at the point **p**. 5 marks

5. In fact **all** multivariable polynomials are differentiable at every point in the domain. Sketch an argument to show this. Either

   - write a formal argument,
   - draw an illustration,
   - record an audio explanation or
   - record a video explanation.

   Imagine you're trying to convince a mathematician of this fact.

   > Pro-tip:
   >
   > You may recall last year we demonstrated all single variable polynomials $f : \mathbb{R} \to \mathbb{R}$ were differentiable.

   3 marks

6. We are using a polynomial to approximate the altitude of terrain near Middlesex University. In general would you expect the true altitude of terrain to be differentiable? If not, provide a mathematical description of a non-differentiable geographic feature. Either

   - write an explanation,
   - draw an illustration,
   - record an audio explanation or
   - record a video explanation

   to justify your answer. 4 marks

Figure 5. Example coursework with inclusive submission options for students.

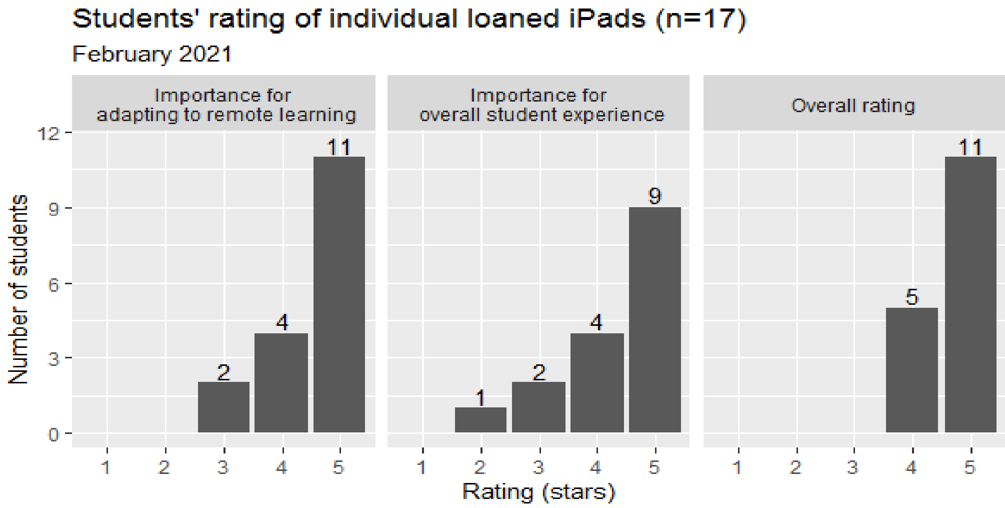


Figure 6. Results of student survey (Feb. 2021) following iPad loans (Oct. 2020).

Table 1. Response rate of student survey.

Programme	Year	Students enrolled	Responses	Response rate
MSc Financial Mathematics	1	2	1	50%
BSc Mathematics	1	6	3	50%
	2	8	6	75%
	3	10	3	30%
BSc Mathematics with Computing	1	2	1	50%
	2	1	1	100%
	3	3	2	67%
Totals:		32	17	53%

In Figure 6 we report the result of the numerical rating questions. Table 1 shows the response rate, indicating a fairly uniform sampling across the mathematics cohort. A comparison across programmes and year groups wasn't conducted due to small sample sizes. Perhaps unsurprisingly the students gave a high overall rating to the free loan of an iPad. However, we also see that students regard the iPads as being an important tool in adapting to remote learning, but have a more mixed view on the importance for the overall student experience.

The free-text survey questions sought to identify the challenges students faced that iPads could resolve, to investigate how students were using the iPads and the effect it had on their learning, how we could continue using iPads on the return to campus, and whether the devices could improve staff and student interactions and equality of experience. The responses were evaluated and the following themes identified.

Theme 1: iPads facilitated better communication in online sessions.

14/17 respondents reported that "writing" or "whiteboard" usage had been helped by the iPads, including the ability to "scribble down quick proofs" and "writing simultaneously with [the] lecturer". 6/17 respondents reported that iPads improved "interactivity" or enabled "interaction".

Theme 2: iPads enabled easier collaboration.
7/17 respondents reported that “sharing”, “collaboration” or “working closely [with] others” was improved.

Theme 3: iPads enabled quicker, more relevant feedback.
4/17 respondents mentioned a benefit with “feedback”. Staff have been using iPads for a number of years to give feedback on summative assessment. The novelty here perhaps is that students are completing formative tasks in a format they can more easily share to get detailed feedback.

Theme 4: iPads enabled students to be more organised, work faster and more efficiently and reduce stress.
5/17 respondents made reference to time saved through using the iPad compared to traditional approaches, in one case “shortening my courseworks by hours”. 5/17 respondents mentioned increased efficiency, organisation or ease in working. 2/17 respondents mention a reduction in stress due to iPad use in learning.

Theme 5: a universal iPad loan scheme encourages equality of access.
2/17 respondents explicitly mentioned that the learning experience was more equitable. 7/17 respondents reported that the iPad loan scheme helped them overcome technical issues that had previously restricted their access to learning such as slow laptops, malfunctioning cameras and microphones and the use of small screens (such as mobile phones) as their primary device.

7. Conclusions and recommendations

Supporting the recommendations of Mang and Wardley (2012) we argue that it’s not enough to simply give students access to the technology and the ability to collaborate, learning sessions must be designed so that we are integrating these digital pedagogies seamlessly into the learning. This approach is best realised with a common hardware and software platform such as iPads otherwise problems of inclusivity and equality of access become apparent.

We recommend that learning sessions be designed so that:

- Significantly less time is spent on presentation of content, ideally in small 10–20 minute chunks at a time. The collaborative virtual whiteboards and student iPads use make it easy to punctuate sessions with problem-based activities to consolidate learning: students following the lecture on their iPad simply start writing on the device next to the question while enjoying immediate access to the lecture material. Even remote sessions can be delivered in this way if students have equal access to tablet devices.
- Lecture slides and/or notes are available before and during the session. Tablet devices allow lectures to be given directly ontop of these notes through annotation. Students can use iPads to review and contribute to the notes in the same format they were presented.
- Time is given to students to collaborate on activities. This can be effectively managed using the virtual whiteboard timer tool and breakout rooms to separate group discussion if the session is remote. Students can use their iPads to immediately share their ideas as well as highlighting and commenting on each others’ work, which helps to stimulate discussion.
- Separate areas of a common whiteboard are devoted for groups to work on, both during and after scheduled sessions. Each group can then see the lecture material as well as the work of other groups for inspiration. Individual student iPads ensure that everyone has effective access to group work outside of scheduled lecture time.

- Whiteboards are persistent – we use one per learning session and do not delete it afterwards so that students can come back later to see a record of the work done.

This approach allows the team to teach collaboratively in a way that isn't possible even in a face-to-face session where teamwork can be more stilted and less integrated in teaching, and interaction between groups can be difficult to harness. That students and lecturers are working on the same whiteboard provides an invaluable and unique resource beyond the classroom (virtual or physical) and ensuring persistence of the whiteboard means students have a record of their learning and their peers' learning.

We plan to continuing to offer iPad loans to all specialist mathematics undergraduates but will incorporate more sessions to familiarise students with the devices to help develop the necessary digital skills.

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CASE STUDY

Writing mathematics collaboratively in online workshops

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Abstract

This case study examines the changes that were made to workshops for first year mathematics students when moving from in-person to online in the 2020/21 academic year. In the workshops, students tackle unfamiliar problems in small groups, with a focus on group work and mathematical communication skills. Transitioning to online workshops presented several difficulties around how best to enable students to have meaningful mathematical discussions and collaborate in writing their solutions when working online. We discuss the changes and mitigations we implemented in order to move the workshops online and how this will inform future in-person workshops.

Keywords: Mathematical writing, group work, study skills, online learning.

1. Introduction

Mathematical Investigations is a first-year unit at the University of Bristol which is compulsory for single-honours mathematics students. The unit focuses on developing teamwork and study skills through a combination of weekly two-hour workshops alongside multi-week group projects. In the workshops, students work in groups of around four on worksheets which lead them through a problem in steps. The topics are chosen to reinforce concepts from other units that they have typically found challenging. Emphasis is placed on communication skills: discussing the problems and learning how to write mathematics well. We discuss the goals and structure of the workshops in Gunns, et al. (2020). At the end of each workshop each group submits a collaborative solution which is marked for feedback. Students must attend and engage with the workshops but the work is formative only; the grade for the unit comes from the project component.

2. Aims of the workshops

2.1. Mathematical writing

Precise mathematical writing, the ability to clearly articulate complex ideas and an ability to work in teams are important competencies for UK mathematics graduates (QAA, 2019). Writing mathematics well is frequently challenging for students; an overview can be found in Schleppegrell (2007). Part of the difficulty is that it is not necessarily clear to students what is meant by "good" writing. In Lew & Mejía-Ramos (2019) fourteen common types of errors in undergraduate students' mathematical writing are identified and a following study (Lew & Mejía-Ramos, 2020) finds differences of opinion among mathematics teachers as to which of these are reasonable expectations of students writing. Common types of improvements we focused on in workshops included having students write grammatically in full sentences; that they ensure notation was defined when introduced and used correctly; and that equations were connected with an explanation of how to get from one step to the next, where all assumptions were clearly stated. These correspond to the three main themes identified in Lew & Mejía-Ramos (2019), and were also informed by tutors' experiences of marking student work.

2.2. Group work

The workshops were designed as a group task. In part this was to help students build teamwork and communication skills, with students working together both to solve the mathematical problems and to decide how to clearly express their arguments in writing. Group work has well established benefits including allowing students to become more actively engaged in learning, share ideas and build supportive learning communities (e.g., Healey, et al., 1996). For mathematics, the introduction of communication skills can aid students in the development of mathematical problem-solving skills (Taylor & McDonald, 2007). An aspect of this is that students' roles in their learning are shifted, as summarised in Nilson (2016), to include students as active participants in problem solving and as sources of feedback for their peers. The collaborative nature of the workshops allows these benefits to be realised in this unit as the collaborative writing process allows students to check that their arguments are understood by peers as they go. Group work also permits students to be able to tackle more challenging tasks that are difficult to do alone. We believe that learning to write mathematics well is the type of challenging task that becomes more approachable in a group setting.

3. Moving the workshops online

When planning for the 2020/21 academic year during summer 2020, it was decided that the Mathematical Investigations workshops would be held online. The requirements on social distancing and mask wearing meant that group work would have been difficult, verging on impossible, in person, and so it was necessary to explore options for online platforms and formats that would best allow us to transfer the workshop experience online.

The workshop team began by considering the key functions that online platforms would need to be able to effectively host workshops. These included breakout room functionality, so that students could work together in small groups; the ability for an instructor to easily move between breakout rooms to check on and help students; the ability for students to signal to the instructor if they required help; screenshare functionality that could be annotated by all students in the room to aid discussion of the worksheet; the ability to save annotations to revisit and submit work; and considerations of the user-friendliness of the platform. After testing platforms including Zoom, Bluejeans, Blackboard Collaborate and Microsoft Teams, it was decided that Zoom was the best option for satisfying these requirements. While testing the various platforms, members of the workshop team also accessed test sessions using a range of different devices.

While testing the different platforms, it became clear that moving the workshops online was introducing a conflict between two of the key goals: working collaboratively and writing well. In person, these goals were often aligned: by working in groups students would have to communicate their arguments to their teammates clearly, and when writing their solutions could discuss the wording and get feedback from their group easily. Online, this became much more difficult. The most effective way for students to write their solutions clearly would be for them to write individually using pen and paper, then scan and upload their solutions. However, this would make it difficult for students to discuss the wording of their solutions, as it would be time-consuming to upload and share their solutions with other students, especially if multiple iterations were required. Additionally, if students were not working on writing their solution together it seemed likely that many would not discuss their work either, especially as they would not necessarily know the other students in the breakout room. On the other hand, prioritising collaboration and requiring students to use online annotations would encourage discussion and allow all students to contribute more easily to each question, but using annotations is more difficult than writing on paper and so adds an additional barrier to the writing process. This might lead to students taking shortcuts or including fewer details because of the additional difficulty, or to unequal contributions depending on different equipment. After weighing up these contrasting options, we ultimately decided to prioritise the collaborative aspect of the workshop and encourage students to use the annotation function to submit their work. This was because there were few opportunities for students to talk to each other in general, so this was a good opportunity for them to both learn to discuss mathematics and to get to know each other.

4. Changes made to online workshops

In order to effectively move the workshops to an online setting, we implemented a number of changes, some of which have led to longer-term changes that will be carried forward to future in-person workshops.

4.1. Structural changes

The initial change that was made to all workshops was a practical one to make the worksheets easier to use in the online setting. To facilitate the process of using annotations, we altered the format of the worksheets provided to students. Instead of a question document to be answered on separate paper, we created slides from each worksheet containing one question per slide, and lots of white space for the students to annotate. Each group then screenshared so they had a shared workspace to discuss and answer the question. Students could then take a screenshot of each slide, and then submit these to be marked. This avoided the need to keep switching between a whiteboard and the question document. Notation was altered in several workshops (for example, changing Greek characters to Roman characters where possible) to reduce the amount of drawn (rather than typed) input required, and some questions were altered to make annotated answers easier. In workshops where students were asked to read proofs, we ensured that the entire proof could be displayed on one slide. This helped students focus on seeing the big picture and write notes about the proof without needing to delete the notes to change slide. We repeated previously calculated answers and pictures from previous slides so the students didn't have to make a note of all their answers outside their submission. These changes, though useful in the online setting, were no longer needed when the workshops returned to taking place in-person.

4.2. Induction activity

To help students get used to the online annotation tools and Blackboard submission process, we replaced the first workshop with an induction session. This consisted of several short activities designed to introduce students to the various Zoom tools and also to promote some initial discussions. For example, one activity asked students to consider some common misconceptions about mathematics and annotate a scale on the screen to indicate how much they agreed with the statement. These misconceptions were based on a list from Alcock et al. (2015), building on the work of Schoenfeld (1992). Another activity separated students into breakout rooms and asked them to produce some simple annotations, screenshot these, and practice uploading them. A short guide about using Zoom (including how to annotate, screenshare, save screens and use the 'raise hand' function to get the instructor's attention) and how to submit work on Blackboard was also provided to students.

The induction activity provided a good opportunity to provide a clearer introduction to the workshops, stimulate some discussions regarding students' expectations of university mathematics, and outline our expectations of good mathematical writing in advance, as well as getting students to practice the technical skills they would need. We have retained the discussion-based elements of this activity now that in-person workshops have resumed.

4.3. Different question types

Even with the changes to the worksheet structures discussed above, it was clear that some questions would not translate well to the online setting, as it was likely that students would have to spend too much time thinking about how to deal with complicated notation online rather than focusing on the content of their answers. To deal with this, we implemented new types of question.

Some questions which involved long but standard calculations we changed to ‘fill in the blank’ style questions, where we gave students a large part of a sentence or calculation, but asked them to fill in the justification. By giving them the framework and asking for the details, the students still had to work through the problem, but not write down all the awkward notation. This also allowed us to model the format of solutions we were looking for, by indicating where we would expect students to provide links and justifications.

For some questions where we had previously asked students to prove a result but where notation may have been difficult to write online, we instead provided examples of poorly constructed proofs, and asked students to identify the problems. As discussed in Selden & Selden (2003), students often struggle to determine the validity of a proof. The examples we gave students contained errors that we had commonly seen students make in previous years' workshops. By discussing these errors, we hope to more easily address the misconceptions and help students to improve their proof validation skills, and these questions can be used at the end of the session for a whole class conversation about what ‘good writing’ means.

4.4. Markschemes

Another change was to introduce marking criteria for each week in the second term. Online, it was harder to informally discuss and direct students than it had been in person, so it became more important to clarify the goals of each workshop. These generally involved 4 elements, each marked out of 2, with an aim to reinforce to students the writing aims for that week. They also served as reminders that students should continue to do certain things, e.g. “All answers are written in complete sentences.” was a criterion used every week. At least one criterion usually focused on the mathematical content for that week e.g., “Demonstrates understanding of conservation laws”. These marking criteria provided a simple framework for students to check that they had completed the key aims for each week, reminded them of the writing focus for that week, and allowed for more focused feedback. These marks did not contribute towards the assessment for the unit, and if full marks were not awarded for these criteria, then some comment on the script would be made by the tutor. The role of assessment on student learning has had long-standing interest (e.g., Gibbs & Simpson, 2004). Here they highlight conditions which assessment should aim to achieve, such as “Feedback is acted upon by the student” (this is their Condition 10). Our marking criteria seemed to achieve this, by focusing students on the aims for that week. Relevant for the pandemic, Gibbs & Simpson (2004) also say that “[students] can cope without much, or even any, face-to-face teaching, but they cannot cope without regular feedback on assignments.” This year we have applied marking criteria to every assignment, carefully considering how these requirements build on those from the previous weeks.

4.5. Online submissions

The way that students received their feedback after online sessions also changed. Originally, submissions were handed in and marked on paper and returned to the students in the next workshop. Within this setting, students could see their feedback immediately before starting their next assignment, and hopefully this encouraged them to act on their feedback. However, if they missed the session, were late, or otherwise did not get a chance to see the comments on their written work, then this could lead to them missing their feedback. Submissions were made as a group in 2020/21, as in previous years, but the submission was done online through Blackboard. Students were then notified of their feedback and score by emailing all students in each group and including the feedback for their work either through annotations to their electronic submission or by typing it into the email. This had the advantage that students could get their work back soon after the session rather than having to wait until the next week. This has the benefit that the feedback is more timely (see Condition 6 of Gibbs & Simpson, 2004). It also meant that there were no issues with legibility of comments, and they could be done in relation to the specific marking criteria (which were discussed in the previous paragraph). Of course, in the online setting, it was unclear whether students even read the comments! Student submissions did appear to satisfy the criteria relating to earlier assignments, but whether this was due to the feedback would be difficult to judge accurately. Sending the comments by email could be seen as a way to start a direct dialogue with a student, since replies or clarifications could be asked for, but no instances of this occurred in 2020/21. Use of discussion boards where the groups and the tutor could comment on work might be one way to encourage such a dialogue, and to check that students are actively engaging with their feedback. In the future we will continue to provide feedback via email, due to the advantages we saw whilst the workshops ran online.

5. Conclusions

The move to online workshops for the 2020/21 academic year necessitated a variety of changes to be made to the format in order to run the sessions smoothly. Some of these changes, such as reformatting the worksheets into slides, were purely practical changes, and will not be retained for in-person workshops. However, many of the other changes (such as the induction session, some new question types, mark schemes and the feedback procedure) arose from the move online but appear to be generally beneficial to students' learning, and so will be continued now that in-person workshops have resumed. In addition, we have extended some of these ideas further in the in-person workshops. As well as marking criteria, we begin each workshop with a more in-depth reminder of pitfalls to be aware of in the current workshops, and highlight particular writing features that we want students to focus on each week. Building on the induction activity and variety of question types, we also end each workshop with a whole-class discussion, asking students to reflect on the work they have just done, discuss examples of proofs and look for flaws, and consider extensions of the topic.

6. Acknowledgments

We would like to thank Lee Butler, Thomas Jordan, John Mackay and Sebastian Müller for their help testing online platforms.

We would like to dedicate this paper to the memory of Lynne Walling. Her leadership was an integral part of the development of this course and we are all grateful for her support in designing and running these workshops.

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RESEARCH ARTICLE

The impact of remote teaching on statistics learning and anxiety

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Abstract

In March 2020, staff and students at UK universities had to suddenly transition from on-campus teaching to remote learning as a result of the pandemic, which continued throughout the 2020/21 academic year. Unlike traditional online learners, students may lack the motivation or confidence to learn as effectively online particularly for modules such as statistics which students often find difficult or stressful face to face.

This paper uses survey results from students studying elective and compulsory statistics modules in the 2020/21 academic year to gain an insight into remote learning of statistics from the students' perspective.

When compared to previous face to face teaching of statistics, students were less likely to actively engage with material, ask for help or work with peers remotely. Emotional wellbeing, motivation to learn, statistics anxiety and having a suitable learning environment all impacted on being able to learn statistics remotely. Although statistics anxiety in online teaching situations was generally lower, there was no evidence to suggest anxious students would benefit from online learning going forward.

Keywords: statistics anxiety, motivation, remote learning.

1. Introduction

The term 'online learning' refers to courses which are delivered in a fully virtual environment relying on the use of the internet for teaching material and interactions with teaching staff and other learners. Studies into online learning often look at the attributes of a successful e-learner (Wighting, Liu and Rovai, 2008) with motivation (Horzum et al, 2015) and engagement (Martin and Bolliger, 2018) being key factors of both retention and success. Motivation was described as the 'engine' of learning by Paris and Turner (1994), highlighting its importance in effectively undertaking learning of any kind. Intrinsic motivation, where students are innately interested in the topic and see value in learning it, is particularly important (Pintrich et al, 1991) and has been shown to relate to both performance and statistics anxiety (Marshall et al, 2021). When a student is motivated by the desire to perform well this is known as extrinsic motivation.

Whilst research suggests that online learners generally have higher intrinsic motivation (Schroff et al, 2007), Park and Choi (2009) found that online courses had higher drop-out rates compared to face-to-face courses of a similar nature. Key factors for high drop-out rates in online courses include lack of motivation (Artino, 2008; Keller, 2008), technology issues (Hara & Kling, 2003), feelings of isolation and loneliness (Paulus & Scherff, 2008), and problems with time due to external responsibilities (Keller, 1999).

Hodges et al. (2020) suggest that it typically takes six to nine months to plan, prepare and develop a fully online university course, but in Spring 2020 both staff and students had to transition almost overnight from on-campus teaching to enforced remote learning. Most academic staff lacked the pedagogical or technological experience to adapt quickly to online delivery and many students lacked 'online readiness', which includes a lack of suitable technology or learning environment at home (Horzum et al, 2015; Hung, 2010; Hodges et al., 2020; Ching et al., 2018).

Statistics anxiety is the worry felt by an individual towards statistics in any form and at any level and is thought to 75%-90% of students (Onwuegbuzie & Wilson, 2003; Marshall et al, 2021). Students often have negative attitudes as well as anxiety towards statistics (Sese et al, 2015) which impact on self-efficacy (Finney & Schraw, 2003), motivation and perseverance. Whilst there is very little literature on statistics anxiety and online learning, Devaney (2010) found that the online students had suffered from higher levels of statistics anxiety compared to their on-campus counterparts.

This paper uses survey data collected from students studying both compulsory and elective statistics modules during 2020/21 to get an overview of the students' experiences of remote learning, investigate the factors impacting most on being able to learn remotely and whether learning online may benefit or hinder students with statistics anxiety.

2. Methods and cohort differences

Two separate online surveys were carried out at the start and end of the academic year 2020-21 for specific cohorts studying statistics at Sheffield Hallam and the University of Glasgow although apart from the section on statistics anxiety, this paper focuses on the results from the second survey from Spring 2021.

Most questions used were 7-point Likert and scale means were calculated to represent key academic factors which are described within the relevant sections of the results as well as the Appendix (with Cronbach's alpha scores). A key scale used throughout is students' perception of their ability to learn remotely was assessed using three questions: whether they felt able to learn statistics remotely, software remotely and complete assessments to a good standard. The Motivated Strategies for Learning Questionnaire (Pintrich et al, 1991) was used to measure motivation, asking students or staff for help when needed and task value (intrinsic motivation).

Table 1. Students participating in the two surveys.

Institution	Survey taken	1st year	2nd/final year
University of Glasgow	October 2020	90	
	Feb/March 2021	44	22
Sheffield Hallam University	October 2020	151	
	April-July 2021	54	33

Although the focus of the paper is not an evaluation of teaching methods, a brief overview of the cohorts taking part in the study at Sheffield Hallam and Glasgow Universities and teaching delivery in 2020/21 is given here.

The first-year statistics modules at the University of Glasgow are elective whereas statistics is compulsory for first- and second-year Mathematics and Psychology students at Sheffield Hallam. At Glasgow, the first-year modules are open to students from any program of study conditional on sufficient grades in mathematics at school and the second-year modules are open to students with sufficient first year maths grades.

In Glasgow the first-year students had assigned reading and online exercises in preparation for online live interactive lectures which were recorded and made available to those who could not attend. All Sheffield Hallam and second year Glasgow students predominantly had asynchronous recorded lectures to accompany the lecture notes followed by live online tutorials although Psychology students also had a few face-to-face tutorials at the start of the year. Sheffield Hallam Mathematics and Psychology students both have some form of peer support within their first year to encourage communication between students and staff.

3. Results

3.1 Learning statistics remotely, engagement and motivation

Independent t-tests were used to test for cohort differences. As the boxplots in Figure 1 show, there were some noticeable differences between the students studying elective and compulsory statistics modules with Glasgow students significantly more likely to feel they were able to learn remotely ($p< 0.001$, MD=1.2) but less likely to ask for help ($p<0.001$, MD=1.1). The mean differences between cohorts were much larger when comparing first year students and although not significant, first year students felt less able to learn remotely than second years.

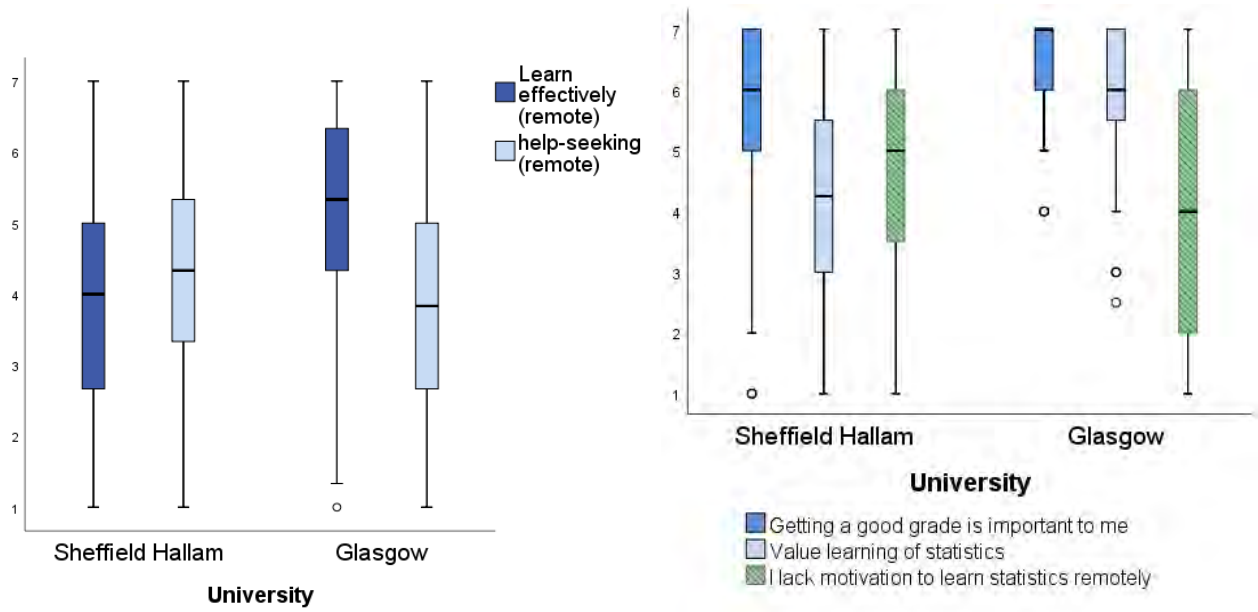

Figure 1. ‘Boxplots of being able to learn remotely, motivation and helpseeking’.

Glasgow students were also more motivated by a desire to do well (p<0.001, MD=0.74), more intrinsically motivated (p<0.001, MD=1.8) and are less likely to lack motivation to study statistics (p=0.005, MD=-1) which given they are studying an elective course is not surprising.

Passive learning was classified as basic engagement with a course such as attendance and watching videos whereas the active engagement scale asked about engaging more with material or others (see Appendix). Sheffield Hallam students' average levels of passive engagement were significantly lower than Glasgow (p<0.001, MD=-1.3) where most students were passively engaging. Active engagement was lower but similar for the two groups. Research suggests that students may perceive passive learning as more useful than active learning (Deslauriers et al, 2019) and if extrinsically motivated, may regard focusing on what they need for assessment with more control over how and when they learn. 64% of Sheffield Hallam students and 73% of Glasgow students agreed that they focused on studying what they needed for assessment when studying remotely.

Students were also asked whether they thought lectures and videos were effective for learning statistics and although 45% and 65% of Sheffield Hallam and Glasgow students felt videos were at least somewhat effective, only 18% and 23% respectively thought they were more effective than on-campus lectures. Further results for Sheffield Hallam showed that 76% of students would prefer videos to be a back up rather than the main source of learning but 45% said they would be less likely to attend if recorded back up was available.

3.2 Comparison with on campus delivery

Students who had experienced learning statistics pre-pandemic were also asked the same questions about learning statistics on campus. An interesting and significant interaction between institution and mode of delivery was observed through a mixed ANOVA, F(1,40) = 5.2, p = 0.028. As Figure 2 shows, there was no significant difference between cohorts when learning on campus but a substantial difference when learning remotely. For Sheffield Hallam students, there was also a significantly higher mean for learning on campus compared to learning remotely (p = 0.014).

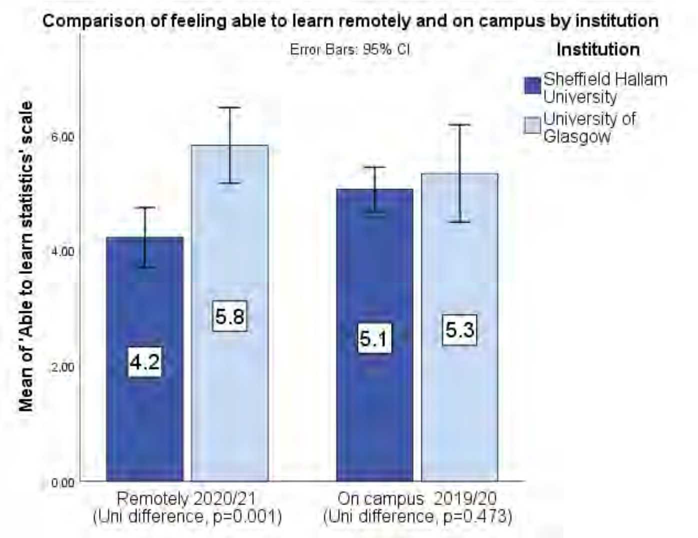

Figure 2. Remote and on-campus differences for being able to learn.

Figure 3 shows the mean paired differences (remote – on-campus), confidence intervals and paired t-test results for other key aspects of learning. Apart from an increased use of the internet, most aspects of learning were worse remotely with active engagement, working with other students and asking for help significantly lower when learning remotely. Sheffield Hallam students had significant differences for all aspects except using the internet more which was significantly higher only for Glasgow students.

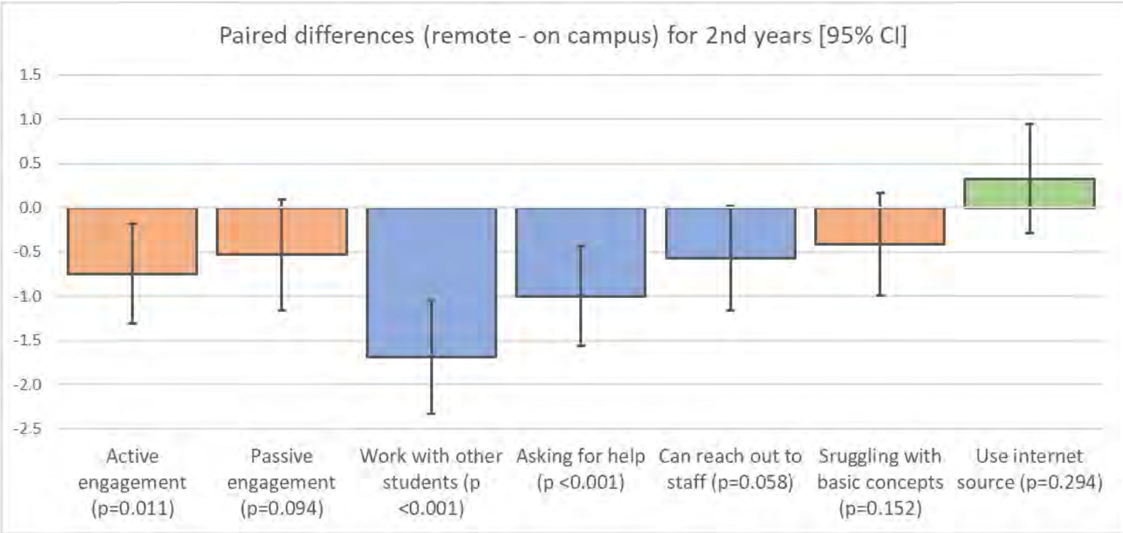

Figure 3: ‘Paired mean differences (remote – on-campus) and t-test results for 2nd years’.

3.3 Online readiness and emotional well-being

It was anticipated that the impact of living through a pandemic, even apart from the move to online learning, may impact negatively on emotional wellbeing. Students were asked how often they felt sad, emotionally exhausted, worn out and had problems concentrating in the past few weeks on a Likert scale ranging from Not at all (1) to All of the time (5).

Worryingly, at the time of taking the survey 83% of Sheffield Hallam students and 59% of Glasgow students had a mean score of at least 3, suggesting that the majority of students were struggling with negative emotional wellbeing at least a part of the time. An independent t-test showed that Sheffield Hallam students had significantly higher levels of negative emotional wellbeing ($t(146) = 3.21$, $p = 0.002$, M.D. = 0.5) than students at the University of Glasgow.

Another aspect explored related to the ‘online readiness’ of students studying remotely. Even though students had been learning remotely for quite a while by the time of the survey, many still did not have a suitable place to study, had technology issues making it difficult to study online, or felt unable to manage their workload online, particularly at Sheffield Hallam (see Figure 4). These issues are likely to impact on learning, stress and motivation and represent an increased risk of falling behind.

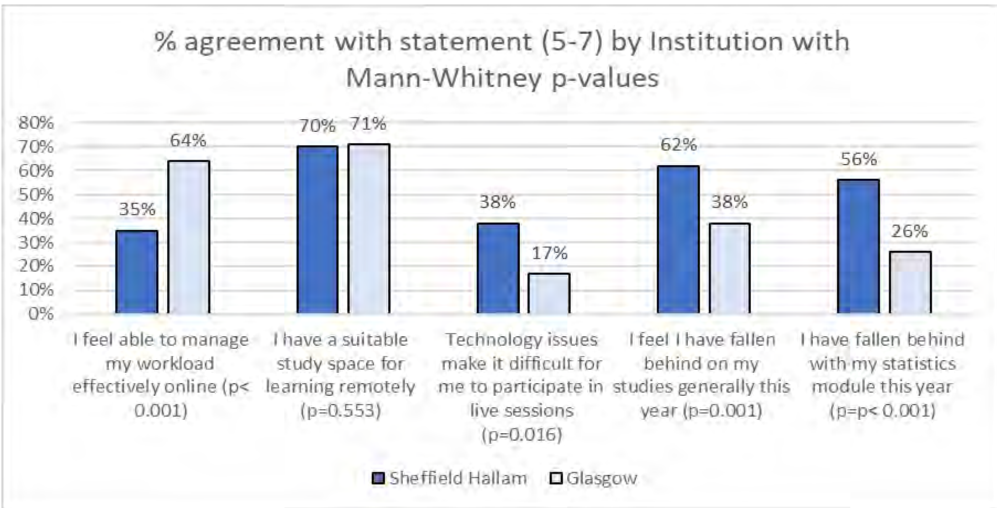

Figure 4. Summary of online readiness and falling behind responses by cohort.

While it was anticipated that students would particularly struggle with statistics following the move to online teaching, this does not seem to be the case. Fewer students reported falling behind with statistics than falling behind with their studies in general.

3.4 Factors impacting on learning remotely

Figure 5 summarises relationships between students' feeling that they can learn remotely and other factors such as motivation, emotional well-being and online readiness to study. Pearson's correlations (r) are given for individual aspects, R^2 for block contribution to explained variance of being able to learn and significant predictors from a final backwards regression model (explaining 60% of the variation in being able to learn) are starred.

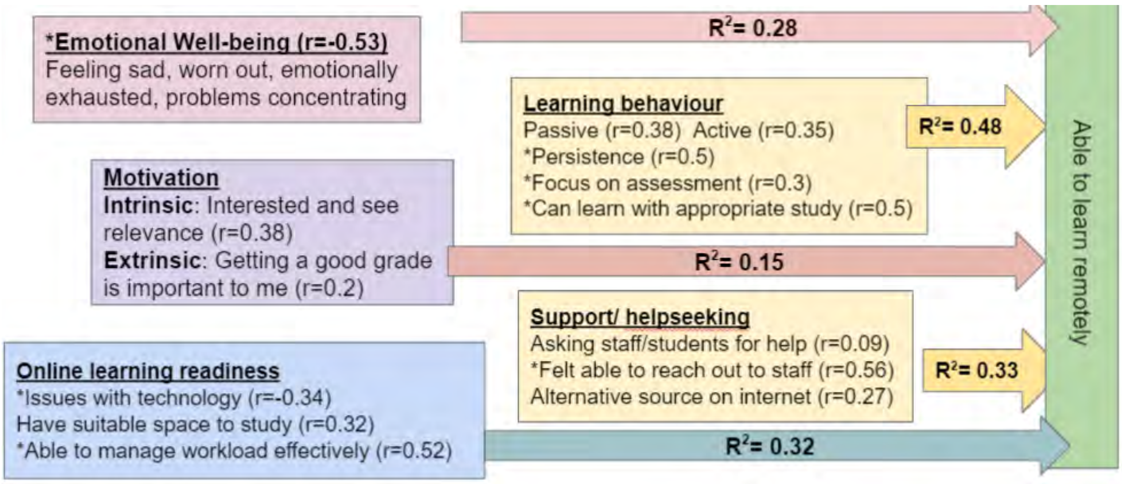

Figure 5. relationships with 'able to learn remotely'.

Negative emotional wellbeing was a strong negative predictor of feeling able to learn remotely and students' motivation (particularly intrinsic) had a moderate positive impact. Online readiness explained 32% of the variation in being able to learn with reported ability to manage workload the strongest predictor.

Regarding learning behaviour and help-seeking, the strongest positive predictors were belief they can learn statistics if they study appropriately (control of learning), persistence and feeling able to reach out to staff if struggling. Students who focused on assessment were more likely to feel they were able to learn remotely but this may not be reflected in their understanding of the topic or final grade. There was no significant difference in feeling able to learn between the two institutions after controlling for motivation and negative wellbeing and learning behaviour. These results suggest that students on elective courses are more motivated to learn which leads to better engagement and learning outcomes.

Statistics anxiety was measured using the Statistical anxiety measure (SAM; Earp, 2007) which gives a general measure of statistics anxiety and the help-seeking anxiety subscale from the Statistics anxiety rating scale (STARS; Cruise, Cash and Bolton, 1985). In addition, subscales relating specifically to online and on-campus situations and software were created (see Appendix). Students were asked how anxious they felt in a variety of situations from 1 (Not at all anxious) to 7 (Extremely anxious) for all subscale items.

All students taking the October survey were asked these questions including the first-year cohort from Glasgow who later took part in the second study. Although some students took both surveys, the results in Table 2 are not paired due to small numbers.

Table 2: Comparison of mean scores at the start and end of the year

Survey details				Type of statistics anxiety								Other factors		
Survey	Institution	Year Group	N	Task	Software	Help seeking	Online class	Face to face class	Watching videos	Working with students online	Working with students face to face	Getting help when needed	I lack motivation to learn statistics	Task value
Start of year	Sheffield Hallam	1	151	4.3	4.2	3.5	2.7	3.5	1.9	3.4	3.4	5.0	3.5	4.7
	Glasgow	1	90	3.6	3.6	3.8	2.8	3.4				4.2	2.7	5.8
End of year	Sheffield Hallam	1	54	5.0	4.6	4.6	4.1	4.6	3.0	4.4	4.5	4.3	4.9	4.0
	Glasgow	1	44									3.1	3.6	6.0
	Sheffield Hallam	2	33	4.9	4.3	4	3.7	3.1	3.2			4.4	4.4	4.4

At the start of the year, class anxiety in an online setting was significantly lower than perceived anxiety in a class on-campus with watching recorded content having a very low mean value. Despite choosing to study statistics, some students at Glasgow do experience anxiety about statistics particularly around help-seeking which is significantly higher than at Sheffield Hallam ($p=0.006$). This higher level of anxiety about seeking help may help explain why Glasgow students were less likely to ask for help and more likely to use the internet as a source compared to Hallam students.

By the time of the second survey, the means for the first-year students had increased particularly in an online situation. For second year students who experienced both face to face and online learning actually feel less anxious about face-to-face teaching than they are about online. Students were less likely to ask for help or value the learning of statistics and more likely to lack motivation at the end of the year.

Further investigation of relationships between emotional wellbeing, intrinsic motivation, and statistics anxiety with key learning behaviours and opinions on the use of videos for teaching statistics are contained in Table 3. Partial correlations, controlling for emotional wellbeing, were used for statistics anxiety to separate statistics anxiety from emotional wellbeing as emotional wellbeing was related to help-seeking anxiety in particular and was likely to be increased due to the pandemic.

Negative emotional wellbeing and general lack of motivation to learn statistics are generally the strongest predictors but intrinsic motivation has a bigger impact on a student's belief that they can learn statistics if they study effectively (control of learning) and how effective they think videos are for learning statistics. Even after controlling for emotional wellbeing, students anxious about statistics are less likely to feel they can learn statistics, reach out to staff if struggling, learn statistics remotely or find videos an effective method for learning statistics. These findings and those in Figure 5 suggest that motivation, emotional wellbeing and statistics anxiety are impacting on learning behaviour and beliefs which in turn affect being able to learn effectively remotely and opinions on the effectiveness of videos.

Table 3: Partial stats anxiety correlations after controlling for emotional wellbeing

Cohort	All students			Sheffield Hallam students only			
Statistic	Pearson's correlations			Partial correlations (controlling for emotional wellbeing)			
Factor	Negative emotional well being	Lack motivation to learn	Intrinsic motivation	Statistics task anxiety (SAM)	Software anxiety	Help seeking anxiety (STARS)	Class anxiety (online)
Negative emotional well being		0.46	-0.27	0.31	0.24	0.49	0.37
Lack motivation to learn statistics			-0.3	0.30	0.14	0.25	0.33
Intrinsic motivation (task value)				-0.31	-0.27	-0.19	-0.35
Able to learn remotely	-0.53	-0.47	0.38	-0.51	-0.49	-0.46	-0.40
Videos are effective for learning statistics	-0.29	-0.33	0.37	-0.25	-0.31	-0.27	-0.31
Effectiveness difference (video - lecture)	-0.25	-0.21	0.19	-0.06	-0.07	-0.13	-0.15
Able to manage workload effectively	-0.55	-0.5	0.24	0.08	0.05	-0.08	-0.06
Control of learning	-0.32	-0.3	0.51	-0.41	-0.34	-0.32	-0.41
Persistence	-0.32	-0.62	0.26	-0.13	-0.10	-0.19	-0.18
Can reach out to staff if needed	-0.39	-0.31	0.31	-0.24	-0.31	-0.36	-0.39

4. Conclusions and discussion

This study has given an overview of the student experience of learning statistics remotely during a pandemic for elective and compulsory cohorts. Many students experienced problems with emotional wellbeing, their study environment or motivation which impacted on learning. Although some students felt able to learn online and thought videos were an effective method of learning, the majority thought lectures were more effective than videos for learning and would prefer recorded content to be a back up. When compared to experiences on campus prior to the pandemic, students were much less likely to actively engage with learning material, work with other students or seek help when needed with students on compulsory statistics modules being more affected by the switch to remote learning.

In line with previous literature on students studying online by choice, this study has shown that motivation, online readiness and engagement are key predictors of being able to learn remotely. In addition, negative emotional wellbeing and statistics anxiety have a negative impact on learning behaviour, motivation, opinion on the effectiveness of videos and being able to learn remotely. Students studying elective statistics modules felt more able to learn but this difference was explained most by a higher level of intrinsic motivation impacting positively on learning behaviour.

Students surveyed at the start of the year generally felt less anxious in online situations but average statistics anxiety increased and the gap was reduced by the end of the year. For second years who had also been taught statistics face to face, anxiety in online settings was actually higher which is more consistent with Devaney (2010).

Looking forward to future education of statistics, some online learning could be considered for elective cohorts, who have suitable environments to learn remotely and are motivated to learn independently but has been detrimental to those with negative emotional being, statistics anxiety or lacking motivation so is not generally recommended for teaching statistics. Further research could investigate when and how to use recorded content effectively particularly regarding anxious students and why students at Glasgow are less likely to seek help.

5. Acknowledgements

The data collection had ethical approval from Sheffield Hallam and University of Glasgow.

6. Appendix

The following questions were contained in the scales used in the survey for the end of the academic year at Sheffield Hallam and Glasgow Universities. They are grouped by subscale where appropriate with Cronbach's alpha. Most questions were taken directly or adapted from the Motivated Learning Strategies Questionnaire.

Scale	**Engagement with statistics learning activities remotely (7 point SD - SA)**
Passive engagement (Cronbach's alpha = 0.88)	I generally attended live teaching sessions online
	I regularly watched all recorded content
	I generally attended online statistics tutorials
Active learning (Cronbach's alpha = 0.88)	I regularly attempt the weekly exercises or tasks associated with learning content
	I actively participate in learning in live online sessions e.g. take notes, complete exercises/tasks, communicate with others
	I actively participate in online tutorials e.g. complete exercises/tasks, communicating with staff/students about class work
Learn effectively remotely (Cronbach's alpha = 0.85)	I am able to learn statistical software effectively remotely
	I am able to learn statistics material effectively remotely
	I feel able to complete any assessments to a good standard this year
MLSQ: Help-seeking (Cronbach's alpha = 0.64)	If I don't understand something, I ask a member of staff for help
	If I don't understand something, I ask another student for help
MSLQ: Effort regulation/ Persistence (Cronbach's alpha = 0.57)	I work hard in my statistics module even if I didn't like what we are doing
	When coursework is difficult, I give up or only complete the easy parts (REVERSED)
Single questions using same 7-point strongly disagree - agree	
I focus on studying what I need for assessment when learning remotely	
I lack motivation to learn or continue learning statistics remotely	
If I don't understand something, I search for an alternative source on the internet	
I felt I could reach out to statistics staff if I was struggling	

Second year students were asked all of the above questions again but for last year when they had face to face classes for most of the year.

The questions for the ‘emotional wellbeing’ scale were taken from the from the English version of the Third Copenhagen Psychosocial Risk Assessment Questionnaire (COPSOQIII)

Negative wellbeing (Cronbach’s alpha = 0.9)	Thinking generally about how you have been feeling over the past month, how often have you: 1=Not at all; 2=A small part of the time; 3=Part of the time; 4= A large part of the time; 5=All of the time
	Felt worn out
	Emotionally exhausted
	Had problems concentrating
	Felt sad

Statistics anxiety was measured using the Statistical anxiety measure (SAM; Earp, 2007) which gives a general measure of statistics anxiety and the help-seeking anxiety subscale from the Statistics anxiety rating scale (STARS; Cruise, Cash and Bolton, 1985). In addition, subscales relating specifically to online and on-campus situations and software were created. Students were asked how anxious they felt in a variety of situations from 1 (Not at all anxious) to 7 (Extremely anxious) for all subscale items. The items used for each statistics anxiety subscale are given below with Cronbach’s alpha.

Subscale	Individual items
Online class anxiety (CA=0.9)	Attending an online statistics lecture
	Attending an online statistics tutorial
Face to face class anxiety (CA=0.9)	Being in a statistics lecture in person on campus
	Attending a statistics tutorial in person on campus
Statistical anxiety measure (CA=0.95)	Sitting an exam in person on campus
	Studying statistics generally
	Reading statistical studies
	Calculating probabilities
	Formulating and testing hypotheses
	Developing conclusions based on mathematical solutions
	Interpreting statistics
	Explaining your statistical findings
Software anxiety (CA=0.93)	Inputting/manipulating data in statistical software
	Using statistical software to carry out analyses
	Summarising results from the statistical software output
Help-seeking anxiety (CA=0.91)	Going to my statistics lecturer for individual help with material I am having
	Asking a statistics lecturer for help understanding computer output
	Asking a fellow student for help in understanding statistics material

	General preferences for videos (irrelevant of what they had this year)
SHU and Glasgow	Videos are an effective way to learn statistics
	Lectures are an effective way to learn statistics
SHU only	Doing examples is an effective way to learn statistics
	I prefer to learn statistics primarily from recorded content rather than lectures
	I prefer to learn statistical software primarily from recorded content rather than face
	I would like videos to be a back up for missed sessions or for revision purposes
	If I knew lectures were recorded, I would be less likely to attend in person

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CASE STUDY

Facing Challenges in Remote Mathematics Education and Support by building a Community of Practice around Student-Staff Partnership

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Abstract

Our active community of practice, developed from a weekly book club, formed through a shared interest in student-centred learning. Inspired by the book *The Power of Partnership*, this interdisciplinary group fostered and nurtured two practical mathematical applications through a partnership approach. In Munster Technological University academic support, including Mathematics and Statistics support, is offered to students through the Academic Learning Centre (ALC). Project 1 explored, through student-staff partnership, how best to promote the ALC. The project challenged our assumptions of how students would like to interact with the service. Project 2 focused on teaching mathematical analysis to a large cohort of year two students at the University of Southampton during the Covid-19 pandemic, elaborating on how small study groups, facilitated by student partners, were utilised to maintain a sense of connection and belonging, when possibilities for in-person teaching were extremely reduced. This paper discusses how a dispersed community of practice collaborated to enhance learning and teaching of mathematics.

Keywords: Partnership, Engagement, Inclusivity, Student centred learning, community of practice.

1. Introduction

This paper discusses two partnership projects that emerged from and were supported by weekly book club discussions. In Munster Technological University academic support, including Mathematics and Statistics support, is offered to students through the Academic Learning Centre (ALC). Project 1 explored, through student-staff partnership, how best to promote the ALC. The project challenged our assumptions of how students would like to interact with the service. Project 2 involved teaching mathematical analysis to a large cohort of year two students at University of Southampton. Reduced in-person contact time presented additional pressures to maintain a sense of connection and belonging. The design and implementation of these projects were guided and supported by weekly conversations with the book club community.

In April 2020, amid the turmoil of emergency remote teaching and learning, a book club formed to discuss student-staff partnership. The book club discussions centred around *The Power of Partnership* (Mercer-Mapstone & Abbot, 2020), a text comprising individual case studies about partnerships developed across varied educational settings. These discussions were enriched by the diversity of book club participants - two from lecturing (mathematics, engineering), one from mathematics support, and two from central learning and teaching functions with a focus on student engagement and academic/educational development. Personal experiences fed into discussions of the chapters, sometimes leading us in unexpected and serendipitous directions. This book club developed into a community of practice (Wenger, et al., 2002) which has continued to meet weekly throughout three lockdowns, periods of remote and hybrid teaching and gradual return to campuses. The community of practice discussions grew into concrete plans. There was reciprocal benefit between the book club conversations and the evolution of the projects described in this paper to inform our understanding of partnership and to inform project development. Figure 1 shows how the timeline of the book club and the projects ran in parallel.

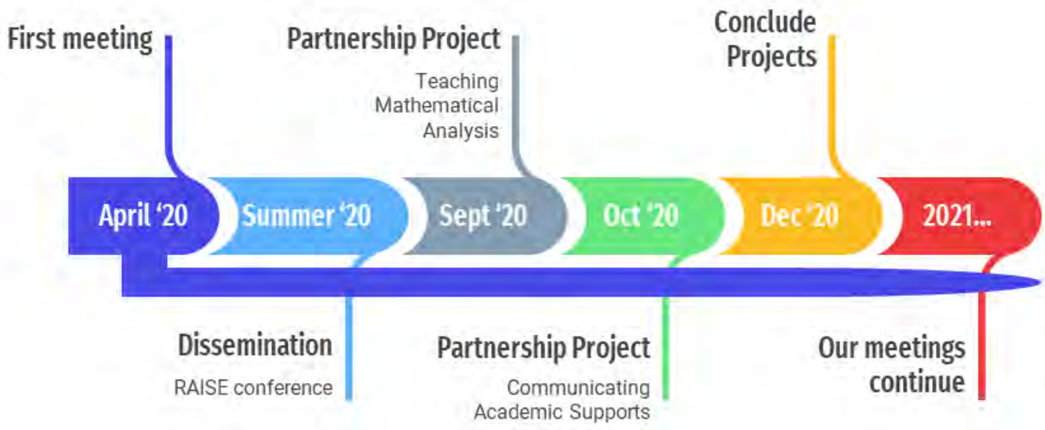

Figure 1: Timeline showing how the projects and the book club ran in parallel

The remainder of this paper is organised as follows. The next section discusses the literature in relation to student staff partnership and applications of student voice work in mathematics. Next, two parallel case studies are presented where student-staff partnership was applied to two Mathematics-related contexts. The final section discusses the implications of the work to date and plans to deepen and broaden the work of our community of practice.

2. Literature Review

2.1 Partnership

Student partnerships, student-staff partnerships and a range of interrelated, yet distinct, terms (student involvement, student engagement, student voice) have gained increasing attention over the last 20 years, particularly in the UK, Australia, USA, and Scandinavia (Klemencic, 2011; Bovill & Bulley, 2011; Healey, et al., 2014; Ashwin & McVitty, 2015). Indeed, partnership was identified by the (Higher Education Academy, 2015) as core to the vision for its future as a process to enhance learning and teaching practices through meaningful and shared dialogue with all members of the academic community.

Healey, Flint and Harrington (2014) define partnership as a process rather than a product, one that supports student engagement, whilst recognising there are many other potential opportunities to engage students. Their definition is that partnership "represents a sophisticated and effective approach to student engagement...the potential for a more authentic engagement with the nature of learning itself and the possibility for genuinely transformative learning experiences for all" (Healey, et al., 2014, p. 55). Partnership extends traditional views of feedback from students about their learning experiences, defined by Cook-Sather et al. (2014, pp. 6-7) as:

"a collaborative, reciprocal process through which all participants have the opportunity to contribute equally, although not necessarily in the same ways, to curricular or pedagogical conceptualization, decision-making, implementation, investigation, or analysis".

Importantly, a series of values underpin partnership practices including trust, authenticity, reciprocity, inclusivity, empowerment, challenge, respect, and community (Higher Education Academy, 2015).

Partnership is a commitment to open, constructive and continuous dialogue and can often challenge preconceptions of how students and staff can work together by "navigating the difficult terrain of power hierarchies" (Verwood & Smith, 2020, pp. 30-31). It challenges the "consumerist relationship presenting a constructive alternative in which opportunities arise to foster positive behaviours, and to develop communities within and between students and staff" (Ody & Carey, 2016, p. 33). By challenging the traditional roles - staff as holders/givers of knowledge/decision and students as recipients of such - and encouraging greater active participation from students to engage in decision making, staff relinquish some of the power and control to make space for the process of partnership to take place (Bovill & Bulley, 2011). This is however a process and exploring shared goals and objectives is a key in achieving shared equity of power.

However, adopting a new way of working can be uncomfortable and unsettling. These roles often have blurred boundaries and no clear expectations. Through accepting this process, students and staff are stepping into a new, neutral and brave space (Cook-Sather, 2016) which may feel disorientating and risk-filled. Cook-Sather argues that using the terminology *brave space* as opposed to *safe space* sets the tone and mode for this type of engagement. Being brave requires active engagement, stepping out of your comfort zone and therefore, potentially risky. By choosing to be vulnerable, and open to entering a new, open, and risky environment is where the potential of a transformational learning experience may occur (Healey, et al., 2014).

2.2. Rhizomatic Pedagogy

Partnership can be a slow and challenging process, with its outcomes not always clearly seen. Consequently, rhizomatic pedagogy (Deleuze & Guattari, 1987) and rhizomatic growth (Mathrani & Cook-Sather, 2020) have been used as an analogy for partnership, given a rhizomes limited upward growing shoots do not convey the significant amount of activity beneath the surface, defined as nodal relationship and barriers/branching. This analogy also resonated with the book club members in the development of the two projects. Although the projects had outputs, it was the experiences beneath the surface where the projects were developing and adapting where much of the growth happened.

2.3. Partnerships in Mathematics

Whilst partnership in institutional Learning and Teaching (L&T) activity has been well documented (Healey, et al., 2014; Cook-Sather, et al., 2014; Ody & Carey, 2016), the disciplinary perceptions of students as active or passive participants in learning is varied. Students in STEM were likely to self-identify as passive learners (Bunce, et al., 2016), which will impact the scope and nature of students as partners-type activity (Matthew, et al., 2017). Using the view of partnership noted earlier, there is little evidence of well-developed partnership activity in mathematics, yet innovations in learning and teaching, and student engagement practices are wide ranging including flipped classroom (Lo & Hew, 2021), tilted classroom (Alcock, 2018) and peer-learning programmes (Duah, et al., 2014; Malm, et al., 2021), but not explicitly partnership activities (Duah & Croft, 2011). Therefore, whilst examples of specific partnership activity are low, the hallmarks of partnership in other areas of mathematical pedagogy exist. The book club projects seek to add to the mathematics partnership literature describing new practice for those engaged in innovative, mathematical learning and teaching activity.

3. Project 1: Communicating with students around Academic Subject Support, Munster Technological University

3.1. Context

This section discusses a project with a partnership team involving academic support staff, academic teaching staff, undergraduate and postgraduate students. Academic support, including Mathematics and Statistics support, is offered to students through the Academic Learning Centre (ALC). The ALC Coordinator wondered if students in remote emergency teaching mode were aware of the service. The project was conceived to ask the question – How can we communicate our message to students to improve student uptake of the service? Informed by the partnership book club it was decided that the logical step was to ask students, engage them in partnership discussions and work together to figure this out. The project looked at the communications methods used to engage students with subject support through the Academic Learning Centre.

The project team was recruited by sending an email to all students and inviting applications through a Microsoft form. The final team was composed of six students from first year to third year undergraduates, along with one postgraduate, one Academic Learning Centre member of staff, one Academic Success Coach, and two library staff. The student partners were drawn from all the Cork campuses and from a spread of disciplines. The project was supported by the Student-Staff Partnership Project Officer (AnSEO - The Student Engagement Office, 2021) who chaired the first meeting and gave some background into what partnership was as well as being available for on-going support of the project. The team committed to two hours per week for six weeks. There would be a one-hour meeting plus an expectation of one hour of independent work per week between meetings.

3.2 How was this partnership? What did partnership look like in this project?

From the outset the project aimed to be as democratic as possible. The initial project brief was not prescriptive about how the partnership team would reach its conclusions and how these would be gathered or reported. Following discussions about how to communicate with students, the team decided to survey the student body. The co-created survey garnered over 300 responses. These responses were discussed in subsequent team meetings which further deepened our understanding of the students' perspectives. The final ‘product’ of the project was a Padlet with recommendations for future communications with students. However, the real product was the increased understanding of each other's perspectives.

3.3 Outcomes, lessons learned and future work

As well as the final ‘product’ we learned a lot through these discussions. Some things we learned from the students included the following.

Many students hate emails and feel a sense of email overload. Many students learn to skim their inbox and only open emails that appear directly relevant to them. Even when an email is opened it is often skimmed for important information. For this reason, we realised that emails should be short and snappy. We need to make the ‘What is in it for me!’ (WIIFM), very relevant as soon as a student opens their email.

Messages about the ALC were not reaching other campuses in the way that we expected. Students from those campuses may feel that ALC is not relevant to them and therefore not pay attention to the information.

The main source of university information for students on social media was Instagram. Despite the emphasis on social media and electronic communications, it is the recommendations from Academics that really matter to students. Students fed back that they listened to recommendations made from real people that they already interact with, and hopefully trust. Academics are the main source of contact and information about the university. This is where students take their news from, and the source that they trust.

As well as lessons directly related to the project mission, we also learned a lot about partnership work through this first partnership project. We realised that students are a huge untapped resource. The students have lots of expertise to bring. Not just their experience of being a student but also unique individual skills from other life experiences and from their studies. The project challenged our assumptions of how students view the service and how they would like to interact with it. This has informed our thinking around promoting the service to students and has been especially valuable in the current climate of remote teaching and learning.

In future if we are wondering what students think we will just discuss it with some students.

4. Project 2: Students as Partners in building learning community through study groups, Mathematical Sciences, University of Southampton.

4.1. Context

To teach a compulsory second year Analysis module to a large cohort is always a challenging task. Achieving this during the global pandemic added extra layers of difficulty. There are always students who do not appreciate the mathematical rigour and abstraction of the module, and therefore, may struggle and disengage. Pre-Covid module design allowed for constructive dialogue with students, reviewing their performance and potentially identifying a need for additional individual help. It was not clear how to mimic this provision in the pandemic. The Analysis module, with one-hour face to face teaching per week, was the only second year module with an in-person teaching component, aiming to maintain a sense of connection and belonging to the University community.

Our task was to identify and introduce an intervention into the established module delivery practice that would make learning and teaching less isolating and create a caring atmosphere where everyone matters, is supported, and nobody feels abandoned. To achieve these goals a research-informed approach suggested utilising the collective strengths of students themselves, an underutilised resource (Biggs & Tang, 2011). Encouraged by support from academics and informed by pedagogical research, we decided to create study groups, to divide the cohort into groups of five who would work together on their homework submitting it as group work. In their groups, students marked each other's work, gave and received feedback, as well as reflecting on how receiving feedback helped to improve their own work. However, the implementation and overseeing of 36 groups was a huge task. Drawing on regular discussions with the book club, the situation gave rise to an opportunity to engage with student partnership pedagogies. We recruited volunteers from year three and year four student cohorts, who in partnership with the module Academic, would oversee and support the study groups. The concepts, study groups as well as student partnerships, represented a novelty to both groups of students and were carefully introduced and motivated. Both concepts emphasise process over product, learning over performance, time requirements and messiness, kindness to one another (Campbell & Bokhove, 2019). The third year and fourth year student volunteers were open to engaging with this new concept of pedagogical partnership, the role beyond their traditional student role.

4.2. Student Partnerships

The notion of ‘student helpers’ is well established within our institution. However, the notion of the student partnerships is less established and to build an authentic partnership beyond the declaration and good will, is a non-trivial task. Whereas in the ‘student helpers’ approach, students are told what to do, in a ‘students as partners’ approach, students are positioned differently. The partnership concept draws a different picture blurring the demarcation line between the student-staff sides. Our partnership evolved from having cameras off during the online meetings to lively discussions with cameras on towards the end of the project. The student partners were highly motivated and very conscious about their role, as one of them put it: *“Group work and discussion has played a huge role in my mathematical education, and I think it should always be encouraged especially now that it has become slightly more challenging to do so.”*

To build and deepen our partnership we created a time and space for reflection. We held bi-weekly meetings that allowed us not only to discuss how to support the students in line with their needs, but also how to support each other.

4.3. Some outcomes and future work

Evaluating the intervention from the student learners’ perspective, the project achieved its objectives to connect students and to enable them to support one another.

The feedback from students, learners and partners, was overwhelmingly positive. Groups reported increased self-confidence, improved organisation, time management, communication skills, leadership, and the experiences valued beyond job applications.

Despite our intentions, it is not completely clear whether this example represents a truly authentic partnership. It was difficult to overcome perceived power dynamics, replacing students’ agreeable nodding, with more meaningful contributions when making decisions developed slowly.

In 2021/22, the project entered its second iteration with the 2020/21 group work participants, volunteering in the role of student partners.

With the support of book club, the project has a huge potential to continue supporting the development of students as reflective learners and educators.

5. Discussion

The creation of the book club was timely amid concerns of keeping the ’heart’ in our interactions with students during the Covid-19 pandemic. Creating a shared experience of lockdown and, as we were almost strangers, a freedom in discussion around the impact partnership opportunities have had and could have on our own practice gradually emerged. Weekly focus on the book emboldened each member to develop stronger partnership mindsets (Peseta, et al., 2020) and stimulated implementation of the case study projects discussed above. Authenticity was a common thread as we wanted to work with the projects in a way that would resonate positively for ourselves and the project teams. The support each member received from the group was particularly critical and informed the group ethos in the ensuing partnership projects, each member mutually enabling based on individual prior experience.

Through our book club, and the projects discussed above, we experienced that student-staff partnerships have the potential to begin challenging the roles students and staff can play in the learning environment, allowing some traditional hierarchies in higher education to be questioned with respect to power and how it is exercised, and identifying ways of working that build a more open student learning experience. It caused us to review our preconceptions of partnership and to reframe our own language. We began to view partnership as a way of operating rather than an outcome, Adopting the partnership mindset gave confidence to those undertaking the projects, otherwise processes and experiences would have remained static and continued unchallenged. We became attentive to our assumptions and those of our institutions, valuing process over product. Emboldened by the knowledge that we were not alone in these thoughts we grew braver in conversations outside the book club. We started to see the beginnings of a ripple effect (e.g. in the second iteration of Project 2) that encourages a sense of community, and its potential to impact student success and retention.

6. Conclusion

This paper discusses how two student-staff partnership projects grew out of an online book club during the Covid-19 pandemic. The book club members came from a diverse range of backgrounds within Higher Education Institutions across Ireland and England. The weekly discussions drew on the varied experiences of the participants to deepen our understanding of student-staff partnership. The first project looked at communicating with students about Mathematics support services while the second focused on creating community in online delivery of an Analysis module. Both projects had positive outcomes for staff and students. Experiences from these projects fed back into the book club discussions, which facilitated a space for reflection and synthesis. As the book club approaches its second anniversary it continues to enrich our understandings of student-staff partnership work and drive us forward to work more in this space. Our hope is that readers find inspiration to take a brave step to try a partnership activity of their own.

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