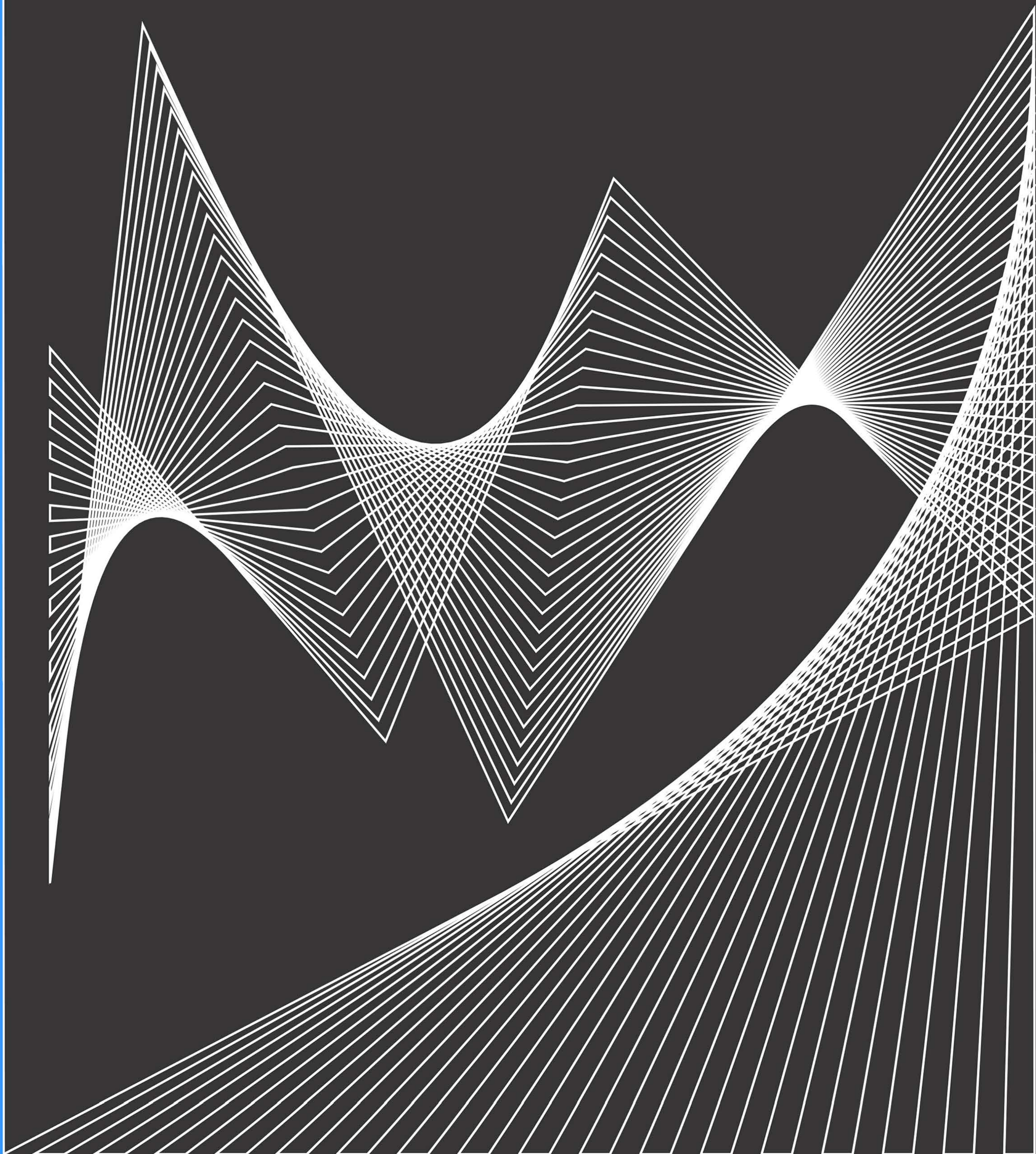
MSOR connections

Articles, case studies and opinion pieces relating to innovative learning, teaching, assessment and support in Mathematics, Statistics and Operational Research in HE.

Volume 16 **No.** 1



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This journal is published with the support of the **sigma**-network and the Greenwich Maths Centre.

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Editorial

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This latest issue of Connections continues to reflect the broad range of mathematics education activity taking place across Higher Education in the UK.

We start with a workshop report from Mehbali and Roberts on a **sigma** Network event at London South Bank University. The report summarises the key points raised and discussed in relation to embedded provision of mathematics support (as opposed to extra-curricular drop-in support). The following case study by Cornock highlights the various ways in which the Mathematics department at Sheffield Hallam University have been trying to develop their learning community. With questions relating to community now included in the National Student Survey, this is a topic that is likely to get increasing attention across the sector.

The next paper provides a rationale and review of an innovative approach to student personal and mathematical development. Burrell et al. outline their *Activity Guide* which aims to support students making the transition to university level mathematics.

The next three case studies each discuss alternative approaches to develop student understanding in their respective topics; Khan outlines how a simple card game can be used to explore strategy in game theory; Deshpande describes the "*10 steps*" to developing financial computing literacy; while Xu and Lenton highlight some of the difficulties students face in understanding fractions along with ways in which this might be addressed.

In the final paper of this issue, Rowlett describes a resource produced in partnership with students to support game play in the popular Maths Arcade; a range of games and puzzles aimed at developing mathematical thinking.

WORKSHOP REPORT

Maths Support provision through embedded classes at LSBU

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Abstract

The present report summarises the communications presented at the **sigma** Network event on 'Maths Support through Embedded Classes' held at London South Bank University (LSBU). Two keynote speakers Professor Jon Warwick, from the LSBU Business School and Dr Lesley Roberts, Head of Skills for Learning, contributed valuable presentations. They outlined how maths support went through various stages and demonstrated the experience of embedded classes at LSBU. Five other delegates communicated their experiences of Maths Support through an embedded approach in their respective institutions.

Keywords: Maths support, embedded classes, learning development.

1. Introduction

As a part of the Centre for Research Informed Teaching (CRIT), Skills for Learning hosted the **sigma** network event on the theme of 'Maths Support through Embedded Classes' at London South Bank University. Participants have been involved in the mathematics learning support in Higher Education; most of them came from South-East England. Professor Shân Wareing, Pro Vice Chancellor (Education & Student Experience) opened the event and Dr Saranne Weller, Head of CRIT, welcomed the attendees and highlighted the importance of the service provided in helping students improve their learning experience.

Support in Mathematics and Statistics at LSBU is provided across the whole University to both undergraduate and postgraduate students. The support covers a range of topics such as Numeracy skills, General Maths, Advanced Maths for Engineering, Quantitative Methods for Business, Psychometric tests, Statistics and Research Methods. The delivery makes use of four different learning formats: workshops, one-to-one consultations, drop-in sessions and embedded classes.

Mathematics and statistics support centres in many universities are actively developing learning programmes that go beyond the traditional drop-in or one-to-one formats. They closely collaborate with faculties and departments to put in place 'embedded' support to meet the needs of specific courses and to reach larger groups of students.

Both keynote speakers Professor Jon Warwick, from the Business School and Dr Lesley Roberts, Head of Skills for Learning, commented how maths support went through various stages and exposed the experience of embedded classes at LSBU. Five further communications were presented by the delegates from University of Bath, Brunel University, Middlesex University and University of East London, followed by discussion. Participants were pleased to share their respective experiences on support provision through an embedded approach.

2. Keynote speakers

2.1. Jon Warwick, Professor of Educational Development in the Mathematical Sciences. Mathematics Support Experiences at London South Bank University.

Professor Jon Warwick gave the first plenary lecture; he described some of the work conducted within the School of Business supporting students who have to study mathematics as non-specialists. In his introduction, he focused on the challenges facing the teaching of mathematics in universities, a task made more difficult by intakes of students of mixed mathematical abilities. It is recognised that poor educational experiences in mathematics learning and poor contextualisation of the mathematics taught contributes significantly to this problem. The teaching of mathematics is a real challenge recognised by employers who complain about graduates' weaknesses in mathematical skills, lack of competency and confidence.

Engagement. Using his extensive experience in teaching mathematics at LSBU, Warwick has developed a teaching model which relies on knowledge acquisition, understanding, assessment and engagement. It is a cyclical model in which the student engagement acts as a feedback to the learning process. The model also takes into account parameters such as mathematical anxiety and self-efficacy. Mathematical anxiety is defined as a feeling of tension that hinders the manipulation of numbers and impedes the solutions of mathematical problems across a wide range of life and academic situations. This anxiety has the potential to cause a student to lose self-confidence in the subject of mathematics. Mathematical self-efficacy is defined as a person's judgement of their capabilities to organise and execute courses of actions that are required to attain specific types of performances. Self-efficacy of an individual is believed to be influenced by a number of factors such as previous success or failure in mathematical performance, comparison with peers' performance, received comments from persons in position of authority such as teachers, parents, and psychological pre-disposition that may give rise to feelings of anxiety and tension.

Changing attitudes. Warwick stressed the importance of taking into account these concepts of mathematical anxiety and self-efficacy in any meaningful endeavour of teaching mathematics. These factors are intimately linked to achievements in mathematics, and modules requiring substantial mathematical input. They will impact employability, improve wider career and occupational prospects and will also feed into academic institutions' performance and standing.

Assessment. Student evaluation constitutes an important stage in Warwick's cyclical learning model. Assessments and feedbacks are used with the aim of furthering student's progress and development. The objective is to allow the student to demonstrate mastery in the subject. Mastery is defined by improvement rather than outcome, sustained learning effort and satisfaction gained from hard work and gradual learning.

Mastery entails encouragement of 'risk taking' in the classroom which is conceived as a safe environment where it is acceptable to make mistakes and learn from them. It is also predicated on knowing the students' backgrounds, capabilities and anxieties early in the academic year so that specific help can be provided to address any issues that hinder students' progress.

Support processes. The learning model requires support processes to be put in place in order to explore students' expectations, enhance self-efficacy and reduce anxiety. The aim is to foster a belief that skills can be nurtured and developed through regular and developmental feedback, encouragement for peer interaction, teamwork and continuous personal development outside the classroom. To make independent learning possible academic institutions should provide electronic resources and specific online help.

Figure 1: Participants at the Maths Support through embedded classes event

Implementation of teaching models. Professor Warwick suggested two models of support that have been implemented with first year undergraduate students (Warwick, 2008 and 2010). Then he summarised some interesting results achieved through these two support models implemented within Business school.

Model 1: Embedded support tutorials

1. Support sessions were timetabled i.e. not optional;
2. Once a mathematics ‘driving test’ was passed, attendance at support sessions no longer required;
3. Driving tests could be attempted without penalty until passed.

Model 2: In-curriculum support

1. Students worked in small groups, compared and discussed problems and worked with their dedicated tutor;
2. Teaching activities were designed around the Kolb learning cycle;
3. Feedback was constant from peers and tutor;
4. Sessions included theory, worked examples, practice questions and reflections on progress;
5. Completion of a portfolio demonstrating professional competency skills;
6. A richly populated VLE.

The outcomes achieved are summarised below:

- Since implementation the average pass rate increased from just below 60% to 86%;
- 86% of students rated the quality of the module as ‘acceptable’ or better, with corresponding figures of 86% for the quality of lectures, and 83% for the difficulty of assessment;

- As a crude measure of engagement, 70% said the module held their attention over the year;
- Impact was sustained with evaluation regularly meeting university KPI's for progression (70%) with student feedback scores above sector norms.

The speaker ended his presentation by posing a series of questions for further discussion (subsequently considered by the audience during the last session of the event).

2.2. Dr Lesley Roberts, Head of Skills for Learning. Embedded Learning at London South Bank University.

Dr Roberts' presentation covered the embedded learning at LSBU. She started her talk by briefly describing the role of the Skills for Learning team in supporting learning at LSBU. The service is based on a centralised learning development structure providing support in Academic skills & English and Maths. The support is provided through four different formats namely drop-in sessions, workshops, one-to-one consultations and embedded classes. The following key points were then addressed.

Embedded teaching at LSBU takes place through timetabled classes and seems to fit with the discipline specific approach (i.e. it is tailored to the discipline the student is studying). The embedded lessons are planned in close collaboration with the modules leaders and are delivered at critical times in the course of the modules to produce maximum learning impact.

Roberts raised the following question: Why embed? Embedded learning is a form of "*Opt-out rather than opt-in*" approach (Thomson, 2012). All the students, whether vulnerable or able, benefit from the provided support. This approach is a mainstream one, targets all students and thus avoids some of the stigmatising effects of other forms of remedial learning.

In mainstream embedded classes, it is assumed that the students are not broken. The emphasis here is on motivating and encouraging them to take ownership of their education. Classes provide a learning context enabling students to apply academic practices to their specific subject and assessments as opposed to a bolt-on generic 'skills' class. Beach (2003) states that skills may not be transferable unless the dots are joined or a bridge has been built. By providing embedded classes, impediments to learning and transferring previous skills acquired in different contexts to the students' current learning condition can be investigated and remedied to break with what Roberts called the "*non-transferability of transferable skills*".

Roberts highlighted the experience gained by the Skills for Learning department from a pilot programme covering 39 modules spread across the seven Schools of the University. The support included targeting low performing modules and the extent of the intervention varied from 1 to 5 sessions. Approximately 1,700 individual students were involved with 2,685 registered attendances.

The results of the pilot showed first attempt pass rates for the year 2015/16 improved by an average of 10.76%. On average, modules with one support session produced an improvement of 3.95% whereas support of two or more sessions improved the results by an average of 21.7%. In the case of low performing targeted classes, the first attempt pass rate improved by an average of 53%.

Maths support poses a particular conundrum as far as embedding is concerned. Embedding classes for modules where maths is not the primary content is not problematic. This is easily done for modules, which require drug calculations, or the statistics needed for final year empirical projects. It is when maths is the primary content such as in engineering subjects where the problem arises. Typically two situations arise, either to take the ‘broken’ students approach or to teach a whole module. In other words, either to provide a curative response, or to substitute the module lecturer. As the current role and mission of Skills for Learning do not allow the luxury to implement either option, we tend to opt for a compromise solution. It is that neither these options are mainstream Learning Development, because the first option is remedial and the second is simply not Learning Development. For instance, teaching a couple of topics and running revision classes on the topics that the students find most difficult.

In conclusion, Roberts asked the audience to reflect on the two following important questions:

- Is there a role for embedding in modules where maths is taught as the primary content?
- Are there different approaches that could mark out embedded maths learning development from traditional module teaching?

3. Contributed presentations

3.1. Cheryl Voake-Jones, MASH Coordinator and Teaching Fellow. Embedded Support at the University of Bath.

Cheryl Voake-Jones began her presentation by giving a brief overview of MASH, the mathematics and statistics help and advice service at the University of Bath. Support includes drop-in sessions (some of them peer led), Statistics Advisory Service (SAS) appointments, and embedded sessions in three Faculties (Engineering, Social Sciences and Science) and School of Management.

In the main content of the talk, Cheryl recounted candidly on her experiences of providing embedded sessions on a variety of courses. Her first example was teaching statistics (including SPSS labs) on a Biology and Biochemistry core module where the content was determined by the module staff. There were three sessions in Year 1 followed by a refresher session in Year 2 and in Year 3. Other examples followed including key skill sessions for Year 1 Chemistry students, statistics and Excel for MSc students in the School of Management, and for the Sport and Exercise programme, and bespoke sessions for final year Management and Economics students on graduate numeracy assessment. MASH were asked to teach first year Civil Engineering students weekly compulsory maths classes to individuals who had obtained low scores in a diagnostic test. Here the content was determined jointly between Faculty staff and MASH staff.

A ‘one size fits all’ approach certainly does not work for embedded support. It is not just the content that varies by department or programme, but also the background knowledge of the students, staff expectations, student expectations, mode of delivery and time available to cover material.

Whilst describing these examples Cheryl discussed the issues that have arisen along the way, as well as the benefits. Departments can be demanding beyond the resources of MASH with staff-time limited and demand for sessions clustering around particular times of year. Is assessment part of the deal? If it isn’t, do you know how the students will be assessed? Making teaching materials discipline-specific can be very time-consuming but is worthwhile. Another issue is that sometimes you can inherit teaching materials not of own choosing, what do you do if you are not happy with the quality of the materials? Additionally, course staff can have unrealistic expectations of what can be covered in one session and it is not always clear on the level of students and assumed knowledge. These issues need to investigated and resolved with the aid of the department. Cheryl emphasised the need for timely delivery in relation to a course structure to ensure material taught is experienced as helpful by students.

3.2. Mohamed Mehbali, Learning Development Adviser for Mathematics, Skills for Learning. Maths Support through Embedded Classes at London South Bank University.

Mehbali's presentation revolved around one aspect of the maths support provision at LSBU, namely embedded classes. He started by giving a brief description of the role of Skills for Leaning department which is part of the Centre for Research Informed Teaching at LSBU. The service is dedicated to helping students develop their learning skills particularly in two areas: Mathematics, English and Academic skills. Mr Mehbali who leads the Maths Support team, talked about his experience in embedded classes and how to run them in such a way as to maximise benefit to all the students attending these sessions. He then focused on two case studies of embedded learning, one concerning first year Nursing students, and the other, second year Product Design Engineering students.

<u>Case Study 1.</u> Mehbali collaborated with course directors from the Health & Social Care School (HSC) to deliver embedded lessons in drug dosage calculations to the first year nursing students. He worked with the module leaders to select the topics to be covered. The embedded lessons were then incorporated into the learning modules and the students' attendance of them was monitored.

The Head of Maths Support planned the sessions and specified the required resources such as staff, time, rooms, learning materials and announcements via Moodle (Virtual Learning Environment). The teaching sessions were delivered over six consecutive weeks and focussed on drug dosage calculations.

The lessons were interspersed with tasks designed to test students' progress and to allow formative assessment to be made on the performance of each student. At the end of each session, homework tasks were set to consolidate the educational experience and promote independent learning.

As the course leader wanted an assessment to be carried out, the Maths Support team developed an online assessment tool (namely Kahoot activity) for this purpose. The students were asked to complete a test paper then they logged on to the Kahoot website (https://kahoot.com) through their smartphones or tablets. Next, they were required to enter their individual answers one by one using their devices. A detailed report on the students' responses was subsequently generated. The students engaged with the activity and valued the prompt feedback on their performance. Gibbs and Simpson stressed how the provision of feedback can affect student learning behaviour (Gibbs and Simpson, 2004).

The Maths Support staff are not directly involved in the summative assessment but do contribute to the summative process. The team is heavily involved in the formative assessment through providing feedback on students' learning. Formative assessment and feedback are crucial for students to learn effectively (Black, 1998 & 2003). The assessment experience was shared with other lecturers in the HSC School. After the examination results, the course leader forwarded some feedback: "*Thank you for your support of the students' numeracy education. The pass mark was high for all groups at above 98% which is brilliant."*

<u>Case Study 2.</u> A similar embedded learning experience was repeated with second year Product Design Engineering students. A collaborative approach was adapted to encourage group-work and to stimulate cooperation. Students learn better when they work together and interact with their peers (Race, 2009). Six timetabled embedded lessons were delivered to the students over 3 weeks. Three topics were covered (Algebra, Differentiation & Integration).

Key outcomes were:

- High attendance rate (87%);
- Students engaged and interacted well with the lessons having realised their importance to their course;
- Students showed interest by asking to have their homework checked;
- After the lessons, students made regular visits to Skills for Learning willing to further improve their mathematical skills.

An evaluation questionnaire was produced and handed to the students to be filled in to record their experience and feedback. The results were communicated to the course leaders.

The following remarks can be made on the embedded lessons.

- Positive feedback from students, on their evaluation form;
- After the embedded sessions, more students became regular visitors to the Skills for Learning department for extra help;
- Partnerships established with module leaders.

In conclusion, the speaker mentioned that embedded classes:

- May contribute to effective teaching, in reference to Biggs who challenged traditional methods of teaching and suggested seven characteristics of effective teaching contexts (Biggs and Tang, 2011);
- Have a wide impact on students;
- Enable us to effectively support more students;
- Offer an opportunity for building partnerships with module leaders.

Figure 2: David Bowers providing details on the sigma network

3.3. Lois Rollings, Maths, Statistics & Numeracy (MSN) Lecturer at Middlesex University. Embedding Maths Support – Some Thoughts.

Lois Rollings reflected on embedding maths support in her presentation and wanted to share some thoughts with the audience. Lois stressed the importance of communication between the stakeholders involved in the support provision. If embedded sessions are going to be successful then good communication between all those involved such as maths support tutors, lecturers, their respective managers and the students themselves, is vital. She reported that problems could arise if managers are speaking to each other without adequately informing staff and vice versa.

The speaker added that the maths support tutor and course lecturer need to be clear about the expected content and timing of a session as well as what can be expected of the students. For the MSN tutor, having access to the course resources available on the virtual learning environment (VLE) site can be extremely useful as it can then be seen what else students have covered and the style of the lecturer, and maybe more.

Rollings noted that it is also important to ensure that what is requested is feasible – MSN tutors can only be in one place at a time, and their overall workload limits need to be observed. MSN tutors also need a reasonable notice to prepare sessions.

Rollings remarked that students needed to recognise that an embedded session is relevant to their course – and preferably will be assessed in some way. The session also needs to be at the right time, so that the content will be needed in the near future.

Rollings concluded the presentation by posing two questions:

- What about students taking maths courses? Perhaps they need 'attached' rather than embedded sessions.
- Does maths need a different model from academic writing?

3.4. Inna Namestnikova, Academic Skills Adviser - Mathematics and Numeracy. Brunel Educational Excellence Centre. Academic Skills at Brunel at University – A Short Overview of the Service.

The Academic Skills (ASK) service provides support to Brunel students on academic skills, maths and numeracy, statistics and SPSS. The ASK also coordinates Peer-Assisted Learning (PAL) schemes at Brunel which during 2016/17 have been operating in Computer Science, Maths, Occupational Therapy, Business Studies, Civil Engineering and Mechanical and Aerospace Engineering. ASK is part of the Brunel Educational Excellence Centre (BEEC).

ASK offers drop-in sessions, one-to-one appointments, central workshops, in-school workshops and run several special events throughout the academic year, e.g. ASK week, Undergraduate dissertation week and Maths Café. Four full time members of the team are responsible for academic skills support and two members of team are full time advisors responsible for maths and statistics support. Some lecturers and PhD students are also employed on a part time basis to assist the team.

<u>Academic skills and SPSS support.</u> The ASK team has been running many embedded classes around the university. The pilot project was for this was in Politics, History and Law students. At the moment they deliver sessions for many departments such as social sciences, sport science, maths and engineering etc. There is a similar situation with statistics and SPSS support. Upon staff request, they run embedded sessions to help students with using SPSS for statistical analysis in their studies and final year projects.

Maths and numeracy support. In the past, ASK focused only on core maths and numeracy support but in recent years it was decided to try running some embedded maths and numeracy revision sessions. They delivered such sessions for computer science (level 1 and 2), sport science (level 1 and 2), bioscience (level 1) and economics (masters' level). All these sessions were timetabled and not optional. By running these sessions, they definitely increased the number of students using their service and were able to advertise it.

However, all of these sessions (except master level) were for large groups of students (more then 100) with a wide range of ability in maths and numeracy skills. This variation in knowledge level sometimes made it simply impossible to keep all students engaged and active during these sessions and to work with them in an effective way.

ASK repeated these sessions for several academic years, but it was eventually decided to stop doing so. Students were advised to attend the one-to-one service and use subject related materials developed and placed on Blackboard (Virtual Learning Environment).

In recent years, they have only run timetabled revision sessions for level 2 maths students to help them to prepare for their mid-term test but these sessions are not compulsory and were attended only by students who feel they need the support being offered (usually about 40 - 70 students). This year they also offered an additional online session, which was attended by 14 students.

ASK recorded around 700 student visits (224 unique student visits) to the Maths Café this year, which is the main event during the revision and exam period. At the same time, they have been looking for ways to encourage students to work throughout the academic year and not to leave maths preparation and revision until the last moment. ASK try to help them to develop the skills they need to learn independently and thrive academically, whatever their level or subject.

3.5. *Andrea Didier, Head of Academic Skills. Embedding Maths Support at the University of East London (UEL).*

Andrea Didier delivered the last presentation of the event. In her introduction, she provided information about her Skillzone team structure, the Learning and Language Support, and its operation over two university campuses. She then focussed on the philosophy of embedding maths support at UEL. The support is open to all students across the University. The team has achieved good results despite facing challenges.

The Learning and Language Support is part of the Library and Learning services and includes:

- Academic Writing and English for Academic Purposes (EAP) tutors;
- Learning Achievement Assistants;
- Maths tutors.

It has a two-strand approach. The first covers the university as a whole. The support is delivered from virtual and physical spaces which are branded "*Maths Space*" in both the Stratford and Docklands campuses. It operates continuously and is accessible online 24 hours a day and 7 days a week throughout the year. The Maths tutors have designed and developed Maths diagnostics assessments for the Business & Law; Health, Sport & Bioscience; Architecture and Computing & Engineering schools.

The second covers embedded provision for a tailored support targeting specific Schools and Programmes on particular topics such as Business & Law; Health, Sport & Bioscience; Architecture, Computing & Engineering. The topics covered are Numeracy, Algebra, Calculus and Statistics. Tailored assessments and learning materials for specific programmes and modules are currently being developed. Collaboration with lecturers is essential in order to work out and schedule the best times for incorporating these sessions.

The maths tutors ran a successful pilot project for the pre-entry programmes to identify which provision is needed in the main foundation short course. There is ongoing interest for information advice and guidance for outreach programmes such as summer schools and workshops for mature learners, specific workshop sessions on Numeracy and Psychometric Tests for employability enhancement. This results in increased demand on the time of colleagues in the university's schools.

The speaker gave statistics on the numbers of face to face sessions per school. The data presented showed an increased exposure of students to maths through embedded lessons. There is also ongoing work which seeks to develop the best ways of approaching students. Maths support through workshop sessions is more popular than embedded classes for Health, Sport & Bioscience School.

The positive outcomes of embedding Maths support at UEL are summarised as follows:

- Establishing the importance of Maths as a legitimate part of Academic Skills Development;
- Developing relationships and gaining champions within both schools and the service;
- Encouraging the take up of Maths support within numerate and non-numerate subject areas;
- Using internal and external networks for information;
- Growing interest in big data and all things statistical;
- Leveraging the importance of being numerate.

Finally, Andrea mentioned some of the challenges that maths tutors are facing.

- Managing expectations while meeting increased demand for maths support;
- Exploring different modes of delivery/platforms;
- Developing materials for different modes of delivery;
- Remaining agile enough to align with changing institutional priorities

The speaker concluded her talk by setting the following question to the audience: "*How do you measure the impact of your embedded interventions and non-embedded interventions when maths is not the key area being examined?*"

Figure 3: Left to Right - David Bowers (Chair of sigma Network), Lesley Roberts (Keynote speaker), Jon Warwick (Keynote speaker), Mohamed Mehbali (Event organiser).

4. Conclusion

Skills for Learning at LSBU is a centralised structure, which strives to help students develop their Mathematics learning. Mathematics Learning Support is already provided under traditional formats such as one-to-one, drop-in and workshop sessions. The objective sought by organising this **sigma** event is to explore the possibility of offering further support through an embedded classes approach. All participants agreed that Mathematics Learning Support deals with mathematics content knowledge and therefore presented challenges. The agreed suggestion was to contribute to the disciplines where Mathematics is not a major part of the programme but its impact is crucial to students' achievement. The delegates shared their respective experiences and raised some interesting questions for future consideration, for instance:

- How do we see the role of a mathematics support centre?
- What does 'success' look like for those engaged in providing mathematics support?
- How do we measure it?
- How should we deal with diverse student intakes?
- Should one-size fit all?

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CASE STUDY

Development of a Course Community

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Abstract

There are feelings of belonging amongst those involved with the mathematics course at Sheffield Hallam University. A number of factors contribute to this, including the use of year tutors, a peer assisted learning scheme, a shared working space, a Maths Arcade, an induction week programme, a final year de-stress day, and other social events. In addition to communication with peers in their own year group, interactions with staff and students in other year groups are encouraged. The students are given opportunities to form friendship groups, which has a large impact on their university experience. The desire from staff to work alongside students and support activities has resulted in students feeling part of a community.

Keywords: Course community, belonging, student support, partnership.

1. Introduction

McMillan and Chavid (1986) define the "*sense of community*" as being "*a feeling that members have of belonging, a feeling that members matter to one another and to the group, and a shared faith that members' needs will be met through their commitment to be together*" (originally from (McMillan, 1976)). A main aim on the mathematics course at Sheffield Hallam University is to make the students feel at home and give them a sense of belonging. As stated by Thomas (2012), there is "*powerful evidence of the importance of student engagement and belonging to improve student retention and success.*" Zhao and Kuh (2004) present that a learning community is an "*effective educational practice*", with benefits including improved student success and increased satisfaction. This is also indicated by Rovai (2002) who states that "*one strategy to help increase retention is to provide students with increased affective support by promoting a sense of community*".

It is believed that a community has not been fostered through just a single initiative, but it is thought that a combination of factors have contributed to its development. These include having year tutors and academic tutors to be main contacts and who get to know the students, as well as there being support from other staff. A peer assisted learning (PAL) scheme results in the formation of peer support groups and initiates inter-year communication. Having a shared learning space means that staff and students from all year groups work alongside each other, and encourages further interaction. Running the Maths Arcade adds to this as students are able to use the space to play strategy games with each other and staff. Identified as a key week, an intensive induction week programme is run. This introduces the first year students to staff, their PAL group, their PAL leader and the Maths Arcade. Other social events, such as final year de-stress days, Rubik's cube championships, quizzes, and film nights have also given staff and students the chance to spend time with each other outside formal teaching time. An important element is the willingness of staff to work with students, particularly taking an interest in each individual, having an open door policy, and contributing to events.

There is evidence to suggest that the approaches are successful. It was found that 86.8% of final year students who filled in a survey in a lecture in 2014-15 said that they "*felt part of a mathematics community*" (Cornock, 2016). Evaluation for different activities will be given throughout this case study.

2. Personal support and year tutors

The staff who teach on the course are very student focussed. When asked what drives them in their job, one member of staff said the desire to provide "*a secure environment in which both staff and students are able to grow and develop their professional practices*". When asked about the key priority in their job, a member of staff said that it was "*the students, both their overall education but also their well-being*" and another said it was "*providing a positive learning experience for students*". There are approximately 28 members of staff attached to the course and typically 100 students in each year group. The group is keen to limit student numbers to make it possible for staff to get to know the students.

A great deal of personal support is provided to the students in addition to the usual university support services. The students have a year tutor who acts as a main contact for all the students in a year group. Students tend to get drop-in support from their year tutor as they make themselves the most visible to that group of students. In 2015-16, a survey was carried out with 29 of the 66 final year students. They were asked how their year tutors had supported them. They indicated that year tutors do the following as part of their role:

- Help with future plans;
- Provide encouragement and motivation;
- Provide general help and support;
- Make themselves available for chats;
- Check the students are ok;
- Provide advice regarding the course;
- Take time to get to know everyone;
- Provide lots of reminder emails;
- Help students settle into university life;
- Provide information of opportunities;
- Provide strategies to approach work;
- Collect feedback;
- Answer questions.

General comments made by the students about year tutors included:

"*It's great having a first port of call! Like a safety net.*"

"*It is good to know that there is someone to talk to if you have any issues.*"

Having a year tutor makes a big difference to some students on the course as indicated by the following remarks:

"*The support I have received during difficult times has been brilliant and helped me progress in the course. Without this, I believe I wouldn't have come this far.*"

"*Would not have coped as well with personal issues without them.*"

"*I could potentially be on another course had I not spoken to my first year tutor.*"

Students also have an academic supervisor, with each member of staff supervising fewer than 10 students in each year group. Students and their academic supervisor meet for a one-to-one meeting at least once a semester. The course already had year tutors before the university introduced academic supervisors, so the addition gave the students another point of contact. The students also have the option of seeing their student support officer, who is not an academic.

The students were asked what forms of personal support they had used whilst at university. Figure 1 shows the responses. A very large proportion of the students indicated that they had received support from their year tutor. A lot had received support from their academic supervisor and other members of staff, but in comparison very few of the students had received support from services outside the group.

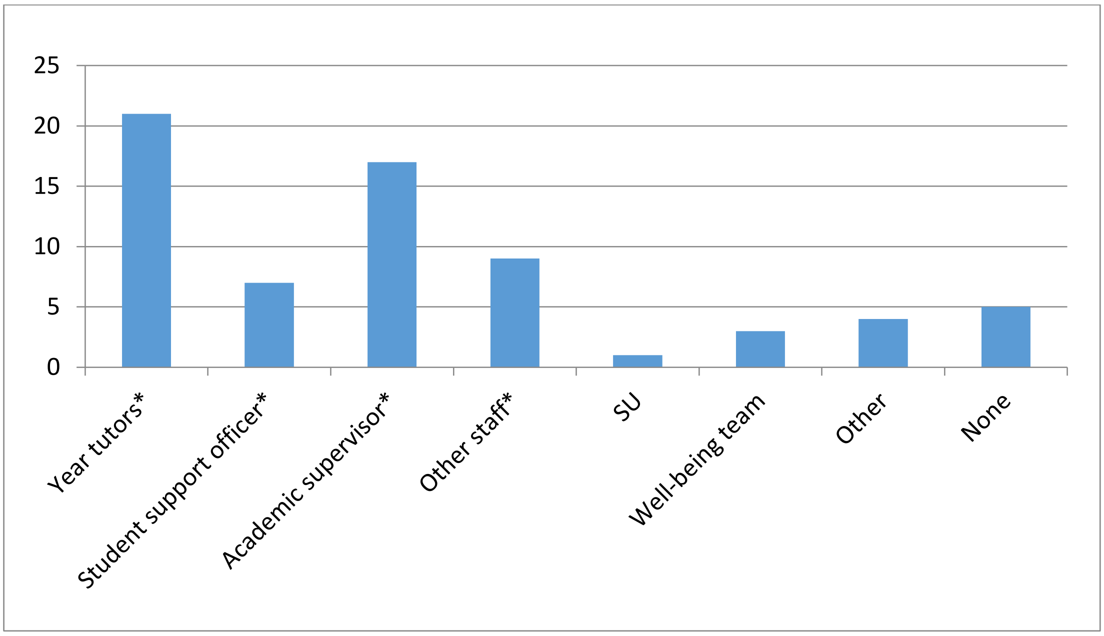

Figure 1: Personal support used whilst at university (*based within the maths team)

The students were also asked where they usually went for help first. These results are displayed in Figure 2. It can be seen that year tutors and other members of staff play a large role when it comes to personal support. It was found that 65.5% of the students surveyed indicated that they would usually go to a member of staff within the group as their first point of call for help, with 37.9% of the students saying they would usually go to their year tutor first.

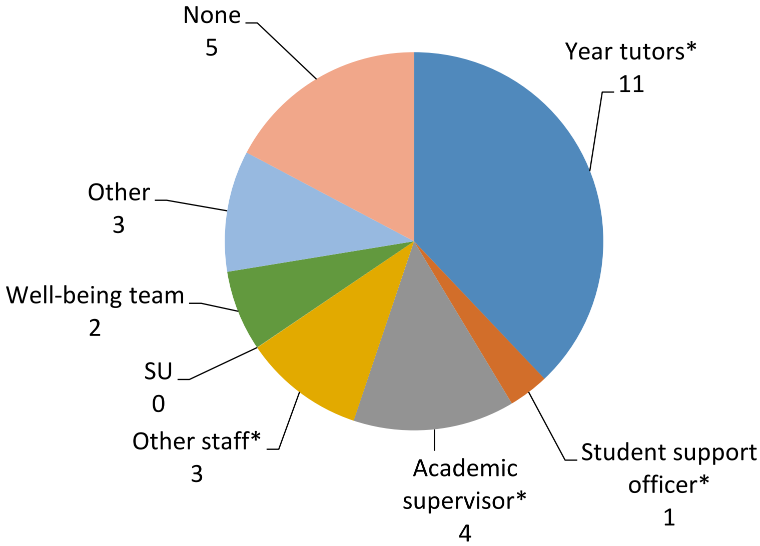

Figure 2: Where students usually go first when needing personal support (*based within the maths team)

3. Peer assisted learning scheme

Having a peer assisted learning scheme has a large impact on the students on the course. As presented within Waldock (2011) and Cornock (2016), the scheme consists of groups of first year students being supported by pairs of students further on in their studies. Group work fosters "*feelings of belonging to a community of learners of mathematics*" in general (Challis, 2015), so the group work itself plays an important role. Through peer-led academic learning "*partnership is built between students, peer leaders [and] staff*" (Keenan, 2014). The PAL scheme encourages students to communicate and work together, both within support groups and between year groups.

Figure 3: Images of the peer assisted learning scheme

The importance of the scheme is reflected in how the timetable is now arranged around the peer assisted learning scheme to maximise involvement and make it easier for everyone. Initial consequences include the students having a support group from their first day on the course, and inter-year communications. The advantages of the scheme extended far beyond the scheme itself as these friendships often continue and groups tend to stick together in classes. As presented by Keenan (2014), students taking part in peer-led sessions "*experience easier transition in HE with greater belonging and participation*", and "*build community cohesion*".

During an evaluation in 2014-15 in Cornock (2016), it was found that students were generally very positive about the scheme. A large theme that emerged in the evaluation was how it gave the students a group of friends. Some students said that they would have found it difficult to make friends without the scheme and that it gave them peers to talk to from the start. The students liked how PAL gave them a support group for all parts of the course and outside university, and how easy it was to meet people in this way. The PAL leaders and first year students commented about how they enjoyed working with each other.

4. Working environment

As discussed by Waldock (2015) and Waldock et al. (2016), students on the course have a place to work which is alongside staff offices. A floor plan is given in Figure 5. This includes lots of working spaces, group tables, and meeting rooms.

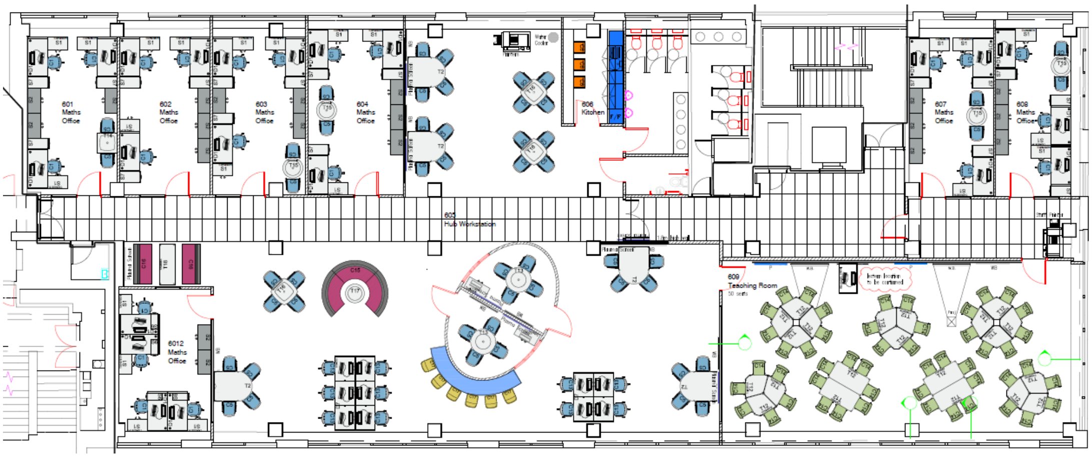

Figure 5: Floor plan of maths space (Waldock et al., 2016)

As presented by Waldock et al. (2016), the working space gives the students a home, a feeling of belonging, and promotes students working together. In the research, it was found that there was a strong indication that "*increased accessibility of both staff and other students has contributed to the students feeling like they are part of a mathematical community*". Student comments, as presented by Waldock et al. (2016), included the following:

"*Having such a wealth of knowledge just a knock away is brilliant – it is so much easier to approach staff than previously.*"

"*Having a home for the discipline makes the maths department seem more united.*"

"*As the area is purely maths it is easier to find someone who also studies a module you do and promotes students to help one another and interact."*

"*Really like this idea, it's made everything generally a better atmosphere rather than being lost within the uni not having a home.*"

"*Its spacious design has led to a great social atmosphere as well as providing excellent study facilities. Intermingling between year groups has also been created and the extra interaction between student and staff will no doubtably aid in the provision of work and assignments.*"

5. Maths Arcade

Following on from the success of the original Maths Arcade at the University of Greenwich (Bradshaw, 2011), Maths Arcades were rolled out to several other universities (Bradshaw and Rowlett, 2012). Initial information about the Maths Arcade at Sheffield Hallam University was provided by Cornock and Baxter (2012). One of the main aims was to add to the development of a mathematics community. The Maths Arcade now runs all day on one day during the week in the open access area, so staff and students can come and go as they please. Gaps are built in the timetable to ensure that students have the opportunity to take part. As presented by Cornock (2015), "*the Maths Arcade is enjoyed by students in all the year groups*". Also presented in the paper, the evaluation in 2014-15 showed that it helped students to make friends, they got to know staff better, and inter-year interactions increased. In the evaluation student comments included that the space "*creates more of a mathematical community*", that its "*spacious design has led to a great social atmosphere*" and that it brings a "*sense of 'home'*". Staff comments included that it "*gives everybody a nice feeling of community and partnership*", that it creates a "*good atmosphere amongst all maths students of different years*", and that staff "*can easily say a quick hello to students as [they] walk through*".

Figure 6: The Maths Arcade

6. Induction week programme

As the first part of a degree programme is so important, an extensive induction week programme is run. The initial time at university is of particular interest as it has an effect on the rest of the students' degree programme (Lawson, 2015). The week, which was mostly designed by colleagues, features an induction lecture where they meet their year tutor and other staff. During the lecture, the PAL scheme is introduced and students meet their PAL group. In the rest of the week, the students take part in a quiz, meet their PAL leaders at the Maths Arcade as presented in Cornock (2015), meet their academic supervisor, take part in a team building event, and carry out other activities.

7. Final year de-stress day

Since 2014-15, a de-stress day has been held for the final year students. The first de-stress day was briefly mentioned in Cornock (2015). Each year the date is carefully picked to maximise the benefit to the students. It takes place on a day when all the final year students are at university for a compulsory module, in a busy period of the year. It usually runs for 4 hours immediately before a lecture takes place. The aim of the day is for the students to spend time together, as a break from studying.

Figure 7: The de-stress day in 2014-15 (Cornock, 2015)

Figure 8: The de-stress day in 2015-16

In 2015-16 a survey was carried out with 52 of the 66 final year students in a lecture immediately after the de-stress day. Out of those students, 31 attended the de-stress day and 21 did not. The remaining final year students did not attend the lecture. Those who attended the de-stress day spent an average time of 1.36 hours there. Figure 9 shows which activities and resources the students used. When asked whether they enjoyed the de-stress day, 28 out of 31 said that they did, 2 did not answer, and 1 did not enjoy the event.

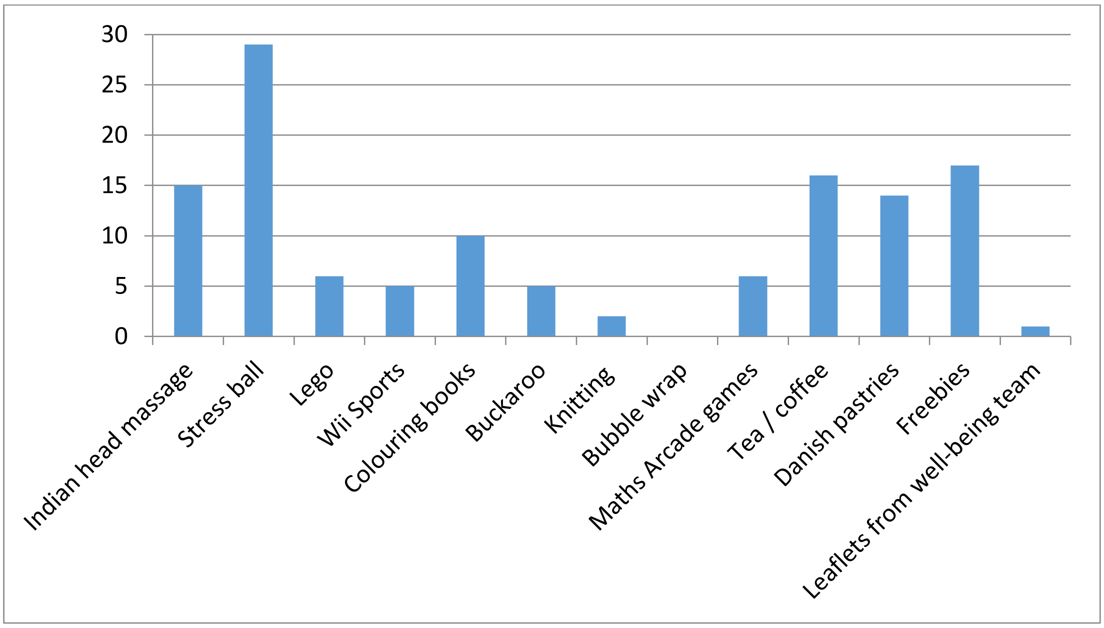

Figure 9: Number of students using the activities/resources at the de-stress day

When asked how they found the day, comments included that it gave them "*time to chill*", it was generally a *"calm and fun atmosphere"* and there was "*plenty of variety."* One student said that they *"will use [their] stress ball daily."* They said that *"it was great to spend time relaxing without feeling guilty about not doing work"* and *"it was fun without worrying about work / deadlines for a while."* Another comment was that *"everyone was able to enjoy themselves and not talk to each other about work."* One student said that they *"had fun playing with friends"* and that they *"talked to others on course."*

When asked why they attended, the reasons included the free stuff on offer, the activities, for them to relax, because they felt very stressed, to have a break, because it sounded fun, to forget about work for a while, and because of their heavy workload. The reasons for not attending included doing assignments, commuting, and still being in bed.

8. Other social activities

There are a number of other activities that take place. As presented by Cornock and Baxter (2012) and Cornock (2015), Rubik's cube championships have been run since 2012. They are enjoyed by participants and spectators. In 2014-15, a couple of final year students ran an inter-year quiz. There was at least one group from each year group and a staff team. In 2016-17, film nights were introduced, which take place approximately once a month. So far, there have been students from every year group and staff at the evenings. Plans have been made to incorporate a film night into the induction week programme from 2017-18.

Figure 10: The 2015-16 Rubik's cube championship

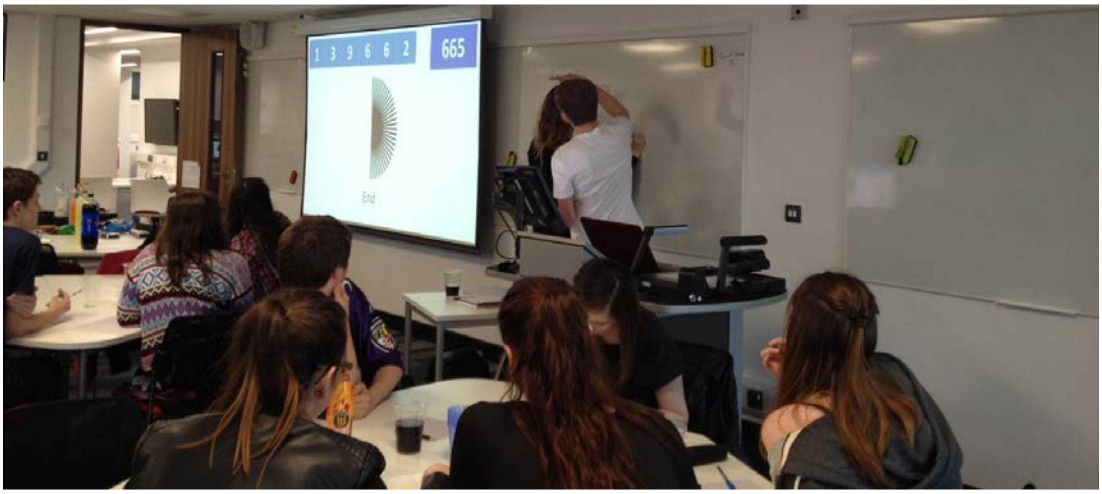

Figure 11: The 2014-15 quiz

Figure 12: The 2014-15 quiz

9. Conclusions

A course community has been built amongst staff and students on the mathematics degree at Sheffield Hallam University. This has been because of the attitudes of staff and having a mixture of opportunities to spend time together outside of formal teaching time. Staff are willing to work alongside students in a shared working space, and there is lots of support provided by year tutors and other members of staff. The opportunities to spend time together include a peer assisted learning scheme, a Maths Arcade, a de-stress day, and other social activities. The result is the formation of support groups and discussions between students in different years. The response to various evaluations is that there is a sense of home, that friendships have formed and that there is a large amount of support. Despite this, new activities and initiatives will be sought to encourage further development of the community.

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RESOURCE REVIEW

Developing an 'outdoor-inspired' indoor experiential mathematical activity

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Abstract

The issue of poor retention and achievement rates is one that plagues many British universities. While well documented and researched, there is still need for innovative practices to address this problem. This article outlines the theoretical underpinning of the *Activity Guide*, a tool the authors developed to support mathematics departments in order to make the transition to university easier for students and thus increase retention and attainment. Some of the topics covered here include reflective practise, experiential learning and independence; topics adapted from an outdoor frontier education course that had been specifically tailored by the authors to target and develop study skills particularly important for mathematics subjects. To allow for transferability and use by the entire higher education mathematics community the *Activity Guide* was produced to bring a similar course on university campuses, or even in classrooms, to better cater for resources and the scale the institutions' facilities allow. The *Activity Guide* contains all that lecturers will need to plan, set up and deliver a range of activities to their students.

Keywords: Experiential activities, outdoor, transition to H.E., activity guide, mathematics skills.

1. The issue of transition to university.

Modern universities are facing a well-documented problem of students entering Higher Education lacking adequate Mathematics preparation and study skills (LMS, 1995; Hawkes, 2000), for example, the concept of proof is not explained under the current A-level curriculum. The gap in academic requirements between A-level mathematics and the Higher Education courses (Hawkes, 2000) and the inability to cope with the new requirements of independence in their studies have a negative impact on their ability succeed in their university career (Cook, 1999). A summary on which factors influence retention in first year students engaged with mathematics modules can be found in Anthony (2000). A prevalent finding of research (Cook, 1999) is also that students often arrive at universities to find that the reality of the courses differs greatly from their expectations. Such students become hard-to reach, or "*inert*" (Krause, 2008); those who struggle to socially engage with peers, or lack motivation share a similar attitude. Whilst teaching the curricular knowledge of mathematics as a subject is mostly confined to the classroom, it is worthy focusing also on the other factors that can improve retention and attainment. An experiment of Cardelle-Elawar (1992) showed how teaching "*metacognitive skills*" (such as developing systematic strategies in problem solving, reflection and monitoring one's own progress) (Flavell, 1979) increased the success rate of low-achieving students. We will refer to metacognitive skills simply as skills later on, to distinguish from what is commonly understood as 'mathematical skills' (i.e. numeracy, knowledge of the subject…).

Universities employ various support mechanisms and tools in an attempt to bridge the gap in knowledge, in study skills and ability to be an independent learner between sixth form study and what is expected of an undergraduate. Some examples are a PASS (Personal Academic Support System) mentoring system, mathematics support centres, organised study groups and personal tutors.

We want to add a new tool to this pool of resources, which can be employed to improve retention and help students in mathematics courses and courses with mathematics components in this transition: an outdoor leadership inspired crash course. Such a course will make students experience some of the new demands of higher education and make them reflect on how they can best approach the problems (difficulties in understanding, organizational, etc.) which inevitably will appear during their course of study (Hawkes, 2000).

2. Why experiential learning and an outdoor course?

Experiential learning was introduced by Kolb (1984), who hypotheses that learning is not a static process but follows a cycle of having a concrete experience, reflecting on it, conceptualizing the experience (and thus realise the possible implications or developments stemming from this reflection), and applying this newly gained knowledge.

The use of the outdoors as an educational tool is not a new phenomenon. Tracing the lineage of using the outdoors as a learning tool is rather difficult. The conception of organised outdoor learning is often credited to the Scouting movement in the early nineteenth century, led by Lord Robert Baden-Powell. Other instrumental figures in the development of outdoor education (i.e. experiential learning in the outdoors) of individuals include the writer John Muir (active in the second half of the 19^{th} century), Kurt Hahn (founder of outdoor-based educational organization Outward Bound in 1941) and Joshua Miner (who brought Outward Bound to the US in 1961), to name but a few. These individuals have been incredibly influential in the acknowledgement and development of transferability of outdoor education to the workplace.

However, some of the greatest human endeavours of physical exploration (in the very name of furthering the knowledge of the human race) have come as a grander form of what one might consider traditional activities that are associated with outdoor education.

For example, if Sir Edmund Hillary (himself a keen mathematician) had not gained a simple understanding of rock climbing and geographical navigation he certainly would not have become the first human to summit Mt. Everest and reach both the North and South Pole in his lifetime. While seeming a rather far-fetched and farcical example nothing more perfectly exemplifies the fact that the outdoors, much like mathematics, will always be filled with boundaries that exist to be pushed.

The University of Central Lancashire has the advantage of an outdoor division that is structured to provide its students with high quality Frontier Education courses that are specifically aimed at using experiential learning in outdoor settings. The Frontier Education courses exist to afford undergraduates not only the opportunity to test and push their own physical boundaries in activities such as canoeing, climbing or gorge walking, but to challenge their intra and interpersonal boundaries also. These activities are alternated with experiential games, facilitation sessions and lectures on working in a team (such as Belbin’s theory of team roles) (Belbin, 1981). What was not considered, though, was whether these types of courses could be tailored to target subject specific skills that would by extension lead to a greater element of transferability from the outdoor setting into the classroom, thereby maximising the impact of the experiential education. It emerged in the facilitation that some of the activities could highlight mathematics-specific aspects, which would be useful to the students during their degree (for example how to go from solving a problem for a simple specific case to describe an abstract general solution).

In the past year the authors of this article have devised a frontier education course that targets key skills, detailed in the next section, that were identified through primary research by the authors. The experiential activities, which were most significant for the skills, were identified and the three days restructured to cater for more facilitation sessions on identifying problems students are encountering, reflecting on the activities and seeing how methodologies developed in the residential course could be brought back in the classroom. The importance of the topic of team work, although still present, was reduced as not as prominent for mathematics courses as for other subjects offered. This new course is now being delivered to the first year mathematics undergraduates.

3. The skills of a successful mathematics undergraduate

The skills which mathematics students need to develop once they start a degree is a field that has been extensively researched (Whimbley 1984, Silver 1987, Schoenfeld 1992). However, it would seem that there is still a very real issue of mathematics students not succeeding in graduating. (In the report of the National Audit Office (2007), mathematics and computer science subjects are identified as a major contributor to low retention rates). While the instance of students not achieving due to personal, financial or medical reasons is a major factor in this, there is still certainly instances of students failing to gain the academic standards required in a Higher Education setting (Johnston, 1997).

This incongruence of educational literature and what actually happens in universities indicates that either universities are failing to equip all students with the appropriate skills with which to graduate, or some students are simply not engaging with the mathematical education and therefore fall behind. Either way it is the role of educational establishments to afford its students the greatest opportunity to develop the skills needed to be able to gain knowledge of the subject and thus to succeed.

It is impossible, however, to give a universal recipe, which will enable students doing mathematics modules to develop the needed set of skills. There is no ticking-the-box exercise to guarantee success. We therefore asked both the lecturers and the students at the later stages of their mathematics degree at the University of Central Lancashire the following questions:

1. What characteristics do you think you need as a mathematician?
2. What did you struggle with when you first came to university?
3. What do you do when you approach a new problem (mathematical or otherwise)?

(For lecturers, the second question was reworded “What do you think first year students struggle with?”). The questions were presented in the form of a questionnaire, or an informal interview, and allowed the authors to gather some data from students who had recently terminated their school studies in UK; whilst much of the literature is based in other countries, which have a different school system. The experience of these students whom had almost completed their journey from A-Levels to graduation, staff opinions and literature such as (Anthony, 2000; Johnston, 1997; Shaw, 1997) were pivotal to gauging what was essential to the successful mathematics undergraduate. We applied Interpretative Phenomenological Analysis (IPA) to the surveys and the interviews, and then compared the results with those found in (Anthony, 2000; Johnston, 1997; Shaw, 1997).

The resulting skills list is as follows:

Abstract thinking. New students starting a mathematics degree at university commonly expect computationally harder versions of the standard problems that they are familiar with from secondary and further education (findings from the surveys). They therefore struggle to work with abstract ideas and the concept of proof, which were absent in their previous education. Higher levels of abstractions (pattern recognition, capacity of developing memory schemata) (Silver 1987) are fundamental to approach complex problems and to fully understand theorems.

Thinking out-of-the box. Students need to not only be able to solve problems with previously explained methods, but also to create new methods to solve unfamiliar problems. First year students generally have the habit of looking for similar problems in notes and tutorial sheets and then replicate the resolutions found to what they have been presented with, giving up when they cannot find the supporting material. (Findings from the surveys).

Resilience (Gavriel, 2015). Working on new problems without a resolutive guideline requires time and persistence despite inevitable failed attempts; it is therefore needed to develop resilience at an early stage of the studies. (Findings from the surveys).

Ability to understand threshold concepts (Meyer and Land, 2005). The development of resilience in a student aids in comprehending a threshold concept. Threshold concepts refer to students understanding or comprehending an idea or theory that is necessary to the progression of their studies, but which is somewhat counterintuitive or new to the students' experience, and therefore requires a "*liminal phase*" of questioning their own knowledge and a final mental shift to fully grasp the concept. Classic examples are the concept of 'limit' and 'imaginary number', which might be a struggle to understand to begin with but become tools underpinning the entire mathematics degree.

Team work/Collaboration. Students need to be open and willing to work with, or take advice from, other students. Generally, all people tend to learn better from people of the same intellectual level as themselves. Therefore, students need to be encouraged and prepared to work together, or even to discuss issues, as this will help them learn.

Independence (Haemmerlie, Steen and Benedicto, 1994. Field, Duffy and Huggins, 2015). As well as being willing to work with other people, students need to have the skills and ability to work individually without any help. Students that rely heavily on notes, lecturers and fellow students tend to struggle with independent thought and originality. There is a fine balance that students must find between working together and being willing to take advice and help, and having the ability to work and think on their own. (Findings from the surveys).

Resourcefulness. Being aware and prepared to use all of the resources available, even if not instructed to do so by staff. New students are accustomed to being told where to look for answers or what to use and when. The drive and ability to independently find resources besides what has been provided is what sets apart the students that are more likely to succeed in higher education. Staff also exist as a resource for students, and often students do not feel comfortable enough to interact with staff on a one to one, out-of-class hours basis. (From the collected data we inferred that students were more proactive in asking for help to the lecturers after the outdoor course, also a prevalent issue in the questionnaires to students and course leaders of Johnston (1997)).

Communication (Ellis, 2003). The ability to express ideas and concepts correctly to people of different backgrounds, age and expertise is essential to the undergraduate. Students often come into a mathematics degree assuming that they will only be working with numbers or letters. They do not expect to be involved with wordy proofs or reports. If the students are unable to articulate properly, they can fail despite having understood the concept or idea. Students need to be able to produce work that is fluent and coherent so their understanding can be accurately gauged.

Critical thinking/Mathematical thinking (Kun, N.D.). Students need to be able to analyse and evaluate a problem in order to create a sound and mathematical judgment on how to approach it. Being able to see a path through a new problem and working systematically on finding a solution is what is wanted by many employers from a mathematics graduate.

Curiosity/Being inquisitive (Sparks, 2014). Without curiosity, students will lack the drive to look beyond what is taught in the lectures. It is curiosity that encourages a student to find new and innovative methods to problems, and even motivates them to work instead of procrastinating. This curiosity also has an impact on their willingness to approach others in the search for help or collaboration and their propensity to be resilient. (Findings from surveys).

Organisation. Organisation is not only necessary for assignment deadlines and exam revision, but also the ability to maintain a good balance between work/study and enjoyment. As an undergraduate student, the ability to manage more than one module, project, assignment, and exam at once is crucial. A lack of organisation will result in deadlines being missed, lectures being unattended, notes not being reviewed, revision not starting soon enough and even maybe missing paying the rent. All these things can lead even a mathematically gifted student, to fail the degree. (Johnston, 1997).

Building on previous knowledge (Kimmerle, Moskaliuk and Cress, 2011). Being able to build on topics that students have already learned and make links across subject areas is incredibly important. Students that are unable to expand on topics that have already been covered or discussed will find it difficult to succeed in the later years in the mathematics degree. The different modules in mathematics cannot be separated completely, for example, it is very common to use general ideas from pure mathematics to solve an applied mathematics problem.

Optimization/Efficiency. Finding realistically applicable and useful methods, requiring the least number of steps/time or occupying the least amount of computer memory is a requirement for many mathematics graduates, especially is finding a job in industry. Solving problems by ‘brute force’ or ‘trial and error’ can only be a first stage and the art is in going back to the problem and obtain the result in a more efficient way.

Accuracy/Precision (Whimbley, 1984). Students need to be able to find answers that are correct or accurate to a certain degree despite time or exam pressure. This often leads to mistakes and errors that are the result of increased pressure, clumsiness or even laziness rather than the result of a lack of understanding the concept or idea. Students forget that although speed is important, accuracy is more so.

4. The creation of an indoor ‘outdoor course’.

While the new maths specific frontier education course worked in its delivery within the University of Central Lancashire’s infrastructure, its application as a student support tool for the mathematics community remained limited due to its lack of transferability across institutions that find themselves lacking the investment in activity resources, facilities and practical expertise required to execute a full three-day residential frontier education course. Feedback from **sigma** and colleagues at the 2015 CETL-MSOR conference suggested the need for an adaptation of this Frontier Education course, in order to allow it to be delivered by lecturers on campus or in a classroom environment without use of specialist equipment. The intent here is to scale down the size of the activities (by means of resources and space required) and giving lecturers the tools with which to engage students in quality reflections. The *Activity Guide* is thus a guide on how to deliver an outdoor-inspired course in a campus environment, with a list of experiential games, and guides on how to set up, how to organise the schedule of the event, and how to run facilitation sessions. The authors believe it will act as an enjoyable but effective ancillary to traditional methods of teaching.

5. A handy overview on the theory of coaching

5.1 Progressive learning

When structuring a series of activities, much like learning new concepts, one must take great care to give students a solid foundation (Bush and Smith, 2010) of multidimensional learning. These are the cognitive, affective and skill capacities dimensions (Kraiger, Ford and Salas, 1993). These aspects should be introduced, logically, from the very beginning of the programme. It is this progressive schedule that affords students the opportunity to formulate their own self-direction (Knowles, Holton and Swanson, 2012) and fosters a keenness for learning, which is key to the students 'buying in' to the process of the activities. Once solid foundations have been established then the programme can see the introduction of activities requiring greater complexity and organisation.

It is with this in mind therefore that the example timetables within the *Activity Guide* are designed in a progressive manner. All programmes will begin with icebreakers and lower level activities to firstly break down any social barriers (Mertes, 2015) between both students and staff that may hinder productivity later in the programme and academic year. This will then see the programme progress to activities that develop maths specific skills alongside student skills focussing on varying instances of independent work and collaboration.

5.2 Independence

Undergraduate education is characterised by independence. The surface occurrence of independent study fosters the development of independent thought and as a result, professional autonomy. Noble and Hames's (2012) articulation that independence is an accelerant for development appears to be confluent with the Transition to Independence Process model (Kalinyak, Gary, Killion and Suresky, 2016). Within this model, the enhancement of young individual's competencies that assist them in becoming self-sufficient is essential to this end, and as mentioned earlier, remains a necessity for tertiary education to deliver due to the gap from the demands placed on previous levels of the educational process.

The instance, therefore, of students operating away from lecturer stimuli should be commonplace in the deliverance of the *Activity Guide.* Activities have been carefully devised to enable students to do work independently and create a learning environment in which the lecturer exists simply as a resource to augment their learning process, not as the sole source of information. As such, the lecturer's conduct becomes pivotal to the development of independence. Spoon feeding students information and help may increase task achievement but as a result will increase the learner dependency (Daily and Landis, 2014) which is the exact opposite of what the *Activity Guide* is designed to do. Careful consideration therefore should be given to the Facilitation section (see section 5.5) to ensure effective deliverance of the *Activity Guide.*

5.3 Collaboration

The extent to which learning is affected by the ability of a Mathematics student to collaborate should not be underestimated. The ability to collaborate and learn from peers is confluent with the notion of active learning. Petress (2006) articulates that this is effective for the enhancement of the learning process. Not only this but the encouragement of the students to act as a community of practise (Kimble, Hildreth and Bourdon, 2008) can lead to the body of students acting independently from the lecturing staff.

The *Activity Guide* affords students the experience of working as a team to foster this sense of community. A mixture of activities whereby working together is required and optional both ensures that students collaborate but also have choice as to whether or not collaboration is appropriate to meet the demands of certain activities. This further places the responsibility for ownership of learning onto the students, making them accountable for their learning but hopefully also more confident in their ability to approach activities in a more adventurous manner.

5.4 Exploration

This desire to discover new frontiers is pivotal to the notion of creativity. Defining creativity is often one of the most difficult tasks for educators, and so its development in students often misses the metaphorical mark. Marquis and Henderson (2015) identified that individuals often contextualise creativity based on the line of work that they do. With this in mind it is fair to assert therefore that in mathematics creativity is essential to not only the development of new solutions to unfamiliar problems, but also to the transfer of knowledge across subject areas (Kirwan, 2008), the understanding of abstract concepts, development of more efficient methods for problem solutions and often the articulation of the self. All of which are essential to a successful mathematics graduate.

Therefore, the exercises within the *Activity Guide* are structured to allow the greatest amount of creative input from students as possible. It is for this reason that there is no offering of ‘ideal’ solutions to the activities. Success in the activities is not necessarily the most important outcome from the *Activity Guide*, engagement in the learning process that it enables however is. Part of this process is being creative and experimenting with ideas, exploring new concepts and ideas and making valuable contributions to attempting a solution.

5.5 Facilitation and reflection

It is appropriate that any lecturer attempting to utilise the *Activity Guide* accommodates for a shift in ethos from Teacher to Facilitator (Justice and Jamieson, 2006.Wilkinson, 2012). The facilitation comes in the form of the lecturer leading their students through a change process, and is due to the intended holistic development of the student. This therefore means that the lecturer should be focussed on creating an environment whereby this development is enabled. Part of this is an explicit focus on experiential learning, with reflective practise playing a key role in students exploring and learning meaning from their experiences.

The *Activity Guide* not only draws on academic theory surrounding reflective practice from academics such as; Borton (1970), Dewey (1963), Gibbs (1988) and Kolb (1984), but also draws on the experience of skilled outdoor professionals that make a living by delivering university level thinking from frontier education. The *Activity Guide* contains an adaptation of Rolfe, Jasper, Freshwater and Rolfe’s (2011) model for reflection both in and on action and is therefore a product of sound theory based practise. This is designed to streamline the reflective process for novice facilitators. The *Guide* provides a framework for structured reflection (in the form of a prompt sheet) and points to consider for during the activity to maximise the impact that the facilitation can have on the students.

6. Conclusion

What is clear from this project is that mathematics education is as complicated as the subject itself is. The *Activity Guide* is designed to provide lecturers with a resource and tool with which to engage students in a manner that is not usually done in universities. While fun, innovative and ‘something a bit different’, the *Activity Guide* is completely dependent on effective delivery and a conscious effort from staff to ensure that much of the focus remains on quality reflections. Without reflecting on the activities, Kolb’s cycle (Kolb, 1984) is interrupted and the possibility to help the students develop new skills from the experiences is wasted. It is imperative to understand, however, that the *Activity Guide* cannot solve all of the problems of progression. It should be used in conjunction with a range of student support tools to augment an effective student experience.

The progression of mathematical education is certainly in the hands of the higher education mathematics community. Innovative and creative ways of educating students should be prevalent at every level of mathematical education. However, universities must lead the way in these endeavours. It is our hope that, if successful in universities, the *Activity Guide* will be adapted to suit the needs of Further Education and assist in the process of producing individuals that start undergraduate study with a set of skills that allow them to hit the ground running.

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CASE STUDY

Using a simple poker game to introduce mixed strategies in game theory

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Abstract

This article presents a classroom activity that introduces the idea of mixed strategies in game theory. In the activity, the students investigate a simple two-player poker game with the aim of determining the best strategies for both players. It comprises a mixture of hands-on playing, analysis using game theory, and simulation using computers.

Keywords: Practical Activity, Game Theory, Poker, Simulation.

1. Introduction

At Coventry University, the mathematics undergraduate students, as part of a Problem Solving first year module, were taught some introductory game theory over two sessions. The first session involved the students engaging in a Prisoners' Dilemma activity. This is a scenario first formulated by Tucker (1950), and it is briefly outlined in Section 2 of this article. In that activity, the students met concepts such as maximin, saddle point, and pure strategy; ideas in game theory originated by Von Neumann and Morgenstern (1944).

This article concentrates mainly on the students' second session. This built on their knowledge from the first session, by introducing a situation where there was no saddle point, and a mixed strategy was required. Section 3 describes a simple poker game that the students played, and measures empirically their resulting profits. Section 4 analyses the game theoretically and produces recommended playing strategies. In Section 5, the recommended strategies are implemented by the students and a comparison of profits is made. In Section 6, the theory is generalised so that recommended strategies can be obtained for different versions of the game. Then, in Section 7, a computer simulation is used to test the effectiveness of the theory.

2. Prisoners' Dilemma

This is an often-used example as an introduction to game theory. There are many different versions, but they all roughly follow along similar lines as detailed below:

- Arry and Butch are gangsters. The police have arrested them, but do not possess enough information for a conviction. Following the separation of the two men, the police offer both a similar deal;
- If one testifies against his partner, and the other remains silent, the former receives a 1 year sentence and the latter receives 10 years;
- If both remain silent, both are sentenced to only 3 years in jail for a minor charge;
- If each testifies against the other, each receives a 6 year sentence;
- Each prisoner must choose either to betray or remain silent. Whilst choosing, they do not know the other's decision. What should they do?

The actions and outcomes can be displayed in a 'payoff matrix' (Table 1):

Table 1: Payoff matrix for Prisoners' Dilemma

Prison sentences (Arry, Butch)		Butch	
		Betray	Silent
Arry	Betray	(-6,-6)	(-1,-10)
	Silent	(-10,-1)	(-3,-3)

- Arry's line of reasoning could be as follows. If Butch betrays, then Arry should also betray to maximise his own outcome. However, if Butch is silent, again Arry should betray to maximise his own outcome. Therefore Arry should betray whatever Butch does. This is called a maximin strategy as in essence he maximises his minimum gain;
- Similarly, Butch could follow the same line of reasoning, and conclude that he himself should betray;
- The result would then be that both would betray and consequently both end up with a 6 year prison sentence. The cell (-6,-6) is called the saddle point as neither prisoner can improve their outcome by changing their mind if the other doesn't change;
- However, if they both agree to cooperate and both keep silent, they would both be better off with only a 3 year sentence. But, this does rely on trust as one of them could renege on the deal to personally benefit - hence the dilemma.

3. Simple Poker Game

The students were split into groups of three with each group given a standard pack of playing cards and 40 casino style chips. Two of the group played the game (Player 1 and Player 2), and the third dealt the cards and recorded the number of hands played.

Player 1 (P1) and Player 2 (P2) were given 20 chips each and the rules were as follows:

- P1 and P2 both put 1 chip into the pot.
- P1 is given a card. He can either:
  - Fold – in which case P2 wins the pot; or
  - Bet 1 (i.e. put another 1 chip into the pot).
- If P1 bets, P2 can then either:
  - Fold – in which case Player 1 wins the pot; or
  - Call (i.e. put another 1 chip into the pot).
- If P2 calls, P1 shows his card:
  - If P1 has 9 or higher, P1 wins the pot;
  - If P1 has 8 or lower, P2 wins the pot.
- Shuffle the cards and play again several times.

Figure 1: Cards and chips for simple poker game

Before they started playing, the students were asked which player they thought was most likely to be in profit after several hands. The majority who responded suggested that it would be P2, as there were only 6 winning cards (9 to ace) for P1 but 7 winning cards (8 to 2) for P2.

The game was played for three or four minutes, after which the profit/loss was recorded for P1 along with the number of hands played. The process was repeated twice more with the students swapping roles within each group so that each had a turn at each role.

There were 33 students in the class. Some of the results are shown in Table 2.

Table 2: Students' P1 Profits

Student No.	P1 Profit	Number of Hands
1	4	20
2	-8	16
3	-3	22
etc.	…	…
Total	-8	455

The total number of hands played was 455 and the P1 total profit was -8 chips. Thus the mean profit per hand was -8/455 = 0.018 i.e. P1 lost on average 0.02 chips per hand.

The students were asked which strategies they employed, and it was evident that those P1's who ended up in profit were mainly those who employed 'bluffing' i.e. Betting on a losing card (an 8 or lower) in the hope that P2 would Fold.

4. Game Theory Analysis

Let Strategy 1 (S1) for P1 be 'Fold if he has a losing card i.e. 8 or lower, Bet if he has a winning card i.e. 9 or Higher' (F if L, B if W).

Let Strategy 2 (S2) for P1 be 'Bet if he has a losing card i.e. 8 or lower, Bet if he has a winning card i.e. 9 or Higher' (B if L, B if W).

Let S1 for P2 be Fold, and S2 for P2 be Call.

Thus for each hand played there are 4 possible strategy pairs for the two players – (S1, S1), (S1, S2), (S1, S1), (S2, S1), and (S2, S2). (For each pair, the first number represents the strategy that P1 employs and the second number represents the strategy that P2 employs).

Consider (S1, S1). This means that P1 will only Bet if he has a winning card and P2 will always Fold if P1 has Bet. Indeed this was a common pair of strategies amongst the students, particularly at the beginning. Cautious P1's didn't Bet if they had a losing card, and cautious P2's assumed that if P1 had Bet then he must have a winning card. Here the expected profit for P1 can be calculated as follows:

P1 Folds if he has 8 or lower, so

$$P(P1\ Folds) = \frac{7}{13}, P(P1\ Bets) = \frac{6}{13}.$$

If P1 Folds, his profit is -1. If P1 Bets, his profit is 1 (because P2 will Fold). Hence

$$E(P1\ Profit\ if\ (S1, S1)) = -1 \times \frac{7}{13} + 1 \times \frac{6}{13} = -0.077.$$

Now consider (S2, S1). This means that P1 Bets when he has a winning card, and also Bets when he has a losing card. In the classroom session, as the playing progressed many P1's who were employing S1 tended to realise that they were Folding a lot of hands, and thus started to bluff some hands – this was either through reasoning that they might have more chance of obtaining a profit, or through getting a bit bored with sticking with the same approach.

$$E(P1\ Profit\ if\ (S2, S1)) = 1$$

because here we are assuming that P1 is always Betting and P2 is always Folding.

For (S2, S2), P1 Bets whatever his card, and P2 Calls. In the activity, as the playing progressed it was found that P2 Called more often, as he became more aware that P1 was bluffing.

$$E(P1\ Profit\ if\ (S2, S2)) = -2 \times \frac{7}{13} + 2 \times \frac{6}{13} = -0.154$$

because P1 wins 6/13 of the time.

Finally, consider (S1, S2), i.e. P1 only Bets if he has a winning card, and P2 Calls. Here,

$$P(P1\ Folds) = \frac{7}{13}, P(P1\ Bets) = \frac{6}{13}.$$

If P1 Folds, his profit is -1. If P1 Bets, his profit is 2 (because P2 has Called). Hence

$$E(P1\ Profit\ if\ (S1, S2)) = -1 \times \frac{7}{13} + 2 \times \frac{6}{13} = 0.385.$$

These profits are displayed in a payoff matrix in Table 3:

Table 3: Payoff matrix for Poker Game

P1 Expected Profit			P2	
			S1	S2
			F	C
P1	S1	F if L, B if W	-0.077	0.385
	S2	B if L, B if W	1	-0.154

Key:
S1 – Strategy 1,
S2 – Strategy 2,
F – Fold, B - Bet, C – Call,
L – has losing card,
W – has winning card.

There are some differences between this payoff matrix and the one from the Prisoners' Dilemma (Table 1). Firstly, it can be seen that there is only one value in each cell here, as opposed to two in Table 1. This is because the poker game is a 'zero-sum game'. That means that a gain for one player corresponds to a loss of equal magnitude for the other player. Hence for (S1, S1) the payoff for player 2 is 0.077, for (S1, S2) it is -0.385, and so on. As these values are assumed, it is not necessary to display them in the matrix. Also, as there is some uncertainty in the outcomes in terms of the probabilities of the card being dealt, the entries are expected values. These can appropriately be interpreted as the expected profit in the long run, because the game is repeated. The Prisoners' Dilemma however would often be perceived as a game that would only be played once if in a realistic context.

Being a zero-sum game, there is no possibility of cooperation helping both players, as a change in strategy could not increase the pay-out for both players. Moreover, there is no saddle point here i.e. there is no cell in the matrix with a value representing the optimal strategy pairing for the two players. Hence we would say that there is no 'pure' strategy solution.

Nevertheless we can obtain a 'mixed' strategy (Maschler, Solan and Zamir 2013) solution i.e. to maximise his expected payoff, a player can use S1 some of the time and S2 some of the time. P1's optimal mixed strategy solution is to use S2 a proportion, x, of the time so that his expected payoff is the same whatever P2 does, and P2's optimal mixed strategy solution is to use S2 a proportion, y, of the time so that his expected payoff is the same whatever P1 does. This will result in a mixed strategy equilibrium.

First, consider P1's mixed strategy. Using the values from Table 3;

$$E(P1\ Profit\ if\ P2\ Folds) = -0.077(1 - x) + 1x,$$

$$E(P1\ Profit\ if\ P2\ Calls) = 0.385(1 - x) - 0.154x.$$

Equating these to obtain the optimal mixed strategy for P1:

$$-0.077(1 - x) + x = 0.385(1 - x) - 0.154x$$

$$x = 0.29.$$

This means that P1 should employ S2 29% of the time and S1 71% of the time.

For P2's optimal mixed strategy; again using the values from Table 3;

$$E(P2\ Profit\ if\ P1\ uses\ S1) = -0.077(1-y) + 0.385y,$$

$$E(P2\ Profit\ if\ P1\ uses\ S2) = 1(1-y) - 0.154y$$

Equating these gives:

$$-0.077(1-y) + 0.385y = 1 - y - 0.154y$$

$$y = 0.67.$$

Thus, P2 should use S2, 67% of the time, and S1, 33% of the time.

In Table 4 these proportions are added to the payoff matrix.

Table 4: Optimal Mixed Strategies and Payoffs in Poker Game

P1 Expected Profit				P2	
				0.33	0.67
				S1	S2
				F	C
P1	0.71	S1	F if L, B if W	-0.077	0.385
	0.29	S2	B if L, B if W	1	-0.154

If these mixed strategies are used, then

$$E(P1\ Profit) = -0.077(0.71)0.33 + 0.385(0.71)0.67 + 1(0.29)0.33 - 0.154(0.29)0.67 = 0.23$$

So, the optimal mixed strategies are that P1 should bluff 29% of the time, and P2 should Call 67% of the time. This would result in a 23% profit for P1. If one of the players deviates from this, his expected profit would decrease.

5. Playing the Game again

The students were then asked to play the game again several times, utilising the optimal strategies that they had just learnt. In order to randomly bluff approximately 29% of the time, P1 would glance at their watch or phone and bluff if the last digit of the seconds of the time was a 3, 6 or 9. Similarly P2 would Call if the first digit was 0-3. (Alternatively, appropriate sided dice could be used but these would not be as easy to conceal from the opponent and may inhibit the flow of the game.)

The result for the class was a mean gain of 0.36 chips for P1, a lot better than the 0.02 average loss that P1 had the first time they played the game, highlighted in Section 3. The result was also considerably higher than the 0.23 predicted by the theory in Section 4. This is likely attributable to the fact that, when questioned afterwards, many of the P2's admitted that they hadn't stuck to the optimal 67% for Calling. They had taken it upon themselves to try and 'spot' when P1 was bluffing, resulting in a higher Call rate.

6. Generalising the Theory

The students were then asked to find optimal strategies in the general case where, in the poker game, the probability that P1 gets dealt a winning card is p. Using the same reasoning as in section 4,

$$E(P1\ Profit\ if\ (S1,S1)) = -1(1-p) + 1p = 2p - 1;$$

$$E(P1\ Profit\ if\ (S2,S1)) = 1;$$

$$E(P1\ Profit\ if\ (S2,S2)) = -2(1-p) + 2p = 4p - 2;$$

$$E(P1\ Profit\ if\ (S1,S2)) = -1(1-p) + 2p = 3p - 1.$$

These expected profits are displayed in a payoff matrix in Table 5:

Table 5: Payoff Matrix where P(Winning Card) = p

P1 Expected Profit				P2	
				1-y	y
				S1	S2
				F	C
P1	1-x	S1	F if L, B if W	2p-1	3p-1
	x	S2	B if L, B if W	1	4p-2

Finding the equilibrium point, P1 should bluff a proportion, x, of the time, where

$$(2p-1)(1-x) + x = (3p-1)(1-x) - (4p-2)x$$

$$x = \frac{1}{3}\left(\frac{p}{1-p}\right) \qquad (1)$$

P1 should Call a proportion, y, of the time, where

$$(2p-1)(1-y) + (3p-1)y = 1 - y + (4p-2)y$$

$$y = \frac{2}{3}.$$

Interestingly, P2 should Call 2/3 of the time, whatever the probability of P1 receiving a winning card. This results in

$$E(P1\ Profit) = (2p-1)\frac{1}{3}\left[1-\frac{1}{3}\left(\frac{p}{1-p}\right)\right] + x + (3p-1)\frac{2}{3}\left[1-\frac{1}{3}\left(\frac{p}{1-p}\right)\right] + (1)\frac{1}{3}\left[\frac{1}{3}\left(\frac{p}{1-p}\right)\right] + (4p-2)\frac{2}{3}\left[\frac{1}{3}\left(\frac{p}{1-p}\right)\right]$$

$$= \frac{8p-3}{3}$$

Using the value of $p = 6/13$ from the poker game, from Eq. (1) the proportion of time that P1 should bluff,

$$x = \frac{\left(\frac{1}{3}\right)\left(\frac{6}{13}\right)}{1-\frac{6}{13}} = \frac{2}{7} = 0.29,$$

and from Eq. (2),

$$E(P1\ Profit) = \frac{8\left(\frac{6}{13}\right) - 3}{3} = 0.23$$

confirming the values found in Section 4.

The students then found the optimal strategy for P1 if the rules of the game were changed slightly, so that P1's winning cards were reduced to 10 or higher:

Here $p = 5/13$, so

$$x = \frac{1}{3}\left(\frac{5/_{13}}{1 - 5/_{13}}\right) = \frac{5}{24} = 0.21$$

and

$$E(P1\ Profit) = \frac{8 \times \frac{6}{13} - 3}{3} = 0.03.$$

Hence P1 should bluff 21% of the time and will end up on average with a profit of 3%.

Many students were surprised or sceptical of this finding, in that P1 only has five winning cards to eight losing cards, but will still end up in profit through strategic game play.

7. Computer Simulation

To convince the sceptics, the students were asked to test the optimal strategy for the '10 or higher' rules. This time, rather than deal cards, to save time they played the game on a computer simulation created in Excel. An example of a screenshot is shown in Figure 2.

The spreadsheet operated as follows: The student clicked on the Deal tab and a card was randomly dealt. In the example in Figure 2, this was an 8. The program then recommended to Bet or Fold according to the optimal strategy i.e. it randomly recommended 21% of the time to Bet on a card 9 or lower. The student then clicked on the Bet tab or Fold tab. If the student selected Bet, they then clicked on the P2 Decision tab. The program would then Call 67% of the time and Fold 33% of the time. The number of chips won or lost for that hand were then displayed. The students then repeated the process, playing as many times as they wished, and the program displayed their mean profit and the percentage of the time that the student had bluffed.

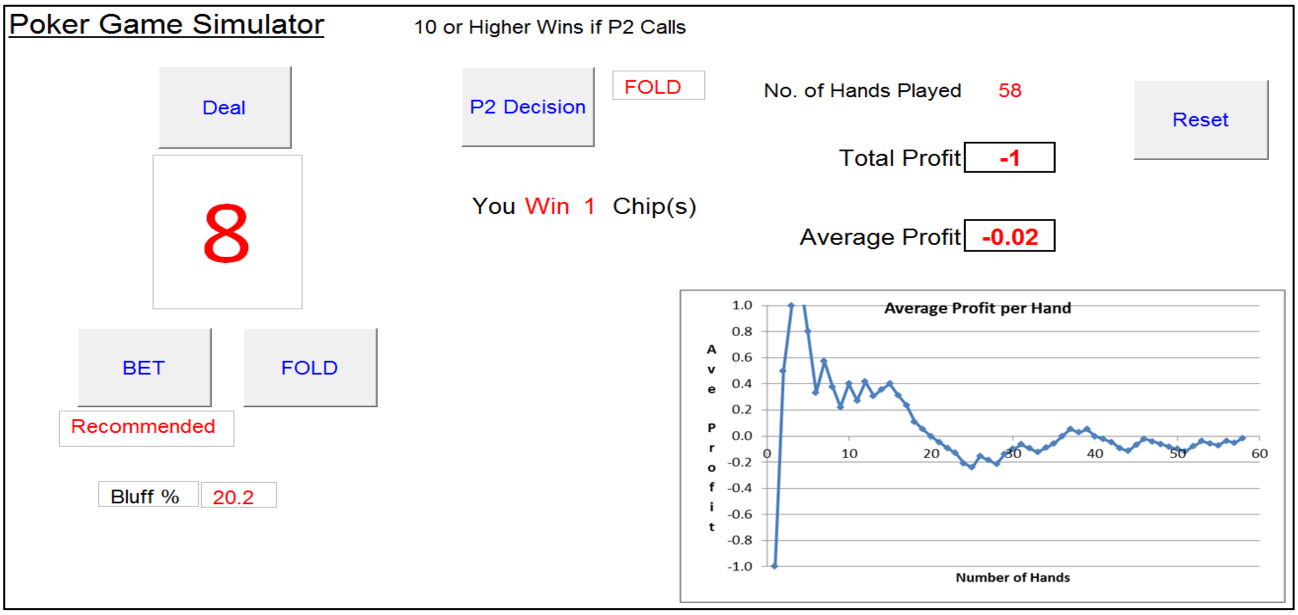

Figure 2: Simulation of ’10 or Higher’ Poker game

In Figure 2, the student’s bluff percentage was 20.2 which was close to the recommended 21%. This suggested that the student had likely followed the automatically generated recommendations by the program. Consequently the mean profit here of -0.02 was close to the expected profit of 0.03 calculated in Section 6. Obtaining results from the other students in the class, those who had deviated from the recommended action generally ended up with a lower average profit. This helped to convince the sceptical students of the validity of the theory.

8. Conclusions

Through participating in a hands-on activity, the students investigated and applied mixed strategies in game theory to see them work in practice. There is much research that activity led learning is a successful learning mechanism, for example see Freeman et al. (2014). Hence the activity outlined in this article could be considered a useful tool when teaching an introduction to mixed strategies in game theory.

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CASE STUDY

Financial computing literacy: 10 steps

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Abstract

It is often the case that in a financial engineering/mathematics master's curriculum, computer programming if taught, is before the start of an important course typically titled ‘Numerical analysis of financial derivatives’. Typically in this computer programming course, C++ is taught, and the material spans from basic constructs to an introduction to advanced programming such as design patterns. A major reason for its early introduction is that students would subsequently be able to use the skills to computationally solve problems occurring in numerics. We believe this curriculum strategy to be an element of ‘computer literacy’ which has been criticized in our context as one that discourages students with lower programming abilities who otherwise may have predisposed mathematical abilities. We aim to make fundamental financial computing inclusive via a series of 10 steps thereby leading to what we refer to as financial computing literacy.

Keywords: Computing literacy, financial computing, 10 Steps.

1. Introduction

MSc courses in Financial Engineering/Mathematics are popular amongst students for its job-market appeal. A crucial component of its curriculum is a course on numerics. It consists primarily of finite difference methods (FDM) for partial differential equations (PDE), implicit and explicit schemes, stability, consistency and convergence, and applications to option pricing via computer implementation. Typically, student backgrounds range from business, economics, engineering and mathematics. Enrolment is normally double digit. Unfortunately not all of these have even primary exposure to computer programming, like the students which we investigate for this case study. On the other hand, C++ being a favourite language of financial quants (or financial quantitative analysts), is taught in the programming class. The material therein spans from basic constructs to an introduction to advanced C++ programming like design patterns. The objective is that students would then be able to use the skills to computationally solve problems occurring in numerics. We believe this curriculum strategy to be a part of what Luehrmann (1981) calls “*computer literacy*". Luehrmann implies that the word “*literacy*" after the word “*computer*”, must also mean the ability to do computing, and not merely recognize, identify, or be aware of alleged facts about computing. Inherent in this definition, is a fact that there is something fundamental in computing that every student must be aware of. However, Harvey (1983) argues that the word “*literacy*" is unnecessarily broader in scope. Harvey in fact propositions that computers should be made available to students as a serious tool and more importantly in a pointed and embedded way, not as something they'll need later, which in our context happens if programming is taught preceding a numerics course. This may risk discouraging students who have weak programming skills but otherwise predisposed mathematical abilities. We believe that if standard computer programming is embedded within numerics, it can only enthuse students to tune their minds to learn and use advanced financial computing concepts later on. It also results in a deeper conceptual understanding of the involved numerics. Additionally, this embedded teaching pattern modulo few changes may also qualify for its early introduction into an undergraduate curriculum. In summary, the newness of this approach is its embeddedness within numerics and inclusivity to financial computing via simple steps through MATLAB. Expectedly, implementing this case study is to aid rather than substitute an important course on C++ programming.

To achieve this objective, we provide 10 steps that should lead to 'any slightly motivated' financial mathematics student interested in pursuing further advanced financial computing. These steps start from teacher or tutor-led lab demonstration (to teach students basics of MATLAB) and culminate with student self-driven projects. These steps must be followed in ascending order and may or may not contribute to final grade. With no prior assumed coding or any sort of programming background, the choice of the programming language becomes crucial. We choose MATLAB instead of C++, for the following reasons: availability, generality, engineering usefulness, stability, simplicity to use and learn, and hence importantly its inclusivity. This also has been cited as the reason in Öhström et al. (2005), and Higham (2004) and we concur with that.

We believe that there are three possible ways to introduce computing to students in a lab. These are:

1. Assign projects while you watch them do. Help them out if needed;
2. Have teaching assistants (TAs) who will join the instructor in helping in-need students in the lab session;
3. Provide stand and deliver demonstration and craft the lab tutorials in such a way that it makes them yearn to code.

The first method expects financial computing literacy while the second expects it from the TA's. Also, it is costly and restrictive. Hence we followed the third approach. In it lab instruction is carried out in a wireless internet enabled classroom with desktop and projector screen. We needed internet to access MATLAB from the university's software hub. We required wireless room because students were going to use their own laptops. Brown and Lippincott (2003) have appealed to the use of laptops in pedagogy, popularly termed as Bring Your Own Device (BYOD). This was not a constraint since all students usually own laptops or may get them on loan from the university library. Interestingly, my observation when I once took the lab in a wireless enabled class with desktop computers was that students found comfort in using their own laptops. We believe the primary reason being that this way they could sit with their colleagues and discuss the exercises. However we wish to point out that we envisage that the pedagogy developed here is as effective if implemented in a room with desktop computers, a computer with projector and a screen. In fact, we recommend mix use of desktop first and laptops later, since the first nine steps expect independent work, followed by the last one which encourages group activity in a classroom. We note here that laboratory teaching activity is also not just about designing the learning task but also the physical arrangement of the space and the roles of the teachers/students within it (Pretto, 2011).

The idea was to have a stand and deliver approach to lab teaching which had to be carried out. However, Ramsden (2003) argues that this instruction style encourages surface learning as the approach in his opinion is narrow and minimalist. Echoing Ramsden, Davies (2008) argues that lab instructors must ponder as to how to inculcate creativity and independence which could be fostered via designing lab questions that makes students understand the uncertainties and inaccuracies of computing experiments from the outset; which we believe is what our 10 steps achieve. In that spirit, the 10 steps that we designed are based on the pedagogy model of Hazel and Baillie (1998), and are obviously arranged based on increasing autonomy of these ten tasks. These activities are based on the classic book of Wilmott et al. (1995). The main reason behind the choice of this book is that, it has pseudo codes on financial computing using PDEs that can be made easily compatible with MATLAB or C++, and are also close in sprit to the classic Numerical recipes book, Press et al. (2007) that we will use at the end.

This laboratory activity resonates with the objective of the Quality Assurance Agency for Higher Education (QAA) and the UK Standard for Professional Engineering Competence (Engineering

Council, 2014) who state that students should have succinct exposure to hands-on laboratory work followed by project work to achieve satisfactory understanding (Davies, 2008). Given that such an objective needs to be covered in a short intensive period of a semester, on an average of three months that also includes teaching the mathematical theory involved, it is imperative to realize it as efficiently as we can. This can be achieved, we believe, by focussing only on the very basic programming tenets needed for implementing numerical pseudo codes as what these 10 steps do. Table 1 describes numbered exercises in conjunction with expected level of autonomy desired.

Table 1: Levels of autonomy for types of laboratory activity corresponding to exercises

Autonomy	Laboratory activity	Aims	Material	Method	Answer	Exercise
0	Demonstration	G	G	G	G	1, 2, 3
1	Test	G	G	G	O	4, 5
2	Structured Enquiry	G	G in w/p	O in w/p	O	6, 7, 8
3	Open Ended Enquiry	G	O	O	O	9
4	Project	O	O	O	O	10

Here G, O, G. in w/p, and O. in w/p stands for Given, Open, Given in whole or part, and Open in whole or part respectively. In the next section we describe a set of MATLAB based exercises. This set of 10 exercises forms our ten steps. In the table above we map the type of laboratory activity to these 'exercises' as the last column depicts.

2. The 10 steps

Exercise 1. Type a vector u with elements u = (1; 2; 3). Check if you can extract the first element i.e. 1, by typing u(0) in the command prompt. Comment. Now, extract its third element.

Exercise 2. Write a function that prompts the user to input two real valued numbers. It returns a sum and a product of these numbers.

Exercise 3. Now write a second function that has a real valued input argument. Call the first i.e. the earlier function in Exercise 2, in this second function. The product value returned by the first function is again multiplied by the new real value supplied to this second new function. Now return the new value of the computed product.

Exercise 4. Write a separate function to compute the Black-Scholes-Merton price. Use the standard command for the same. You can use MATLAB help or just Google to search the inbuilt command. Compare your answer with the one generated by this inbuilt MATLAB function.

Exercise 5. Note the grid drawn in Figure 1.

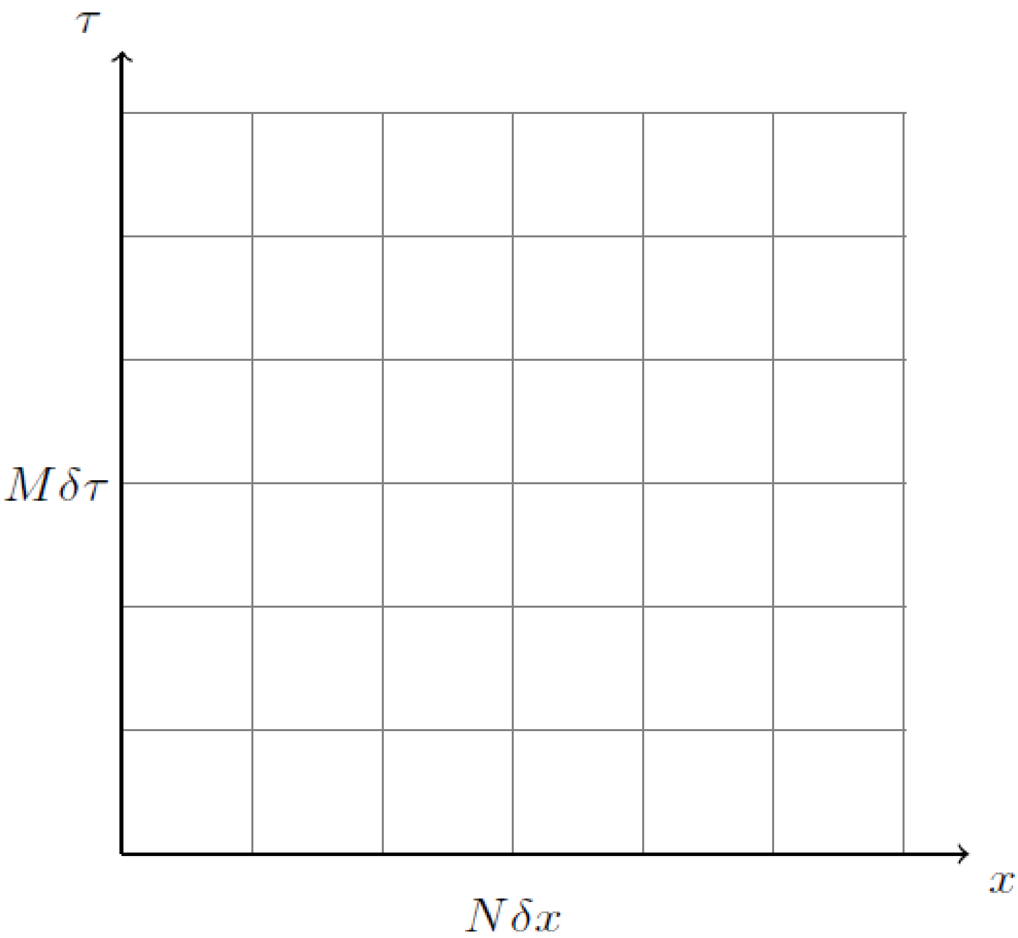

Figure 1: Discretized x – τ grid

In the already available MATLAB code (see Figure 2), replace FILL by appropriate statements that codes the grid defined as below. Run the code.

u=0, at x=0 and x=1 ∀ t belongs to [0,∞) ; u=1-4(x-$\frac{1}{2}$)^2 for t=0 ∀ x ∈ [0,1].

```
function FILL = gridxtau(x,tau,M,N)
deltax=x/N;
deltatau=tau/FILL;
k=0;
% populate u at time 0 and all x i.e. u(.,1)
for n=0:1:FILL,
    k=k+1;
    u(k,FILL)=1-4*(n*FILL-1/2)^2;

end
% populate u at boundary points
for t=0:1:M,
    u(FILL,FILL)=0;
    u(FILL,t+1)=0;
end
```

Figure 2: Editable code provided to students

Exercise 6. In the below MATLAB code of this exercise, for Nminus<0 and Nplus>0 integers, substitute the word FILL so that the code, if typed on the command prompt in MATLAB, will run.

```
FILL
for n=Nminus:1:Nplus,
FILL
    oldu(k)=n;

end
```

Figure 3: Editable code to run in MATLAB

Hence, if you want to further code the following for loop pseudo code following the above MATLAB one, how will you do so?

```
for(n=Nminus+1; n<Nplus;++n)
b[n]=oldu[n]
```

Figure 4: Additional for loop pseudo code

Exercise 7. Use pseudo code Figure 8.5 (Wilmott et al., 1995) for the explicit FDM and compute output u at terminal time for Exercise 5 when the stability factor is in the range of [0,0.5] and otherwise. Comment.

Exercise 8. Use pseudo code Figure 8.11 (Wilmott et al., 1995) for the fully implicit FDM using SOR and compute output u at terminal time for Exercise 5 when the stability factor is in the range of [0,0.5] and otherwise. Comment.

Exercise 9. Use the codes developed in Exercise 7 or 8 to compute a Black Scholes Merton price for European Put.

Exercise 10. Use any pseudo codes in the book on Numerical recipes in C++ (Press et al., 2007) to compute related mathematical idea in MATLAB.

3. Reasoning

We discuss here the reasons behind exercises enumerated in the 10 steps above.

Exercise 1. This tests whether one can type an array of numbers in MATLAB. Also it tests whether the user understands that the first element of any array is accessed with counter 1 and not zero. That is, one should type u(1) in command prompt to get the answer which here is 1 and not u(0). This also helps the same user deal with a similar issue in C++ where the counter, unlike MATLAB, does start with 0. Since further FDM pseudo codes use vector operations instead of matrix, this exercise on vector serves well for later.

Exercise 2. This exercise prompts the user to first write a simple mathematical program in a function format akin to what "*Hello World"* is.

Exercise 3. Next, we ask the user to call a function inside a main function. This is what we will do in the FDM codes, where we write ancillary MATLAB functions that are called into the main function.

Remark A: Note that as per Table 1, Exercise 1, 2 and 3, being fundamental, are demonstration based. Though this pedagogy is primarily being 'stand and deliver', we expect pro-active participation from the students.

Exercise 4. This exercise tests whether students understood the demonstration held for Exercise 2.

Exercise 5. This exercise makes the student understand how to plot the grid (see Figure 1).

Remark B: Exercise 4 and 5 are test based laboratory type, as noted in table 1, since they are 'just' applications of what is covered in the theory class viz. Black Scholes pricing equation and demos of Exercises 1, 2 and 3.

Exercise 6. This exercise summarizes Exercise 1 on vector indices. The code is also useful while answering Exercises 7, 8 and 9. In this exercise they will learn to think as they write loops with arrays.

Remark C: One can heavily utilize MATLABs vectorization facilities instead of loop based vectors. Hence this may lead to eschewing 'for' loops. As argued by Higham (2004) who uses them in actual codes that he provides, the resultant codes are snappier, shorter and less daunting. We rightly believe this to be so. However we still introduce loops as these are often seen in the pseudo codes in Wilmott et al., (1995) and in the classic numerical recipes book of Press et al. (2007). Hence we introduce this exercise on 'for' loops to let the students become comfortable with loops.

Exercise 7. This is our first main FDM code. Students use the pseudo code given in Figure 8.5 of Wilmott et al. (1995). It defines a grid as in Exercise 5 and imports it in the main file that forms Figure 8.5 of the said book.

Exercise 8. This is our second main FDM code. The objective is similar to Exercise 7. They are expected to benefit with better understanding of the SOR method used in the fully implicit FDM.

Remark D: Exercise 6, 7, 8 all come under the pedagogy of what is known as laboratory based 'structured approach' in the sense that, as seen by the wordings of these questions, students are presented with a problem and are suggested some book/reference(s). The output is produced by an individual primarily in the classroom. Group work is encouraged only outside class. This is primarily to foster a deep approach to laboratory learning by coding in MATLAB solely via personal initiative.

Exercise 9. This is our third main FDM code. Student's use the codes developed in Exercise 8 to compute a Black Scholes Merton price for European Put.

Remark E: Exercise 9, expectedly, is a tricky one. Students are aware that analytic options pricing formulas are available in the book of Wilmott et al. (1995). They also have relevant FDM codes for both explicit (see Exercise 7) and fully implicit scheme (see Exercise 8). Using either of these to finally compute the Black-Scholes price is not that easy as they need to carefully synchronise codes in Exercise 7 or 8 with several analytic expressions related to options price. This ensures more decisions and experimental design considerations with students. Hence it is "*open ended enquiry*" as per Hazel and Baillie (1998).

Exercise 10. We next make the student do their first major independent project which uses any pseudo codes from the book in Numerical recipes.

Remark F: Exercise 10, is by nature an independent project. Group activity is encouraged for team building spirit. Students are expected to become comfortable with the Numerical Recipes book that is usually used in the quantitative industry.

4. Findings and conclusion

Being a small class, we found by observation that students were undoubtedly much more connected with numerics since they could visualize the numerical analysis concepts like stability and convergence using a computer. We believe that these ten steps could also be used to teach financial computing even to a typically large cohort of 2nd or 3rd year mathematics students who may only be exposed to smattering understanding of numerical analysis of PDEs. Effectiveness of these steps correlated strongly with the motivation level of the students. With no contributing credit, lack of motivation can be remedied by providing contributing credit to at least the 8th / 9th and 10th exercise. We aim to utilize this study to a large cohort of university undergraduates in financial mathematics and understand and report their response.

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CASE STUDY

Understanding of fraction and its application in unit conversions

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Abstract

Many British university students lack confidence in manipulating fractions. In this paper, we take a detailed look into the difficulties that students are experiencing across disciplines. We also introduce a method on how to effectively convert units via manipulating fraction operations. Though this approach to unit conversions is widespread in the United States, particularly in the discipline of Chemistry, it is not well documented or applied within the UK Higher Education sector to the authors' knowledge. The method has been frequently used by the authors in Coventry's Mathematics Support Centre with very positive feedback.

Keywords: Fraction, Unit conversions, Mathematics support.

1. Introduction

In England fractions are introduced in Key Stage 1 & 2 and students are expected to grasp its four operations by Key Stage 3 (Department for Education). However, many university students still lack proficiency to some extent with fraction manipulations. Out of 40 students with grade A in A-Level Mathematics who took Coventry's diagnostic test in 1998, 17% incorrectly answered basic fraction questions such as $\frac{3}{5}\times\frac{7}{8}$ and $\frac{x}{3}+\frac{y}{2}$ (Lawson, 2003). Some mathematics undergraduates at Coventry created easily avoidable mistakes similar to those presented in Figure 1 while taking the diagnostic test.

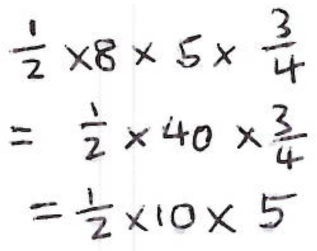


Figure 1: Example of student misconception

Though some students are able to correctly answer such questions, their approach can be inefficient, demonstrating a lack of confidence with this basic skill as well as a reliance on rote methods. Here is how a mathematics undergraduate computed $\frac{3}{5}\times\frac{7}{8}$ in the diagnostic test in 2016.

$$\frac{3}{5}\times\frac{7}{8}=\frac{24}{40}\times\frac{35}{40}=\frac{840}{1600}=\frac{84}{160}=\frac{42}{80}=\frac{21}{40}.$$

Although the working is correct, understanding fractions would have provided a much easier solution.

Many students who enter third level STEM programmes have problems with core mathematical skills (MathTEAM 2003), which includes basic fraction manipulations. Questions involving fractional indices create further difficulties, such as those presented in Figure 2.

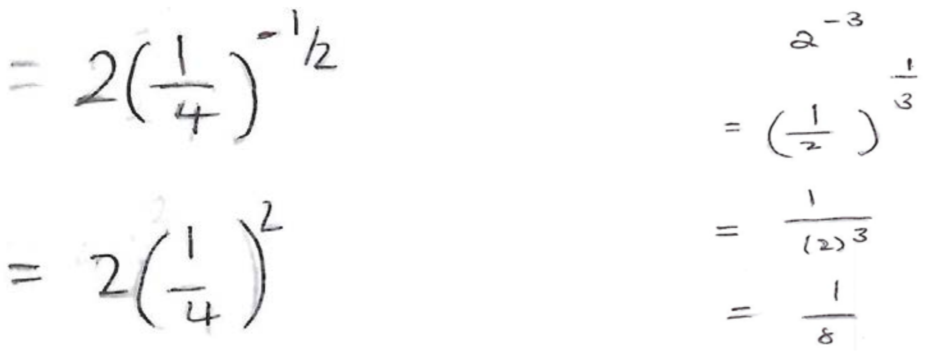


Figure 2: Student calculations involving fractional indices

Sometimes engineering students incorrectly answer calculator questions such as $\frac{20\times 9.81}{2.3\times 1.5\times 1.6}$, because although they interpret fractions as division (correct!) they input $20 \times 9.81 \div 2.3 \times 1.5 \times 1.6$ (this is incorrect), and are often unable to reason why this gives the wrong answer.

2. Typical phenomena

2.1. Addition and subtraction of mixed numbers

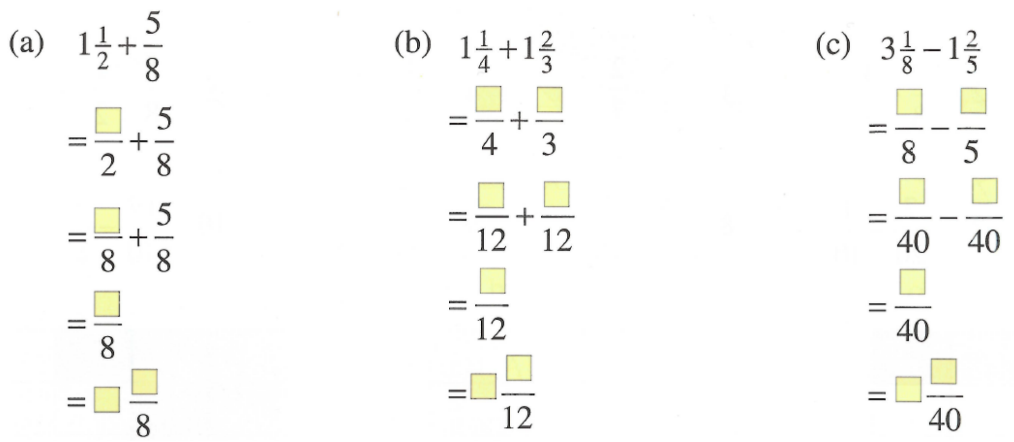


Figure 3: Examples of addition and subtraction of mixed numbers (Rayner and White, 2008)

Figure 3 above is taken from Rayner and White (2008). The idea is to guide students to first convert mixed numbers to improper fractions then add or subtract them as would be done with proper fractions. Many students have no doubts about following this method in all situations. However, splitting a mixed number into the addition of a whole number and a proper fraction creates an easy option; taking part (b) above as an example:

$$1\frac{1}{4} + 1\frac{2}{3} = \left(1 + \frac{1}{4}\right) + \left(1 + \frac{2}{3}\right) = (1 + 1) + \left(\frac{1}{4} + \frac{2}{3}\right) = 2 + \frac{11}{12} = 2\frac{11}{12}$$

Similar procedure can be followed for subtraction. The difference here may be considered negligable, however, such decomposition involves conceptual understanding and is far more effective when working with larger numbers such as $123\frac{1}{5}+247\frac{3}{4}$. Students too dependent on calculators or rote algorithms may fail to appreciate what is actually being done by such operations, missing opportunities for simpler solutions and developing greater understanding. Encouraging students to be explicit in their working whilst learning will help ensure that they really understand the concepts thus gives them greater confidence using such methods in the future.

2.2. Multiplication and division

Table 1 below represents some typical methods for computing multiplication and division of fractions.

Table 2: Methods for multiplication and division of fractions

Multiplication	Multiply the numerators	Multiply the denominators	Reduce if necessary
	$\frac{2}{5}\times\frac{3}{4}=\frac{6}{\ }$	$\frac{2}{5}\times\frac{3}{4}=\frac{6}{20}$	$\frac{6}{20}=\frac{3}{10}$
Division	Invert the fraction	Multiply terms	Reduce
	$\frac{2}{3}\div\frac{4}{5}=\frac{2}{3}\times\frac{5}{4}$	$\frac{2}{3}\times\frac{5}{4}=\frac{10}{12}$	$\frac{10}{12}=\frac{5}{6}$

Although these procedures will give the correct results, larger numerators and/or denominators become cumbersome to work with and are more prone to error. Whenever possible, cancellation (division) should be done first (before multiplication). For example,

$$\frac{\cancel{2}}{5}\times\frac{3}{\cancel{4}2}=\frac{3}{2\times 5}=\frac{3}{10}.$$

Such cancellation is also very effective when applied to unit conversions.

Regarding fractional division, students generally know that they need to invert the divisor fraction then multiply without comprehending why. Fractions are equivalent to divisions and students should be able to fluently interchange between them. The following might facilitate students' understanding:

$$\frac{a}{b}\div\frac{c}{d}=\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}=\frac{\frac{a}{b}\times\frac{d}{c}}{\frac{c}{d}\times\frac{d}{c}}=\frac{\frac{a}{b}\times\frac{d}{c}}{1}=\frac{ad}{bc}.$$

Reasons:

- The first equality: division is the same as a fraction of the terms;
- The second equality: we do not want to keep a fraction as the denominator so we multiply it by its reciprocal $\frac{d}{c}$. To keep the fractions value unchanged, we should multiply the numerator by the same value.

Or we could apply the four operations of integers:

$$\frac{a}{b} \div \frac{c}{d} = (a \div b) \div (c \div d) = (a \div b) \div c \times d = \ a \div b \div c \times d = (ad) \div (bc) = \frac{ad}{bc}.$$

When being addressed in an algebraic context, this becomes more problematic if they lack conceptual understanding. A typical example is the following common mistake:

$$\frac{7x^2}{14x^2 + 21x} = \frac{7x^2}{14x^2} + \frac{7x^2}{21x} = \frac{1}{2} + \frac{x}{3}$$

The working out is wrong but can be hard for students to discover, especially when it happens as part of a larger problem. This question should be addressed as:

$$\frac{7x^2}{14x^2 + 21x} = \frac{7x^2}{7x(2x + 3)} = \frac{x}{2x + 3}$$

2.3. Complex numbers

The following question is taken from the engineering maths book (Singh 2003):

Express $\frac{3+j4}{j5}$ in the form of $a + jb$ where a and b are real.

(Some worked solution is provided via the online answers accompanying the Singh (2003) text at https://he.palgrave.com/resources/CW%20resources%20(by%20Author)/S/Singh/worked-solutions/Solutions_10a.pdf.)

We need to find the complex conjugate of $j5 = 0 + j5$, which is $0 - \ j5 = \ -j5$. Hence,

$$\frac{3 + j4}{j5} = \frac{-j5(3 + j4)}{5^2} = \frac{20 - j15}{25} = 0.8 - j0.6.$$

This is also a typical method for both mathematics and engineering undergraduates. However, if they could freely manipulate fractions, they could achieve the goal in an easier way:

$$\frac{3 + j4}{j5} = \frac{1}{5} \times \frac{3 + j4}{j} = \frac{1}{5} \times \frac{(3 + j4) \times j}{j \times j} = - \ \frac{-4 + j3}{5} = 0.8 - j0.6.$$

Note that we have multiplied the numerator and denominator with j. We can also use its conjugate $(-j)$, which would not make much difference.

2.4. Simplification of closed-loop transfer function

Students in Control/Mechanical Engineering at Coventry are expected to work out the closed-loop transfer function if given the open-loop transfer function. They are familiar with diagrams similar to those presented in Figure 4, and the formula $\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$.

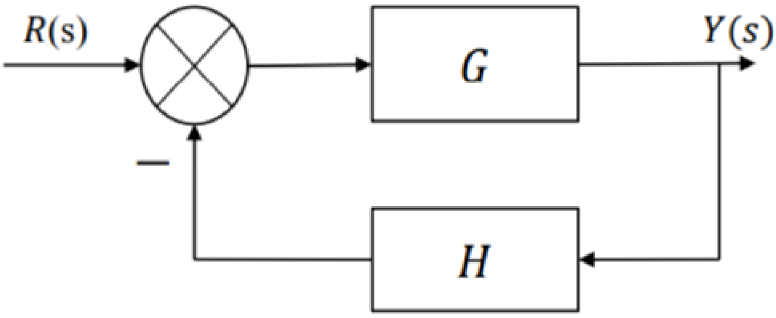


Figure 4: Closed-loop block diagram

Derivation of the formula is typically unproblematic. However, while dealing with concrete examples, many find it hard to simplify the closed-loop transfer function due to its complexity involving fractions. For example, an open-loop unstable system is to be stabilised by a feedback control with the control gain K_p in the feedback as shown in Figure 5.

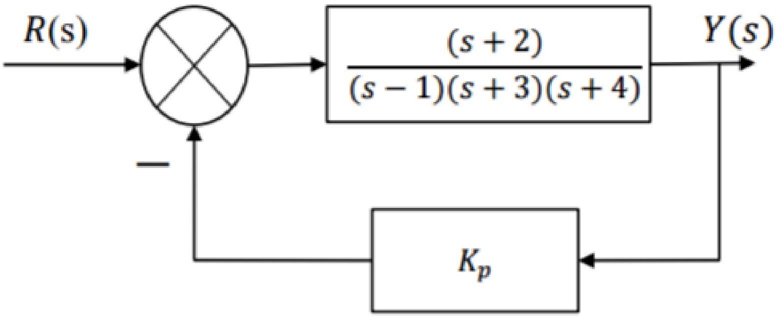


Figure 5: Diagram for closed-loop transfer function example

Obtain an expression for the overall closed-loop transfer function in its simplest form.

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{\dfrac{s+2}{(s-1)(s+3)(s+4)}}{1 + \dfrac{s+2}{(s-1)(s+3)(s+4)}.K_p}$$

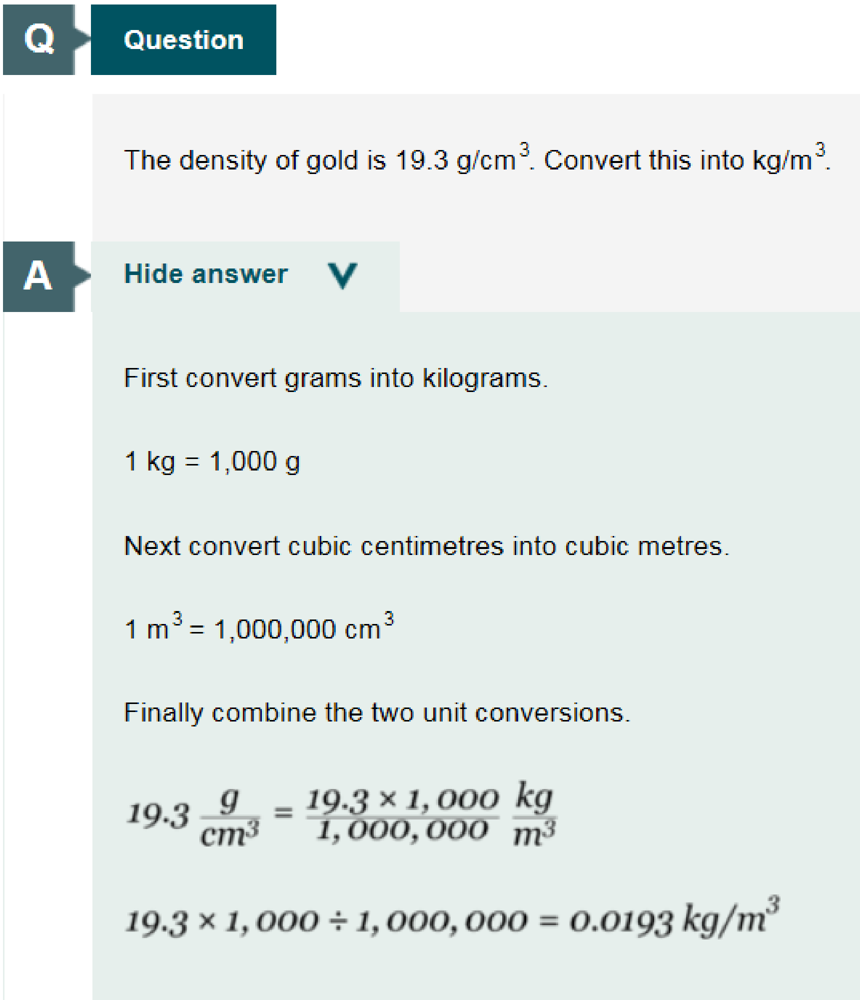
$$= \frac{s+2}{(s-1)(s+3)(s+4) + K_p(s+2)}$$

$$= \frac{(s+2)}{s^3 + 6s^2 + (5 + K_p)s + (2K_p - 12)}$$

Many students find the goal unachievable.

3. Unit Conversions (Dimensional Analysis)

Mistakes in related areas are everywhere. Figure 6 illustrates an example extracted from the BBC Bitesize website http://www.bbc.co.uk/education/guides/z8b9d2p/revision/10.



Q Question

The density of gold is 19.3 g/cm^3. Convert this into kg/m^3.

A Hide answer

First convert grams into kilograms.

1 kg = 1,000 g

Next convert cubic centimetres into cubic metres.

1 m^3 = 1,000,000 cm^3

Finally combine the two unit conversions.

$$19.3\,\frac{g}{cm^3} = \frac{19.3 \times 1,000}{1,000,000}\,\frac{kg}{m^3}$$

$$19.3 \times 1,000 \div 1,000,000 = 0.0193\ kg/m^3$$

Figure 6: Conversion example from BBC Bitesize website

Similar mistakes happen at British universities across the range of science disciplines.

Cancellation can be very effective when being applied to unit conversions, which play a very important role in Sciences and Engineering courses. There is a multitude of units for measuring most quantities and students are often required to convert from one to another. Students are frequently unsure whether they should multiply or divide some numbers. Dimensional Analysis, also called Factor-Label Method or the Unit Factor Method, is a problem-solving method that uses the fact that any number or expression can be multiplied by one without changing its value (Texas A&M University, n.d.; Rice University, n.d.). It is a useful technique and may be widespread in the United States, particularly in the discipline of Chemistry. The idea is to interpret numbers with units, e.g. 16.2 meters or $32ft/sec^2$, exactly the same as coefficients with variables. Consider a simple example of changing from m to cm, we need to decide which ratio equal to '1' we should multiply m by to get cm. If we use the fraction with cm on top and m on bottom, m will 'cancel out' when we simplify as follows

$$4m = 4\cancel{m} \times \frac{100cm}{1\cancel{m}} = 400cm$$

When converting units student should bear in mind the following: anything can be multiplied by '1' — a carefully chosen form of '1' such as $\frac{100cm}{1m}$, without altering its value. This can be applied to all unit-conversion questions, regardless of the units' complexity. We will discuss this further with examples from different backgrounds. However, the authors have not seen anything like this here in the UK.

3.1. Nursing numeracy example

All universities and some nursing jobs require students to pass a nursing numeracy test in order to gain a place on a nursing course, or to be offered a job. This numeracy test normally consists of some drug calculation questions, which could be treated as unit conversion.

Question: How much penicillin mixture should be given for a dose of $500mg$ if the mixture comes as $250mg$ per $5ml$? (sigma Coventry University, n.d.)

Students are advised to use the formula given in Figure 5:

$$\frac{\text{What you want}}{\text{what you have}} \times \frac{\text{the stock level}}{1}$$

Figure 5: Drug calculation formula

Solution:

- What you want is $500mg$
- What you have is $250mg$
- The units are the same
- Stock volume is $5ml$
- Volume to be given = $\frac{500}{250} \times 5 = 10ml$.

Such method is frequently recommended at universities across the UK (University of Leeds, n.d.). However, many nursing students find it hard to remember formulae and they can get confused when required to place numbers in the correct places. If we apply the special '1' method (Straight A Nursing, 2017), the approach here becomes: $5ml = 250mg$, we can use $'1' = \frac{5ml}{250mg}$ because we need to cancel out 'mg', consequently

$$500mg = \cancel{500mg}2 \times \frac{5ml}{\cancel{250mg}} = 10ml.$$

3.2. Bioscience example

Each year at Coventry about 200 students are enrolled into Bioscience courses and they are required to do a module on Quantitative Skills, where many questions are based around different unit conversions. Students can use the same technique and multiply by a well-chosen fraction that equals '1', only they need to do it for each unit to be converted. For example:

Question: What is the molar concentration (in mM) of a 0.4% (w/v) $NaOH$ solution? (The molar mass of $NaOH$ is $40g/mol$)

Solution: To achieve our goal we need to perform a series of unit conversions: $(g/ml \rightarrow g/l \rightarrow mmol/l \rightarrow mM)$

$$0.4\% = \frac{0.4g}{100ml} = \frac{0.4g}{\cancel{100ml}} \times \frac{\cancel{1000ml}10}{1l} = \frac{4g}{1l} = \frac{\cancel{4g}}{1l} \times \frac{1mol}{\cancel{40g}10}$$

$$= \frac{0.1mol}{1l} = \frac{0.1\cancel{mol}}{1l} \times \frac{1000mmol}{1\cancel{mol}} = 100mmol/l = 100mM$$

3.3. Engineering example

Last year the Mathematics Support Centre at Coventry had over 11,800 student visits with the majority (74%) coming from the faculty of Engineering, Environment and Computing. Many engineering students struggle with unit conversions. Some questions are similar to:

Rank the following values of stress in INCREASING order of magnitude

- $5\ kN/mm^2$
- $6000N/mm^2$
- $200\ kN/m^2$
- $80000N/m^2$

All the quantities are given in different units, we need to convert them into the same one, for example kN/mm^2, the others should then become

$$6000N/mm^2 = \frac{6000N}{1mm^2} = \frac{\cancel{6000N}6}{1mm^2} \times \frac{1kN}{\cancel{1000N}} = \frac{6kN}{1mm^2} = 6kN/mm^2$$

$$200kN/m^2 = \frac{200kN}{1m^2} = \frac{200kN}{\cancel{1m^2}} \times \frac{\cancel{1m^2}}{(10^3mm)^2} = \frac{200kN}{10^6mm^2} = 2 \times 10^{-4}kN/mm^2$$

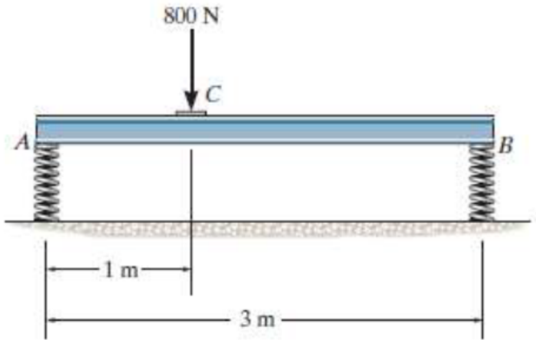
$$80000N/m^2 = \frac{80000N}{1m^2} = \frac{\cancel{80000N}80}{1m^2} \times \frac{1kN}{\cancel{1000N}} = \frac{80kN}{1m^2}$$
$$= \frac{80kN}{\cancel{1m^2}} \times \frac{\cancel{1m^2}}{(10^3mm)^2} = \frac{80kN}{10^6mm^2} = 8 \times 10^{-5}kN/mm^2$$

As long as we have the same unit, we can easily order the four quantities in increasing order as $80000N/m^2$, $200\ kN/m^2$, $5\ kN/mm^2$, $6000N/mm^2$.

4. Accuracy

Another advantage of helping students become more comfortable using fractions is to improve the accuracy of their calculations. At Coventry, around 300 students per year join the Mechanical Engineering course, the following question (Figure 6) comes from their first-year module:

The horizontal beam is supported by springs at its ends. Each spring has a stiffness of k=5 kN/m and is originally unstretched so that the beam is in the horizontal position. Determine the angle of tilt of the beam if a load of 800 N is applied at point *C* as shown.



Answers
a) 1.02°

Figure 6: Mechanical Engineering example from Coventry University

Solution: We label the spring forces F_A and F_B at points at A and B respectively. To work out F_A, we take B as pivot so we have the moment equation

$$F_A \times 3 = 800 \times (3-1)$$

Consequently,

$$F_A = \frac{800 \times (3-1)}{3} = \frac{1600}{3} \text{ N}$$

Similarly we have $F_B = \frac{800\times1}{3} = \frac{800}{3}$ N.

If we label the compressed distance X_A and X_B at points at A and B respectively, then apply Hooke's law, we have $F_A = k\,x_A$ and $F_B = k\,x_B$

$$x_A = \frac{F_A}{k} = \frac{\frac{1600}{3}\text{ N}}{5\frac{\text{kN}}{\text{m}}} = \frac{1600}{3}\text{ N} \div \frac{5\text{kN}}{1\text{m}}$$

$$= \frac{1600}{3}\text{ N} \times \frac{1\text{m}}{5\text{kN}} = \frac{1600}{3}\,\cancel{\text{N}} \times \frac{1\text{m}}{5\,\cancel{\text{kN}}} \times \frac{1\cancel{\text{kN}}}{1000\text{ N}} = \frac{8}{75}\text{m}$$

And similarly $x_B = \frac{4}{75}$m.

Thus the angle is $\Theta = \tan^{-1}\left(\frac{\frac{8}{75}-\frac{4}{75}}{3}\right) = \tan^{-1}\left(\frac{4}{3\times75}\right) = 1.02°$.

Some students do not consider fractions as numbers so they work their results with decimals. If they keep three decimal places in their working, they would have had

$$F_A = 533.333 \text{ N} \text{ and } F_B = 266.667 \text{ N}$$

$$x_A = 0.107 \text{ m and } x_B = 0.053 \text{ m}$$

As a result,

$$\tan(\theta) = \frac{0.107 - 0.053}{3} = 0.018$$

$$\theta = \tan^{-1}(0.018) = 1.031°$$

They then doubt the correctness of their working out. Greater confidence with fractions would enable students to employ them throughout the problem, increasing the accuracy of their results.

5. Conclusion

Many students still struggle with fractions at university level. They take basic operations on fraction as procedural rather than conceptual. If students could gain a deeper understanding of fractions they could answer subject-related questions much more confidently and efficiently. Confidence in handling fractions opens up further methods to the student such as the described approach to unit conversion, as well as improving their overall accuracy by utilising them throughout the problem solving process. The stated method of unit conversion has been repeatedly used by the authors in the Mathematics Support Centre at Coventry, which has been proven to be very successful. In spite of students' backgrounds, they find the method easy to understand and accept. Consequently students know what they should do so have largely boosted their confidence in applying mathematics.

6. Acknowledgements

The authors are extremely grateful to the anonymous referee for the useful comments and references on the manuscript.

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CASE STUDY

Game information sheets: a student-produced resource to help you run a Maths Arcade

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Abstract

The Maths Arcade is an extracurricular activity to play strategy games and puzzles, aiming to provide an informal student support environment, improve staff-student interaction and develop mathematical thinking. Running a Maths Arcade presents some need for information about the games. Three projects have hired students to create game information sheets to support using the games with new players and to encourage development of strategic and logical thinking.

Keywords: Maths Arcade, Students as partners, mathematical thinking, games, student experience.

1. The Maths Arcade

The Maths Arcade is an activity operated at several U.K. universities which aims to improve the student experience by providing informal student support and an opportunity to develop mathematical thinking. It is a regular, optional drop-in session to play strategy games and puzzles. The first Maths Arcade was developed by Bradshaw (2011) at the University of Greenwich in 2010. This was expanded to eight universities in 2012 as part of the National HE STEM Programme (Bradshaw and Rowlett, 2012), with other university maths departments starting their own Maths Arcades since. The Maths Arcade has also been used in other disciplines and in schools. A Maths Arcade can be started with a fairly small grant, for example Webster and Rowlett (2013) started one with a grant of £400 from the Institute of Mathematics and its Applications which was used to buy games that are commercially available.

The games are chosen to have simple rules that appeal to strategic and logical thinking. This provides a supportive environment for students to engage with each other and staff outside of the formal curriculum on a discipline-adjacent activity. That is, the activity uses logical and strategic thinking without being reliant on prior mathematical knowledge. This makes it ideal for induction and transition activities because students with different mathematical backgrounds are able to engage equally. Those who attend can be encouraged to think about the strategy behind the gameplay and this provides an opportunity to develop mathematical thinking.

Croft and Grove (2015) highlight the Maths Arcade as "*encouraging more opportunities for staff/student interactions*" (p. 181). At Sheffield Hallam University, the Mathematics group recently moved to a new learning space "*designed around the principle of co-location, with an open learning space surrounded by staff offices to encourage informal contact between staff and students*" (Waldock et al., 2017; p. 589). This has had some success, including indications that students feel like part of a "*mathematics community*" (p. 600). Students are positive about the integration of the Maths Arcade into this new space, with positive feedback and increased popularity of the activity reported by Cornock (2015).

Carpenter (2011), giving a student view, wrote about how attending the University of Greenwich Maths Arcade was "*of enormous benefit to [him] both on an academic and social front*" and how all maths students also benefited from "*fun, enjoyment, queries, banter and a light hearted approach to all aspects of mathematics*" (p. 30). An evaluation of the Maths Arcade at five U.K. universities found that students who attended made friends, valued staff attendance and, though numbers of responses were small, “*students who attend more frequently are drawn to the more challenging two-player games*”. As such games are more open to analysis (via combinatorial game theory), this may indicate that regular attenders develop “*from simply playing games towards analysis of strategy*” (Rowlett, et al., in press).

2. Game information sheets

One issue with running a Maths Arcade is information about the games. In an induction or outreach situation, there might be many participants new to the games all trying to play at once, making it impractical to personally teach everyone how to play a game. Experience shows that the instructions that come with some games are quite impenetrable. This indicates a need for games to have a simple, clear set of instructions.

Another issue is trying to encourage deeper strategic and mathematical thinking when staff may not have investigated each game themselves, particularly when one is trying to encourage a wide range of staff to take part in a staff-student community activity. This indicates a need to develop prompts to act as “*starting points*”, “*introductions to potentially rich mathematical explorations*” (Schoenfeld, 1994; p. 45), to encourage players to move beyond simply playing the games to analysis and strategic thinking.

In 2013/14, James Hind and I obtained funding from our institution (Nottingham Trent University) to hire an undergraduate student, Kingsley Webster, to work on producing game information sheets. These have two aims:

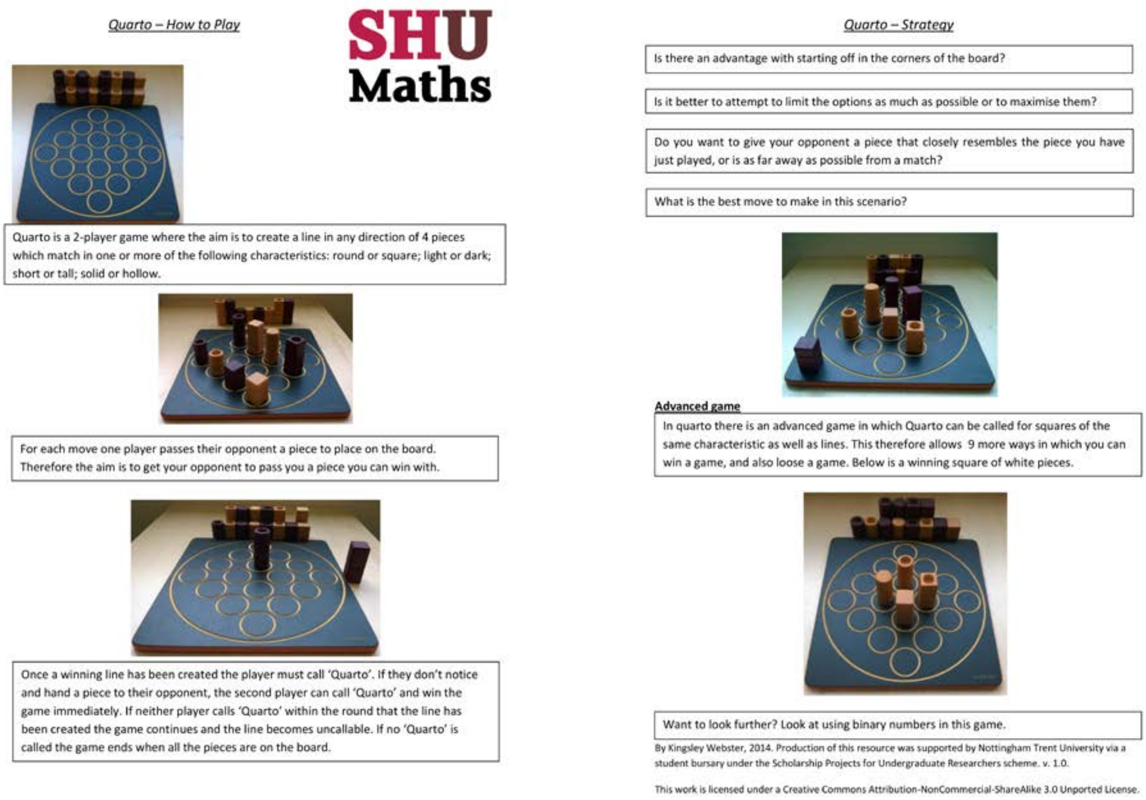
1. to provide a clear description of how to play each game, to make the games easier for new players to pick up, including in busy environments such as induction, open days and outreach activities;
2. to provide prompts to encourage players to think about strategy and logic behind the games.

Sheets are designed around a two-page layout, with instructions including instructive photos on the front and strategy hints on the back (see Figure 1). The strategy hints are designed as a series of prompting questions appropriate for the game, such as “*Is there an advantage to going first?*”, “*Is it easier to win if you keep your pieces close together or separated across the board?*” and “*Is it better to remove lots of stones quickly early on in the game?*”. Most sheets also include a photo of some game scenario with a question asking what would be a good move to make in that circumstance. These are designed to encourage discussion around interesting points of strategy.

This first project produced information sheets for the 15 games in the Nottingham Trent Maths Arcade. In 2015, I moved to Sheffield Hallam University where the Maths Arcade has a greater selection of games, not all covered by the existing sheets. Funding from Sheffield Hallam in 2015/16 allowed me and Claire Cornock to hire undergraduates Lisa Eccelston and Peter Tonks to produce 12 missing sheets and also investigate suitable games, recommending new games to buy. With further funding from Sheffield Hallam in 2016/17, Claire and I hired undergraduates Daniel Arnold and Joseph Sugrue to make game information sheets for the remaining 9 games.

All game sheets developed are being made available by the Institute of Mathematics and its Applications website[i]. These are made available under a Creative Commons Attribution-Non-commercial-ShareAlike license, meaning they can be edited for non-commercial use under the same license provided the original author is attributed. For example, the version of the Quarto game sheet in Figure 1 has been edited to include the Sheffield Hallam logo.

I encourage you to consider running a Maths Arcade and making use of the game information sheets provided to do so. Advice on setting up a Maths Arcade, including a list of appropriate games, is given in the booklet edited by Bradshaw and Rowlett (2012) and in the Maths Arcade pages on the IMA website[1].


Quarto – How to Play

SHU Maths

Quarto is a 2-player game where the aim is to create a line in any direction of 4 pieces which match in one or more of the following characteristics: round or square; light or dark; short or tall; solid or hollow.

For each move one player passes their opponent a piece to place on the board. Therefore the aim is to get your opponent to pass you a piece you can win with.

Once a winning line has been created the player must call 'Quarto'. If they don't notice and hand a piece to their opponent, the second player can call 'Quarto' and win the game immediately. If neither player calls 'Quarto' within the round that the line has been created the game continues and the line becomes uncallable. If no 'Quarto' is called the game ends when all the pieces are on the board.

Quarto – Strategy

Is there an advantage with starting off in the corners of the board?

Is it better to attempt to limit the options as much as possible or to maximise them?

Do you want to give your opponent a piece that closely resembles the piece you have just played, or is as far away as possible from a match?

What is the best move to make in this scenario?

Advanced game

In quarto there is an advanced game in which Quarto can be called for squares of the same characteristic as well as lines. This therefore allows 9 more ways in which you can win a game, and also loose a game. Below is a winning square of white pieces.

Want to look further? Look at using binary numbers in this game.

By Kingsley Webster, 2014. Production of this resource was supported by Nottingham Trent University via a student bursary under the Scholarship Projects for Undergraduate Researchers scheme. v. 1.0.

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Figure 1. Two-page game information sheet for Quarto, edited to include SHU logo.

3. Acknowledgements

The work described in this article to hire students to produce game information sheets was funded by Nottingham Trent University via its Scholarship Projects for Undergraduate Researchers scheme in 2013/14 and Sheffield Hallam University via two Teaching Enhancement grants in 2015/16 and 2016/17. The Maths Arcade at Sheffield Hallam University was initially funded by the National HE STEM Programme and further developed by Sheffield Hallam University. The Maths Arcade at Nottingham Trent University was funded by the Institute of Mathematics and its Applications. Webspace to share the game information sheets is provided by the Institute of Mathematics and its Applications. I am grateful to my collaborators on these projects, James Hind and Claire Cornock, and to the students who completed the work, Kingsley Webster, Lisa Eccleston, Peter Tonks, Daniel Arnold and Joseph Sugrue.

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[i] https://ima.org.uk/865/maths-arcade-selected-games-and-resources/ or search at ima.org.uk for 'Maths Arcade'.