MSOR Connections

Articles, case studies and opinion pieces relating to innovative learning, teaching, assessment and support in Mathematics, Statistics and Operational Research in higher education.

Volume 21 No. 1.

Special Issue: CETL-MSOR Conference 2022 Part 1

This page left intentionally blank

Contents

For information about how to submit an article, notifications of new issues and further information relating to *MSOR Connections*, please visit https://journals.gre.ac.uk/index.php/msor.

This journal is published with the support of the **sigma** network and the Greenwich Maths Centre.

sigma Σ
network for excellence in mathematics and statistics support

UNIVERSITY of GREENWICH

EDITORIAL

Anne Savage, School of Applied Sciences, Abertay University. Email a.savage@abertay.ac.uk
Alun Owen, **sigma** Mathematics and Statistics Support Centre, Coventry University. Email aa5845@coventry.ac.uk

In September 2022, over one hundred delegates from across the globe gathered at Abertay University in Dundee, Scotland for the CETL-MSOR 2022 conference. We were delighted to welcome delegates in person and the conference themes reflected the advent of innovative technologies in teaching and learning, widening access and our recent emergence from the restrictions imposed by the covid 19 pandemic. In March 2020, the Higher Education sector was faced with the task of changing overnight, from traditional in-person teaching to delivering degree level programmes remotely without compromising access or quality. Online delivery of both teaching and assessment became the norm and while this initiated a rich period of innovation and creativity for the sector, issues such as digital poverty and metal health became more apparent. Since the end of the pandemic, we have had the pleasure of seeing our campuses full of students once more but whether our institutions have adopted 'new normals' such as Blended Learning or returned to in-person teaching, it cannot be denied that higher education teaching has changed.

In this edition of MSOR Connections, we present Part 1 of a collection of ideas, issues, solutions and opinions on the teaching, support and assessment of mathematics and statistics, that were presented at CETL-MSOR 2022. Part 2 of this special edition will hopefully be published by the end of March 2023.

Assessment features strongly in this edition. The impact of assessment methods as a barrier to learning is discussed by Mann, whilst issues arising from the use of non-invigilated online examinations are shared by Walker. The use of smartphone quizzes in the classroom is then described by Berrington et al., whilst Fairfax reports on their experiences of authentic assessments to enhance student engagement, and the use of Numbas for automated assessments of coding in R and Python is reviewed by Graham et al. Also reported are the results of research by Sikurajapathi et al. on the need to identify common student errors (CSEs) to improve mathematical e-assessments, and an approach to allocating students' contributions to group work by Shaw. Other articles include an examination by Gratwick and O'Hagan on the use of STACK workbooks to teach complex analysis, whilst Russell shares their experiences of adapting online activities to create an in-person flipped approach in the classroom. Finally, given, the additional problems created during the pandemic for students already facing other barriers to learning, we close this edition with a timely workshop report by Hand et al. on the sigma Accessibility Special Interest Group. These clearly illustrate our sector's flexibility and effectiveness in addressing its greatest challenge in decades.

MSOR Connections can only function if the community it serves continues to provide content, so we strongly encourage you to consider writing case studies about your practice, accounts of your research into teaching, learning, assessment and support, and your opinions on issues you face in your work. We welcome submissions to the journal at any time.

Another important way readers can help with the functioning of the journal is by volunteering as a peer-reviewer. When you register with the journal website, there is an option to tick to register as a reviewer. It is very helpful if you write something in the 'reviewing interests' box, so that when we are selecting reviewers for a paper, we can know what sorts of articles you feel comfortable reviewing. To submit an article or register as a reviewer, just go to http://journals.gre.ac.uk/ and look for *MSOR Connections*.

This page left intentionally blank

OPINION

Assessment as a barrier to inclusion

Tony Mann, School of Computing and Mathematical Sciences, University of Greenwich, London, UK. Email: A.Mann@gre.ac.uk

Abstract

I argue that the methods used to assess mathematics in higher education too often prevent people from achieving their potential and have the (presumably unintended) consequence that the diversity of the mathematical community is reduced as a result.

Keywords: Assessment, inclusion, academic integrity

1. Introduction

This paper presents my personal views. It has been prompted by two relatively recent developments, which I feel should lead the higher education mathematics community to reconsider some of its assessment policies in order to provide more meaningful assessments and promote inclusion and diversity.

In the first part of the pandemic universities had to move teaching and assessment online, at short notice. This enforced innovation will have long-term consequences as we use the experience to improve the way we teach and the learning resources we provide. Initiatives such as TALMO (2020), conferences such as the 2021 CETL-MSOR conference (**sigma**, 2021) and journal special issues such as those of *MSOR Connections* (Hodds and Rowlett, 2022a and 2022b) helped disseminate the resulting good practice.

The switch to online examinations at the end of the 2019/20 academic year coincided with the biggest recorded drop in the white-Black, Asian and minority ethnic awarding gap (the difference in proportions of white and Black, Asian and minority ethnic students awarded a first/2:1 degree), between 2018/19 and 2019/20 (Codiroli Mcmaster, 2021). Despite this apparent success (to which other factors may have contributed) and the concern over the awarding gap expressed both by regulators and by many HE institutions, to my disappointment many mathematics departments in HE seem keen to revert to traditional examinations for mathematics degrees.

The second development was the tragic case of Natasha Abrahart, a student at Bristol University who committed suicide in 2018 on the day when she was expected to participate in a presentation as part of her course (The County Court at Bristol, 2022. The court judgment shows how university assessments can, despite the best intentions of all those involved, cause considerable stress for some students: it is revealing to discover, from the judge's analysis, how higher education activities appear to someone outside the system.

2. Assessment in Mathematical Sciences

While other disciplines have broadened their assessment diets, Iannone and Simpson (2022) have shown that, for mathematics, universities in the UK continue to rely heavily on traditional closed-book examinations (the very slight decrease over the last decade being attributable to an increase in the use of modules with a less mathematical focus rather than a broadening of assessment in mathematics modules).

This contrasts with the working practices of today's mathematicians, at least in my experience. Whether in industry or academia: mathematicians do not work under exam conditions without access to digital and other resources, generally work collaboratively, and are rarely subject to the unrealistic time pressures imposed by traditional examinations. It is also arguable that the skills tested by these examinations are not very relevant to working mathematicians: for example the keynote talk at the 2021 CETL-MSOR conference by Neil Sheldon (a former Vice-President of the Royal Statistical Society (RSS)) discussed how exams in statistics tend to focus on mathematical calculations rather than statistical understanding, and a very distinguished applied mathematician told an Institute of Mathematics and its Applications (IMA) meeting in February 2022 how the assessment regime he experienced as an undergraduate failed to prepare him for research in mathematics (my undergraduate experience was similar).

Some university mathematicians argue that traditional examinations preserve "academic integrity", suggesting that other forms of assessment are exploited by students who cheat. (I suspect that this view is based in part on an underestimate of the potential for cheating in examinations, especially as technology develops). I would have thought that "academic integrity" would be better served by assessment which meaningfully engages with the professional practice for which universities are preparing their students, rather than the very artificial examinations which, as I will suggest below, do not accurately measure the mathematical ability of many undergraduates.

The IMA, the London Mathematical Society and the RSS have issued a statement about assessment in mathematical sciences (IMA, 2022), which is endorsed by the Edinburgh Mathematical Society and the Heads of Departments of Mathematical Sciences (HoDoMS). This statement suggests that some mathematics departments are under pressure from their institutions to reduce their use of traditional invigilated closed-book examinations.

Assessment is also discussed in the Subject Benchmark Statement for Mathematics, Statistics and Operational Research published by the Quality Assurance Agency (QAA) (QAA, 2019): at the time of writing a revised subject benchmark statement is in preparation to be released in 2023. The 2019 subject benchmark statement indicates that a range of assessment methods is appropriate.

At a time when, for many reasons, assessment methods in mathematics are under discussion, their impact on inclusion should be one of the factors considered.

3. Issues with assessment types

An inclusive community welcomes people from all backgrounds. Bradshaw and Mann (2021) identify obstacles which might affect students' sense of "belonging" as mathematicians. The assessment students' face during mathematics degrees is potentially a barrier.

Each form of assessment in mathematical sciences has strengths and weaknesses. The following discussion can only indicate some of the issues which affect the inclusive nature of our subject. My own experience, as a privileged straight white cis male from an academic family, provides the context for my discussion, which draws heavily on my more than 30 years' experience of teaching mathematics in higher education.

As a student I was very good at traditional examinations – my school had prepared me very well in exam technique – and I enjoyed and looked forward to my university exams. (In fact, I was so good at school exams that I got the highest mark in my year for French, despite not being able to make myself understood in the language or to understand spoken French, which says something about the value of exam marks as an indication of ability!) Nevertheless, at university I lost over a stone in weight every year in the run-up to the examination period, and during my final exams my doctor put me on Valium because I was experiencing chest pains due to nerves. (It didn't occur to me at the time that this might not be good for me!) Students who suffer from mental health difficulties, or physical illness or disability, may struggle in exams in ways for which arbitrary allowances of extra time may not fully compensate. Time of day, time of the month (for half the population), and time of year (for example for those suffering from hay fever or taking medication for it) may affect exam performance.

Of course, from a student's perspective, which topics in an exam come up is a matter of chance. (In my father's university scholarship exam, the unseen translation question happened to be the passage he had tried as a practice the day before.) But this luck doesn't affect everyone equally – some students are more adept than others at question-spotting or picking up hints dropped by their tutors, and students' background is a big factor in that skill.

I did well at exams because I was well prepared in examination technique from an early age, by family and teachers, and was trained from my youth to revise and focus on exams. Not all students have my privileged background, and during my teaching career I have seen many able students whose exam performance did not reflect their mathematical competence. When Nobel laureate Sir Roger Penrose has talked about his slowness in doing mathematics exams (Fry and Penrose, 2018), it seems to me hard to argue that time-constrained exams are a good way to assess mathematical ability.

Some students suffer from exam nerves, and some may be unable to focus on exam preparation because of family or caring commitments. A single parent who has to take children to school on the way to the exam, or who has to worry about leaving the exam room promptly to collect their children, will not be able to focus on the exam as I could. A student caring for an elderly dependent won't be able to revise as single-mindedly as I used to. Students who rely on part-time work to support their studies may not be able to spend the time preparing for their exams as their more privileged peers do.

So, in a number of ways traditional examinations may disadvantage some students, particularly those from less privileged backgrounds.

Of course, other forms of assessment can also present barriers to inclusion. For example:

- Presentations may be terrifying for some students and will potentially be more difficult for students for whom the language of presentation is not their first language. They can be difficult for some students for reasons relating to neurodiversity or mental health.
- Writing reports can feel intimidating for students not confident in their language skills.
- Group assignments might be problematic for students whose work, caring commitments or health makes it difficult to attend meetings with other group members. Working with others may be intrinsically difficult again, because of neurodiversity or mental health factors.
- Coursework deadlines create stress and are difficult to manage for many students.

The fact that such assessments are valued by employers and develop skills that will benefit students in their careers, while a strong reason for including such elements as part of the assessment diet, should not cause us to overlook the barrier they may pose for some of our students, not affecting everyone equally, and thus reducing the potential diversity of the future mathematics community.

4. Mitigations

So, how can we design our mathematics assessments to encourage inclusion in our courses? I don't claim to have any answers but I tentatively offer the following suggestions.

- In designing assessment strategies the impact on inclusion should be considered.
- A variety of different assessment methods will to some extent mitigate the drawbacks of each assessment type.
- Explaining the rationale for each chosen form of assessment to students might be beneficial. For example, helping students understand the relevance of presentations or groupwork for their future careers might motivate an anxious student to address their fears.
- Providing purely formative or low-stakes opportunities to practice each assessment style would help develop the desired skills and allow students to build their confidence. (In this context it is perhaps unfortunate that institutional assessment policies designed to reduce the assessment burden on students sometimes have the effect of reducing opportunities for low-stakes assignments.)

In my opinion the excessive focus which degree courses put on assessment has the unfortunate consequences of causing great stress to students and disadvantaging those who do not perform well in the forms of assessment they face. While students as well as tutors assign great importance to assessment, that can sometimes obscure that the primary objective of degree study is learning. It is particularly unfortunate therefore that assessment may not correlate with students' attainment, causes stress and may affect mental health, and may disadvantage those whose backgrounds or circumstances do not suit the way in which they are assessed.

5. Conclusion

We aspire for inclusion in our mathematics degrees but assessment issues can discourage and/or disadvantage some students, especially those from non-traditional academic backgrounds. To mitigate this, consideration should be given to this aspect of assessment. As Iannone and Simpson (2021) report, higher education institutions in the UK rely heavily on traditional forms of assessment such as time-constrained examinations. The community should consider whether changes in our assessment strategies might make our subject more inclusive.

6. Acknowledgements

I am very grateful to the many colleagues, students and former students whose views and experiences have helped me appreciate some of the issues around assessment. I would also like to express my appreciation of the helpful comments and suggestions made by an anonymous reviewer, and to Professor Chris Sangwin for drawing my attention to the judgement at Bristol County Court in 2022

7. References

Bradshaw, N. and Mann, T., 2021. Welcoming students to the mathematics community: obstacles to "belonging". *MSOR Connections*, 19(3), pp. 75-86. Available at https://journals.gre.ac.uk/index.php/msor/article/view/1268 [Accessed 29 November 2022]

Codiroli Mcmaster, N., 2021. Ethnicity awarding gaps in UK higher education in 2019/20. York, AdvanceHE. Available at https://www.advance-he.ac.uk/knowledge-hub/ethnicity-awarding-gaps-uk-higher-education-201920 [Accessed 29 November 2022]

The County Court at Bristol, 2022. Judgment between Dr. Robert Abrahart and the University of Bristol. Available at https://www.judiciary.uk/wp-content/uploads/2022/05/Abrahart-v-Uni-Bristol-judgment-200522.pdf [Accessed 29 November 2022]

Fry, H., and Penrose, R., 2018. Interview. Available at https://www.youtube.com/watch?v=iZ8GaK2YzWo [Accessed 29 November 2022]

Hodds, M. and Rowlett, P. (editors), 2022a. CETL-MSOR Conference Special Issue 1: Responding to the COVID-19 pandemic. *MSOR Connection,* 20(1). Available at https://journals.gre.ac.uk/index.php/msor/issue/view/93 [Accessed 29 November 2022]

Hodds, M. and Rowlett, P. (editors), 2022b, CETL-MSOR Conference Special Issue 2: Innovations in teaching, learning, assessment and support. *MSOR Connections*, 20(2). Available at https://journals.gre.ac.uk/index.php/msor/issue/view/94 [Accessed 29 November 2022]

Iannone, P. and Simpson, A., 2021, 'How we assess mathematics degrees: the summative assessment diet a decade on'. *Teaching Mathematics and its Applications: An International Journal of the IMA* , 41, pp. 22-31. Available at https://academic.oup.com/teamat/article/41/1/22/6271028 [Accessed 29 November 2022]

Institute of Mathematics and its Applications, London Mathematical Society, and Royal Statistical Society, 2022. Statement on Methods of Assessment in the Mathematical Sciences. Available at https://ima.org.uk/20287/statement-on-methods-of-assessment-in-the-mathematical-sciences-2/ [Accessed 29 November 2022]

Quality Assurance Agency, 2019. Subject Benchmark Statement Mathematics, Statistics and Operational Research. Available at https://www.qaa.ac.uk/docs/qaa/subject-benchmark-statements/subject-benchmark-statement-mathematics-statistics-and-operational-research.pdf [Accessed 29 November 2022]

sigma, 2021. The CETL-MSOR Conference 2021 Homepage. http://sigma.coventry.ac.uk/cetlmsor2021 [Accessed 29 November 2022]

TALMO, 2020. Teaching and Learning Mathematics Online. http://talmo.uk/ [Accessed 29 November 2022]

WORKSHOP REPORT

Collusion, Rackets, and Plagiarism in Assessments

Alan James Walker, School of Computing, Engineering, and Physical Sciences, University of the West of Scotland, Paisley, Scotland. Email: alan.walker@uws.ac.uk

Abstract

Recently, due to the global pandemic, some higher education institutions moved from formal closed-book examinations to emergency virtual assessments (EVAs). These EVAs normally comprised open-book, remote, short time-frame assessments. Most institutions are moving back to formal examinations as effects from the pandemic reduce, but some institutions have created a "new normal" regarding assessments and have opted to remain with open-book, remote, non-invigilated assessments. With these enforced changes, the mathematical sciences assessment setter is tasked with creating assessments which are resistant to collusion, plagiarism and other forms of academic malpractice. Here we discuss some recent examples of issues encountered in the assessment of science and engineering topics without formal invigilated examinations.

Keywords: assessment, cheating, collusion, malpractice, plagiarism,

1. Introduction

Following the spread of the Covid-19 virus in late 2019 and early 2020, the World Health Organisation (WHO) (2020) declared a global pandemic in March 2020. The Scottish Government (2020) reported the first confirmed case of COVID-19 in early March. The first community transmission of the disease was recorded on March 11th (Scottish Parliament, 2020) and the first death just two days later. Citizens of the U.K. were subjected to a "Stay at Home" order by Prime Minister, Boris Johnson, soon after.

Many universities across the world closed their buildings to most staff and students, in line with their government's regulations. Emergency procedures concerning teaching and assessment were enacted. Scheduled teaching was done online, where possible, and in-person class tests and examinations were replaced by online assessments: open-book, remote, examinations. These have been dubbed emergency virtual assessments (EVAs) and created issues concerned with digital poverty, lack of appropriate assessment environments, and lack of support for students with additional learning and assessment requirements (Khan, 2021). Further, the emergence of non-invigilated examinations meant that reported cases of plagiarism and collusion rose significantly (Lancaster and Cotarlan, 2021).

With the effects of the pandemic easing, many higher education institutions are reverting to assessing via formal, invigilated examinations. In most cases, the formal examination removes most of the temptation and ease for collusion, plagiarism, and other academic malpractice. However, some institutions are extolling the virtues of 'authentic assessment' which is sometimes translated to meaning 'no more formal, invigilated examinations'.

For the mathematical sciences assessment setter (and others), it could be asked how does one draft an open-book, non-invigilated, remote, short-time assessment, which is fair, engaging, 'authentic', meets the learning outcomes, and is resistant to plagiarism, collusion, and other forms of academic malpractice? This paper considers some of the varieties of assessment types used in higher education today, how they are being abused, and what form assessments might take in the future.

2. Forms of Assessment and Associated Malpractice

1.1. The Essay/Report

Not just the vehicle for humanities assessment, the essay can also be used to assess science and engineering topics. From a short 1000-word essay on explaining a cryptographic protocol, to the Honours Year dissertation on Diophantine equations, scientific writing can be used to significant effect to tease out students' knowledge on a particular topic. Often, a drawback for science and engineering students is that they are not repeatedly asked to write substantial documents such as a dissertation and thus their writing skills, and ability to cite appropriately,other sources, can raise issues. Programme teams should be mindful of this when considering their assessment maps. Normally, higher education institutions have excellent central support teams who can run bespoke sessions on scientific writing and referencing. It is important that these are advertised and advertised at the appropriate times.

Sometimes, however, the essay-type assessment can provide opportunities for academic malpractice in the form of plagiarism. Module coordinators and Academic Malpractice Panel (AMP) members are sometimes provided with essays with substantial sections of text which have simply been lifted from internet sources, often without appropriate referencing. Academics tasked with marking these pieces of work should be on the look-out for text or citations which do not fit the subject matter at hand, changes in language, and changes in writing level.

Assistance is available in spotting suspected plagiarism through the software Turnitin, an internet-based plagiarism detection service. It is of little use for scientific calculations but is an excellent tool for uncovering cases where similar text has appeared previously in student submissions, academic papers, or online. Suspected plagiarised text is highlighted by the Turnitin software. The text is then connected with proposed original source later in the Turnitin report.

Care must be taken with the software. A recent example made available to the author illustrated that Turnitin had reported around 30% similarity with a student submission at the same institution. However, upon analysing the Turnitin report fully, it transpired that this 30% similarity was across over 100 students at the same institution, with a maximum of 9% similarity from any individual student. Further, parts of this similarity were accounted for in standard title pages and prescribed section headers. It is worth attending any Turnitin training that institutions may provide.

Further, Turnitin is not fool proof, and there are instances where students have attempted to circumvent the software checks by paraphrasing sections of text (there are online tools available for this – sometimes with unintended humorous consequences), using synonym replacement software (also not without possible unintended meaning changes), changing document types, using translation software, and, more concerning, using essay mills.

Essay mills are companies that provide original pieces of writing to students. Some companies will offer students the option of a piece of writing which will score a certain grade to not arouse suspicion. Recently, the U.K. government have sought to make the use of essay mills illegal via the Skills and Post-16 Education Bill (Department of Education, 2021). Nevertheless, academics should still be vigilant to uses of these companies. This can be done by considering changes to language used (from previous assessments), being aware of sudden grade/level jumps, or being aware of strange consistencies in assessments from a student cohort.

In a recent example made known to the author, the referencing for each figure in a series of reports seemed reasonable when viewed as a single submission. However, when viewed as a collective, the consistency of having figures referenced in such a manner aroused suspicion. Investigating these specific cases further uncovered that citations did not match the subject matter, and often the subject matter was not aligned with the assessment brief. In this example, it was uncovered that each essay had been procured from a third party. Unfortunately, many markers may miss small tell-tale signs of misconduct, especially when a high volume of marking has been allocated.

More recently, academics have been made aware of ChatGPT, a chatbot which can provide real-time answers to simple questions, short essays, and even poetry. The freely available (at the time of writing) prototype software has been created by OpenAI, an artificial intelligence company founded by, amongst others, Elon Musk. Artificial Intelligence is a multi-million-pound industry, and it has recently been reported that Microsoft have announced a multibillion-dollar investment in OpenAI (BBC, 2023). Currently, not without potentially inaccurate answers, the software can be mistreated for the creation of sentences or paragraphs which answer specific questions. For example, the author asked the software "What is a cryptographic protocol?". Within a minute, the software had made the following answer available:

> "A cryptographic protocol is a set of rules and procedures for secure communication in the presence of third parties, known as adversaries. These protocols use various cryptographic techniques, such as encryption and digital signatures, to provide confidentiality, integrity, authenticity, and non-repudiation for the communication. Cryptographic protocols are widely used in various applications, such as secure communication, secure electronic transactions, and secure computer networks."

This response seems appropriate, but when ChatGPT was asked to integrate x*ln(x) with respect to x, the software provided the response x*ln(x) – x + C, complete with a step-by-step breakdown of how this "solution" was achieved.

Academics have already begun testing the software further, with it being reported that the software could pass an MBA examination (Terwiesch, 2023).

Whilst software like this poses a problem for academics using short (or long) essays in assessment, it is not necessarily a new problem, as students could always hire others to write for them. However, the speed (and current lack of associated cost) of the prototype software is worrying. The solution to the simple mathematics question perhaps more so, but for different reasons.

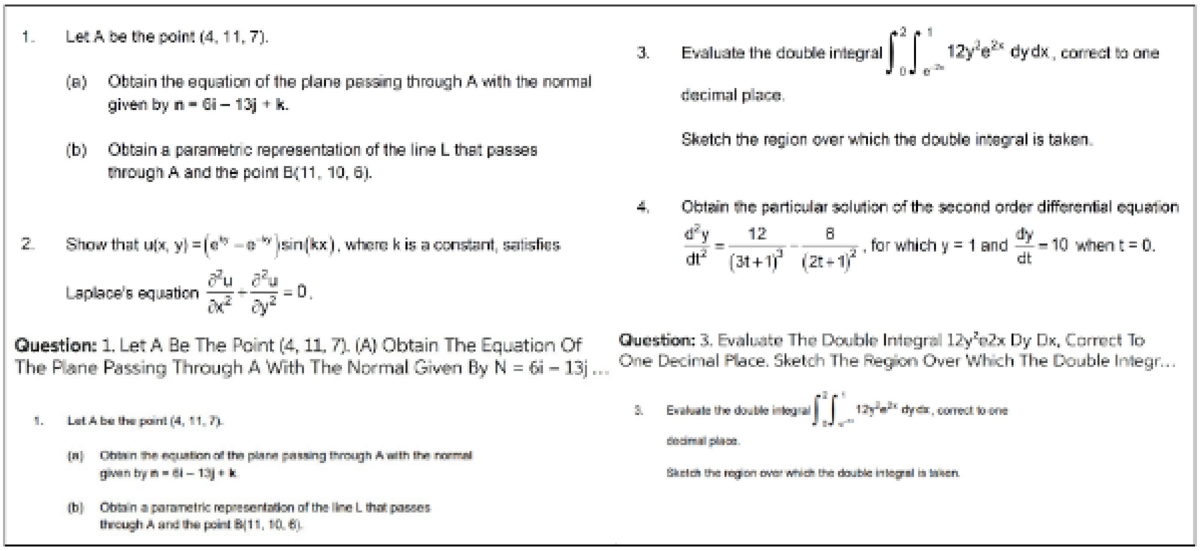
Of course, not all topics and courses can be appropriately assessed via an essay or dissertation. We now consider examinations and coursework which are more numerical in nature.

1.2. The Remote Examination

In most science and engineering topics, material has been assessed via formal, closed-book, invigilated examinations for many years. Assessment questions would be centred around regurgitating subject knowledge in short descriptive questions or longer essay-type questions or applying required knowledge in contextualised problem-solving questions. The opportunities for collusion and plagiarism in these events is reduced, and with invigilation, the opportunity for malpractice in other forms (such as the use of pre-prepared notes or sharing of solutions) is also reduced. That being said, invigilators must be mindful of developing technology which may be used to circumvent the invigilation process.

With the move to EVAs and, in some cases, non-emergency virtual assessments, the science and engineering examination questions which are not essay-based can be subject to malpractice. Standard questions in low-level "service-teaching" calculus assessments, for example, where the knowledge and application of techniques is being assessed, can simply be answered via a host of computer packages (such as Maple, MATLAB, Mathematica, *et cetera*) or even websites such as Wolfram Alpha. Examiners should be on the look-out for “skipped working” which suggests that the student has not worked through the solution without external aid.

However, issues with remote assessments run deeper. For higher-level scientific assessment questions, which cannot simply be inserted into a computer algebra package, some students have sought alternative workarounds. Students have reported working together on examinations, either face-to-face, or have shared solutions via WhatsApp, Snapchat, Facebook Messenger, and Discord. Further, websites which offer “assistance with homework” such as Chegg, CourseHero, and AnswerHappy can provide worked solutions, for a small fee, to scientific questions within as little as 30 minutes. This means that students can upload remote examination questions to these types of websites and receive worked solutions in plenty of time for rewriting and submitting as their own work (or even sharing via the aforementioned social media platforms). An example of such potential malpractice can be seen in Figure 1.


1. Let A be the point (4, 11, 7).

(a) Obtain the equation of the plane passing through A with the normal given by n = 6i – 13j + k.

(b) Obtain a parametric representation of the line L that passes through A and the point B(11, 10, 6).

2. Show that $u(x, y) = (e^{ky} - e^{-ky})\sin(kx)$, where k is a constant, satisfies Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

3. Evaluate the double integral $\int_0^2 \int_{e^{-2x}}^1 12y^2 e^{2x}\, dy\, dx$, correct to one decimal place.

Sketch the region over which the double integral is taken.

4. Obtain the particular solution of the second order differential equation $\frac{d^2y}{dt^2} = \frac{12}{(3t+1)^3} - \frac{8}{(2t+1)^2}$, for which y = 1 and $\frac{dy}{dt} = 10$ when t = 0.

Question: 1. Let A Be The Point (4, 11, 7). (A) Obtain The Equation Of The Plane Passing Through A With The Normal Given By N = 6i – 13j ...

1. Let A be the point (4, 11, 7).

(a) Obtain the equation of the plane passing through A with the normal given by n = 6i – 13j + k.

(b) Obtain a parametric representation of the line L that passes through A and the point B(11, 10, 6).

Question: 3. Evaluate The Double Integral 12y²e2x Dy Dx, Correct To One Decimal Place. Sketch The Region Over Which The Double Integr...

3. Evaluate the double integral $\int_0^2 \int_{e^{-2x}}^1 12y^2 e^{2x}\, dy\, dx$, correct to one decimal place.

Sketch the region over which the double integral is taken.

Figure 1. (Top) Four questions which appeared in a remote mathematics examination in Academic Year 2019-2020, and (bottom) questions which appeared on Chegg.com minutes after the aforementioned questions had been released.

Again, vigilance from the assessment setters and markers is required here. A simple, but effective strategy is to be on the look-out for non-standard solutions, perhaps using techniques which have not been discussed in class. Another is to diligently check “study help” websites for uploads. However, it should be questioned whether markers should be trawling through these websites on examination days to see if questions are being uploaded. Further, if questions are being uploaded, should individual academics be paying for the solutions to ascertain if provided solutions are matching submissions, or should this be the purview of institutions’ AMPs? It may be a concern of some academics that other academics, or even host institutions, have a ‘hear no evil, see no evil’ approach.

After the introduction of EVAs, and the concomitant rise of reports of plagiarised materials, some academics have introduced individualised coursework and individualised remote examinations. This has a three-fold attack on combating plagiarism and collusion. Firstly, each student has a distinct set of assessment questions and hence the ability to collude is reduced. Secondly, if students are aware that assessments are individualised, they may be less tempted to submit their questions to “study help" websites. Thirdly, if students do submit their individualised questions to "study help" websites, then the vigilant marker can identify these to a particular student and then take the appropriate action. Such an example of semi-individualisation can be seen in Figure 2. A naïve student could upload such a question to one of the aforementioned websites, providing their matriculation number at the same time.

2 a A coin with diameter d=20+X mm is floating in a glass of water.

Calculate its maximum mass (in grams) for the coin not to sink.

X is given by the seventh digit of your Banner number.

Figure 2. A question posed in a remote examination which is individualised, in part, using the student's Banner (matriculation) number.

Again, with remote examinations, the onus here is on the assessment setter to provide an examination which is resistant to plagiarism and collusion, but there is a significant amount of time which must be spent on this task. Using the example in Figure 2, the assessment is semi-individualised (there must be up to ten different possible questions), which at least gives some variety, but the questions themselves are not completely individualised. However, even with this limited example, we have ten different questions and ten different answers. Extrapolate this over an entire assessment and we have multiple versions of each assessment which simply adds to the workload of the assessment marker. Alternatively, students may be provided with completely independent assessments using a unique data set to be considered, or a unique parameter set used in a Mail Merge (for Word users) or \Merge command (for LaTeX users). An example of a rudimentary parameter set can be seen in Figure 3.

Additional time allocation aside, which is not inconsequential, the assessment setter must be careful to include appropriate parameter bounds so that questions, and solutions, are appropriate, applicable, and error-free.

Individualisation can help reduce the temptation of collusion, but only if the students have been made aware that their assessment has been individualised. The assessment setter must think carefully on the question *do I let my students know of individualisation*? By not doing so, the assessment setter is simply providing means of recognising collusion, rather than removing the temptation. Consider Figure 4, where two students are provided with similar, but individualised questions. What happens when both students provide a solution to the same question? What is more important here, that collusion is reduced, or that identified collusion is increased?

	A	B	C	D	E	F	G	H	I	J
1	Master	B00402412	B00309856	B00992681	B00388237	B00387575	B00365214	B00402229	B00390352	B00310726
2	A_1	2	3	4	5	6	7	8	9	10
3	A_2	19	18	17	16	15	14	13	12	11
4	(A_1+A_2)	21	21	21	21	21	21	21	21	21
5	A_1A_2	38	54	68	80	90	98	104	108	110
6	A_2+1	20	19	18	17	16	15	14	13	12
7	B_1	19	18	17	16	15	14	13	12	11
8	B_2	2	3	4	5	6	7	8	9	10
9	a	3	4	5	6	7	8	3	4	5
10	b	8	7	6	5	4	3	8	7	6
11	\frac{(a-1)}{a}	\frac{2}{3}	\frac{3}{4}	\frac{4}{5}	\frac{5}{6}	\frac{6}{7}	\frac{7}{8}	\frac{2}{3}	\frac{3}{4}	\frac{4}{5}
12	\frac{b}{a}	\frac{8}{3}	\frac{7}{4}	\frac{6}{5}	\frac{5}{6}	\frac{4}{7}	\frac{3}{8}	\frac{8}{3}	\frac{7}{4}	\frac{6}{5}
13	e	2	-2	2	-2	2	-2	2	-2	2
14	c0	2	2	3	4	4	6	6	8	6
15	c1	6	14	10	6	16	2	19	9	10
16	+(C_0+C_1)	+8	+16	+13	+10	+20	+8	+25	+17	+16
17	+C_0C_1	+12	+28	+30	+24	+64	+12	+114	+72	+60
18	h	260	1042	702	420	1690	260	2742	1292	1122
19	C_0	C_0	C_0	C_0	C_0	C_0	C_0	C_0	C_0	C_0
20	+C_1	+C_1	+C_1	+C_1	+C_1	+C_1	+C_1	+C_1	+C_1	+C_1
21	+C_2	+C_2	+C_2	+C_2	+C_2	+C_2	+C_2	+C_2	+C_2	+C_2
22	+C_3	+C_3	+C_3	+C_3	+C_3	+C_3	+C_3	+C_3	+C_3	+C_3
23	i	1	1	-1	-1	1	1	-1	-1	1
24	j	1	-1	1	-1	1	-1	1	-1	1
25	k	\alpha_1	\alpha_1	\alpha_1	\alpha_1	\alpha_1	\alpha_1	\alpha_1	\alpha_1	\alpha_1
26	l	\alpha_2	\alpha_2	\alpha_2	\alpha_2	\alpha_2	\alpha_2	\alpha_2	\alpha_2	\alpha_2
27	C_5	2	4	6	2	4	6	2	4	6
28	C_6	4	6	8	4	6	8	4	6	8
29	\frac{1}{2}C_5	\mbox{}	2	3	\mbox{}	2	3	\mbox{}	2	3
30	C_7	\frac{1}{3}	\frac{4}{3}	-\frac{2}{3}	-\frac{5}{3}	\frac{1}{3}	\frac{4}{3}	-\frac{2}{3}	-\frac{5}{3}	\frac{1}{3}
31	C_8	\frac{2}{3}	\frac{5}{3}	-\frac{1}{3}	-\frac{4}{3}	\frac{2}{3}	\frac{5}{3}	-\frac{1}{3}	-\frac{4}{3}	\frac{2}{3}
32	C_9	\frac{2}{3}	\frac{5}{3}	-\frac{1}{3}	-\frac{4}{3}	\frac{2}{3}	\frac{5}{3}	-\frac{1}{3}	-\frac{4}{3}	\frac{2}{3}

Figure 3. A rudimentary example of an Excel file created to provide parameter sets for inclusion into LaTeX-created coursework assignments.

Question 1 (10 marks)

The surface S has the equation

$$z = 6x^2y^2 + 12xy^2 - 8y + 35.$$

Question 1 (10 marks)

The surface S has the equation

$$z = 3x^2y^2 + 6xy^2 - 12y + 30.$$

Figure 4. Introductions to two versions of the same question provided to Year 2 calculus students. What happens when one student submits a solution to a different student's question?

Some institutions have considered the use of software which monitors the actions of students during remote examinations, namely remote examination proctoring. However, there remain significant issues with this route when considering cost, digital poverty, bandwidth, and security/privacy concerns. Further, it has also been reported that such software may produce disparities in reports when race, skin tone, and sex is considered (Yoder-Himes *et al.*, 2022).

Crucially, the assessment setter and marker must be aware that, no matter the vehicle of assessment, they must be aware of the temptation of plagiarism, collusion, and other academic malpractice; the opportunities available to counteract this temptation (and associated time-costs); and the opportunities available to discover occurrences of academic malpractice (and associated time-costs).

1.3. Presentations

HEIs are tasked with producing world- and work-ready graduates who are prepared for the 21st century workplace and the fourth industrial revolution Given this; degree programme leaders aim to instil in their graduates the skills required to flourish in this environment. Communication and presentation skills are often highlighted as being sought after by employers. For this reason and recognising that presenting material can illustrate a level of knowledge of a subject, formal presentations of subject matter are also used in HEI assessment settings. Further, presentations can also be paired with groupwork, so that students are also assessed on working with others, leadership, managing conflict, and other useful, relevant skills.

Presentations provide an excellent vehicle for evidencing subject knowledge and application. Pre-prepared (and submitted) slides can be checked for plagiarism via Turnitin, and level of input into group presentations can be assessed via the presentation, or via student proformas where they are asked to rate members' contributions. It should be noted that the latter vehicle can provide issues in itself.

Whilst presentations can seem to provide an assessment vehicle which is less obviously open to plagiarism, the marking of presentations can be very time-consuming for the academics involved. Further, assessment setters must take care to ensure that presentations are fair for all students involved, especially when considering aspects such as anxiety, social or otherwise.

1.4. Group work

The aspect of group work is introduced above, in the context of assessing via presentations, but group work can also be used in other assessments such as reports and simple coursework assignments. Whilst an excellent vehicle for practicing the aforementioned graduate skills, there is one drawback to consider when plagiarism arises in a group work submission: what happens when one group member includes plagiarised work in a submission and it hasn't been known to the other group members? Should all be punished equally, since it is the duty of all group members to be aware of what is being submitted under their name, or should there be different levels of punishment? Issues such as these should be discussed in, and, if possible, procedures set in place by the institution's AMP, as discussed in the following section.

3. The Academic Malpractice Panel

The issues surrounding academic malpractice do not stop at how to counteract and how to discover occurrences. Academic question setters and markers must also be aware of their institutions policies on how suspected occurrences of academic malpractice are processed. Further, the policies must be enacted by all concerned in a simple, coherent, and consistent manner.

Occasionally, it is found that an assessment setter/marker may try to deal with a case of suspected academic malpractice "in-house". This could be due to a number of reasons, including sympathy for the student, disdain for paperwork and associated hassle, and recognition on the potential negative effect on module performance statistics. This can often lead to issues further down the line, especially if a student believes that the assessment setter is unfairly treating them. The simplest route, when suspecting possible academic malpractice, is to report to the faculty/institutional AMP.

The AMP takes a variety of forms across higher education institutions and can offer a variety of penalties for students found guilty of malpractice. Penalties include resubmission of material (with or without loss of attempt), notifications on official records, suspension of studies, and expulsion from degree programmes. Due to the stakes involved, AMP meetings which involve assessment setters and (independently) students suspected of academic malpractice can be stressful environments for all. Students often claim to not be aware of their institution's rules on collusion and plagiarism, they often do not realise the cultural differences in assessments, and often simply do not realise what is expected of them.

Much of this can be combated by having clear and concise information presented to students at the earliest opportunities by senior officials of the institution such as Deans of Faculties, or Chairs of AMPs. By illustrating to students that academics are aware of malpractice, then temptation can be reduced.

Occasionally, students may appeal the decision of the Panel, and for this reason it is important that policies have been enacted, by all, and at all stages, to the letter. Further, it is important that the constituent members of AMPs are representative of the faculty or institution. As noted earlier, being summoned to the AMP can be a stressful experience, which can be mitigated with an appropriate choice of panel members. Recognition of cultural differences can often be key.

4. Next Steps?

Many academics (and higher education institutions) were caught sleeping at the wheel when EVAs were introduced during the pandemic. Due to the move to remote assessments, some academics have become acutely aware of occurrences of academic malpractice, including the use of "study help" websites and essay mills. It is important that occurrences of suspected malpractice are reported through the official channels and that executive members of institutions are made aware of the scale of the problem.

Government officials have made steps to combat the issue, but whilst their focus is currently on essay-mills, companies offering "study help" are still able to assist students in malpractice. Some institutions have taken steps to block such websites from being accessed on campus, but that is of little assistance when students are able to access off-site. It causes further issues for staff members who are willing to monitor these sites for exam-time uploads. Further, due to the vast sums of money involved, (Financial Times, 2021) "study help" companies could be seen to be a little reluctant to work closely with institutions with malpractice concerns.

Suggestions have been made here on individualisation of student assessments, but these can come with an associated time cost. Further, a consistent approach within university departments could be difficult to garner, especially if some colleagues do not agree with the time cost versus benefit argument.

Finally, whilst many science and engineering academics stand steadfastly to the opinion that the formal, closed-book, invigilated examination is the best [and only useful] method of examination, there are an increasing number who are willing to experiment with different vehicles for assessment (e.g. essays, presentations, reports). Whilst there is not the time available to afford every student a *viva voce*, other methods of assessment are available, and do not necessarily need to be vectors of malpractice.

5. References

British Broadcasting Company, 2023, ChatGPT: Microsoft to invest billions in chatbot maker OpenAI, Available at https://www.bbc.co.uk/news/technology-64374283. [Accessed 24 January 2023].

Department of Education, U.K. Government, 2021, Press Release: Essay mills to be banned under plans to reform post-16 education, Available at https://www.gov.uk/government/news/essay-mills-to-be-banned-under-plans-to-reform-post-16-education. [Accessed 28 November 2022].

Financial Times, 2021, Student cheating is now a multibillion-dollar business, Available at https://www.ft.com/content/3fcfcf72-4838-49e5-b9ce-a360b5b5ab6b. [Accessed 30 November 2022].

Khan, M. A., 2021, COVID-19's Impact on Higher Education: A Rapid Review of Early Reactive Literature, *Education Sciences*, 11, 421, https://doi.org/10.3390/educsci11080421 .

Lancaster, T. and Cotarlan, C., 2021, Contract cheating by STEM students through a file sharing website: a Covid-19 pandemic perspective, *International Journal for Educational Integrity*, 17(3). Available at https://edintegrity.biomedcentral.com/articles/10.1007/s40979-021-00070-0.

Scottish Government, 2020, Coronavirus (COVID-19): First positive case in Scotland, Available at: https://www.gov.scot/news/coronavirus-covid-19/ [Accessed 28 November 2022].

Scottish Parliament, 2022, Timeline of Coronavirus (COVID-19) in Scotland, Available at: https://spice-spotlight.scot/2022/11/18/timeline-of-coronavirus-covid-19-in-scotland/ [Accessed 28 November 2022].

Terwiesch, T., 2023, Would Chat GPT3 Get a Wharton MBA? A Prediction Based on Its Performance in the Operations Management Course, Mack Institute for Innovation Management at the Wharton School, University of Pennsylvania. Available at https://mackinstitute.wharton.upenn.edu/wp-content/uploads/2023/01/Christian-Terwiesch-Chat-GTP.pdf. [Accessed 24 January 2023].

World Health Organization (WHO), 2020, WHO Director-General's opening remarks at the media briefing on COVID-19 - 11 March 2020, Available at: https://www.who.int/director-general/speeches/detail/who-director-general-s-opening-remarks-at-the-media-briefing-on-covid-19---11-march-2020 [Accessed 28 November 2022].

Yoder-Himes, D. R., Asif, A., Kinney, K., Brandt, T. J., Cecil, R. E., Himes, P. R., Cashon, C., Hopp, R. M. P., and Ross, E., 2022, Racial, skin tone, and sex disparities in automated proctoring software. *Frontiers in Education*, 7, 881449, https://doi.org/10.3389/feduc.2022.881449 .

CASE STUDY

An Updated Show of Hands

Helen Berrington, Learning Development/Department of Mathematics & Statistics, Lancaster University, Lancaster, United Kingdom. Email: h.berrington@lancaster.ac.uk
Anna Karapiperi, Learning Development, Lancaster University, Lancaster, United Kingdom. Email: a.karapiperi@lancaster.ac.uk
Eleftherios Kastis, Learning Development/Department of Mathematics & Statistics, Lancaster University, Lancaster, United Kingdom. Email: l.kastis@lancaster.ac.uk

Abstract

It is a tried and tested technique to gauge the overall understanding of a class: a multiple-choice quiz with a show of hands for who thinks the answer is a, b or c. Although quick and easy, how much does it really measure the students' understanding? On top of that, how useful is it as an informal formative assessment? A few students usually dominate the class and less confident students may not put up their hand, or may follow what their classmates are doing, and hence both the learner and educator may never know the individuals' true answer.

Here we discuss "an updated show of hands", whereby students scan a QR code to take them to a real-time quiz hosted on the Moodle Virtual Learning Environment (VLE), that they can answer on their smart device. All students answer the same question at the same time, and after a set time, the correct answer is revealed and the class results for that question are then displayed to everyone as an anonymous percentage. Whilst this updated method has the obvious advantage of anonymity and the obvious disadvantage of potential technical problems, in this case study we provide a full description of the implementation and an in-depth discussion on the pedagogy and practicalities of the updated show of hands – the real-time smart device quiz.

Keywords: Smart Device Quiz, QR Code, Interactivity, Digital Technology, Student Engagement.

1. Introduction

As educators, we are continuously looking for ways to update teaching methods and approaches. Whilst the move to online teaching in 2020 brought with it many challenges, it also introduced new ways to embrace technology in teaching, many of which can also be used in a face-to-face classroom setting. Almost all students at Lancaster University bring a smart device to teaching sessions (since attendance recording also requires the use of a smart device) and hence we can make use of this technology within the session itself.

A traditional show of hands is a well-used technique when teaching groups of students to assess the group's understanding of a topic. It might be a multiple-choice quiz, or a true or false question, where students are asked to raise their hands in favour of a particular response. However, there are reasons why this technique may not truly assess the group's understanding, nor act as a useful learning exercise for the students themselves. For example, stronger or more confident students may dominate, with less confident students waiting to see what others answer before raising their own hand. Some students may not even raise their hand at all, especially if they are worried about answering incorrectly in front of their peers. Cold calling techniques may not be appropriate in many teaching sessions, where there is time pressure that hinders students' self-confidence (Lemov, 2021) as well as many other anxiety inducing factors related to cold calling. Therefore, we may not truly be assessing the understanding of all students as a group.

In the following section, we describe the implementation of “an updated show of hands”, using a real-time smartphone quiz, as used in an in-person teaching session. Whilst we recognise that the use of a smartphone quiz is not novel in itself (see for example Licorish et al., 2017 and Zainuddin et al., 2020) this particular implementation is noteworthy for its simplicity and efficiency. We then discuss the pedagogy and practicalities of its usage, including recommendations for future use.

2. Implementation

1.1. Class Setting

The Maths and Stats Hub (MASH) at Lancaster University provides additional workshops for students for a number of modules. In this case study, we focus on a face-to-face workshop on differentiation for first year Accounting & Finance students. This workshop is optional for students to attend and usually has 10-15 students attending per week. These may be students who have not studied A-Level Mathematics or may be less confident in their maths skills and wish to have more practice and support with topics in maths. Since these classes are not compulsory, note that from week to week we may have different students. This brings extra constraints in the attempt to build student engagement.

In this particular workshop, the focus was on applying the chain, product and quotient rule to differentiate functions. Before asking students to apply these rules, there was a focus on recognising when to use each rule, which is the topic of this example.

1.2. Use of Technology

In the classroom, a PC connected to a projector screen is used to display material to the class. To fully interact in the real-time quiz, students require a device that can connect to the internet. A QR code to the quiz is generated using a web browser.

Students can either scan the QR code to take them to the web address or can enter the URL on their laptop. For those students who for whatever reason cannot access the quiz in this way, they can still take part in the activity via a paper handout or reading from the projector screen. Although they will not experience the full interactivity, they can still attempt the quiz at the same time.

The quiz is a “Realtime Quiz” on the MASH Moodle page, which students are enrolled on. Therefore, when connecting, students are prompted to login using their University login details.

1.3. Execution

Students are asked to scan the QR code displayed on the projector screen to take them to the quiz. It is clearly labelled as a “non-assessed real-time quiz”, so that students are aware that although it is conducted through Moodle, their results will not contribute towards their grade in any way. It is also reiterated verbally that this quiz is to test their own understanding as part of their learning and is not a formal assessment.

Once students have reached the page, the class leader starts the quiz. Students are given a function and have 30 seconds to choose which rule they would use to differentiate this function. Figure 1 shows the projector view that the whole class can see, and the smart device view that the individual student sees when answering the question.

After the 30 seconds have elapsed, the correct answer is displayed both on the projector screen and the smart device. Everyone can see how many students chose each of the options, but importantly, students cannot see others individual results. For example, if three students had selected the incorrect answer, those students themselves would know their result, but the rest of the class would not know which three students they were.

Before proceeding to the next question, the class leader can take time to answer any queries or add any explanation to the question, depending on the needs of the students and their responses.

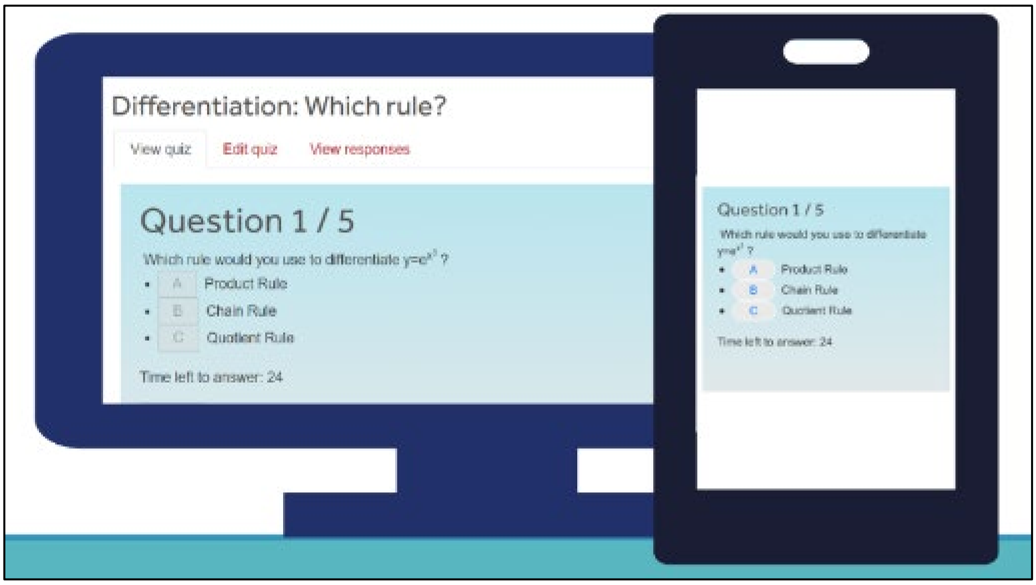


Figure 1. Projector view (left) and smart device view (right) of Realtime Quiz.

Questions continue to proceed in this way until the end of the quiz, whereupon the students can see their individual score and the overall class score. Again, students cannot see each other's individual scores.

1.4. Results

The overall results for the quiz are available after the quiz has finished and can be viewed by the class leader (not students). Figure 2 shows a table of results for this quiz. In this way, the class leader can quickly and easily see which questions were answered in which way and therefore can adaptively plan for topics that may need more focus for this particular group of students. A breakdown of individual results may also be viewed by the class leader privately after the class.

1	Product Rule	**Chain Rule**	Quotient Rule	
Which rule would you use to differentiate y=ex3 ?	1 ✗	8 ✓	0 ✗	88.89%
2	**Product Rule**	Chain Rule	Quotient Rule	
Which rule would you use to differentiate y=3x2ln(x) ?	8 ✓	0 ✗	1 ✗	88.89%
3	Product Rule	Chain Rule	**Quotient Rule**	
Which rule would you use to differentiate y=ex/x4?	0 ✗	1 ✗	7 ✓	87.5%
4	**Product Rule**	Chain Rule	Quotient Rule	
Which rule would you use to differentiate y=(2x5+6)(ex-x)?	5 ✓	1 ✗	0 ✗	83.33%
5	Product rule	**Chain Rule**	Quotient Rule	
Which rule would you use to differentiate y=ln(x4+(1/x)) ?	3 ✗	5 ✓	1 ✗	55.56%

Figure 2. Table of results of Realtime Quiz, as viewed by the class leader.

3. Discussion

The decision was made to host the quiz using the University's VLE software, Moodle, as this gives many advantages over using a third-party software. It means that there is no cap on the number of participants, no subscriptions or additional accounts needed for staff or students, and it provides a seamless student experience within the VLE. It also gives more consistency to be used year after year, since third-party companies can often update their software without warning, which hinders forward planning. The VLE also allows for formatting of mathematical equations, which is of upmost importance in our application. The layout of the software is familiar to students, as they use it to access course materials. It is also very useful as a formative assessment, since the formatting is similar to the summative assessments used in the VLE.

One of the main advantages that the updated show of hands brings as opposed to a traditional show of hands is the anonymity of responses between the students in the class. In not seeing the responses of fellow students, it allows each student to use their own knowledge and reasoning to answer the question, giving an independence that the traditional show of hands does not allow. This also allows students to answer the question without being influenced by others that may have answered earlier. In addition, the fact that others do not see their response, gives the individual more confidence to attempt the question, with less fear of embarrassment if they do not select the correct answer, which unfortunately can be ingrained from past school lessons (Royer & Walles, 2007). It has been shown that maths anxiety has a negative relationship with performance in mathematics (Zhang et al., 2019), and therefore anything we can do to reduce maths anxiety by boosting confidence is a positive step forward.

In implementation, the updated show of hands clearly is much more time-consuming to plan. Questions and answers must be thought of in advance and a QR code must be generated. One may wonder that given how stretched many teaching staff are for planning time, if the time consumed to plan a quiz is worth it, when a traditional show of hands can be done in an impromptu manner. However, once set up, such a quiz may be used year after year. There is also the very real issue of temperamental technology in the teaching session itself. Whilst digital technology can be extremely helpful in pedagogy, it must be used with caution when there is the potential of using valuable contact time solving computer problems and we must also ask ourselves if this is the best use of our time. Especially in larger class sizes, there may be many more technical issues.

When implemented, the updated show of hands also brings with it the advantage that the results are recorded. This not only makes judging the proportions of correctly answered questions much easier but gives a useful record that can be revisited when planning further activities.

4. Aspects to Consider for Future Implementation

Here we presented the updated show of hands as used in our optional small group workshops. When considering whether to use the updated or traditional method, a number of factors must be considered.

Firstly, it is important to consider the subject matter of the quiz. Here we presented a choice of rules to use for differentiation, but we have also used it for choosing whether to use a paired or non-paired t-test in statistics. In mathematics and statistics at university level, there are limitations on what you can feasibly answer in 30 seconds. Therefore, this is a great opportunity to consider activities solely based on the solution strategy to answering questions, and not necessarily doing the individual steps to the solution (Suurtamm, et al., 2016). This can really help students understand their approach and thought processes in attempting questions; a great exercise in assessment for learning. By using a setup that appears similar to summative assessment, but has no consequence on grading, such a quiz is an opportunity for formative assessment where students can see their progress without worrying about their marks. It also gives immediate interactive feedback (Sambell, et al., 2013) after each individual question, which can be extremely helpful for learners to develop their skills in approaching mathematics exercises.

The class size and setting are important to consider, as well as any additional support needs. We anticipate the updated method works best with small to medium size classes and may be more difficult to implement smoothly in a large class, although we have not yet tried it in the larger classes. The updated method was very useful in our MASH workshops as many of the students that come lack confidence in mathematics, and in implementing the updated version, they can gain confidence in their ability. Many students have been pleasantly surprised that their intuition guided them correctly, when they may otherwise have been afraid to raise their hand. We believe that the most important aspect to consider when choosing whether to use the updated or traditional show of hands is the benefit to learning that each activity can bring. If the updated show of hands can truly enhance the students' learning experiences by giving a confidence boost without the risk of embarrassment, then it is a worthwhile venture.

5. References

Lemov, D., 2021. *Teach Like a Champion 3.0: 63 Techniques that Put Students on the Path to College*. 3 ed. San Francisco: Jossey-Bass.

Licorish, S.A., George, J.L., Owen, H.E. and Daniel, B., 2017, December. "Go Kahoot!" enriching classroom engagement, motivation and learning experience with games. In *Proceedings of the 25th international conference on computers in education*, pp. 755-764. Asia-Pacific Society for Computers in Education.

Royer, J. M. and Walles, R., 2007. Influences of Gender, Ethnicity, and Motivation on Mathematical Performance. In: D. B. Berch & M. M. M. Mazzocco, eds. *Why Is Math So Hard for Some Children?*. Baltimore: Paul H. Brookes Publishing Co., pp. 349-369.

Sambell, K., McDowell, I. and Montgomery, C., 2013. *Assessment for learning in higher education*. 1 ed. London: Routledge.

Suurtamm, C., Thompson, D.R., Kim, R.Y., Moreno, L.D., Sayac, N., Silver, E., Ufer, S., and Vos, P., 2016. *Assessment in Mathematics Education: Large-Scale Assessment and Classroom Assessment*. Cham: Springer

Zainuddin, Z., Shujahat, M., Haruna, H. and Chu, S.K.W., 2020. The role of gamified e-quizzes on student learning and engagement: An interactive gamification solution for a formative assessment system. *Computers & Education*, *145*, p.103729.

Zhang, J., Zhao, N. and Kong, Q.P., 2019, The Relationship Between Math Anxiety and Math Performance: A Meta-Analytic Investigation. *Front. Psychol.* 10(1613)

CASE STUDY

Improving student engagement through employability themed group work

Dr Simon A. Fairfax, Department of Mathematical Sciences, University of Liverpool.
Email: simon.fairfax@liverpool.ac.uk

Abstract

In an ideal world, universities and their departments are able to reach out to employers for collaborative, employer-set, authentic assessment which align industry expectations with an assessment that tests the intended learning outcomes of a module. This is a large and ambitious undertaking for practical reasons. The author identified three practical challenges as: sourcing willing employers, relevance and level-setting, and scalability, i.e., use in modules with large numbers of students. As module leader, each of these challenges were addressed and solutions identified allowing the employability project to be embedded into a module with 150 participating students contributing 30% towards the overall module mark.

Keywords: confidence, engagement, embedded employability, digital story, group work.

1. Introduction

The ultimate goal for the average mathematics graduate is to land that dream job setting the foundation of their career. However, to what extent does their studies prepare them for that leap into industry? Employers perceive there to be a graduate skills gap in global recruitment markets and suggest universities do not provide enough opportunities for students to develop valued skills; see the QS Global Employer Survey 2018 & the CBI's 2017 Education and Skills Survey. In this case study, the author, acting as the Module Leader, strived to introduce an authentic assessment which simulates a real-life assignment into Level 5 Financial Mathematics, giving students an opportunity to develop professional skills within their studies. Students' sense of belonging at their home institution and engagement with their studies are key contributors to student success; (Thomas, 2012). The year under consideration had an equal split between local student and overseas student from the Chinese partner university which presented challenges from the point of engagement, and opportunities for a culturally rich experience. As part of the evaluation of the assessment, the Module Leader analysed the impact on students' change in confidence following completion of the assessment task. The methods used align with those in York, 2016**,** which considers the importance of belongingness, student engagement and self-confidence.

The Module Leader made attempts to connect with employers in North West England. However, it became increasingly apparent that their proposals required extensive knowledge of financial mathematics beyond that of the module, advanced knowledge of software packages such as MATLAB including use of toolboxes, and often deviated from the proposed intentions of the assessment.

2. Constructing assessment tasks within a fictional secondment

One solution for aligning assessment aims with employer-set projects is to simulate the workplace. This way, employers' input can be obtained, but it is not essential for them to have an active role. An excellent starting place for designing a simulated work-related project is with MathWorks Inc. They are the software developers for MATLAB providing a specialist package used in industry, particularly finance, and have strong links with many HEIs in the UK supporting scientific research. The Module

Leader was able to connect with MathWorks through their Education Consultants and obtain industry insights, for example, the software needs of MathWorks customer base, understanding routine processes using software, and the typical challenges encountered for which developers support their clients. The Module Leader was able to access real-life stock market data freely available online and run trials based on the input from MathWorks within MATLAB. This became a basis for designing assessment tasks which involved exploring trends in the data, presenting financial measures, running procedures for analysing relationships between variables, and making recommendations to corporate clients using the financial framework of modern portfolio theory. By reviewing the annual reports of financial companies listed on the FTSE100 stock exchange, it was possible to develop a back story to add to the authenticity of the assessment giving context to industry problems. Searching job websites for ‘Financial Manager’, ‘Director of Risk’, ‘Portfolio Manager’ is a useful tool for getting an insight into role descriptions and operational matters such as chains of command. Putting all this together, the Module Leader created the profile for a fictional secondment to a fictional company which he named *Consultio International*. The assessment task was an investigative brief set by the Portfolio Manager at *Consultio International* for which the five successful applicants were seconded. The assessment involved presenting financial analyses and making financial recommendations using the financial framework developed in lectures, incorporating the use of financial software via MATLAB.

3. Group work

Naturally some students lack confidence and to some extent rely on peer support. The assessment task aimed at allowing these students to develop key skills such as problem solving and communication in a supportive environment. Many students use English as a second language and naturally gravitate towards common nationalities due to a lack of confidence. To respect the challenges many students face, and to foster a collaborative environment, they were permitted to buddy up in pairs before wider groups were formed. This allowed diverse groups of students to come together whilst they worked on a common work-related project, including digital story, and have a safety net via peer support. The Module Leader formed groups with five-members (30 submissions for this large module) and each group had to appoint a Project Leader. The Module Leader made clear from the beginning that all team members are responsible for all areas of the project, however, as a group they should decide how best to split up roles. As part of the project, students submitted a written report which addressed the project brief. In addition, each group submitted a digital story; (see below). A range of expertise was required in the areas of leadership, organisation, communication, problem solving, using mathematical software and report writing. The Project Leader represented the group when the Module Leader was involved with group matters, for example investigating non-attendance. This person also ensured the group was functioning as agreed, in line with group agreements and the project instructions. Teams were required to hold at least one meeting with all members every week and ensure minutes of the meeting were submitted to the Module Leader. Groups were not required to persistently follow up on non-attendance; this was handled by the Module Leader signposting relevant information such extenuating circumstances procedures and academic support. Collectively, groups were responsible for assigning roles within the team, agreeing subgroups to work on different sections, assigning responsibilities to individual members, determining timescales, and ensuring support was available to members who needed it. This focussed students’ attention on their area of the project and allowed the Module Leader to monitor progress. These documents were used as part of the peer moderation, facilitated using Buddy Check, when assigning individual marks for the group’s final product.

4. Digital story telling

The author's view is that embedded employability activities should provide students with opportunities to develop and reflect upon key skills. The challenges they encounter within the group environment, and their responses to them, are not only character building, but also opportunities for evidencing specific skills. This is particularly useful at, say, interviews. These experiences will be unique to individual members of the teams and provide a backdrop for demonstrating critical thinking in a specific situation. Digital story telling is one method for supporting students with articulating their impact in the group. As part of a fictional secondment, each group was required to produce a 3-minute digital story using video editing software Canva. Prior to the project, students were directed to resources from the Careers and Employability at the University of Liverpool Careers Hub. The purpose was two-fold, firstly to put many students onto the careers journey, and secondly to allow them to interact with job advertisements to discover skills relevant to appealing industries. At the end of the project, students will have had a clear idea about their priority skills and encountered a simulated employment environment to put them into action. The digital story encouraged group reflection for skills development. As part of the assessment, students were tasked with identifying three common group key skills. Using the STAR reflection model, students developed a script based on the situations that had arisen, the tasks they collectively agreed, the actions they took and the result as a direct consequence. This approach to reflection is recommended widely in the jobs market, for example, it appears as online advice from recruitment agencies due to its effectiveness and ease of use. The Module Leader's aims were to encourage students to: communicate experiences between themselves, reflect upon the group work, view their experiences as examples for demonstrating key skills, practice verbalising their achievements and improve confidence, doing this via the video recording. Canva is especially beneficial for students with low verbalising confidence since these students can record a reflection privately, as many times as they wish without peer pressure, and embed their finished contribution to the main story later. Overall, digital stories add value to embedded employability tasks and prepare students for modern interview methods using pre-recorded videos.

5. Evaluation

At the completion of the employability project, students were asked to reflect upon their change in confidence following the completion of the fictional secondment. This was divided into three categories: communication, using technology and study confidence.

1 - Communication. This category was broken down by:

- Responding to questions asked by a lecturer in front of a full lecture theatre.
- Asking lecturers questions about the material they are teaching during lectures.
- Attending an office hour to ask the lecturer a question.
- Discuss project work in groups with fellow students.

The assessment activity was designed to support students who lacked confidence. In the category of communication this represented 27 (8+19) students; see Figure 1 row 1 and row 2. Of those, 13 (2+8+3) students reported an improvement, 13 (6+7) student who remained unchanged and only 1 student who felt a little worse, demonstrating the project's value. The biggest beneficiaries were the most confident students before the start of the project in the D4 – E5 block of Figure 1. These account for 50 (15+6+4+25) students out of 71 (34+37) who reported an improvement (columns D and E).

After / Before	A A lot worse	B A little worse	C Stayed the same	D A little better	E A lot better	
1 - *Not confident at all*	0	0	6	2	0	**8**
2 - *Not very confident*	0	1	7	8	3	**19**
3 - *Neither*	0	0	9	5	3	**17**
4 - *Fairly confident*	0	1	8	15	6	**30**
5 - *Very confident*	0	0	4	4	25	**33**
	0	**2**	**34**	**34**	**37**	***107***

Figure 1. Students' responses after reflecting upon changes in confidence in communicating mathematics

The author's view is that students felt this project bridged their current studies to potential career pathways, with an appreciation for the development of key skills which could be put into action in this simulated environment.

> *"I really enjoyed the project. Went in to it with little confidence but group environment was very reassuring, and the practicality and timing resulted in a consistent urgency without an overload of pressure. I personally much prefer these types of assignments over class tests".*

Of the 17 students who felt neutral before the project in the third row, 9 students responded with no changes to their confidence. They did, however, take positives from the group environment.

> *"As much as I was already happy with group projects before this one, I really enjoyed working together with this group and it helps when others share a good motivation and work level".*

Confidence being attributed to their preference for less time-controlled assessment and more exploratory open-ended projects with their peers. Overall, the responses show the assessment activity has had a positive impact on approximately 50% of the least confident students (row 1 and 2), 47% for those neither confident nor lacked confidence (row 3), and 80% of the most confident students (row 4 & row 5).

2 - Using technology. This category was broken down by:

- Solving mathematical problems using software, e.g., MATLAB, Maple, R.
- Using other technology as part of your studies, e.g., Teams, Zoom, Outlook, Word.
- Getting satisfactory grades in modules using mathematical software.

After / Before	A A lot worse	B A little worse	C Stayed the same	D A little better	E A lot better	
1 - *Not confident at all*	0	0	1	2	0	**3**
2 - *Not very confident*	0	1	8	5	2	**16**
3 - *Neither*	0	0	7	6	1	**14**
4 - *Fairly confident*	0	0	9	18	9	**36**
5 - *Very confident*	0	0	0	5	33	**38**
	0	**1**	**25**	**36**	**45**	***107***

Figure 2. Students' responses after reflecting upon changes in confidence in using technology in their studies

A total of 19 (3+16) students reported a lack of confidence using technology prior to the assessment task; this was lower than communication (Figure 2). Of those, 9 students (2+5+2) reported an improvement, 9 (1+8) remained the same, and 1 felt worse. Peer support had a noticeable impact on the students who previously lacked confidence using technology. The Module Leader, for example, observed students taking time to explain their ideas when developing approaches to problem solving. Other group members demonstrated patience when alternative ideas were presented from different viewpoints. There was a keen sense of mutual respect during the project work. This can be seen in the open response comments.

> *"I realised other students are in the same boat when struggling with work, so I don't feel so hard on myself that it's just me."*
>
> *"Very good experience for me, especially for the video making."*
>
> *"Really enjoyed the group project :)"*

In the same way as the previous category, the data is heavily weighted towards the bottom-right of the table in Figure 2. In the author's view, this demonstrates the value that can be achieved via the introduction of authentic assessment.

> *"This project allowed me to use my soft skills to my advantage so that I could apply my theoretical knowledge in areas that I felt more comfortable and learn in areas that I was less confident."*

Overall, the responses show the assessment activity has had a positive impact on approximately 47% of the least confident students (row 1 & 2), 50% for those neither confident nor lacked confidence (row 3), and 88% of the most confident students (row 4 & row 5). The positive effect on previously confident students is higher in use of technology than the communication category.

3 - Study confidence. This category was broken down by:

- Producing your best work in coursework assignments.
- Studying effectively on your own in independent/private study.
- Work collaboratively in a group environment.
- Mange your workload to meet assessment deadlines.
- Remaining motivated throughout your studies.

After / Before	A A lot worse	B A little worse	C Stayed the same	D A little better	E A lot better	
***1** - Not confident at all*	0	1	1	0	0	**2**
***2** - Not very confident*	0	1	3	1	3	**8**
***3** - Neither*	0	1	4	4	4	**13**
***4** - Fairly confident*	0	2	11	17	11	**41**
***5** - Very confident*	0	0	5	6	32	**43**
	0	**5**	**24**	**28**	**50**	***107***

Figure 3. Students' responses after reflecting upon changes in confidence with studies

A much smaller group of students reported a lack in study confidence prior to the assessment task; 10 (2+8) out of 107 students (Figure 3). Two respondents felt less confidence, one of which, commented: "I like the module". However, no addition insights were obtained. A total of 85 (41+43) out of the 107 respondents felt more confident with their studies. The author attributes this to the authentic nature of the assessment and the realisation of their progress within their studies. The sense of achievement was supported in open response:

> *"I feel 5 about myself now. Thank you for the project. I love it."*

In the improved category, one respondent commented about their preference for open-ended projects over time-controlled examinations such as class tests.

> *"I feel that this kind of project work makes for a more sensible and meaningful method of assessment than exams. Exams have many luck factors we cannot control such as material coming up that we were less comfortable with than other material. The extra time projects give us helps mitigate these luck factors."*

Overall confidence

Overall, this case study demonstrates that the benefits of authentic assessment cannot be under-stated given the positive impact on a range of students from diverse backgrounds. Also, employability activities do not necessarily require direct involvement from employers. The Module Leader witnessed improved engagement and intriguing collaborations between local and overseas students during timetable sessions. The original intention was to support students who lacked self-confidence. Although students in this category benefited from this process, the biggest impact came from students who were already confident and, in the Module Leader's view, saw this as an opportunity to begin preparations for their careers journey. Unfortunately, some of the students who lacked confidence, and later stayed the same, did not offer any insight into why this might be the case; the author plans to explore this in future years.

6. Outlook

In the future, the author will continue to run the embedded employability project and plans to extend the evaluation by incorporating belongingness within the student community. He also plans to take steps to understand how students can be better supported with their studies. The author would be delighted to discuss this activity further with Module Leaders who have similar aspirations or would simply like to know more about this one.

7. References

Confederation of British Industry, 2017*. Helping the UK Thrive. Education and Skills Survey.* Available from: cbi.org.uk/2017.

Thomas, L., 2012. *Building Student Engagement and Belonging in HE at a Time of Change: Final Report from the What Works? Student Retention & Success Programme*. Available from: https://www.phf.org.uk/wp-content/uploads/2014/10/What-Works-Summary-report.pdf

Yorke, M., 2016. T*he development and initial use of a survey of student 'belongingness', engagement and self-confidence in UK Higher Education*. Assessment & Evaluation in Higher Education. https://doi.org/10.1080/02602938.2014.990415

RESOURCE REVIEW

Automatic Assessment of Mathematical Programming Exercises with Numbas

Chris Graham, School of Mathematics, Statistics & Physics, Newcastle University, United Kingdom
Email: christopher.graham@ncl.ac.uk
George Stagg, School of Mathematics, Statistics & Physics, Newcastle University
Christian Lawson-Perfect, School of Mathematics, Statistics & Physics, Newcastle University
Aamir Khan, School of Mathematics, Statistics & Physics, Newcastle University

Abstract

As programming has become a common feature of undergraduate mathematics degrees, there has been an increasing focus on how to teach and assess the subject to mathematicians. The potential benefits of e-assessment of basic programming exercises have many parallels with assessment in mathematics where e-assessment tools are widely used: the chance to give instant feedback to students offers an opportunity to allow students to work at their own pace, accommodating the disparate background in programming that often exists in undergraduate mathematics cohorts. And the randomisation of question content not only offers a powerful tool for practice, with students able to repeat similar problems over and over, it also can offer some protection against plagiarism in a subject where, just like a solution to some mathematical problems, student answers to identical problems are likely to be very similar. This paper considers an extension to Numbas to automatically assess programming exercises and the successful implementation of the resource in undergraduate modules using the programming languages R and Python.

Keywords: Assessment, E-Assessment, Programming, Coding, Computing, Numbas

1. Introduction

This paper considers the development of the Numbas e-assessment software to automatically mark programming exercises using the R and Python programming languages, and its application to both practice and summative assessment in two modules in the School of Mathematics, Statistics & Physics at Newcastle University. Section 2 gives some background on the use of programming and the motivation for automatically marking programming exercises. Section 3 describes the new extension to Numbas and how programming questions make use of the well-established features of the system. Section 4 gives more detail on how the new programming feature is used in mathematical programming modules, including the format of assessments and feedback from students.

2. Background

2.1 Computing in the mathematics curriculum

Modules dedicated to computer programming have been a compulsory component of the single-honours mathematics degree programme at Newcastle University since 2015. The addition of computing to the curriculum is in common with many other mathematics departments in the United Kingdom (Sangwin, 2017), motivated by the increasing relevance of computers in mathematical teaching and research, and in the future career prospects of undergraduate students.

At Newcastle University, students take dedicated computing modules at stages 1 and 2 of the mathematics and physics programmes, focussing on R and Python, with computing embedded in many modules later in the degree, such as *Mathematical Biology* and *Big Data Analytics*. At stage 1 of the mathematics programme, students take beginner courses in Python, with a focus on problem solving, and in R, with a focus on statistics, before moving on to a module on numerical methods at stage 2. Physics students follow a similar path, focussing purely on Python, following a move away from MATLAB in 2020.

The increasing focus on embedding programming into the curriculum at Newcastle emphasises the need to establish a solid foundation in the early stages of these programmes. Incoming students typically present with very different experiences and competencies with programming and computer skills in general. Some have formal qualifications, or have self-taught themselves one or more programming languages. These students are likely to find some of the content straightforward and effort is required to keep them engaged, though they typically still require a re-wiring of their programming knowledge in the context of mathematics. Other students have no programming experience or may even demonstrate high levels of anxiety about computing. Establishing a foundation requires accommodation of these disparate backgrounds and the related consequence that they work through teaching material at different rates.

For many years, the differing abilities of students has been most evident in practical sessions. The programming modules follow a structure with a one-hour lecture, followed by a two-hour practical each week, run by the module leader and a team of demonstrators. The lectures are used to introduce theory and new ideas, and give worked examples, whilst the practicals offer the chance for students to get hands-on with the programming language under supervision. This is a popular format, with students citing that they particularly benefit from seeing the module leader work through the process of sketching out algorithms, coding, de-bugging and enhancing solutions in the lecture sessions.

The practical computer sessions follow a handout describing programming commands to try out, with embedded exercises to complete as the handout progresses. Though students appreciate being given the freedom to work through at their own pace, those struggling with the content will often rush to the exercises and find it difficult to get started, often manifesting as very ‘low-level’ queries of the form “How do I start?” or “What does this mean?”, which require little more than direction to the relevant part of the handout. Others will side-step their completion of the exercises completely by gathering in a small group around a friend who is more competent. Although a teamwork approach is desirable, in this case the passive students often lose understanding and go off the trail of the handout content as a result.

What we desire is for the exercises to be accessible to everyone in the class to complete individually, and, although some of these issues can be solved with careful wording of the questions and hints, there remains a fundamental question of how you give feedback to students. At a cohort level, the timing of feedback is difficult: Solutions made available immediately can be counter-productive to students completing the work; going through exercises with the class at intervals during the practical, over a room’s A/V is often mentioned in a positive light by students, but is invariably not at the correct time for most, who will either not have reached the relevant exercise, or have gone far beyond it; and releasing solutions after a practical has finished is also of limited benefit, particularly if the mastering of exercises is essential to progressing through the handout material. Automatic assessment of these exercises affords the opportunity to give individual feedback at the correct pace for the student, and to scaffold questions or offer a hint to those struggling.

Early efforts to introduce e-assessment gave moderate success using the Numbas e-assessment system to indirectly mark exercises (Graham, 2020). Questions were presented to students to complete in the programming software, before entering a numerical value to Numbas, which used its own internal functionality to calculate a solution that could be compared to the student’s answer.

Although this still has a place amongst questions asked in the new approach, it is limited by not being able to directly assess code. The following sections build on that work to mark actual student code.

2.2 Motivation for e-assessment

Mathematical e-assessment systems, such as DEWIS (Gwynllyw and Henderson, 2009), Numbas (Foster, et al, 2012) and STACK (Sangwin, 2015) are well established and can automatically mark procedural mathematical exercises and give immediate feedback to the student. For such exercises, it is possible to establish whether a student's answer is correct, either through mathematical equivalence (the same numerical value or expression as the correct answer) or based on its properties (for example, is a root of a given equation).

A similar idea can be applied to programming exercises: though a student's method of approaching a problem may vary, just as in a mathematics problem, the expected outcome of their code is often well-defined. Consider the following exercise:

> Write a function `is_prime` that takes a natural number as input and returns a boolean: `True` if the integer is prime, and otherwise `False`.

Consider a test applied after a student's answer, for example, `is_prime(13)`, which would return the value `True` if the student's code is correct. A similar test can be applied with several different input values, sufficient to be satisfied that we can infer that the student's code is correct or incorrect.

The approach of running individual "unit tests" on an answer in this way can offer a lot more than this single point of feedback though, and the potential for running multiple tests on a student's code opens the door to rich, individual feedback. We might also ask any, or all, of the following:

- Does the student's code run without errors?
- Does a function `is_prime` exist in the workspace?
- Does the function accept a single value as input?
- Does the function check if the input is a positive integer?
- Does the function return a single, boolean value?
- Does the function give the expected answer for some test input?
- Does the function treat special cases correctly, `is_prime(1)`, for example?

Each of these can be verified with a single unit test and therefore each gives an opportunity to contribute to the marking of the exercise, or to offer feedback to the student, or both.

The idea of automatically marking programming exercises is not new, particularly to the teaching of computer science (Ala-Mutka 2005, Ihantola et al 2010). And in recent years, with the increased emphasis on programming in undergraduate mathematics teaching, tools have been adopted by some mathematics departments. These include Coderunner (Lobb and Harlow 2016), which has been used in undergraduate teaching at the University of Coventry (Croft and England 2020) and on a mathematics programme at University of Edinburgh (Sangwin 2019), and nbgrader (Blank et al 2019), which extends the functionality of Jupyter notebooks.

Whilst these pieces of software are relatively well established, we were motivated to extend the functionality of the mathematical e-assessment software Numbas to accommodate the marking of programming exercises, as described in the next section.

3. The Numbas code extension

Fast-tracked by the global pandemic in 2020, efforts to extend Numbas to automatically mark computer code have now developed into an official *extension* to Numbas.

3.1 Motivation for using Numbas

Whilst the software mentioned in section 2 may offer the functionality to mark and give feedback on computer code, there are good reasons to develop the provision in Numbas itself.

Runs on the client: Numbas is able to run and assess Python and R code entirely in a web browser, with no dependence on a server.

Familiar integration: Numbas is a system that is a familiar to students at Newcastle, used in almost every other module of their studies at stages one and two. The implementation in Numbas means that there is no need to introduce an unfamiliar interface. On a practical level, such assessments can be deployed through the Numbas LTI tool, with no further installation, server requirements or demand on the IT support teams at Newcastle.

Mixing mathematics and programming: Questions are often not exclusively based on programming code – this might be a part of a larger question, or students might be asked to interpret the output of their code. Using Numbas allows for marking code alongside other question types such as number entry or multiple choice.

Access to the many other Numbas features: Perhaps the most powerful motivation for developing an extension to Numbas was to take advantage the many features of the system that are already established and well developed, and which the systems mentioned in section 2 do not offer. These are discussed in depth in section 3.3 and include randomisation, scaffolding questions into steps, alternative answers and adaptive marking.

3.2 How the Numbas code extension works

A Numbas extension provides new functionality or changes the behaviour of Numbas questions. The functionality to mark code sits alongside extensions for statistical functions, interactive diagrams and others, as an official extension developed by the team at Newcastle University.

When included in a question, the programming extension presents students a code input, which uses the open-source Ace code editor ((Cloud9 and Mozilla, 2022), allowing syntax highlighting. Apart from this new type of input, the question interface is familiar to students who have used Numbas before, with a question prompt above the input, a submit button and an area for feedback and marking notes.

The code input box can appear as an empty area for the student to enter their solution, or the question author can give some initial content for the code box. This could be the structure of some code outlined in comments to fill in, or the first part of a solution left to complete by the student. This feature is also used to give students a full piece of code which contains one or more errors for the student to fix, to build their ‘debugging' skills.

Figure 1. A basic code part, which asks for the first element of a Python list. The question is set up with a built-in validation test, to check if the Python code runs without error, and then a single user-defined marking test, which checks the output against the expected answer.

Early versions of the extension followed a similar process to the software cited in section 2: the student's code and a set of unit tests were sent to a server, to run the code and return the outcome of the tests. Whilst this version was used extensively at Newcastle, the reliance on a server to carry out the tasks makes it a risk, in terms of robustness (if the server gets into trouble and is no longer able to respond to requests, then the assessment can no longer function), and limits the scalability of the set up: the server can only support a limited number of simultaneous users. It also has another significant disadvantage: the reliance on a server in Newcastle limited the ability to share the extension and question content in the same way as other Numbas material, to the wider community of teachers using Numbas.

In the new programming extension, code in R and Python is run in the web browser itself, with no dependence on an external server. This is both desirable, in terms of speed and robustness, and is in keeping with the Numbas project, which runs assessments entirely on the student's device. The code runners Pyodide (Pyodide contributors and Mozilla, 2019-2021) and WebR (Stagg, 2022) are both built using WebAssembly (WebAssembly Community Group, 2022).. WebAssembly is a binary instruction format compatible with most modern web browsers, allowing complex applications, including interactive programming languages, to run in a web browser environment at near native performance.

When the student's code is submitted, the code runner of the appropriate language is loaded. The student's code is combined with other code defined by the question author:

- variable definitions, allowing the student's answer to be marked according to randomisation of a question, as specified by the question author.
- a preamble, to set up anything that needs to run before the student's code, for example variables or functions that they will use in their answer.
- a postamble, executed after the student's answer, to set things up for the marking tests that follow.

- validation tests check that the student's answer is valid, for example to reject an answer that does not define a specific variable or function. These are run after a built-in validation test, which checks whether the student's code runs without error.
- Marking tests, which decide how much credit to give to the student.

The outcomes of the validation and marking tests feed into the Numbas marking algorithm, to apply credit and give feedback to the student. Figure 1 illustrates a basic question using the programming extension. The student receives immediate feedback on their work. They are also able to reveal a correct answer to the question (this feature can be disabled for summative assessment), and a worked solution or explanation can be provided.

3.3 How the code extension uses Numbas features

The functionality of accepting code, marking and presenting feedback is enhanced by a number of features in Numbas which, even though originally designed for mathematics, have very clear applications to programming exercises.

3.3.1 Alternative answers

The question presented in Figure 1 has a highly anticipated incorrect answer: students on our programme study both Python and R, where the indexing of lists and arrays begins from 1, and not 0, as they do in Python. Numbas has an *alternative answers* feature, which can be used to catch, optionally give credit, and provide feedback for specific answers that the author anticipates. In the case of the question in Figure 1, Numbas can give feedback for the case where the student answer retrieves the list element at index 1, as illustrated in Figure 2.

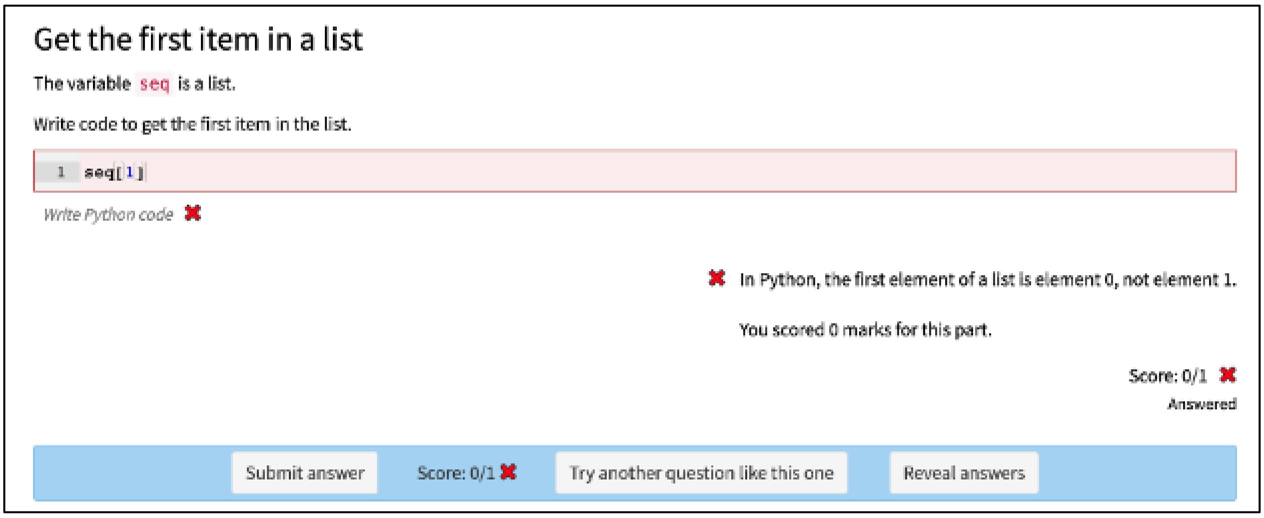


Figure 2. Alternative answer feedback provided for the anticipated incorrect answer to the question in Figure 1, where a student enters `seq[1]` (correct for some other programming languages, including R) instead of `seq[0]`.

3.3.2 Scaffolding using 'steps' and 'explore mode'

Whilst the alternative answers feature can help to give feedback on common errors, other Numbas features assist students struggling to actually get started with a question. These features have been used extensively in the formative material for the modules, including the handout exercises.

The *steps* feature has been a part of Numbas since its inception, inherited from the CALM Project for Computer Aided Learning in Mathematics (Beevers, 2003). Steps allow a basic hint to be presented to the student, for example a reminder of the syntax to use for a particular programming instruction, or of the relevant in-built function to use. Steps can also be used to scaffold a question into smaller chunks, as illustrated in Figures 3a and 3b, for an example which calculates the sum of a series of numbers through operations on numeric arrays. This is particularly useful for questions which require a more substantial block of code to be entered by the student, giving the opportunity to get feedback on each step.

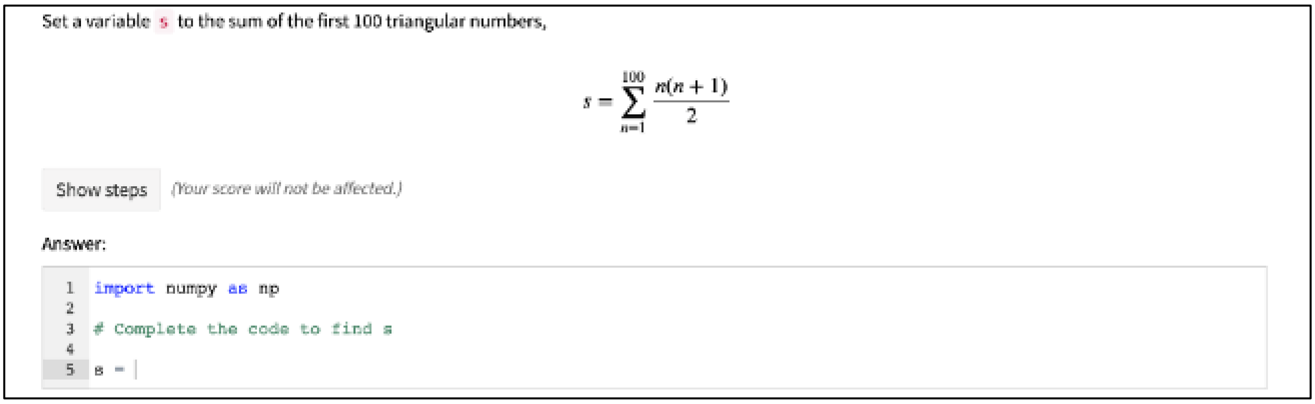


Figure 3a. A question presented as a single answer box, with a *Show steps* button. This sort of question would be typical of a handout exercise where students will often struggle to get started answering the question. The steps offer an “in” to the student and could take the form of a hint or individual answer boxes (Figure 3b).

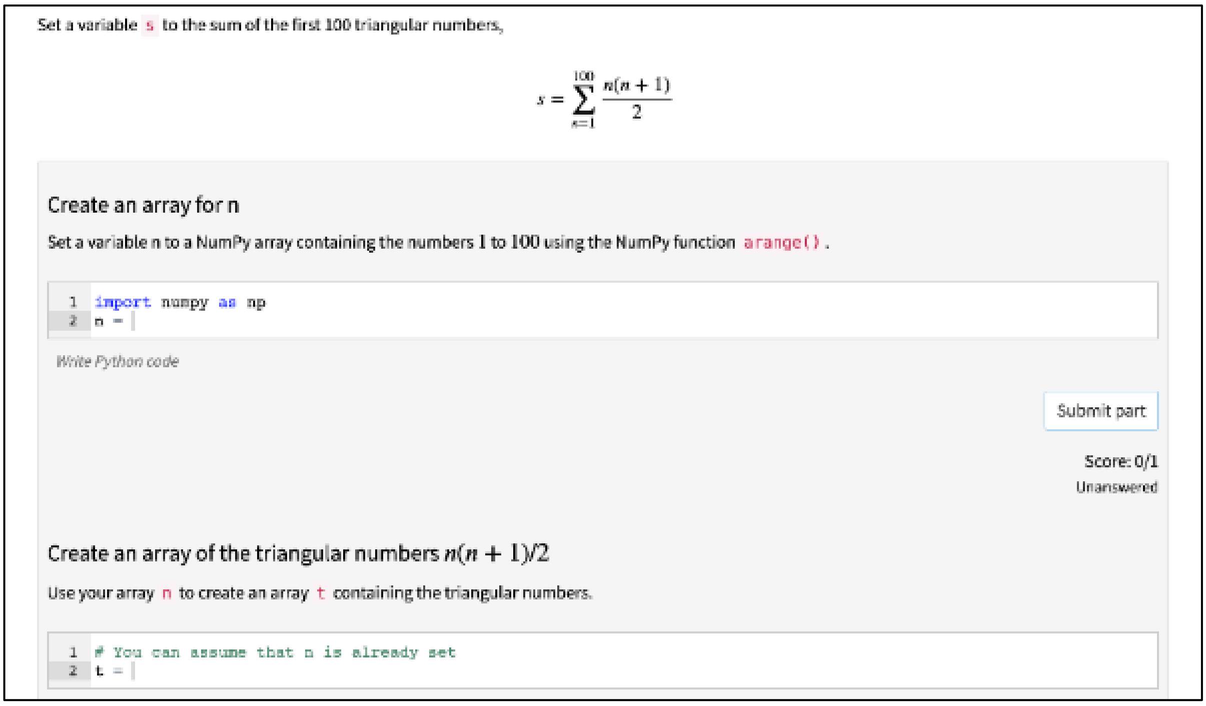


Figure 3b. The steps in this question break the task down into individual one line responses from the student. Each step is a fully-featured code question part which can give feedback to the student using marking and validation tests, and utilise other features such as alternative answers.

A similar approach to scaffolding a question can be made using the *explore mode* feature of Numbas. In this mode, Numbas presents individual parts of a question one at a time to the student, with subsequent parts that can vary depending on the choices made by the student, or their interaction with previous parts. By presenting a question in explore mode, students can be guided step-by-step through a more substantial coding task. In Figure 4, an example is given of an object representing a rectangle, constructed as a *class* in Python, in which the first part of the question asks the student to make a basic class definition. After submitting that part, they can add more code to their question in subsequent parts to add methods to calculate the rectangle's area and perimeter, and to use their class in a practical application. The question uses the variable replacement feature to include the student's code from the first part of the question as the placeholder for the second, and so on, so that they can build up a solution.

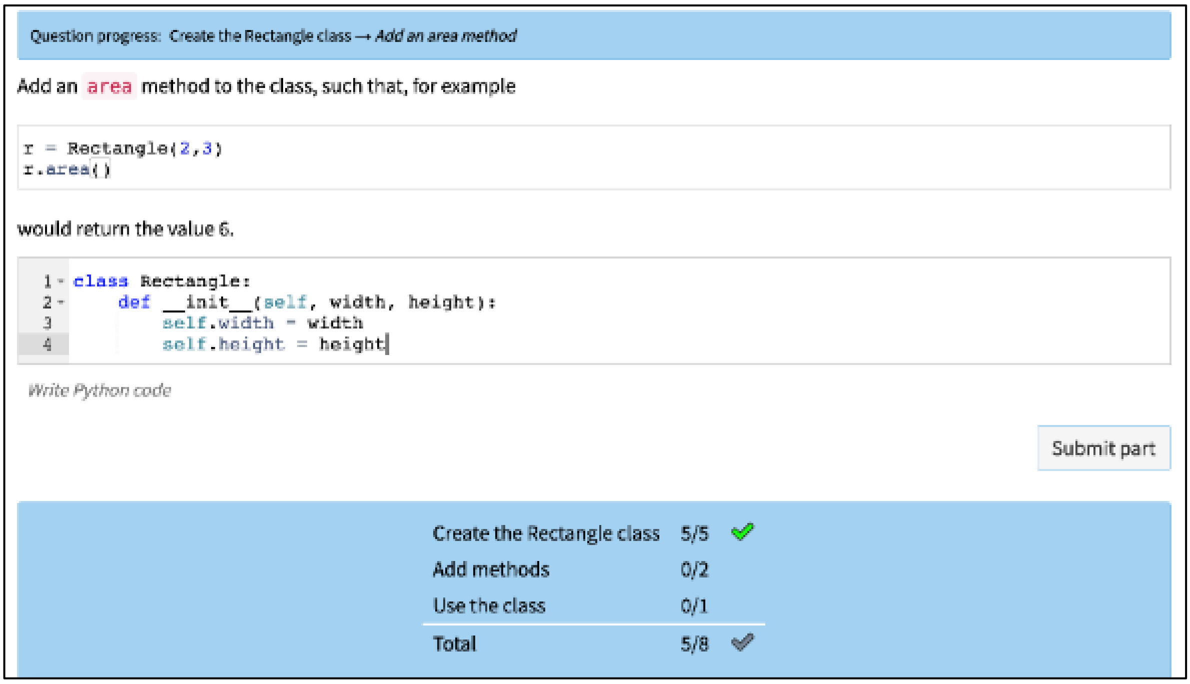


Figure 4. A question on Python classes using explore mode in Numbas. In the first part, the student is asked to create a basic definition of a class for a rectangle. Once they have successfully completed this step, the second part (pictured) asks the student to build on their existing code, adding a method to calculate the area. They can then later move on to add more methods or use the class.

3.3.3 Randomisation

Randomisation is a key feature of mathematical e-assessment, whereby similar questions generated using, for example, a different coefficient of an equation, or numeric value of a property can provide substantial practice for students in a formative mode, or to provide students with different assessment questions, encouraging students to work independently. These motivations are entirely consistent for programming questions, particularly in the context of mathematics.

Randomisation could be different data to work with, or different equations to solve numerically, or even the names of functions or variables. The randomisation of the question itself can make use of the extensive functionality of Numbas, and is passed to the marking test that is applied to the student's code. Figure 5 illustrates a basic example.

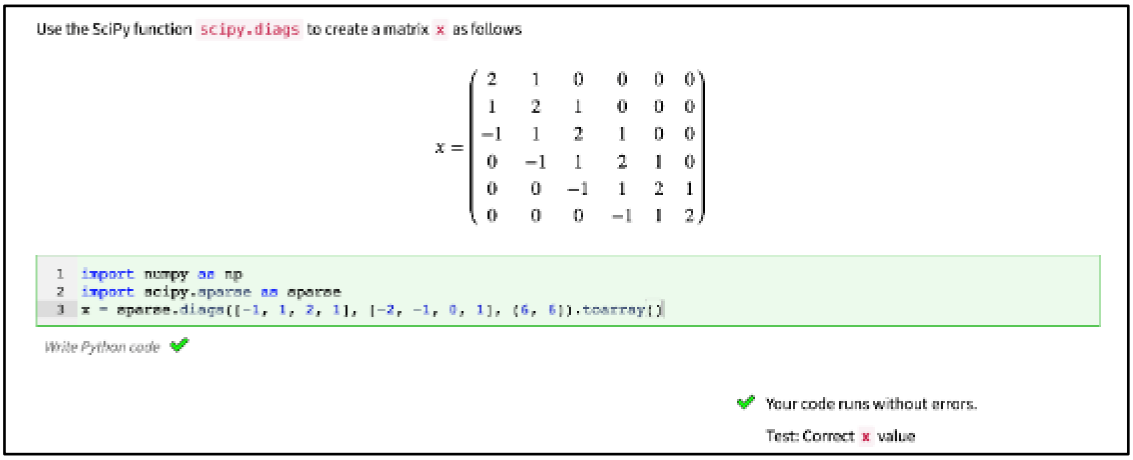


Figure 5. An example of a randomised question. The matrix is randomised to change the values and locations on the diagonals. Different variants of the question can help to reinforce the syntax of the function used to generate the matrix, or to give each student a different version, whilst assessing the same learning outcomes in an equivalent way.

3.3.4 Other part types

Asking for code input is not always necessary or the most appropriate way to assess a programming question. Sometimes a number entry box to accept the output of a computation is a good alternative. In the example in Figure 6, the student is asked for the value of the best fit coefficients of a function, fitted using a Python curve fitting function. In this case, as it is a handout exercise, there is no pressing need to ensure that they have used Python to carry out the task, and by asking for the numeric value it encourages the student to interpret the output of their code. In this case, this requires the student to understand the output of the function, but this could also be critical analysis of whether the code gives sensible values for their problem.

The question in Figure 6 cannot be easily randomised using standard Numbas functionality: it is not practical for a question author to rewrite the algorithm used by the `curve_fit` function to identify the expected answers. There is the option of a fixed question, with hard-coded data values and answers, but the programming extension offers another more sophisticated option: as part of the part's marking algorithm, Numbas can invoke the code extensionprogramming extension to calculate and set the correct numeric answers for the part, by providing it with the code for the correct answer.

In practice, in the application of the programming extension to our modules, many other part types are mixed with the code input, including number entry, mathematical expressions, multiple choice and parts which were marked offline.

Consider the following data:

```
x = np.arange(0,2,0.2)
y = np.array([0, -0.3, -1.0, -1.7, -1.9,-1.6, -0.9, -0.2, 0.0, -0.3])
```

a)

Use SciPy's `curve_fit` to fit the function

$$f(x) = a \exp\left(-\frac{(x-b)^2}{2c^2}\right)$$

What are the best-fit coefficients a, b and c?

$a =$ -1.98 *Round your answer to 2 decimal places.* ✔

$b =$ 0.78 *Round your answer to 2 decimal places.* ✔

$c =$ -0.32 *Round your answer to 2 decimal places.* ✔

```
In [1]:  opt.curve_fit(f, x, y)
Out[1]:
(array([-1.981337  ,  0.78474439, -0.31518832]),
array([[ 9.57167127e-03, -1.09701508e-06, -1.01745329e-03],
       [-1.09701508e-06,  3.22775144e-04,  3.91115264e-07],
       [-1.01745329e-03,  3.91115264e-07,  3.23836523e-04]]))
```

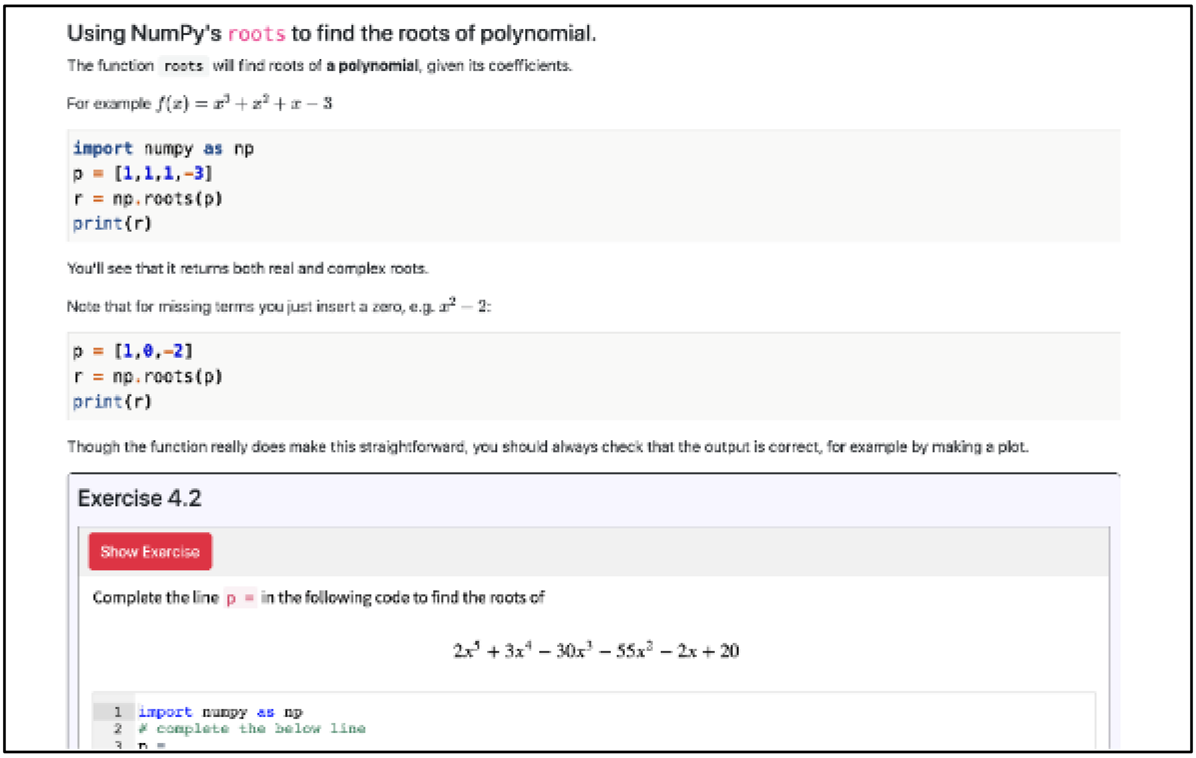
Figure 6. A curve fitting question (top) which asks for numeric values of the best fit coefficients of a given function, rather than the code to obtain them. The question requires the student to interpret the output of the function (bottom), which is a Python tuple, in which the first element is an array of the best fit coefficients, in an order consistent with that specified in their user-defined function (the second value in the tuple being a covariance matrix). Students must understand how the function constructs its output, in order to interpret it and answer the question correctly. In this case, marking the student's code may not be the most approriate means of checking their understanding.

4. Application to programming modules

Used throughout two modules in Python programming in the academic year 2021/22. 100% of question content was delivered via Numbas, though not all assessed automatically.

4.1. Practical handouts

The original motivation for developing a code marking feature to Numbas was to offer feedback on exercise questions in practical sessions. These exercises are embedded inside "virtual handouts", which have in recent years replaced physical handouts and are presented in a web-based format using the Chirun software (Stagg et al, 2022). The format allows students to seamlessly move to an exercise from the relevant handout content.



Using NumPy's `roots` to find the roots of polynomial.

The function `roots` will find roots of **a polynomial**, given its coefficients.

For example $f(x) = x^3 + x^2 + x - 3$

```
import numpy as np
p = [1,1,1,-3]
r = np.roots(p)
print(r)
```

You'll see that it returns both real and complex roots.

Note that for missing terms you just insert a zero, e.g. $x^2 - 2$:

```
p = [1,0,-2]
r = np.roots(p)
print(r)
```

Though the function really does make this straightforward, you should always check that the output is correct, for example by making a plot.

Exercise 4.2

Show Exercise

Complete the line `p =` in the following code to find the roots of

$$2x^5 + 3x^4 - 30x^3 - 55x^2 - 2x + 20$$

```
import numpy as np
# complete the below line
p =
```

Figure 7. A sample of a “virtual handout” used to deliver practical material. The handouts mix theory, commands to try out, and exercises to complete.

Using the new Numbas extension, the handouts provide instant personalised feedback that students can access at their own pace, with the opportunity to get a hint on a question that they cannot start or break it down into more manageable steps. Whilst the move towards Numbas exercises successfully allowed the modules to be delivered without practical sessions in the pandemic-affected 2020/21 academic year, the most noticeable impact has been on the running of practicals since the return to in-person teaching, where students are more self-sufficient, reducing the low-level queries for demonstrators and allowing them to focus on more meaningful conversations and focussed assistance.

Feedback from students was obtained through two evaluations: the first four weeks into the semester, to capture any early issues and suggestions, followed by a second at the end of the module. In both cases, this took the form of qualitiative, free-text feedback. The format is popular with students:

> *“The handouts strike a good balance between being accessible to the students who've never used Python as well as challenging those who have had more practise. I like the freedom of the practical sessions to work through the handout at your own pace.”*

> *“The delivery of the material is interactive and something we can work through and come back to if needs be.”*

> *“I like that I can work through the handout so that I'm learning in the best way for myself, at my own pace.”*

However, students commented that despite feedback and solutions being available in Numbas, they still like seeing the module leader go through the solutions to handout exercises live, or in a video.

4.2 Practice material

Supplementing each week's handout is a set of formative "Test Yourself" exercises available throughout the semester. These are split into three groups of questions:

- 'Warm up' questions allow an easy route into the material. They might focus on some of the key theory from the week's content presented as questions to remove the coding element. They sometimes involve simple tasks focussing on common errors: for example, students are presented with complete code that contains an error and are asked to make a fix, such that the code gives the expected outcome.

- A group of standard questions that are based on the week's handout material. These were designed to be comprehensive, covering the entire week of material even if questions overlapped with the Numbas handout questions.

- 'Bonus questions' which offer an additional challenge for those who are excelling at the module. These would usually stretch the material beyond the module content, or apply the ideas to something completely left field, for example generating pixel art using the knowledge gained from creating and manipulating 2D arrays.

Engagement with the "Test Yourself" practice material was lower than the practical handout exercises, but very high for optional material, in comparison to other modules on the programme: taking the stage 2 Python numerical methods module as an example, 76% of students tried the first set of Test Yourself exercises, with a steady decline to 50% attempting the later sets, which were perhaps superseded by the release of a mock exam.

4.3. Summative assessment

Each of the modules was structured into three assessments: the first was an assessment open for an extended period, covering the foundations of the respective modules; the second a report-style assessment; the third an off-campus class test. In all cases, assessments were open-book, so as to be more authentic, since students will rarely be programming without access to resources.

A key aspect of the summative assessment was their hybrid format, where parts of some questions presented in Numbas were not marked automatically, rather solutions were uploaded to our institution's VLE for human marking at a later date. These parts typically did not lend themselves to online marking. For example, a question on curve fitting, such as that in Figure 6, might go on to ask the student to plot the data and best fit curve. The presentation element of this part of the question is difficult to mark automatically. Similarly, in the report-style assessment, students were asked to upload their code, and feedback was given on the structure, the code efficiency and other aspects such as the use of comments and appropriate variable names.

The hybrid format was very effective in allowing the marking time to be focussed where it is most impactful, and the response from students in the evaluations was favourable:

> *"The feedback from our assignments was detailed and personal to us and gave us information on what we did well and where we can improve."*

Another Numbas feature used extensively in the summative assessments was the re-marking provision in the Numbas LTI tool, which manages student attempts. Since the introduction of programming assessments is fairly new, it was often the case that alternative approaches, deserving of credit, were identified on inspection of student attempts. The re-marking feature allows assessment questions to be updated, in this case to add additional marking tests, before attempts are bulk re-marked, ensuring fairness in marking across the cohort.

5. Future work

The academic year 2022/23 will see a full implementation of the latest client-side version of the Numbas programming extension in our Python teaching, as well as an expansion of its use for R teaching. A more substantial set of example programming questions is planned for the Numbas Open Resource Library.

6. Resources

A demonstration of the Numbas programming extension is available at:
https://numbas.mathcentre.ac.uk/exam/26300/programming-extension-demo/preview/

7. References

Ala-Mutka, K.M., 2005. A survey of automated assessment approaches for programming assignments. *Computer science education*, *15*(2), pp.83-102.

Blank, D.S., Bourgin, D., Brown, A., Bussonnier, M., Frederic, J., Granger, B., Griffiths, T.L., Hamrick, J., Kelley, K., Pacer, M. and Page, L., 2019. nbgrader: A tool for creating and grading assignments in the Jupyter Notebook. *The Journal of Open Source Education*, 2(11).

Beevers, C.E. and Paterson, J.S., 2003. Automatic assessment of problem-solving skills in Mathematics. Active Learning in Higher Education, 4(2), pp.127-144.

Cloud9 and Mozilla. Ace - high performance code editor for the web [Computer software]. https://ace.c9.io/ [Accessed 1 March 2022].

Croft, D. and England, M., 2020, January. Computing with CodeRunner at Coventry University: Automated summative assessment of Python and C++ code. In *Proceedings of the 4th Conference on Computing Education Practice 2020*, pp. 1-4.

Foster, B., Perfect, C. and Youd, A., 2012. A completely client-side approach to e-assessment and e-learning of mathematics and statistics*. International Journal of e-Assessment*, 2(2).

Graham, C., 2020. Assessment of computing in the mathematics curriculum using Numbas. *MSOR Connections*, *18*(2).

Gwynllyw, R. and Henderson, K., 2009, August. *DEWIS-a computer aided assessment system for mathematics and statistics.* In CETL-MSOR Conference 2008.

Ihantola, P., Ahoniemi, T., Karavirta, V. and Seppälä, O., 2010, October. Review of recent systems for automatic assessment of programming assignments. In *Proceedings of the 10th Koli calling international conference on computing education research*, pp. 86-93.

Lobb, R. and Harlow, J., 2016. Coderunner: A tool for assessing computer programming skills. ACM Inroads, 7(1), pp.47-51.

Lawson-Perfect, C., Foster, W., Youd, A., Graham, C. and Stagg, G. 2021. Numbas (Version v6.0) [Computer software]. Available at: https://github.com/numbas/Numbas/ [Accessed 1 March 2022].

Perfect, C., 2015. A demonstration of Numbas, an e-assessment system for mathematical disciplines. In *CAA Conference*, pp. 1-8.

Pyodide contributors and Mozilla. 2019-2021. Pyodide (Version v0.19.1) [Computer software]. Available at: https://github.com/pyodide/pyodide [Accessed 1 March 2022].

Sangwin, C., 2015. *Computer aided assessment of mathematics using STACK.* In Selected regular lectures from the 12th international congress on mathematical education, pp. 695-713. Springer, Cham.

Sangwin, C.J. and O'Toole, C., 2017. *Computer programming in the UK undergraduate mathematics curriculum.* International Journal of Mathematical Education in Science and Technology, *48*(8), pp.1133-1152.

Sangwin, C.J., 2019. *Automatic assessment of students' code using CodeRunner*. IMA Higher Education Teaching and Learning Series. https://maths.mdx.ac.uk/wp-content/uploads/2019/09/2019-6-IMA.pdf [Accessed 1 March 2022]

Stagg, G., Graham C. and Lawson-Perfect, C., 2022, Chirun [Computer software]. Available at: https://github.com/chirun-ncl/ [Accessed 1 March 2022].

Stagg, G. 2022. WebR [Computer software]. Available at: https://github.com/georgestagg/webR/ [Accessed 1 March 2022].

WebAssembly Community Group, 2022, WebAssembly System Interface [Computer software]. Available at: https://github.com/WebAssembly [Accessed 1 March 2022]

RESEARCH ARTICLE

Correct for the wrong reason: why we should know more about Mathematical Common Student Errors in e-Assessment questions

Indunil Sikurajapathi, School of Computing and Creative Technologies, University of the West of England, Bristol, UK. Email: indunil.sikurajapathi@uwe.ac.uk
Karen L. Henderson, School of Computing and Creative Technologies, University of the West of England, Bristol, UK. Email: karen.henderson@uwe.ac.uk
D. Rhys Gwynllyw, School of Computing and Creative Technologies, University of the West of England, Bristol, UK. Email: rhys.gwynllyw@uwe.ac.uk

Abstract

Students may arrive at an incorrect answer when answering a mathematical question due to several reasons, such as random errors, calculation errors or misreading the question. Such errors are sometimes referred to as Common Student Errors (CSEs). This article explains why it is important to know more about Mathematical CSEs in e-Assessment questions, using several examples encountered while conducting the CSE Project at the University of the West of England (UWE Bristol). The CSE Project at UWE Bristol began with an aim of developing a technique to detect CSEs and provide tailored feedback in e-Assessment questions delivered via Dewis, UWE Bristol's in-house e-Assessment system. In this research article, we present one important finding of this project that is related to the parameter selection(s) of e-Assessment questions which have at least one CSE. We highlight why, in this digital era, it is more vital than ever to know more about mathematical CSEs.

Keywords: Mathematical Common Student Errors, Dewis e-Assessment system, e-Assessment Parameters

1. Introduction and Background

Students may make a mistake when answering a mathematical question for a variety of reasons. For example making a mistake in their calculation, misconceptions or misreading the question. When the same error is made by several students, those errors are sometimes referred to as common errors (Rushton, 2014).

Several different terms are used in the literature to refer to either mathematical errors or misconceptions. VanLehn (1982) use the term '*bug*' to refer to a systematic error resulting from wrong steps in the calculation procedure. The term, '*mal-rule*' is used by Rees and Barr (1984) to refer to an understandable but incorrect implementation of a process resulting from a student's misconception. For example, a classic *mal-rule* students make is to answer $a^2 + b^2$ when asked to expand $(a + b)^2$. In this article we use the term Common Student Error (CSE) to refer to an error made by several students.

This article is concerned with CSEs in e-Assessments. Assessment is a key element of teaching and learning and is used widely in higher education. It enables educators to assess the extent of students' skill and knowledge and to ascertain whether students have achieved the desired learning outcomes (Stödberg, 2012). Assessments also give students the opportunity to receive feedback on their work. Race (2014) suggests that, in order for feedback to be effective, it should be available while students still remember clearly the work they were engaged in. Using e-Assessments is one way of achieving this. A comprehensive review of the advantages of e-Assessment to the student, teacher, institution and education aims can be found in Alruwais et al (2018).

The use of E-Assessment for the formative and summative assessment of procedural mathematical techniques has become standard practice in many UK higher education institutions (Sangwin, 2013). Several e-Assessment systems allow the creation of equivalent but different assessments through the use of random variables. One disadvantage is that, typically, in answering an e-Assessment question the student does not enter their intermediate workings, as would be the case for a paper-based assessments. This, together with the fact that each student takes an equivalent but different assessment, makes detecting CSEs in e-Assessment questions harder than for traditional paper-based submissions.

A technique for detecting CSEs and providing tailored feedback in e-Assessment questions has been developed for several Dewis e-Assessment questions used in a first year Engineering Mathematics module (Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021; Sikurajapathi, Henderson and Gwynllyw, 2022a; Sikurajapathi, Henderson and Gwynllyw, 2022b). This research forms part of The CSE Project at UWE Bristol (2019) and further details of the methodology used can be found in the next section.

2. Methodology

2.1 CSE data collection

For the work presented in this article we use Dewis as the e-Assessment system and a first year Engineering Mathematics (EM) Module for the data collection. Dewis (2012) is well-established, was developed at UWE Bristol by a team of mathematicians, statistics and software engineers and uses an algorithmic approach to question generation, marking and feedback. Dewis is lossless, this means that the data for every assessment attempt is recorded and stored on the Dewis server (Gwynllyw and Henderson, 2009). The EM module has used Dewis to deliver e-Assessments since 2009 and as such a huge amount of e-Assessment data is available. This, together with the fact that between 2017 and 2020 the assessment of the EM module included a controlled conditions e-examination were two of the reasons it was selected for the collection of CSEs.

The e-Assessment profile for the mathematical techniques learnt in EM, for the period of interest for the CSE Project, is as follows:

- 22 weekly e-Assessments available throughout the year, with students being allowed unlimited attempts. The e-Assessment coursework mark was calculated from the top 20 marks from these 22 weekly tests;
- A two-hour mid-module e-examination, sat under controlled conditions in January. All of the questions in this e-examination were based on questions students had already encountered in their weekly e-Assessments;
- Formative revision e-Assessments, made available to students a few weeks before the e-examination. Students were allowed unlimited attempts.

Due to a lack of computer rooms, each January e-examination was delivered to a morning and afternoon cohort of students. For each cohort, the parameters of the e-examination questions were fixed, so each cohort sat the same test. Although the official submission was via Dewis, each student was given an examination booklet for their rough workings and these were collected at the end of each e-examination.

A total of 298 and 321 students sat the January e-examination in 2018 and 2019 respectively. Output from the Dewis Reporter was scrutinised in order to select the most common incorrect answers (MCIAs) to each question on the 2018 and 2019 January e-examinations. Once the MCIAs were identified, the rough workings booklets of those students who submitted each of the MCIAs were carefully examined. Having access to the students' workings allowed us to work out what mistake(s) had been made by students resulting in each MCIA.

For each MCIA, the CSE percentage is calculated as follows:

$$CSE\ percentage = \frac{Number\ of\ CSE\ answers}{Number\ of\ incorrect\ answers}\ \%$$

If the CSE percentage is 4% or more, then that MCIA is considered as a CSE in this study.

Through this process, a bank of CSEs has been found and further details of the data collection process and results can be found in Sikurajapathi et al. (2020). Furthermore, this collection of CSEs has been taxonomically classified by Sikurajapathi et al. (2022a) using the taxonomy coding described in Ford et al. (2018) as a guideline.

2.2. CSE capture

In Dewis, the marking of each e-Assessment question, populates performance indicators (PIs). These PIs contain information on how a student has answered a question and are used to allocate marks, report outcomes and provide feedback. For example, for a question that requires one integer input, the three possible PI values would be 1 (correct), 0 (incorrect) and -1 (not answered). In order to capture the identified CSEs within Dewis, each e-Assessment question was amended and an additional PI was introduced, typically taking the value of 1 if the CSE was triggered and 0 if not. This not only allowed Dewis to provide enhanced feedback to the student to address the potential CSE (Sikurajapathi et al., 2021) but also allows the academic, through the Dewis Reporter, to identify all of the students in a cohort that made that CSE.

Since the data for every assessment attempt is recorded and stored on the Dewis server, it is possible to re-mark an assessment, for example, using an amended marking or feedback algorithm for one or more questions in that assessment. The amended CSE capture code for each question was validated by re-marking the e-examinations for the 2017-2018 cohort. This was done by checking that the additional PIs were populated for those students who had already been identified as making CSEs on the e-examination. Once this process had been completed satisfactorily, the weekly e-Assessments and revision tests were also re-marked, using the amended question code. In this research article, we present one important finding from this process, which is related to the parameter selections of e-Assessment questions which have at least one CSE. Details of the prevalence of CSEs made by EM students in e-examinations is available from Sikurajapathi et al. (2022a).

3. Results

During the re-marking of the weekly assessments for the 2017-2018 cohort, some restrictions related to the random parameter selections of the questions which have CSEs were found. Specifically, for some questions, there were particular parameters for which the correct answer and the CSE answer were the same. In these cases, in the marking of the e-Assessment, some students may have been awarded full marks and hence thought that they had answered the question correctly when in fact they had made a CSE.

In this section, several cases in which this happened are presented. For each case, we present a generic form of the question, an example of the parameter selections that lead to the correct and CSE answer being the same, the correct method of solution and the CSE. We use tilde (~) on the CSE answer to differentiate it from the correct answer.

3.1. Case 1

An instance of the first question considered is shown in Figure 3, which requires the student to find the value of the difference between two Unit Step functions at a given point (It should be noted that, the Unit Step function, $u(t)$ is equal to 1 for $t \geq 0$ and 0 for $t < 0$). The generic form of this question involves the function, $f(t) = a\,u(t+b) - c\,u(t+d)$ and the value of $f(p)$ is asked for, where parameters a, b, c, d, p are all integers, and created randomly for each instance of the question.

> The function $f(t) = 2u(t+4) - u(t+2)$
>
> where $u(t)$ represents the unit step function.
>
> Calculate the value of $f(0)$.
>
> Enter $f(0)$: [ ]

Figure 3: An instance of a question on the difference of two Unit Step functions

One CSE has been identified related to this question. This CSE occurs by assuming that the unit step function, u, is equal to 1 and is not a function. Whilst re-marking the weekly tests, it was noted that for some parameter values, the correct answer and the CSE answer for this question were the same. This occurs for example, when $a = 2,\ b = 7,\ c = 5,\ d = 1$ and $p = 4$. For this particular parameter selection, the correct answer and the CSE answer can be calculated as shown in Figure 4 and both are equal to -3.

Correct Answer	**CSE Answer** ***CSE:** taking the unit step function, u, to be equal to 1 and not a function.*
$f(4) = 2u(4+7) - 5u(4+1)$ $= 2u(11) - 5u(5)$ $= 2 \times 1 - 5 \times 1$ $= -3$	$\tilde{f}(4) = 2u(4+7) - 5u(4+1)$ $= 2u(11) - 5u(5)$ $= 2 \times u \times 11 - 5 \times u \times 5$ $= 22u - 25u$ $= -3u$ $= -3$

Figure 4: Workings showing the correct answer and the CSE answer of Case 1

3.2. Case 2

The second question considered here is related to the Geometric Series. Students were presented with an infinite geometric series of the form $a(r) + a(r)^2 + a(r)^3 + \cdots$, where parameters a and r are generated randomly for each instance of the question. The question requires students to calculate the sum S correct to three decimal places. One CSE was identified with this question and it occurs by finding the sum of the first four terms instead of the sum of the infinite series. An example in which the CSE answer is equal to the question's answer was found during the re-marking process and occurs when the sum of the infinite series, $S = 2 + 2(0.1) + 2(0.1)^2 + 2(0.1)^3 + \cdots$ is asked for. This is illustrated in Figure 5. As shown in Figure 5, it can be seen that, to three decimal places, both the correct answer and the CSE answer are the same in this case.

Correct Answer	**CSE Answer** ***CSE:*** *finding the sum of the first four terms instead of the sum of the infinite series*
$S = \frac{a}{(1-r)} = \frac{2}{(1-0.1)}$ $= 2.22222\ldots$ $= 2.222$ (correct to 3 dp)	$\tilde{S} = \frac{a(1-r^n)}{(1-r)} = \frac{2(1-0.1^4)}{(1-0.1)}$ $= 2.222$

Figure 5: Workings showing the correct answer and the CSE answer of Case 2

3.3. Case 3

For this case, students were asked to find the power series expansion, $P_3(x)$, of $f(x) = e^{ax}$, up to and including the cubic term, and to use $P_3(x)$, to calculate an approximate value for $f(x)$ at $x = c$, correct to three decimal places. The parameters a and c are generated randomly for each instance of the question. One of the identified CSEs of this question is to give the exact value of e^{ax} instead of the approximate value of e^{ax} at $x = c$.

It was found that when $a = 2$ and $c = -0.1$, the correct answer and the CSE answer of this question are the same, to three decimal places, namely 0.819, as shown in Figure 6.

Correct Answer	**CSE Answer** ***CSE:*** *finding the exact value of e^{ax} instead of the approximate value of e^{ax} at $x = c$.*
$P_3(x) = 1 + 2x + \frac{(2x)^2}{2} + \frac{(2x)^3}{6}$ $P_3(-0.1) = 1 + (-0.2) + \frac{(-0.2)^2}{2} + \frac{(-0.2)^3}{6}$ $= 0.818667\ldots$ $= 0.819$ (correct to 3 dp)	$P_3\widetilde{(-0.1)} = e^{-0.2} = 0.818731\ldots$ $= 0.819$ (correct to 3 dp)

Figure 6: Workings showing the correct answer and the CSE answer of Case 3

3.4. Case 4

The question for this case, required the student to find the mean value of $f(t) = a\sin(bt)$ in the interval $p < t < q$ correct to two decimal places, where the parameters a, b, p and q are generated randomly for each instance of the question. One of the identified CSEs of this question is to evaluate the mean value of $f(t)$ using degrees instead of radians in the calculation.

During the re-marking process, it was found that when $f(t) = -3\sin(5t)$ and the interval is $3 < t < 7$, the value of the mean, which is $m = -0.02$, is the same as the CSE answer, $\tilde{m}$, correct to two decimal places as shown in Figure 7.

Correct Answer	**CSE Answer** ***CSE:*** *evaluating the mean value of $f(t)$ using degrees instead of radians.*
$m = \frac{1}{(7-3)}\int_3^7 -3\sin(5t)\ dt$ $= \frac{1}{4}\left[\frac{3}{5}\cos(5t)\right]_3^7$ $= \frac{3}{20}[\cos(35) - \cos(15)]$ $= \frac{3}{20}[-0.9037 + 0.7596]$ $= -0.021615 \ldots$ $= -0.02$ (correct to 2 dp)	$\tilde{m} = \frac{1}{(7-3)}\int_3^7 -3\sin(5t)\ dt$ $= \frac{1}{4}\left[\frac{3}{5}\cos(5t)\right]_3^7$ $= \frac{3}{20}[\cos(35^o) - \cos(15^o)]$ $= \frac{3}{20}[0.8192 + 0.9659]$ $= -0.022005 \ldots$ $= -0.02$ (correct to 2 dp)

Figure 7: Workings showing the correct answer and the CSE answer of Case 4

3.5. Case 5

The question in this case involves finding the volume, V, of the solid formed when the part of the curve $y = a\,x^b$ is rotated about the x –axis between $x = f$ and $x = g$ and quoting the answer to two decimal places. The parameters a, f and g are generated randomly for each instance of the question and b is selected randomly from a pre-determined list of possible values. One of the identified CSEs of this question was to calculate V without integrating the required expression, but instead substituting the upper and lower limits directly into the integrand.

During the re-marking process, it was found that for some question parameters, the correct answer and the CSE answer of this question were the same. For example, this occurs when $a = 6$, $b = 1$, $f = 0$ and $g = 3$. In this case, the correct answer and the CSE answer ($\tilde{V}$) can be calculated as shown in Figure 8.

Correct Answer	**CSE Answer** ***CSE:** finding the volume of revolution by substituting for the upper and lower limits without integrating.*
$V = \pi \int_0^3 (6x)^2\,dx$ $= \pi \int_0^3 36x^2\,dx$ $= 36\,\pi \left[\frac{x^3}{3}\right]_0^3$ $= 36\,\pi\,[3^2 - 0]$ $= 1017.88$ (correct to 2 dp)	$\hat{V} = \pi\,[(6x)^2]_0^3$ $= 36\,\pi\,[x^2]_0^3$ $= 36\,\pi[3^2 - 0]$ $= 1017.88$ (correct to 2 dp)

Figure 8: Workings showing the correct answer and the CSE answer of Case 5

3.6. Case 6

Another identified CSE of the question presented in Case 5 was to find the volume of revolution by taking $(x^p)^q$ to be x^{p^q}. The correct answer and the aforementioned second CSE answer of this question are the same when $a = 0.6,\ b = 2,\ f = 1$ and $g = 4$. In fact, this would be the case when $b = 2$ no matter the values of a, f, g since in this case $\left(x^b\right)^2 = (x^2)^2 = x^{(2^2)} = x^4$ and from there on the workings for the CSE answer would be exactly the same as for the correct answer.

4. Resolution

Without rough workings, for the examples presented in Section 3, there is no way of ascertaining whether the student arrived at the final answer by following the correct approach or by making the identified CSE. We have resolved this issue by ensuring that, for future instances of the question, the random parameters are selected so as the correct answer and the CSE answer(s) are different. This was achieved by further amending the CSE question code. For Cases 1-5, at the parameter selection stage of the code, the correct answer and the CSE answer(s) were calculated for each set of parameters. A while loop was then used to re-select the parameters until the correct answer and the CSE answer(s) were all different to each other.

In the original question code for Case 5, b was selected randomly from the following list of values: $[0.25, 0.5, 1, 1.25, 1.5, 2]$. In order to avoid the correct answer being equal to the CSE answer identified in Case 6, the value 2 was removed from the list of possible values for b in the amended code. In addition, for Case 4, a further CSE was identified in which students neglected to divide the integral by the interval $q - p$. To avoid the correct answer being equal to this CSE answer, in the amended code q is randomly selected so that q does not equal $1 + p$.

After finding these cases, all of the other CSE question codes were amended to avoid parameter selections for which the correct answers were equal to the CSE answers. As a further precaution, the question codes were amended so that CSE enhanced feedback is provided only when the PI value of the correct answer is zero and the PI of the CSE answer is one. Thus, the respective CSE enhanced feedback is only given to students making a CSE when their answer is incorrect.

5. Discussion and Conclusion

In this article we have shown why it is important to know more about Mathematical CSEs in e-Assessment questions, using several examples. These examples were discovered while conducting the CSE Project at UWE Bristol. We have shown how a correct answer can take the same value as a CSE answer for certain e-Assessment question parameters. In such cases, there may have been instances where some students were awarded full marks and hence thought that they had answered the question correctly when in fact they had made a CSE. We have described, how we addressed this issue by amending the original question code for all identified CSEs.

There has been a significant increase in usage of e-Assessments in higher education in this millennium. Even before the Covid 19 pandemic (World Health Organization 2020), a JISC report (2020) concluded that the archaic pen and paper assessment process is in need of a technological overhaul by 2025. We believe that, in this digital era, the work presented in our research article demonstrates why it is more important than ever to know more about mathematical CSEs.

6. References

Alruwais, N., Wills, G. and Wald, M. 2018. Advantages and Challenges of Using e-Assessment. *International Journal of Information and Education Technology* 8(1), pp. 34-37. https://doi.org/10.18178/ijiet.2018.8.1.1008

CSE Project at UWE Bristol 2019. *Diagnosing and Remediating Mathematical Common Student Errors in e-Assessment Questions: A Case Study.* Available from: https://fetstudy.uwe.ac.uk/~bin-sikurajapa/dewis/cseproject/ [Accessed 24 February 2022].

Dewis, 2012. Dewis welcome page. Available from: http://dewis.uwe.ac.uk [Accessed 02 November 2022].

Ford, S., Gillard, J. and Pugh, M. 2018. Creating a Taxonomy of Mathematical Errors For Undergraduate Mathematics. *MSOR Connections*. 18 (1), pp. 37-45.

Gwynllyw, R. and Henderson, K. 2009. Dewis: A Computer Aided Assessment System for Mathematics and Statistics. *CETL-MSOR 2008 Conference Proceedings*. pp. 38-44.

JISC. 2020. The future of assessment: five principles, five targets for 2025. Available from: https://www.jisc.ac.uk/reports/the-future-of-assessment [Accessed 02 November 2022].

Race, P. 2014. *Making Learning Happen*, Sage Publications.

Rees, R. and Barr, G., 1984. *Diagnosis and Prescription in the Classroom: Some Common Maths Problems*. London: Harper & Row.

Rushton, N. 2014. Common errors in Mathematics. *Research Matters: A Cambridge Assessment publication*, *17,* pp. 8-17.

Sangwin, 2013. *Computer Aided Assessment of Mathematics*. Oxford, UK: Oxford University Press.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R., 2020. Using E-Assessment to Address Mathematical Misconceptions in Engineering Students. *International Journal of Information and Education Technology*. 105), pp. 356–361.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. 2021. Students' Perceptions of Enhanced E-assessment Feedback Addressing Common Student Errors in Mathematics. *MSOR Connections*. 19 (2), pp. 10-27.

Sikurajapathi, I., Henderson, K., and Gwynllyw, R. 2022a. *Collection of Taxonomically Classified Mathematical Common Student Errors in E Assessments – (CSE Book).* [online]. University of the West of England Bristol, United Kingdom. Available from: https://uwe-repository.worktribe.com/OutputFile/9305581 [Accessed 08 April 2022].

Sikurajapathi, I., Henderson, K., & Gwynllyw, R. 2022b. Gathering and Compiling Mathematical Common Student Errors in e-Assessment Questions with Taxonomical Classification. *MSOR Connections*, 20 (3), pp. 55-71

Stödberg, U. 2012. A research review of e-assessment. *Assessment & Evaluation in Higher Education*, 37(5), pp. 591–604.

VanLehn, K. 1982. Bugs are not enough: Empirical studies of bugs, impasses and repairs in procedural skills. *Journal of Mathematical Behavior*, 3(2), pp. 3-71.

World Health Organization 2020, Coronavirus disease (COVID-19) Pandemic. Available from: https://www.who.int/emergencies/diseases/novel-coronavirus-2019 [Accessed 02 November 2022].

RESEARCH ARTICLE

"It's so unfair" – Can we increase student perceptions of equity in the grading of group assessments by allowing them to declare a distribution of workload?

Laurence Shaw, School of Science & Technology, Nottingham Trent University, Nottingham, UK.
Email: laurence.shaw@ntu.ac.uk

Abstract

One of the most common complaints from students about taking part in group work is that the efforts of those who make the largest contribution are not rewarded fairly. One possible way to combat this is to allow students to agree on and declare a contribution split when submitting group projects, in the knowledge that their grades will be adjusted accordingly. We consider the results of a survey among students who have experienced group work graded both under this format and the standard "everyone in the group gets the same grade" approach. Quantitative analysis reveals that, in general, students may prefer the declaration of workload split approach. However, a closer analysis of free-text comments showed that feelings are often more nuanced than positive or negative. Students with social anxieties seem to be particularly conflicted by this method of assessment, with many reporting feelings of appreciation at the perception that their work is rewarded more fairly, concurrent with heightened stress and anxiety at the idea of approaching the conversation around workload split with their peers.

Keywords: group assessment, equity, anxiety, inclusivity.

1. Introduction

The benefits of group work in higher education are well established. Laal and Ghodsi (2012), described how collaborative learning could have social and psychological benefits as well as increasing academic attributes in students such as critical thinking skills. As graduate outcomes are an increasing priority of HE institutions, the need to embed employability skills into the curriculum has also increased. A study by the Confederation of British Industry found that 20% of graduate employers were not satisfied with the teamworking and problem-solving skills of graduate applicants, while 25% were not satisfied with the communication skills displayed by graduates (CBI 2017). These are three skills that are easily embedded and developed by the use of group tasks in the HE curriculum (Kornelakis 2020).

However, group work can also come with its pitfalls, and it is well established that unequal contributions or non-contributions from fellow group members, sometimes known as free-riding (Hall and Buzwell 2012), is often the biggest source of frustration for students (Aggarwal and O'Brien 2008). The most widespread solution to this issue is to introduce some level of peer assessment, in which students within a group can to some extent determine either the amount of work or level of work achieved by their peers (Topping 1998). This allows for different members of the same group to receive different grades when assessed.

Victoria (2020) provides a review of peer assessment methods and suggests that there are three major models under which peer assessment can take place. The first of these is the additive model, in which an individual's grade is determined by a weighted average of the overall group mark, awarded by the tutor, and a peer awarded mark. Victoria notes that this model has been less prevalent in recent times, perhaps because the peer mark has no relationship to the quality of the final product submitted by the group for assessment and so some academics may view this model to place too much emphasis on process rather than outcome.

The multiplier model is an adaptation that has been used and evaluated more recently (Jin 2012). Under this model, an individual's grade is calculated using the group grade and multiplying it by a ratio of the individual's peer mark in comparison to the average peer mark of their fellow group members. This ensures that the quality of the overall group submission underpins all individual grades. However, this method can be seen as opaque or overly complicated by students, depending on exactly how the multiplying factor is calculated.

The final model outlined by Victoria (2020) is the distribution model, which provides the focus of this paper. For this model, students agree as a group on the distribution of workload that they have completed for the project. An individual's final grade is determined by the group mark, but this is adjusted up or down depending on whether their workload percentage is above or below that of an even split. For example, an individual deemed to have contributed 25% would be given a grade lower, equal to, or higher than the group grade if they were part of a group of 3, 4, or 5 respectively. The extent to which group grades are adjusted if there is an uneven distribution of workload can be determined by the academic.

A study by Planas-Lladó et al. (2018) at Spanish universities found that the distribution model was well-received by students, particularly in terms of providing a fairer and more equitable grade for individuals. However, there are concerns raised around how students reach a consensus on the distribution of workload, especially if no clear guide is given for how students go about doing this. In this paper, we add to the work of Planas-Lladó et al. (2018) by surveying students at Nottingham Trent University (UK) who have completed group work that has been graded using both the standard "one grade for all" method and the distribution model of peer assessment. The aim was to determine whether the distribution model was preferable to students in general but also to find out whether students perceived any specific benefits or causes for concern from the approach. In Section 2, we describe the methods used to carry out this study. Specifically, who was surveyed, what they were asked, and how their results were analysed. Section 3 outlines the results from both a quantitative and qualitative analysis of the survey and Section 4 concludes the paper with a discussion surrounding these results.

2. Methods

The survey was open to Level 4 Forensic Science students and Level 5 Mathematics students at Nottingham Trent University in June 2021. Both sets of students had undertaken group work during the academic year that had been graded using the distribution model of peer assessment and had also had experience of standard group work, with all students receiving the same grade, at some point during their degree. The students were given no guidance about how to go about allocating a distribution of workload in either of the assessments that used this model. For group work under the distribution model, Forensics students were allocated their group, the Mathematics students were able to choose their group.

After determining whether the student was from Forensic Science or Mathematics, the following questions were asked:

1) *If you were set group work in future, would you like to have a declaration form describing the workload split?* **Yes/No/No Preference**

2) *Please rate the following statements from 1 (strongly disagree) to 5 (strongly agree) to describe your experience of using a declaration form compared to standard group work:*

a. *The grade I received for this piece of work was a fairer reflection of my efforts/performance than the grade I would have received in standard group work;*
b. *The percentage that my group agreed on for my individual contribution was fair;*
c. *Agreeing on individual percentages as a group was a significant cause of stress/anxiety.*

3) *Please use the comment box below to describe your feelings towards using declarations forms for group coursework in your own words.* (Free text question.)

Quantitative results, from questions 1) and 2) were analysed graphically. Hypothesis tests were also used to test if the proportion of respondents answering ‘Yes’ to Q1, or 4 or 5, to the Likert scale questions was significantly different from 0.5. These were two-tailed tests with p-values calculated exactly using the cumulative density functions of appropriate binomial distributions.

3. Results

The survey had 35 responses, 14 from Forensic Science, 21 form Mathematics students. Despite the differences in group allocation procedures, no significant difference was detected between the two subject areas when answering any of the questions. Figure 1 shows the results from the first question of the survey.

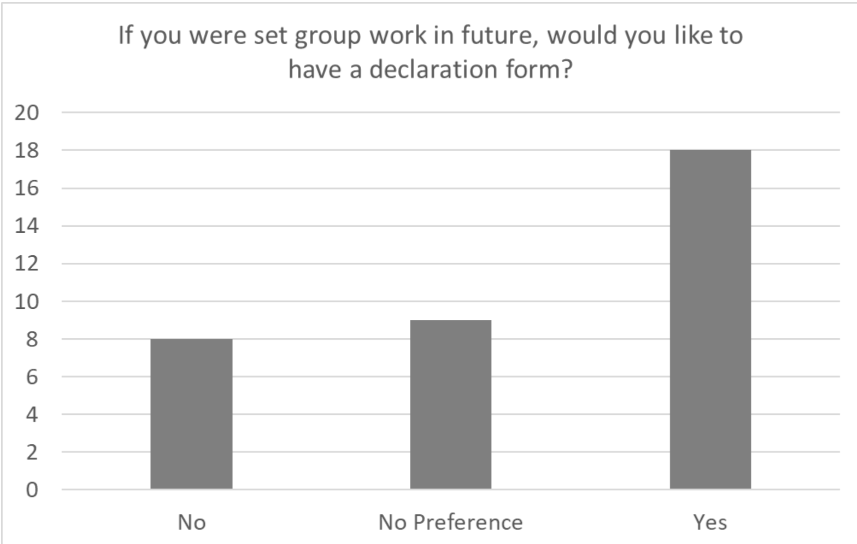


Figure 9. Answers to Question 1 of the survey, regarding whether students preferred the distribution model for assessing group work.

We observe that $18/35$ students actively preferred the distribution model for group work while only $8/35$ were against the use of the declaration form (either through opposition to the distribution model or, possibly, deeming the extra administration unnecessary). Whilst $18/35$ is clearly not significantly more than half of the students, of those who had a preference, $\frac{18}{26} \approx 0.692$ said that they preferred the distribution model to the standard model. However, even this proportion is not significantly different from 0.5 (p-value of $p \approx 0.076$) when tested using the hypothesis test described in Section 2.

Figure 2 shows the results of the Likert scale questions from the survey. Only 2 of the 35 students (5.7%) disagreed with the statement that the distribution model resulted in them receiving a fairer grade for their work than the standard model, with 17 agreeing with the statement and a further 16 not committing either way. However, 29/35 students (82.9%) agreed that the percentage contribution that they agreed with their peers was fair when they completed the declaration form for the distribution model group work. This proportion is significantly different from 0.5, giving a p-value of $p \approx 0.0001$ under our hypothesis testing procedure.

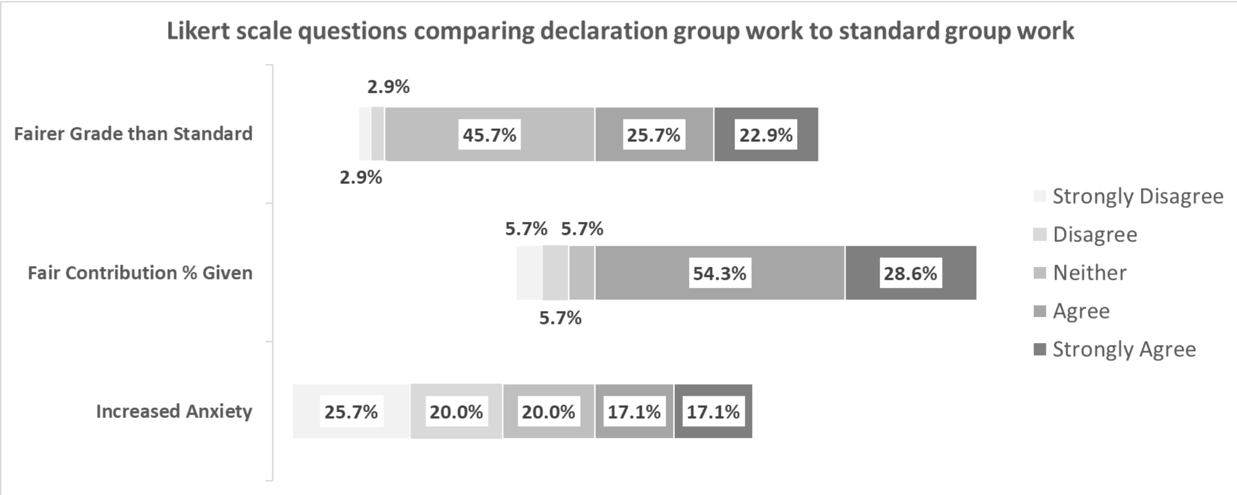


Figure 10. Answers to question 2 of the survey, comparing perceptions of different aspects of group work under the distribution model to standard group work.

The question relating to whether the declaration form led to increased anxiety about group work had a very even split with 16/35 disagreeing with the statement and 12/35 agreeing. Whilst this is slightly skewed towards disagreement, it does suggest significant diversity in the effect of a distribution model of peer assessment on the anxieties of students.

The comments made in response to the final free-text text question reflect this diversity and largely fell under three themes. Of the 28 responses to the free-text question, 13 were wholly positive. Those who had positive feelings towards the distribution model tended to focus on the fairness aspect. with comments such as “thought it was more fair and a more correct reflection on amount of work done”. The other positive was the perception that free-riding became more difficult, with one responder saying that the distribution model “means that everyone has to contribute and we don’t have to chase people to do work”.

A further 5 responses were entirely negative. All of these touched upon a common theme of fellow group members doing extra work in an attempt to gain a higher percentage of the workload distribution. One student stated that the distribution model “allows people to stab you in the back by doing more than agreed/asked for and using that against you”.

The remaining 10 responses were more nuanced. Many perceived that the distribution model was fairer but also noted that approaching the discussion around allocating the distribution was stressful. Interestingly, four respondents in this group explicitly mentioned that they suffered from some level of social anxiety. The following was a typic comment from this subgroup. “I struggle with conflict and telling people if they are doing something wrong. In my group I did a larger share of the work as other people didn't do as agreed. Therefore, I was able to get better credit which was owed. However, approaching this topic is not easy for me.”

4. Discussion

This study has considered the perceptions of UK based Mathematics and Forensic Science students on the inclusion of a declaration for group assessments, in which students agree upon a distribution of workload for a competed project. This is known in the literature as the distribution model of peer assessment (Victoria 2020) and is a means to assign individual grades for a group submission.

In general, students participating in our survey were found to be in favour of the use of this model in comparison to standard group grading. Whilst the extent of this favour was not statistically significant, it does support similar conclusions found by Planas-Lladó et al. (2018) and so this method may have benefits for improving student satisfaction scores for modules and courses containing assessed group work.

Those opposed to the distribution model mainly cited the issue of over-delivery, in which group members did more than agreed, attempting to skew the final workload distribution more in their favour. Some students also declared that they were socially anxious, and they commented that approaching the conversation around workload split was extremely difficult and stressful. However, these students also commented that they liked the aspect of having the opportunity to be fairly credited for their work. The perceptions of these potentially more vulnerable students is perhaps the biggest dilemma for academics considering the use of a distribution model in group assessment. It is easy to understand how the use of this model can create a stressful environment, and that students with anxieties may be particularly affected by this. However, the opportunity to partake in difficult and uncomfortable conversations in a professional environment could be considered valuable in adding to the teamwork, problem solving, communication, and attitude/behavioural skills that employers look for in graduates (CBI 2017). The incentive of such conversations leading to students receiving what they perceive to be a fairer reward for their efforts may mean that assessing group work under the distribution model is one of the best environments for gaining that experience, particularly for those with social anxieties.

Some students suggested in their free-text comments that they would have preferred a model in which they could assign activities to each group member and the assessor work out the exact workload split from there. This approach may remove some of the difficulties around the peer assessment conversation for the students, but it also encourages students to consider the assignment as a series of individual tasks that come together, rather than a collaborative effort, reducing some of the key benefits that group work can provide.

Instead, future research may want to focus on how students can be aided or guided in approaching the conversation around deciding the distribution. Abelson and Babcock (1985), two of the pioneers of the distribution model, suggest that care should be taken to ensure that students may be tempted to evaluate peers evenly. However, this may be seen as a positive since an even distribution may allude to a group appreciating the overall team effort and choosing to overlook minor differences in the strength of contribution. In addition, Gransberg (2010) and Planas-Lladó et al. (2018) found that many groups did declare an uneven workload distribution in their studies of the model.

Where guidance may prove useful is in addressing the over-delivery issue. For example, a tutor may want to advise students that if their peers produce what is agreed of them, or even make a concerted effort to do so, then they should not be given a lower percentage than they would receive through an even split. This may go some way to addressing the perceptions of unfairness raised by students in this study. Research into the effects of different levels of guidance would be an invaluable addition to the literature in this area to help determine whether the distribution model of peer assessment has a future in higher education.

5. Acknowledgements

The survey was carried out with ethical approval under BERS ethical guidelines. The author would like to thank the non-invasive ethics committee at Nottingham Trent University for their recommendations for improving the survey and Dr David Chappell for granting permission for the survey to be conducted on students taking his module.

6. References

Abelson, M. A., and J. A. Babcock., 1985. Peer Evaluation within Group Projects: A Suggested Mechanism and Process. *Organizational Behavior Teaching Review*, 10(4), pp.98–100. https://doi.org/10.1177/105256298601000412

Aggarwal, P., and O'Brien, C. L., 2008. Social Loafing on Group Projects: Structural Antecedents and Effect on Student Satisfaction. *Journal of Marketing Education*, *30*(3), pp.255–264. https://doi.org/10.1177/0273475308322283

CBI, 2017. Helping the UK Thrive: CBI/Pearson Education and Skills Survey 2017. London: Confederation of British Industry. Available from: https://www.cbi.org.uk/media/1341/helping-the-uk-to-thrive-tess-2017.pdf

Gransberg, D., 2010. Quantifying the Impact of Peer Evaluations on Student Team Project Grading. *International Journal of Construction Education and Research*, 6(1), pp. 3–17. https://doi.org/10.1080/15578771003590326.

Hall, D., and Buzwell, S., 2013. The problem of free-riding in group projects: Looking beyond social loafing as reason for non-contribution. *Active Learning in Higher Education*, 14(1), pp. 37–49. https://doi.org/10.1177/1469787412467123.

Jin, X-H., 2012. A comparative study of effectiveness of peer assessment of individuals' contributions to group projects in undergraduate construction management core units. *Assessment & Evaluation in Higher Education*, 37(5), pp.577-589, https://doi.org/10.1080/02602938.2011.557147.

Kornelakis, A. and Petrakaki, D., 2020. Embedding employability skills in UK higher education: Between digitalization and marketization. *Industry and Higher Education*, 34(5), pp.290–297. https://doi.org/10.1177/0950422220902978.

Laal, M. and Ghodsi, S.M., 2012. Benefits of collaborative learning. *Procedia - Social and Behavioral Sciences*, 31, pp. 486-490 https://doi.org/10.1016/j.sbspro.2011.12.091.

Planas-Lladó, A., Feliu, L, Castro, F., Fraguell, R. M., Arbat, G., Pujol, J., Suñol, J. J., and Daunis-i-Estadella, P., 2018. Using peer assessment to evaluate teamwork from a multidisciplinary perspective. *Assessment & Evaluation in Higher Education*, 43, pp.14-30. https://doi.org/10.1080/02602938.2016.1274369

Topping, K., 1998. Peer assessment between students in colleges and universities. *Review of Educational Research*, 68, pp.249-276. https://doi.org/10.3102/00346543068003249.

Victoria, M. F., 2020. Types of peer assessments in group projects. In Sanderson, L. and Stone, S. Proceedings of 2020 Teaching-learning-research: design and environments, 2-4 December 2020, Manchester, UK. AMPS proceedings series, 22.2. Manchester: AMPS [online], Chapter 5, pp.49-58. Available from: https://amps-research.com/wp-content/uploads/2021/11/Amps-Proceedings-Series-22.2.pdf.

CASE STUDY

Using online STACK assessment to teach complex analysis: a prototype course design?

Richard Gratwick, School of Mathematics, University of Edinburgh, Edinburgh, United Kingdom. Email: r.gratwick@ed.ac.uk
Steven O'Hagan, School of Mathematics, University of Edinburgh, Edinburgh, United Kingdom. Email: s.ohagan@ed.ac.uk

Abstract

We describe a new course design, informed by our experience of the pandemic, that we think could be used in other high-level mathematics courses. The course's main resource was a set of interactive STACK workbooks containing the course notes, automatically-marked comprehension and practice questions for self-assessment, and short videos of examples, calculations, and high-level motivation. This freed up synchronous class time to address conceptual understanding using interactive polling. We describe the course and discuss how it worked in practice.

Keywords: university mathematics teaching, blended learning.

1. Introduction

Some courses in the School of Mathematics at the University of Edinburgh have used blended learning techniques for many years (see, for example, Sangwin & Kinnear (2021)). Here, we use the term *blended learning* to mean approaches to teaching that "use multiple methods to deliver learning by combining face-to-face interactions with online activities" (the definition adopted by Advance HE). A blended design offers a variety of approaches to teaching and allows for a range of learning activities to be used throughout a course.

Here, we describe a course redesign which sought to optimize the choice of delivery method for each individual element of the material, exploiting the selection of technology now available, and building on both experiences of teaching during the Covid-19 pandemic and existing practice at Edinburgh. Blended learning can offer students some flexibility and agency in how they engage with a course, promoting independence and self-guided study, and opening up the provision to students for whom solely in-person delivery proves logistically difficult to attend satisfactorily (e.g. owing to caring responsibilities or health conditions). It enables instructors to encourage more active learning, by embedding in the course design regular activities and exercises with which students can engage. Certain efficiencies and better investments of time are gained by re-allocating particular content typically taught in person to recorded asynchronous enabling automatic assessment of student understanding.

2. Course description and design

We describe innovations made to the course *Honours Complex Variables* in preparation for the academic year 2021-22. This is a one-semester, 20-credit, SCQF level 10 course typically taken by students in Year 3 of a mathematics programme. The content is typical of a first course in complex analysis and includes, for example, the definition of holomorphic functions, complex integration and Cauchy's Integral Theorem, Liouville's Theorem, Taylor and Laurent series, analytic continuation, and residue calculus. 239 students took the course in 2021-22; the course was organized and lectured by Richard Gratwick, supported by a course administrator and a team of nine tutors.

In the recent past, the course was run in a fairly traditional way, having three 50-minute whole-class lectures and one 50-minute workshop (tutorial) each week. The content was based on an established set of PDF course notes, developed by Richard Gratwick from materials used by previous lecturers. In addition to the lecture notes, there were weekly problem sheets (each containing between five and ten questions) for discussion during the workshops.

Having taught the course in three previous academic years, the lecturer had identified two possible areas for improvement. First, a tendency of students to attempt only assessed problems from the problem sheets and not to engage with self-directed study expected of students at this level. Second, and related, that students seemed not to do the assigned reading ahead of classes. With this motivation, we aimed to redesign the course for long-term blended delivery in a way that (i) built on established practice within the School, (ii) maintained some positive features introduced in response to Covid-19, (iii) minimized the work needed to create new course materials, and (iv) developed a prototype design that could be used in other courses.

In the redesigned course, the main resource was a collection of Moodle workbooks based on the existing course notes. These followed the model of coherently organized digital exercises and expositions, as discussed by Sangwin and Kinnear (2021). Each section of the existing course notes became an online workbook, making it possible to use different media for different parts of the material, and creating a more active learning resource for students by including multiple-choice and STACK questions. Examples of workbook content are given in the next section.

Course activities for the first four weeks of the course are shown in Figure 1.

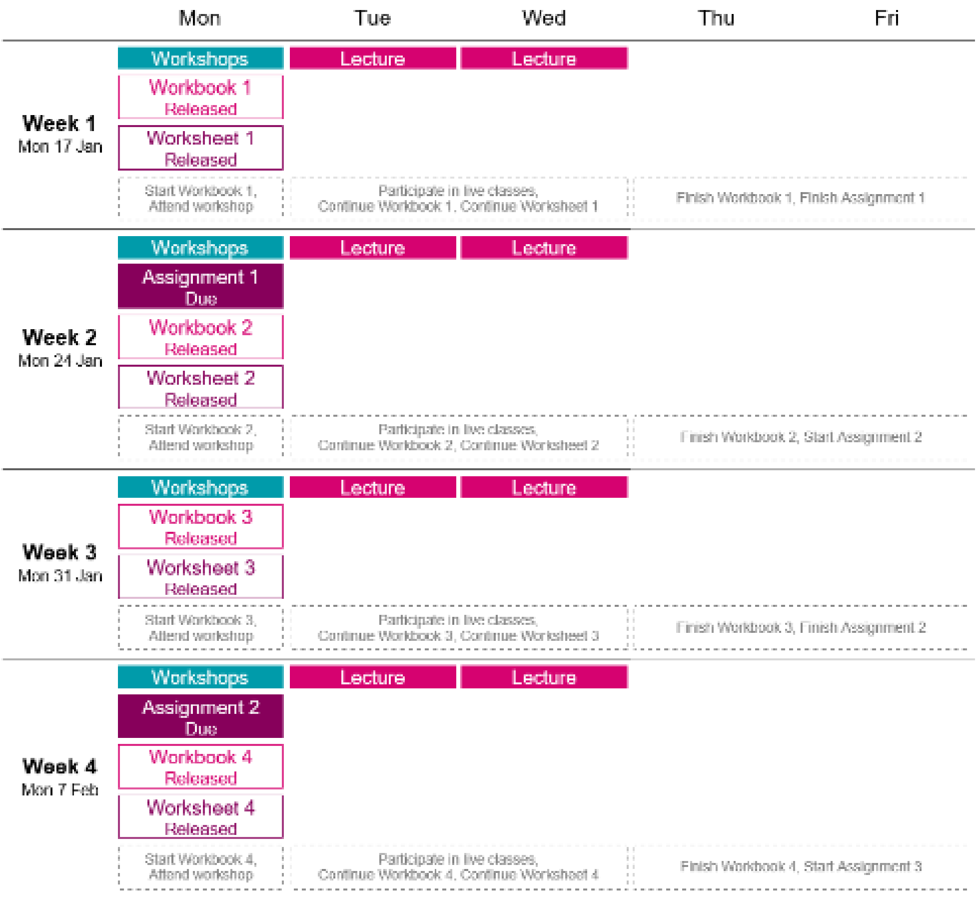


Figure 1: The structure of course activities for the first four weeks, including an illustration of what students were expected to be doing each day.

The course timetable remained largely unchanged. As before, there were weekly worksheets of problems to be discussed in weekly 50-minute workshops, and bi-weekly assignments based on these worksheets. To account for the fact that the workbooks contained computer-assessed exercises, and so students should be spending more time working through these, the number of whole-class lectures each week was reduced from three to two. In 2021-22, these lectures took place online due to pandemic restrictions. It is worth noting that students were simply encouraged to work through the workbooks, and no course credit was given for them doing so.

3. Task design

3.1 Lectures

Two 50-minute lectures were delivered each week. Institutional guidance determined that, given the size of the class and the uncertainty around the Covid-19 pandemic at the point of planning, these were delivered online. This was not a design decision and future iterations of the course will use in-person lectures (which will, as has long been standard, be recorded for students to review later if they wish).

The lectures were timetabled early in the week, so students were not expected to engage with the workbooks substantially before attending (although some did choose to do so, see below). Some content that would in previous years have been presented in lectures was moved to asynchronous content in the workbooks, for example some more routine examples, calculations, and proofs. The content of the lectures could therefore be more conceptual in nature and less involved with technical detail. The lecturer was able to spend more time motivating the subject, and highlighting connections between parts of the material both internally within the course, and beyond to other courses that many students enrolled would likely also be taking. The lecturer felt that students previously did not have much opportunity to appreciate the context of the subject within the wider discipline.

The lectures were not delivered as part of a fully flipped classroom, but rather a tilted one (Alcock, 2018). That is, that the lecturer would indeed spend substantial periods of time presenting content, albeit in limited technical detail, but also some polling was conducted during lectures to encourage active learning. Typically, one or two questions were asked of the students in each lecture. Had the lectures been in-person, these would have been accompanied by appropriate rounds of peer instruction, but we decided not to attempt this online. The chat function was used by students to ask live questions of the lecturer, which were answered either by them or, sometimes, by other students.

We would like to note one element of interactivity that arose under the students' initiative: an ad hoc "watch party" was formed which benignly (we believe) took over some of the social space available to students in the building to watch the lectures in a group of something of the order of twenty students. In this situation students did indeed discuss the polling questions with each other and engage in quite unprompted peer instruction.

3.2 Workbooks

We now discuss the content of the workbooks, which were the main resource of the course. Material was arranged in order to encourage students to be active while studying, and to support them to behave like good students would when reading traditional PDF notes. A typical pattern of content is shown in Figure 2. We see a definition followed a short discussion and video clip of a worked example by the lecturer. An automated and randomized STACK question then gives the student an opportunity to check their understanding of the material.

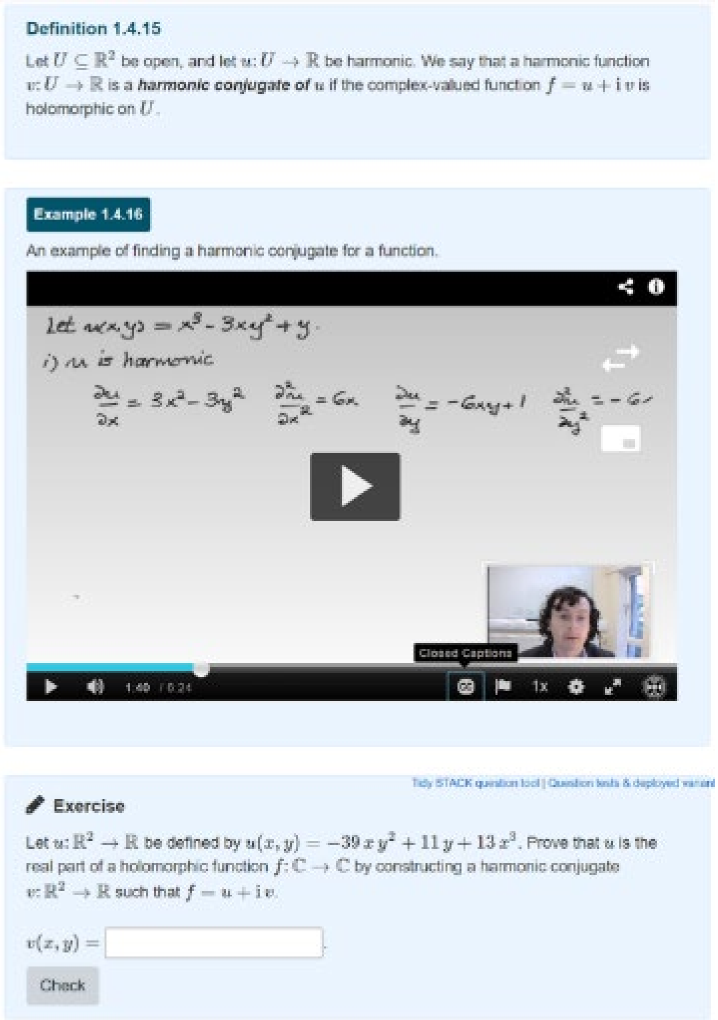


Figure 2: A typical pattern of content from an online workbook, showing a text definition, a video clip demonstrating a standard technique, and an automated STACK question to allow students to practise the technique.

This example demonstrates how an online workbook allowed us to use different media appropriately in the course materials, unlike in a static PDF file. We felt it was important that mathematics students should be expected to read definitions and results, and these were presented as straightforward text, as in traditional lecture notes. The video clip allowed the lecturer to discuss the statement and to demonstrate a method or computation, as would typically happen in a traditional lecture. A more dynamic delivery than text suits explanations of methods and spending live contact time between lecturer and students demonstrating routine computation is not necessarily the best use of that time. Pre-recorded video thus enables class time to be spent in richer and deeper discussion. The automated STACK question encouraged the student to stop and practise working with the new concept immediately. Since the question was randomized, the student could generate another question if they wanted to practise more. While the previous lecture notes regularly included printed exercises, the automatic assessment and immediate feedback of STACK clearly offer a more rewarding engagement with such exercises.

STACK is most commonly used to ask questions where the answer is a number or a mathematical expression, usually resulting from the student carrying out a computation. In the course Honours

Complex Variables, we also wanted to test the student's understanding of concepts and edge cases. Figure 3 shows an example of one way such questions were asked.

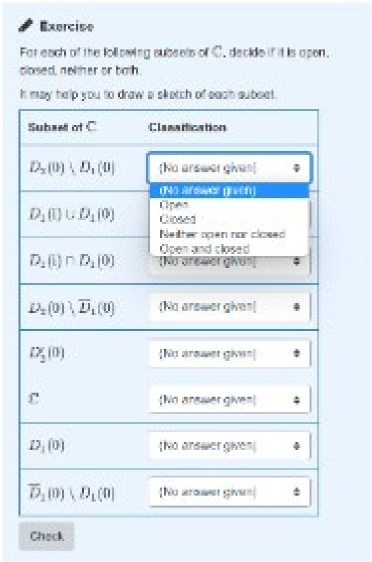


Figure 3: A randomized question designed to help students test their understanding of a new concept.

Another important feature of STACK questions is the ability to generate worked solutions tailored to the question so that students can check their method of solution or remind themselves of standard techniques. An example of a standard question and its worked solution is shown in Figure 4. Again, students had the opportunity to generate another question if they wanted further practice.

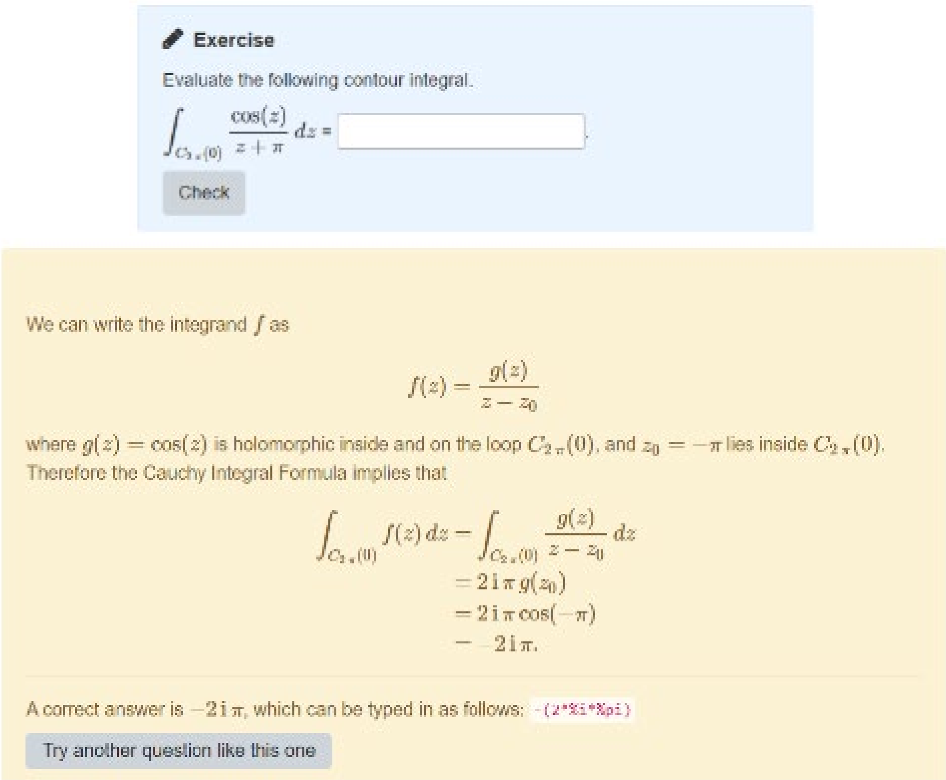


Figure 4: An example of a question and its tailored worked solution.

4. Discussion and conclusions

4.1 Student feedback and behaviour

Student feedback on the course was overwhelmingly positive, with one student responding to the end-of-course survey as follows.

"*Genuinely this course has been the perfect mix of activities for my learning, I'd go as far say to the best organised course I've taken in [the School of Mathematics], certainly this year anyway. The notes being delivered in stack are great and much more engaging than a pdf (the supplementary pdf is much easier to navigate for finding Theorems etc. however), which actually makes me do all the reading before lectures, so I gain so much more from them. Stack is good in part because of the instant feedback on most exercises which are immediately relevant to what you're learning, but also because it breaks the material up well. Stack being the main resource works perfectly with the 2 lectures delivered a week and the tutorial. [The School] should considering delivering all courses in this fashion.*"

Figure 5 shows the average percentage of students in the class attempting questions from each week of the semester. Given that no course credit was awarded for completing the workbooks, we were encouraged by how much they were used by students. Several students also made multiple attempts at a given question, taking advantage of the randomization for extra practice.

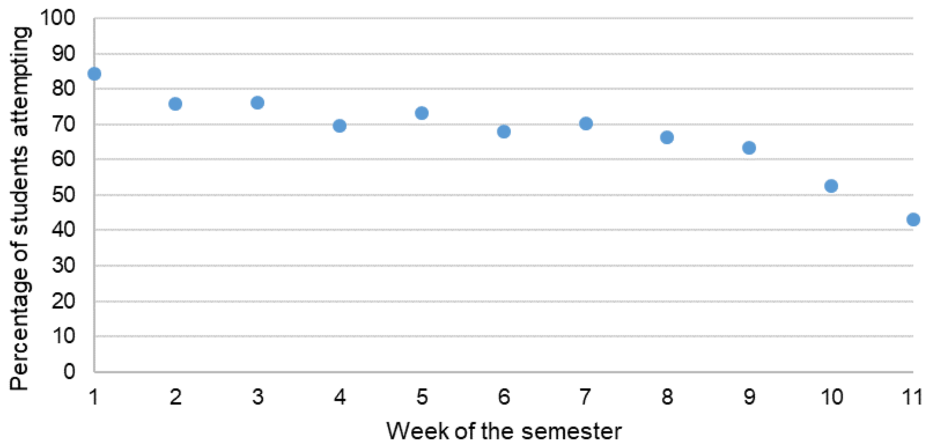


Figure 5: The average percentage of students in the class attempting questions from each week of the semester.

As a remark, we note that Moodle stores detailed data about student interactions with quizzes. It was therefore possible to track student engagement with workbooks, and to produce the plot in Figure 5. Other information beneficial to teaching could also be extracted, such as areas of common misunderstanding in the class or even a personalized report for each student.

4.2 Workload involved in creating the workbooks

We were fortunate to have the assistance of student interns Ivona Gjeroska, Maddy Baron, and Jie Xin Ng to help convert the existing course notes to the new online workbooks. They were employed for some weeks of summer 2021 on this course and other projects. They had the tasks of copying the text from the LaTeX source to the Moodle quiz platform and writing quiz questions as specified by the lecturer. The authors are grateful for the significant amount of time which this saved them on the more mundane tasks involved in the implementation of this redevelopment. This allowed us to invest more time in consideration of the structure and design of the workbooks, recording of the video clips, authoring of new questions or more sophisticated adaptations of existing questions, and rewriting of the live lecture material. The workload involved overall was substantial, but largely it was one up-front investment, and we believe the course to be in a robust position for future delivery.

In order to assist colleagues in other institutions who are interested in making similar changes to their course designs, we intend to publish workbook content online as an open educational resource. This has not happened at the time of writing but readers who would like a copy of the materials may contact the authors directly.

We are grateful to Giampaolo D'Alessandro for sharing pre-existing STACK questions on complex analysis that were created at the University of Southampton.

4.3 Concluding remarks

For the delivery in 2022-23 the online lectures shall move easily to on-campus activities using interactive polling and peer instruction, which had been an established practice in the School before the pandemic. With that modification we believe the redesign of the course to be highly successful and would like to consider the mode of delivery as a prototype for courses of the future.

5. References

Advance HE (2020), *Blended Learning*. Available via https://www.advance-he.ac.uk/knowledge-hub/blended-learning-0 (last accessed 29 November 2022).

Alcock, L. (2018). Tilting the classroom. *London Mathematical Society Newsletter*, 474, pp. 22-27.

Sangwin, C. J. and Kinnear, G. (2021). Coherently Organized Digital Exercises and Expositions. *PRIMUS*, pp. 1–12. https://doi.org/10.1080/10511970.2021.1999352

CASE STUDY

Adapting successful online activities for in-person classes - a new challenge

Ewan Russell, Department of Mathematical Sciences, University of Liverpool, Liverpool, Email: Ewan.Russell@liverpool.ac.uk

Abstract

Over the past few years, discussion across the sector has rightly been concentrated on how to provide a valuable and engaging online experience for students. The shift back to in-person classes has left many practitioners considering whether there are any lessons from the necessary shift to online teaching that can be applied to in-person teaching. This article will cover experiences stemming from a welcome but unanticipated dilemma - the live online classes for the module in question were extremely popular with students in 2020/21. How should the lecturer approach the return to in-person sessions?

Activities for live online classes were designed as consolidation "games" which sought to encourage peer learning and discussion. The positive response to these activities encouraged the lecturer to pursue a flipped classroom model for the 2021/22 academic year.

This article will discuss the various considerations when planning the transition to in-person classes for the 2021/22 academic year. In addition to reflections from the lecturer on the experience, this case study will also present preliminary findings from a formal study aiming to determine whether the activities have any positive effects on student confidence. Specifically, the study will investigate student confidence in areas such as working with peers, preparing for a class using online resources, and communicating mathematics in a written format.

Keywords: playful learning, active learning, student engagement, peer learning, polling software.

1. Background

As with most institutions across the sector, the institution in this case study had a phased return to on-campus teaching. The academic year 2021/22 could certainly be characterised as a transition year where all teaching activity was to be hybrid. Under this strategy, all core material was to be delivered via asynchronous online resources. For on-campus interactions, each module had to offer a two-hour, in-person class delivered in active learning mode. No new material was to be delivered in this in-person session, and this was certainly not be a lecture.

With national COVID-related restrictions easing further as the year progressed, in semester two module leaders were offered the option of sticking with the hybrid model or reverting to a more traditional format with three hours of lectures and one tutorial class per week (all in-person). The hybrid model was effectively a flipped approach and this was appealing to some module leaders based on the online experience of 2020/21.

This case study will focus on the adaptation of successful online activities for a Year 1, semester two module (130 students in 2021/22) into an in-person, flipped approach. The module covers elementary number theory and some initial ideas from group theory. This is a theoretical module, and mostly followed a traditional teaching approach pre-pandemic (a mixture of lectures and tutorials). The module is compulsory for all students on the BSc Mathematics and MMath programmes and optional for students on some other degree programmes offered by the department. For the online sessions in 2020/21, the author developed a playful learning approach centred on three different "rounds" of activity. These were enthusiastically received by students and this left the author in the unexpected position of considering how best to pivot back to in-person classes while retaining the success from the online experience.

The design and success of the three rounds is covered in Russell (2022). The overarching aim of the online live sessions was to consolidate material from the past week, build student confidence, and create an environment where students have ample opportunity to communicate and discuss mathematics with their peers. These activities took inspiration from recreational mathematics (Rowlett et al., 2019 and Sumpter, 2015), learning from errors pedagogy (Tulis et al., 2016 and Metcalfe, 2017), and peer learning (Kuh et al., 2006 and Zepke and Leach, 2010).

2. The in-person flipped approach for 2021/22

For the approach labelled "hybrid", every module in the Department was allocated a two-hour in-person session per week. The playful learning approach for the module in question consisted of three activities (labelled as "rounds") covered in each weekly session. At the beginning of the module, the lecturer explained the approach being taken and the reasoning for this. In particular, it was emphasised that discussing mathematics with peers is beneficial, and making mistakes when learning something new is natural (and expected). At the beginning of each round, the challenge was released on the VLE in PDF format. Students were then encouraged to discuss the particular challenge with their fellow students. Students were given 20 minutes for each of these discussions. After 20 minutes, the whole class came together again and anonymous polling was used to collect thoughts and opinions about the challenges. A summary of the three rounds is given in the table below. The activities themselves were unchanged from those used in the online year 2020/21. Evaluating how these resources work in-person, and reflecting on the additional considerations for this format are the focus of this case study.

Table 1 - breakdown of the three activities in the sessions.

Round	Focus	Format
1	Revision of theoretical ideas and simple examples from asynchronous material for the week	5 or 6 multiple choice questions covering definitions and elementary examples from the weekly material.
2	Presentation of written mathematics related to the weekly material	4 sample answers to typical questions from the weekly material. Each sample answer contains an error. Students are challenged to identify these errors.
3	Consolidation of main ideas from asynchronous material for the week	Students are provided with coordinates in decimal degree format for an attraction in or near to the city. 8 of the digits are missing - students must solve clues relating to the weekly material in order to identify the mystery location. Students must also find out something interesting about the mystery location.

The University in this study uses the Poll Everywhere platform and this was utilised in each of the three rounds to gather student views. This software has many different formats for polls (including multiple choice, open text response, upvoting and clickable image). This range of polling offers the lecturer the opportunity to diversify the methods by which they invite students to engage. The anonymity feature can also encourage student engagement.

There were some issues to consider in the transition to in-person classes and these are outlined below.

Devices for polling

The structure of the sessions is heavily reliant on students using electronic devices for polling. When the live sessions for the module were online in 2020/21, this was taken for granted as students attending were already using such a device. The lecturer made it clear at the beginning of the module that polling using electronic devices was an important part of the activities and so students should ensure that they either have someone close to them who can use such a device (smartphone, tablet, laptop) or that they speak to the admin office to secure a loan. In the end, the structure of the sessions meant that it was not essential for every student to have access to a device as the aim was for the lecturer to get a general sense of any wide-spread issues with the material following on from small-group discussions. Almost all students had a device with them for the sessions and the offer was there for students who wished to use one.

Groups for activities

In the online format, students were assigned to private channels for the activities (which they could choose to go into or not). This gave students a defined set of peers who they could work with, and this was useful. In-person, students naturally chose to sit in their friendship groups. The lecturer gave the class a series of icebreaker activities in the first session to encourage initial discussions and group-forming. The lecturer did not require students to form groups, although the benefits were clearly described. Some students still chose to work alone, and this was also the case when the sessions were online in 2020/21. In future years, the lecturer would probably like to acknowledge and reach out to students in a third category who wish to form a group but don't feel confident reaching out (even after engaging in icebreaker activities). The lecturer could offer a buddy scheme for students to sign up to.

Lecturer intervention

In the online format, the lecturer deliberately left students to discuss the activities in their private channels in an effort to create a safe space (Whitton, 2018) where they could explore the material. The lecturer found that holding back from intervening in discussions was tricky in-person - if students are online in private channels it can be easier to leave them to it while making it clear that they can always reach out for help from their channel. When students are in the room and clearly struggling, it's very difficult for a lecturer to stay away when the natural urge is to support. Students were grateful for these interventions and the lecturer took on a more active role in establishing the "safe space" for the sessions. This was achieved (in part) through a focus on discussion and debate rather than the lecturer only emphasising correct responses. The role of the lecturer in flipped classrooms becomes much more apparent in-person and the responsibility for setting the atmosphere sits firmly with the lecturer (and is essential for success of the flipped model). The lecturer needs to make the class feel comfortable with their presence in the room while students hold their small-group discussions.

3. Evaluation

The in-person sessions for the module were well-attended with an average 60% attendance. For comparison, the average in-person session attendance across all core semester two Year 1 Mathematics modules was under 50%. Students expressed their opinions on the module in two formal surveys (response rate 57%). Students were invited to complete one survey at the beginning of the module before they had engaged in any of the activities (Week 1), and another survey at the end of the module (Week 12). The surveys aimed to establish student confidence in several key areas. The baseline was established with the first survey and any changes in confidence would be observed in the second survey (after students had engaged in the activities). The surveys were a mixture of 5-point Likert scale questions and free-text response questions. For the Likert scale questions, student self-assessed confidence was measured from "1 (not confident)" to "5 (very confident)." The main results are given below.

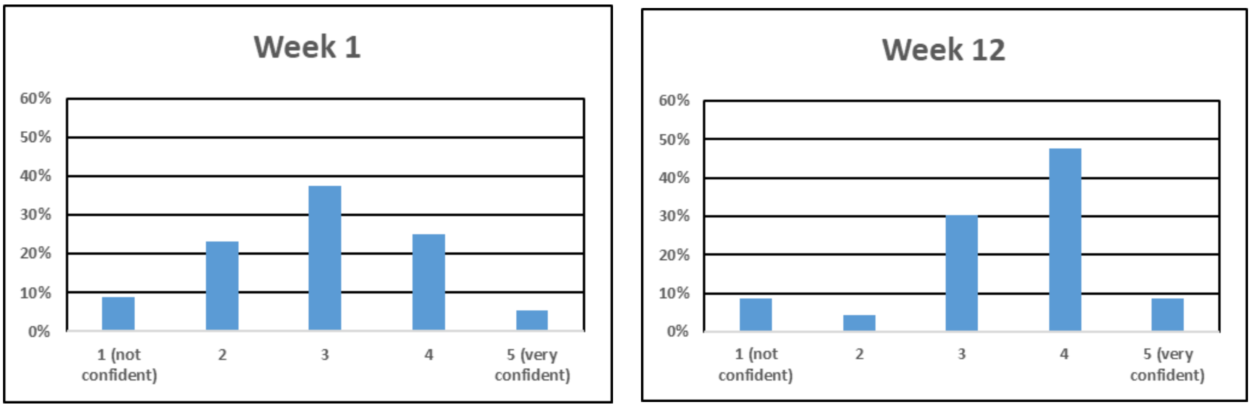


Figure 1. Responses to the question "How confident do you feel tackling a maths problem you have not seen before?" (Week 1 and Week 12)

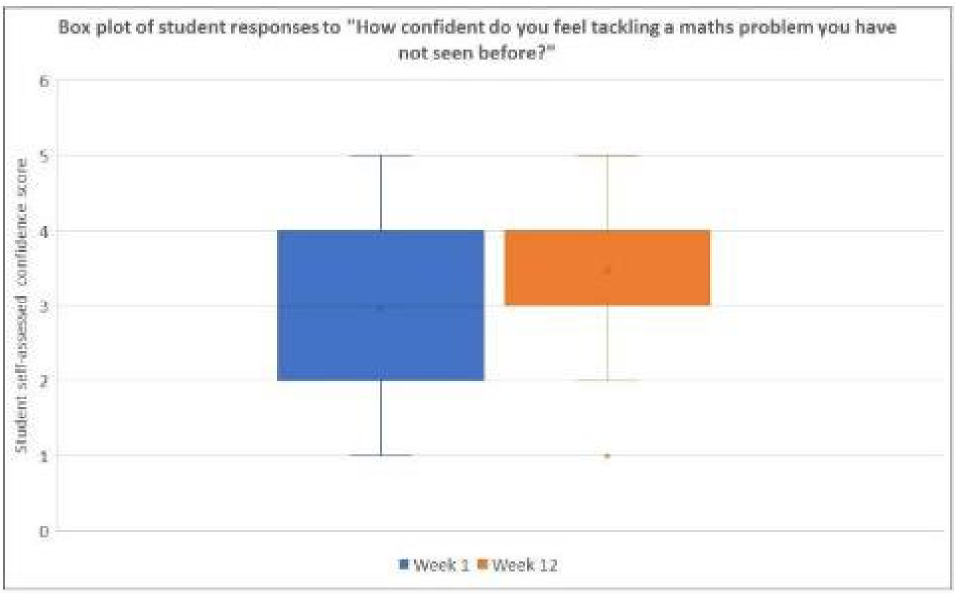
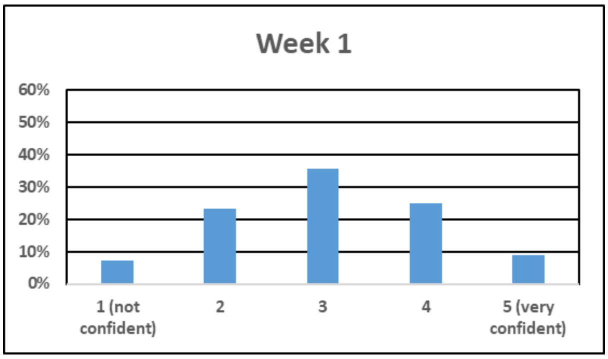


Figure 2. Box plot of student responses to "How confident do you feel tackling a maths problems you have not seen before?" (Week 1 and Week 12).

As can be observed in Figure 1 and Figure 2, there are positive shifts in confidence from Week 1 to Week 12. While over 30% of responses were "1" or "2" (at the lower end of the scale) in Week 1, only 15% responded "1" or "2" in Week 12. The proportion of "4" or "5" responses was 30% in Week 1 and 60% in Week 12. This positive shift is clearly observed in Figure 2. In particular, the first quartile moves from 2 to 3 between Week 1 and Week 12 and the median shifts from 3 to 3.5 over the same period.



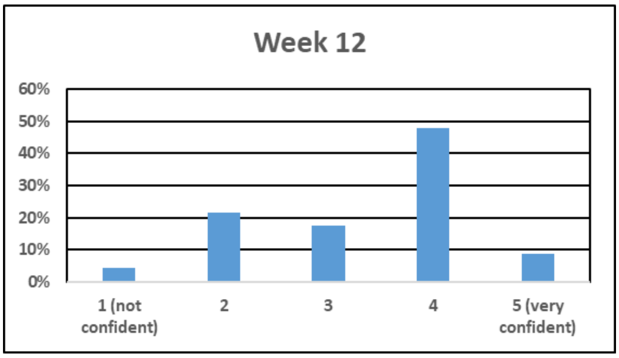


Figure 3. Responses to "How confident do you feel explaining mathematical ideas to others" (Week 1 and Week 12)

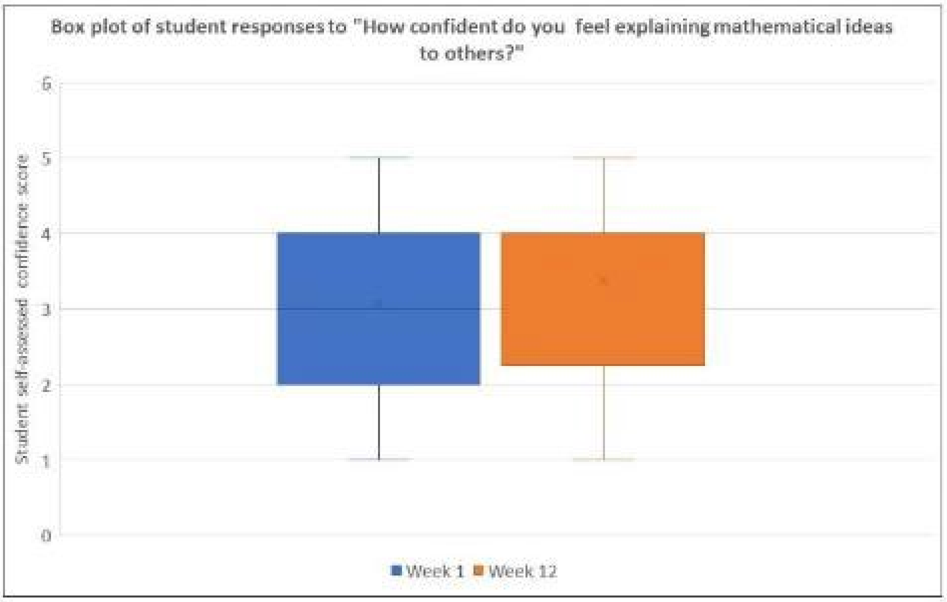


Figure 4. Box plot of student responses to "How confident do you feel explaining mathematical ideas to others?" (Week 1 and Week 12).

Positive shifts are seen in the Figure 3 and Figure 4 responses. - 60% of responses were "4" or "5" (at the higher end of the confidence scale) in Week 12 compared with 35% in Week 1. It should be noted that the responses at "1" or "2" for this question did not seem to move at all (very similar proportions in Week 1 and Week 12). As the format did not require students to engage in group discussions, these responses could be attributed to those working alone in the sessions. Further investigation could confirm this. There could be other experiences outside of this module enhancing this particular skill. Although no other Year 1 modules offered by the Department had the weekly focus on discussion that this module utilised. The box plot (Figure 4) shows that, despite a clear positive change from Week 1 to Week 12, the shift from Week 1 to Week 12 is not as pronounced for this area of confidence. The first quartile and median both show small increases from Week 1 to Week 12.

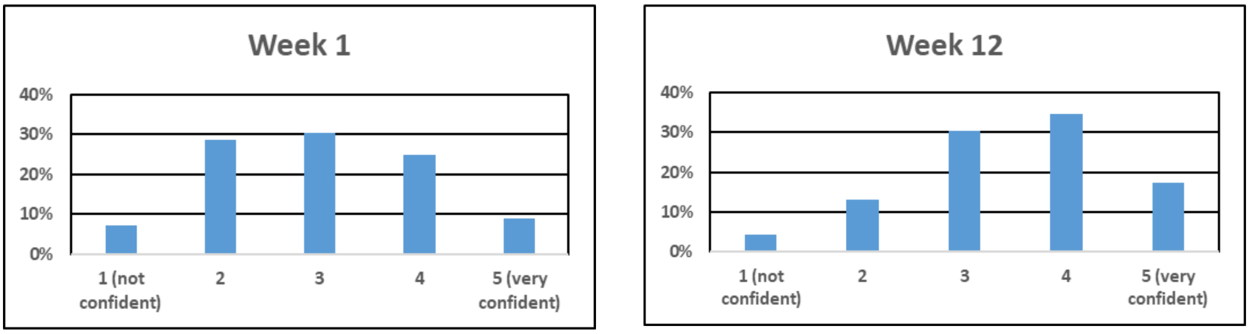


Figure 5. Responses to "How confident do you feel writing out solutions to mathematical problems properly?" (Week 1 and Week 12)

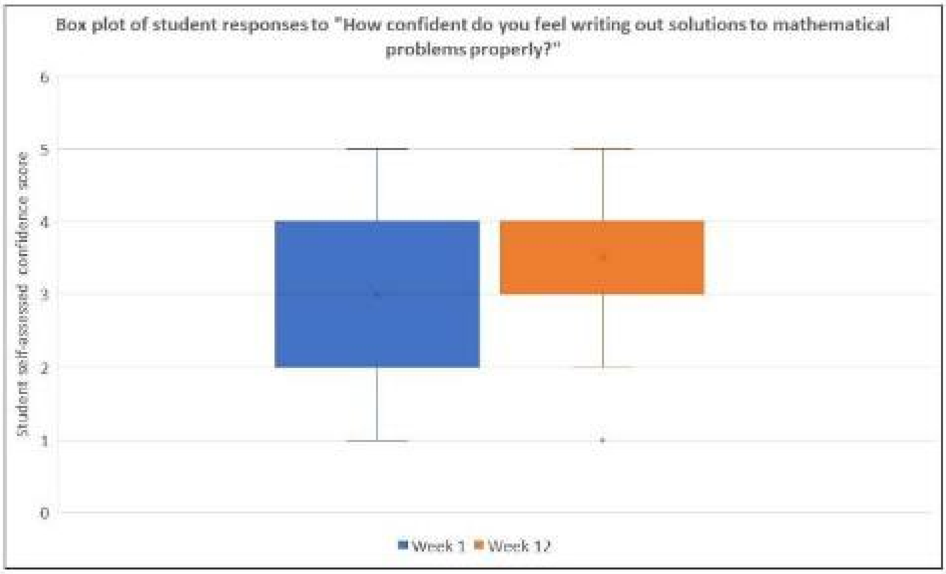


Figure 6. Box plot of student responses to "How confident do you feel writing out solutions to mathematical problems properly?" (Week 1 and Week 12).

Again, some positive changes are observed in Figure 5 and Figure 6 from Week 1 to Week 12. Round 2 had a focus on the presentation of mathematics (identifying errors in mathematical argument or presentation) and so it is pleasing to see that there is some positive change in confidence by Week 12. With this area of confidence, there appears to be movement between Week 1 and Week 12 from the lower confidence end of the scale ("1" and "2") to the higher confidence responses ("4" and "5"). In particular, it should be noted here that the proportion of neutral responses ("3") does not move much between Week 1 and Week 12 so it would seem that there is some shifting here directly from low confidence to high confidence. This is a similar to the results observed in Figure 1 and Figure 2.

In addition, students were asked how much time they spent working with the asynchronous online resources before class each week. The results can be seen in Figure 7.

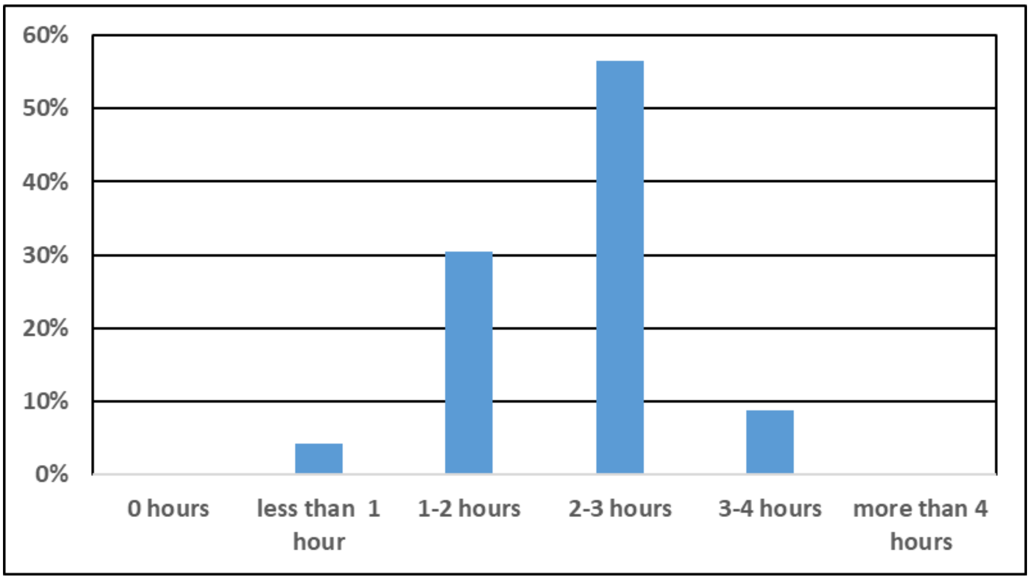


Figure 7. Responses to "How long did you spend studying the online resources before each class (on average)?"

As watching all the video resources alone for a given week takes one hour, it is perhaps concerning, but not necessarily surprising, that very few students (under 10%) acknowledge that they are spending more than three hours studying in advance of the classes.

When it comes to working with others in the sessions, a mixed picture emerged, as can be seen in Figure 8 below.

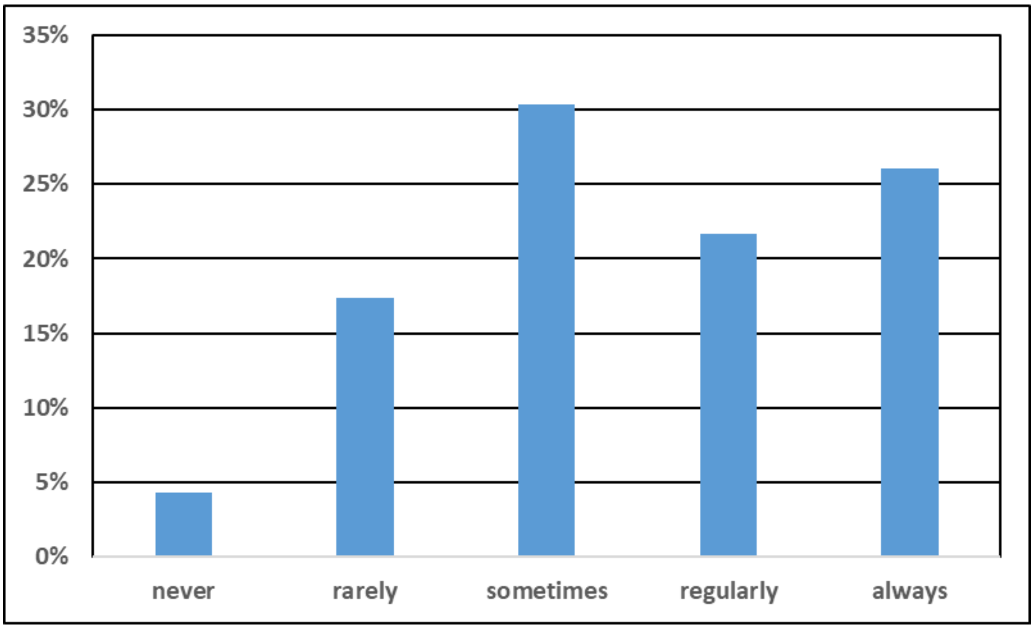


Figure 8. Responses to "I worked with other students in the weekly live sessions..."

With the adopted format on working with others (encouraged but not required), it is unsurprising to see that a mixed picture emerges from the responses in Figure 8. It is interesting to note that the "sometimes" response was given by over 30% of respondents - could this indicate that some students found it useful initially to work with others but then decided to work alone, or vice-versa? Perhaps the students in this category started working with others but their group members stopped attending at some point in the semester and they did not form new groups. It would be interesting to investigate this further.

When asked to consider if they had learned from their peers, the results can be seen in Figure 9 below. Again, a mixed picture was expected from the responses to this question as students were not required to.

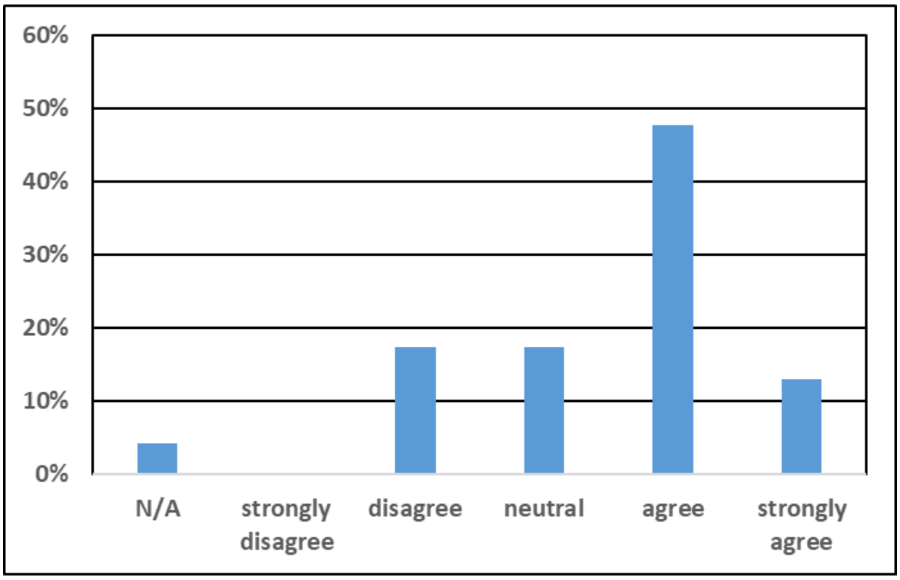


Figure 9. Responses to "I learned from working with other students in the weekly live sessions"

Obviously it is pleasing to see that over 60% of respondents believe that they have learned directly from the experience of working with their peers in the sessions. Given that over 20% of students stated that they did not regularly work with others (Figure 8), this seems like an even more impressive result.

When asked about the activities, students again were very positive (Figure 10). All rounds were popular, with under 10% of respondents believing that any of the three rounds were not very useful. For each round, over 70% of respondents believed that the round was "quite useful" or "very useful".

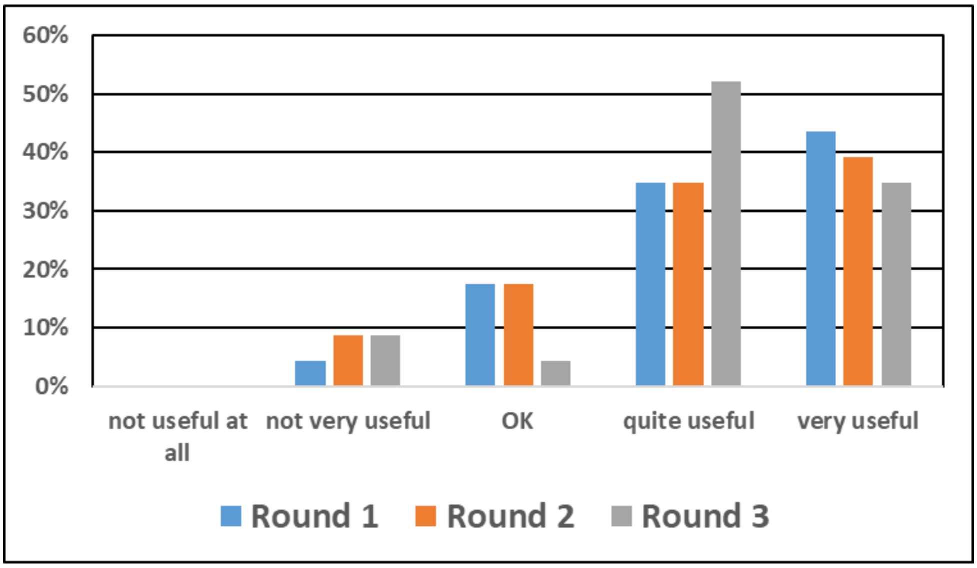


Figure 10. Responses to "Round X was generally..."

Student comments

The free-text comments in the second survey were very illuminating. Some respondents indicated a preference for one round over the others. A couple of particularly interesting comments are given below.

> "(Round 2) was the best because we got to see common mistakes and learn what about these answers was wrong, improving how we tackle questions."

> "Round 1 set me up to tackle the other problems in the next rounds. If you removed Round 1, I wouldn't be able to do the other rounds."

These comments indicate that the structure and order of the rounds was appreciated and necessary for the overall success of the strategy.

Another comment indicated that some students enjoyed one of the rounds more than the others, but also appreciate that liking an activity and finding an activity useful may not always be the same thing:

> "Round 2 was the best. My favourite round was Round 3, but Round 2 was definitely the most useful."

In line with the consolidation and confidence-building aims of the sessions, the comment below underlines what success looks like for this approach.

> "This module has been the best I have taken. After each session I normally feel very confident with the topic unlike in other modules. It is the session I look forward to each week."

Inevitably there are some down sides with this approach. Like all other modules offered by the Department, attendance dropped off towards the end of the semester. As discussed above, there are some students for whom the activities did not work (were deemed "not very useful"). A clear majority of students were very positive about the activities, but it should be noted that there are some students for whom the dial did not shift after engagement with the activities.

4. Reflections on the future of in-person teaching

The department in this case study is moving forward with a "Flipped Classroom Framework". This framework allows module leaders to follow a non-traditional model for their teaching activities utilising asynchronous online resources. For modules operating under the flipped model, there are two 2-hour sessions per week. One of these blocks should be used as an active learning session and the second is an optional supported study session where students can work on problem sheets in small groups or ask questions to the module teaching staff. The structure of the active learning session is not dictated, and the module leader is free to design this as they wish under the proviso that a student-centred approach is adopted and no new material is introduced. Module leaders who do not wish to follow the flipped approach are able to pivot back to the more traditional approach of three hours of lectures and a one-hour tutorial per week.

It should be noted that the future of in-person classes is the subject of much debate across the sector at the moment, with criticism for even considering flipped from some academics (Kapur et al., 2022, for example). Authors such as Nordmann et al. (2021) justify the case for retaining lectures in the "new normal" but on closer examination, the definition which Nordmann et al. use for a lecture may be unfamiliar to some mathematics academics. Although opportunities for student interaction and engagement are encouraged in all institutions and all disciplines, the FILL+ study, for example, found that mathematics lecturers spend over 70% of the class time talking and under 3% of the time asking questions to the class on average (Kinnear et al., 2021). It seems that Nordmann et al.'s definition is in fact that of a "good" lecture with student engagement and interaction as a core aim of the activity. With this in mind, the approach given in this case study could be classed as flipped with active learning in-person sessions, but the lecturer is still talking for around 30% of the class time when the whole room is brought back together to discuss each of the three rounds. Perhaps a more appropriate way to move forward is not to label sessions as "active learning" or "traditional lecture", but to start from the perspective of "what opportunities are there for students to engage with the material in class and how much time is allocated to this?" A binary perspective on "lecture or active learning session" could be unhelpful for the range of approaches and a closer examination of the various interpretations of "lecture" highlights this. There is a similar risk that the "active learning" label indicates to a subset of academics that students are just left to their own devices on a set of problems for the entire session. We should be aware that there are extreme interpretations of "lecture" and "active learning session".

Kapur et. al (2022) argue that there is too much variability in flipped classroom approaches with the classification becoming open to individual interpretation. Kapur et. al emphasise their opinion that similar effects (in terms of outcomes) can be better achieved through a traditional lecture-based approach including student engagement. In addition, Kapur at. al believe that flipped approaches simply perpetuate passive learning. In support of some issues raised by Kapur et al., the author agrees that active learning is the most important component. The nature of flipped requires asynchronous online resources and students have been unanimously positive about the provision of these high-quality resources. This component of a flipped strategy clearly has benefits in terms of accessibility. The author's approach to the class time under flipped is very much focused on consolidation and does not assume that students are already at a pre-determined "baseline" of knowledge after engaging with the asynchronous resources. Students in 2021/22 were attending the in-person classes even when they openly admitted they had barely engaged with the resources for the particular week and were playing catch-up. These students came because they still saw benefits in the classes and felt that these gave them the push to get caught up with the material. From this perspective, an approach to flipped which results in students being less likely to give up on the module seems like a positive outcome. The author's approach places a clear structure on the active learning sessions and ensures that students are constantly engaged and not lingering for too long on one particular activity. The approach here does not accentuate failings but instead encourages discussion / debate and engagement with short "do-able" challenges related to the material. The aim is that the active learning sessions should act as a springboard and confidence-boost for students to tackle more challenging questions on the weekly problem sheets.

Future plans at the institution in this case study include collating and sharing experiences from module leaders who have adopted the flipped model (around half of all mathematics modules at the institution will be delivered in flipped format this academic year). As students will have a mix of more traditional and flipped teaching experiences, it will be interesting to investigate how students are responding to these different approaches.

5. References

Kapur, M., Hattie, J., Grossman, I., and Sinha, T. (2022). Fail, flip, fix and feed- Rethinking flipped learning: A review of meta-analyses and a subsequent meta-analysis, *Front. Educ.*7:956416, https://doi.org/10.3389/feduc.2022.956416

Kinnear, G., Smith, S., Anderson, R., Gant, T., MacKay, J.R.D, Docherty, P., Rhind, S., and Galloway, R. (2021). Developing the FILL+ tool to reliably classify classroom practices using lecture recordings. *Journal for STEM Education Research*, 4, pp. 194-216, https://doi.org/10.1007/s41979-020-00047-7

Kuh, G., Kinzie, J., Buckley J. et al. (2006). What matters to student success: A review of the literature. Commissioned report: https://nces.ed.gov/npec/pdf/kuh_team_report.pdf [Accessed 20 October 2021].

Metcalfe, J. (2017). Learning from errors. *Annual Review of Psychology 2017*, 68, pp. 465-489.

Nordmann, E., Hutchison, J., and MacKay, J.R.D. (2021). Lecture rapture: the place and case for lectures in the new normal, *Teaching in Higher Education*, 22(5), pp.709-716, https://doi.org/10.1080/13562517.2021.2015755

Rowlett, P., Smith, E., Corner, A., O'Sullivan, D. and Waldock, J. (2019). The potential of recreational mathematics to support the development of mathematical learning, *International Journal of Mathematical Education in Science and Technology*, 50:7, pp. 972-986, https://doi.org/10.1080/0020739X.2019.1657596.

Russell, E. (2022). Remote active learning. *MSOR Connections*, 20(1), pp. 46-55, https://doi.org/10.21100/msor.v20i1.1308

Sumpter, L. (2015). Recreational Mathematics - Only For Fun? *Journal of Humanistic Mathematics*, Volume 5, Issue 1 (January 2015), pp. 121-138, https://doi.org/10.5642/jhummath.201501.07.

Tulis, M., Steuer, G. and Dresel, M. (2016). Learning from errors: A model of individual processes. *Frontline Learning Research*, 4(2), pp. 12-26.

Whitton, N. (2018). Playful learning: tools, techniques and tactics. *Research in Learning Technology,* Vol. 26: 2035, http://dx.doi.org/10.25304/rlt.v26.2035.

Zepke, N. and Leach, L. (2010). Improving student engagement: Ten proposals for action. *Active Learning in Higher Education,* 11(3), pp. 167-177.

RESOURCE REVIEW

The sigma Accessibility Special Interest Group: Resources Update

Ruth Hand, Mathematics Resource Centre, University of Bath, UK.
Email: rah202@bath.ac.uk
Ciarán Mac an Bhaird, Department of Mathematics and Statistics, Maynooth University, Ireland.
Email: ciaran.macanbhaird@mu.ie
Peter Mulligan, Department of Mathematics and Statistics, Maynooth University, Ireland.
Email: peter.mulligan@mu.ie
James O'Malley, Department of Mathematics and Statistics, Maynooth University, Ireland.
Email: james.omalley@mu.ie
Rachel O'Neill, Department of Mathematics and Statistics, Maynooth University, Ireland.
Email: rachel.oneill@mu.ie

Abstract

This article contains a short update on the work of the sigma Accessibility Special Interest Group. We announce the release of resources to assist mathematics tutors and coordinators with the support of mature students and those with dyslexia, dyscalculia and dyspraxia. We provide a brief background to the development of these resources and describe their pilot in two institutions, one in England and the other in Ireland. We close with a description of the next stages of work for the special interest group and a call for additional people to get involved.

Keywords: Accessibility, dyslexia, dyscalculia, dyspraxia, mature students, resources, mathematics support.

1. Introduction and Background

The sigma Network for Excellence in Mathematics and Statistics Support has four special interest groups (SIGs) which relate to '…*a number of over-arching themes…that are of strategic interest to the mathematics and statistics support community in higher education.*' (https://www.sigma-network.ac.uk/sigs/). In 2016, the Accessibility SIG was established due, in part, to the increasing numbers of students with disabilities within higher education (HE) (Equality Challenge Unit, 2017) and the fact that the subject area of mathematics and statistics has one of the highest proportions of students with a disability (AHEAD, 2018).

The SIG was led by Emma Cliffe (University of Bath) and Clare Trott (Loughborough University), both of whom had been heavily involved in the development and provision of support for students with disabilities. See Cliffe et al. (2022) for a comprehensive overview of the work in this area prior to the establishment of the SIG. Initially, members of the SIG conducted a survey of mathematics learning support (MLS) practitioners and service mathematics lecturers across Ireland and the UK to determine the main student accessibility barriers that they encountered. For a full description of the findings and an analysis of the results, see Cliffe et al. (2020). Three recommendations arose from this research: the development of resources to assist MLS coordinators and tutors with their support of students with accessibility issues; increased focus on the training of MLS staff in relation to accessibility issues; and improved communication between MLS and corresponding accessibility offices and staff within and across institutions.

The focus of the SIG since then has been on the development of resources for each accessibility issue which would advise MLS coordinators on appropriate provision and offer practical suggestions to tutors on how best to support the students. Two workshops were organised in 2018 and 2019 to start work on the development of the resources. It was decided that the resources should be relatively brief, with a clear and straightforward layout, and with minimal specialist terminology. The tutor resources, designed to be used with students during MLS sessions, would start with a standalone introduction, a brief definition, and then lists of impacts on mathematics with corresponding strategies to help. At the end of the resource, there would be links to further sources of free practical information. The coordinator/manager resources would start with an expanded definition, advice on how to work with other services and recommended reading. The main focus would be on recommendations for MLS provision, e.g. appropriate equipment and software, physical and online environments, additional/alternative provision and tutor training.

Tutor and coordinator resources were completed for dyslexia and, in October 2019, a trial of these resources commenced at Maynooth University (MU). The availability of the resources was advertised via the Disability Office, through the Mathematics Support Centre (MSC), where tutors received training on how to utilise the resource. Each tutor was provided with a pack which contained the necessary materials to implement the strategies outlined during our in-person drop-in sessions. In this trial, the onus was on the student to request the use of the resources, as MSC tutors are not aware of any student's disability, unless they voluntarily disclose this information. While the number of students who engaged was relatively low, their feedback on the use of the resources was broadly positive, and full details are available in Heraty *et al*. (2021).

Due to the time demands required for the implementation of separate accessibility legislation in the UK, the completion of further accessibility resources was paused until the end of 2020. In 2021, resources for dyspraxia, dyscalculia, and mature students were finalised and two pilots commenced at the University of Bath (UB) and MU in the second semester of the 2021-22 academic year. These pilots also included the dyslexia resources. In the following two sections, we briefly outline the details of these ongoing two pilots.

2. The Pilot at UB

At the UB, mathematics support is provided in the Mathematics Resource Centre (MRC) and separated into general provision for any UB staff or student and also Mathematics Department specific provision. Peer Tutors (PTs) are student staff from the Mathematics Department who work with first-year mathematics students. Whilst some one-to-one provision is targeted at students referred from Disability Services, the first-year mathematics support is drop-in without appointment and PTs are not made aware of any specific diagnosed learning needs. The PTs run three open access drop-in sessions per week.

The trial focused on upskilling PTs, offering these inexperienced tutors initial guidance on dealing with student accessibility requirements. The twelve PTs are led by three Senior Peer Tutors (SPTs).

The resources were implemented as follows:

1) The manager of the MRC read the manager resources and provided the equipment from the recommended equipment list.
2) The SPTs were given copies of the four different resources for tutors and asked to run a one hour facilitated discussion with the PTs, in which they could share their thoughts on the ideas presented in the resources. The resources remained readily available to the PTs after the session.
3) Additionally, the PTs were surveyed before the session and two months later, to see if they had felt any impact from the training on their confidence and understanding of the different

accessibility needs. While data analysis is ongoing, and results will be reported on in full in a separate publication, initial findings are very positive.

This suggests that the resources themselves are pitched at the right level. However, it should be noted that almost all tutors indicated that they had no opportunity to use the resources. This presents us with the open question ‘How can we improve the implementation of these resources?’

3. The Pilot at MU

At MU, MAP (Maynooth Access Programme) student is the generic term used for students registered with the Access, Disability and Mature Student offices. The second author is the MAP Academic Advisor for the Department of Mathematics and Statistics, which means that he acts as an academic point of contact for MAP staff and MAP students with regards to students’ specific learning needs. A referral process is in place, which means that the Academic Advisor can meet with students and direct them to MLS as appropriate. For further details, see Mac an Bhaird et al. (2022). The MLS available currently includes one-to-one online sessions for MAP students, if they are required, in addition to the other MSC services.

The pilot took place during the 2021-22 academic year however, unlike the previous dyslexia trial, the dyscalculia, dyspraxia, and mature student resources were not available for use at the beginning of the first semester. Furthermore, in semester 1 of 2021-22, when the majority of lectures for first- and second-year service mathematics students remained online, there was very low engagement from these students with the in-person MSC drop-in. As a result, we decided not to pilot the resources through drop-in but rather to integrate them as part of the referral and one-to-one appointment processes. Ethical approval was received for the collection of student feedback on the use of the resources. Student awareness of the resources was promoted though the MAP Office. Any students studying mathematics or statistics who presented to MAP, and who fell under any of the categories which the resources covered, were made aware of the pilot of the resources and asked if they were interested in using them. Students who were already availing of the MAP one-to-one tuition in semester 1 were also made aware of the resources and invited to avail of them, should any apply. This process identified five individuals: three registered as mature students, one of whom had dyslexia, and a further two students with dyscalculia, though the engagement of one of the students with dyscalculia was sporadic in semester 1. They stopped engaging entirely in semester 2 and did not use the resources.

We did not want to disrupt the relationship and structure of the existing sessions, so students received copies of the appropriate resources and were asked to identify, based on the suggestions in the resource, issues that they would like to address during the sessions. At the beginning of their subsequent meeting, the tutor would then discuss these items with the student, and they collaboratively identified which strategies would be most beneficial. Student feedback was generally not very detailed, but of the four students who participated, the ‘use of colour’ strategy was most often identified as potentially useful. As a result, this was used throughout the sessions. The three mature students also found this strategy useful, though it was not listed on the mature student resource. The tutor involved in providing the one-to-one support reported positive feedback on the ‘use of colour’ with most students finding immediate benefits. Students also reported using it independently with their lecture and study notes and with their other subjects. In particular, they reported better recall and understanding of parts of questions when they came to revise them if they had used different colours. All these students agreed to complete a short survey about their use of the resources, but unfortunately none did so.

Across semester 2, three further students who were referred by MAP to the Academic Advisor also indicated that they would like to use the resources. Unfortunately, none of these students subsequently attended their meeting with the Academic Advisor or continued to engage with MAP in relation to mathematics. This leads us to another important open question: ‘How do we ensure engagement with academic supports from students with accessibility needs who reveal that they are struggling with mathematics?’

Separately to the use of the resources with students, MLS tutors and staff involved in coordinating MLS services reviewed the resources and recorded comments/feedback in relation to their understanding and practical applicability. These were collated, fed back to the SIG lead, and adjustments were made.

4. Conclusion, Recommendations and Future Work

The sigma Accessibility SIG has made good progress to-date during a difficult period, with a completed international survey and four sets of twin resources used across two institutions. Overall, the feedback from students and tutors about these resources has been positive, but the dataset is small.

While both institutions have different systems in place for providing student supports, they faced the aforementioned fundamental issues:

- How can we improve the implementation of these resources?
- How do we ensure engagement with academic supports from students with accessibility needs who reveal that they are struggling with mathematics?

At MU and UB, we are discussing the advantages and disadvantages of the drop-in and referral systems we used to see if we can identify hybrid strategies which may help to address these issues. Whilst these are challenging questions and it will likely always be difficult to ensure that resources reach their intended audience, it is worth being mindful of the widely-recognised benefit to all students in having more accessibility resources available (Rose and Meyer, 2006). Evidence of this was seen at Maynooth, when mature students engaged with ‘use of the colour’. Just getting these resources into MLS is a quick win for many students.

For the next stage in its development, the SIG has decided on a number of measures:

1. The pilot of existing resources will continue over the entire 2022-23 academic year, with the gathering of additional feedback from students and tutors.
2. The SIG will continue to work on the development of further resources, focusing on cognitive disorders, e.g., autism, maths anxiety etc., and sensory impairment, e.g., hearing, sight etc.
3. The JISC Accessible Maths Working Group will be the focal point for the development of technological accessibility. In order to join, see https://www.jiscmail.ac.uk/cgi-bin/webadmin?A0=ACCESSIBLE-MATHS.

A recent survey paper considered articles submitted to this journal over 20 years (Rowlett and Corner, 2021). Perhaps surprisingly, of the 25 topics identified in their analysis, accessibility issues rarely featured. However, they stated that ‘*…accessibility remains a challenge for mathematics and statistics, and hope that this focus will continue to be considered by authors*’ (Rowlett and Corner, 2021, p. 15). Building on this recommendation, to facilitate its continued work, the SIG is looking for more people from across Ireland and the UK to get involved, especially to assist with the development and piloting of resources. Further information, including access to the resources developed so far, is available from the SIG webpage https://www.sigma-network.ac.uk/sigs/accessibility-sig/.

5. Acknowledgements

The pilot of these resources at MU is part of the ICT and STEM Enhancement Project funded by the HEA. Ciarán Mac an Bhaird is the lead on the mathematics strand of this project, and James O'Malley's and Rachel O'Neill's positions are fully funded by the HEA. The authors would like to thank Pádhraic O'Hanrahan and Tara Vivash, both MU, for their time on this project.

6. References

AHEAD Educational Press. 2018 *Numbers of Students with Disabilities Studying in Higher Education in Ireland 2016/17*. Available at: https://www.ahead.ie/data-statistics?id=76 [Accessed 27 June 2022].

Cliffe, E., Mac an Bhaird, C., Ní Fhloinn, E. and Trott, C. 2020. Mathematics instructors' awareness of accessibility barriers for disabled students. *Teaching Mathematics and its Applications: An International Journal of the IMA*, 39(3), 184-200. https://doi.org/10.1093/teamat/hrz012

Equality Challenge Unit. 2017 *Equality in higher education: students statistical report 2017*. Available at: https://www.ecu.ac.uk/publications/equality-in-higher-education-statistical-report-2017/ [Accessed 27 June 2022].

Heraty, C., Mac an Bhaird, C., Mulligan, P., O'Malley, J. and O'Neill, R. 2021. A trial of resources to support students with Dyslexia. *MSOR Connections*, 19(1), 13-21.

Mac an Bhaird, C., Mulligan, P., O'Malley, J. and O'Neill, R. 2022. Access, Disability and Mature Student Opinion on Academic Mathematics Supports. *MSOR Connections*, 20(3), 19-25.

Rose, D.H. and Meyer, A. 2006. *A practical reader in universal design for learning*. Harvard Education Press. 8 Story Street First Floor, Cambridge, MA 02138.

Rowlett, P. and Corner, A. S. 2021. Topics of interest to the MSOR community: evidence from the first 20 years of MSOR Connections. *MSOR Connections*, 19(2), 55-74.