

MSOR Connections

Articles, case studies and opinion pieces relating to innovative learning, teaching, assessment and support in Mathematics, Statistics and Operational Research in higher education.

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Contents

| | |
|--|-------|
| Editorial | 3 |
| – Peter Rowlett | |
| Improving engagement in large undergraduate statistics tutorial classes | 4–17 |
| – Alena Haddley | |
| Maths lecturers in denial about their own maths practice? A case of teaching matrix operations to undergraduate students | 18–28 |
| – Alexander Partner and Alexei Vernitski | |
| Enhancing the Success of College Algebra Students by Incorporating Adaptive Technologies | 29–36 |
| – Alina Stefanov and Barry J. Griffiths | |
| Supporting Students at the Transition to University with a Self-Paced, Online Mathematics Induction Course | 37–47 |
| – Abigail Parkes, Liam Naughton, Nazira Karodia, Jordan Brennan and Emily Rodden | |
| A Trial of the Assistive Technology EquatIO in an Irish University | 48–56 |
| – Ciarán Mac an Bhaird, Rachel O'Neill and Ruchi Palan | |
| Mathematics Learning Support – Linking Practice to Research in the New Normal - A report of the IMLSN Workshop 2022 | 57–60 |
| – Ciaran O'Sullivan and Kirsten Pfeiffer | |

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EDITORIAL

Peter Rowlett, Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield, UK. Email: p.rowlett@shu.ac.uk

Welcome to this issue of *MSOR Connections*.

We open considering undergraduate teaching practice. How many of our readers would consider 180 students a 'small group tutorial'? When your module has close to 500 enrolled students and room availability is restricted, methods for engaging students in tutorial work are complicated, as discussed by Haddley. Following this is a provocative piece by Partner and Vernitski outlining a methodology for evaluating curriculum structure in relation to research practice. In this case, they identify that matrix multiplication is not always presented to students in a manner that reflects its usefulness in later mathematics.

Next, we focus our attention on early stages of undergraduate study with a pair of self-paced introductory courses. Stefanov and Griffiths consider restructuring the teaching of introductory algebra to use adaptive learning technology in hopes of reducing the attainment gap and withdrawal rate. Parkes et al. report on the development of an online, self-paced mathematics induction course for more than 250 undergraduate programmes in science and engineering.

Following this, an interesting account is given by Mac an Bhaird, O'Neill and Palan of their trial of the assistive technology EquatIO, which provides a toolbar for input of mathematical expressions.

Finally, the issue is closed with a report on what happened at the 13th annual workshop of the Irish Mathematics Learning Support Network (IMLSN).

MSOR Connections continues to be a venue for our community to share its innovative practice, and submissions are always welcome via the journal website: <https://journals.gre.ac.uk/>. The journal is also always keen to attract reviewers and we have some experience of supporting those new to reviewing, so please sign up via the website if you are interested in reviewing articles.

CASE STUDY

Improving engagement in large undergraduate statistics tutorial classes

Alena Haddley, Department of Mathematical Sciences, University of Liverpool, Liverpool, UK.

Email: a.haddley@liverpool.ac.uk

Abstract

We investigate whether introducing the mobile polling system Poll Everywhere had a positive impact on student engagement in large tutorial classes of a second year undergraduate statistics module. In particular, a short quiz facilitated by Poll Everywhere was introduced at the beginning of the tutorial in order to promote active participation. Students' perceptions of the Poll Everywhere quiz on their tutorial engagement and learning are also explored. We conclude that the Poll Everywhere quiz seemed to have improved student engagement in tutorials and that students believed that it made tutorials engaging and was useful for their learning.

Keywords: student engagement, mathematics tutorials, large classes, Poll Everywhere.

1. Introduction

For the past few years, the student population in the Department of Mathematical Sciences at a UK university has been growing. This is mainly due to 2+2 programme with a partner university in China where students attend Year 0 and 1 at the Chinese university and Year 2 and 3 at the UK university. More than 400 Chinese students join the mathematics cohort in the second year and thus most Year 2 and Year 3 modules at the UK university typically have 300 to 550 students. It is very challenging to engage with such large cohorts, in particular to actively interact with large numbers of students during tutorial classes. Student engagement in tutorials is generally low across the department and so it is vital to look for new and innovative ways to engage students in these classes, while adhering to University's pedagogical philosophy, such as active learning.

The module in this study is a second-year statistics module which is compulsory on the biggest programme in the department and optional on all other programmes. At the time of the study, the module comprised of 541 students, of which approximately 83% of class were students from the Chinese university. The weekly delivery of the module consisted of three hours of lectures and one hour of tutorial classes. Tutorial classes were split into three groups which meant that there were approximately 180 students allocated to each group. Each tutorial group is supported by a member of staff and 7 to 9 tutors.

There are two types of tutorial classes in this module: computer classes and 'standard' tutorial classes. In computer classes, students learn to operate statistical software by solving practical problems on a tutorial sheet. In 'standard' tutorial classes, students solve problems 'by hand' and some questions on the tutorial sheet are related to theoretical concepts. In both types of classes, students are required to work on problems themselves or with their peers, and ask the member of staff or tutor for help if needed. No material is presented on the board. Students are encouraged to attempt questions in advance of tutorials, however it seems that very few students do so. This tutorial model is similar to workshop tutorials discussed by Sharma, Mendez, and O'Byrne (2005) and Shearman, Rylands, and Coady (2012), however with much larger class sizes.

Over the years, the following trend in tutorial participation was observed:

- Computer classes are usually well attended. There are usually around 180 students assigned to a group and average attendance is roughly 50%. This may be attributed to the assessment

strategy, since statistical software is required to complete the module assessment and so students are more likely to attend these sessions (Oldfield, et al., 2018; 2019).

- 'Standard' tutorials are usually poorly attended. There are usually around 180 students assigned to a tutorial group and average attendance is roughly 25%. However, students attending these tutorials engage very well by working on problems and asking questions about problems and/or course content. In many cases, deep and high quality learning is demonstrated.

It seems that less able students who would benefit most from 'standard' tutorials do not attend. Sharma, Mendez, and O'Byrne (2005) concluded that less able students who attended student-centred tutorials performed, on average, better in the exam than those who did not attend. Since it seems that increased tutorial engagement could lead to improved learning outcomes, it is important to encourage student engagement in tutorials, which is the goal of our intervention.

Disengagement in tutorials is quite common across the department. Although the causes for non-attendance of 'standard' tutorials have not been formally investigated, the following may be possible contributors:

- Tutorial attendance is not compulsory and not monitored. Massingham and Herrington (2006) concluded that lack of interest or motivation are great contributors to missing non-compulsory classes. Burke, Mac an Bhaird, and O'Shea (2013) provided evidence that a scheme to monitor attendance improved tutorial attendance. However, the authors did not consider the quality of engagement and it is unclear whether students present engaged actively or just came for the sake of monitoring. Attendance is a commonly used measure of engagement, however it does not necessarily account for the quality of engagement (Beer, Clark, and Jones, 2010). There are nevertheless many studies showing that attendance policies contribute to increased attendance, and this in turn may improve student performance (Moores, Birdi, and Higson, 2019).
- Lifestyle factors (e.g. students having to work) were also identified as reasons to non-attendance (Massingham and Herrington, 2006).
- 'Standard' tutorials are not directly relevant to summative assessment. As discussed by Massingham and Herrington (2006) and Oldfield, et al. (2018; 2019), students attend classes to gain information required for assessment tasks or exams.
- Solutions to tutorial sheets are available on the VLE after the tutorials. As indicated by Massingham and Herrington (2006) and Moores, Birdi, and Higson (2019), availability of resources may influence student attendance.
- Tutorial groups are too large. Oldfield, et al. (2019) reported that the class size has an effect on attendance. In large classes, student absence is not noticed by a tutor and there are no consequences. Students feel anonymous and the sense of belonging is missing.
- A learning space is not suitable. The 'standard' tutorials were conducted in large lecture theatres with tiered fixed seating. This environment does not promote collaborative learning and there is no easy access to all students. As evidenced by McArthur (2015), the learning space plays a significant role in student engagement and learning. Parsons (2017) also confirmed that active learning spaces (room with round tables) promote interactive learning.

Due to tight budgets and lack of suitable facilities and resources, it is not possible to address the majority of possible causes discussed above. Despite the limitations, a time and cost effective solution to address low levels of engagement in 'standard' tutorial classes is proposed and evaluated in this study. The study took place in the academic year 2019/2020.

2. Review of Relevant Literature

2.1. Student engagement

Student engagement is widely discussed and researched, yet no single definition of student engagement has been agreed. For example, Baron and Corbin (2012) point out multiple different definitions and meanings given in the literature. Kahu (2013) considers student engagement as a complex process and suggests a conceptual framework including institutional and personal factors that influence student engagement and consequences of student engagement. Despite the complexity of this topic, according to Beer, Clark, and Jones (2010), engagement seems to be a combination of several different aspects including active and collaborative learning, participation, communication among students and academic staff, and feeling supported. Student engagement is linked to student achievement and retention, as evidenced in many studies which are summarised by Trowler and Trowler (2010). The importance of student engagement is undeniable and responsibility to improve it relies not only upon the lecturer, but also upon the student, institution and government (Kahu, 2013).

It is important to emphasise that student engagement depends on class sizes and it is much more challenging to engage with large classes, compared to small classes (Exeter, et al., 2010). In particular, Ahlfeldt, Mehta, and Sellnow (2005) show that the smaller the class size is, the more students engage.

In this paper, student engagement will be associated with student attendance and active participation. This study focuses on student engagement in the 'standard' tutorial classes of a large second year undergraduate statistics module. (In the rest of the paper, tutorials will refer to 'standard' tutorials.)

2.2. Tutorials

This study focuses on student engagement during tutorial classes. A tutorial is normally a class in which a small group of students interact with a tutor. It is a learning opportunity in which students can exercise knowledge gained during lectures by asking questions, discussing problems with a tutor and/or peers, challenge other students' arguments or otherwise actively engage. Tutorials can be more interactive than lectures, and provide a personalised way of learning and involve a social aspect (Maharaj, 2012). There are many forms of tutorials, which may depend on discipline, modular or institutional requirements, or the tutor's experience. Tutorials are important and valuable for students' learning since they provide opportunities for students to discuss problems and ask questions in order to consolidate their knowledge and understanding and to deepen their understanding. To allow this, it is essential to promote active and collaborative learning methods in tutorials, which are well evidenced to improve student performance and retention (Beichner, et al., 2007; Freeman, et al., 2014). This is especially important in mathematics tutorials since students learn mathematics by actively solving problems, exploring mathematical concepts and ideas by themselves and/or with peers. Learning mathematics can be compared to learning to play a piano – as Mazur said about learning: "*Suppose you want to learn how to play a piano, you just don't go to a concert hall and listen to a famous pianist playing piano, you've got to play the piano*" (Scherpmedia, 2016). Smith (2008) also concluded that students consider tutorials promoting active learning as the most important part of their learning process.

In recent years, in many universities, the number of students accepted to courses has been growing, which has caused class sizes to become significantly larger, and this in turn has affected student engagement (Ahlfeldt, Mehta, and Sellnow, 2005). Due to a lack of resources in many modules in the

department, a tutorial is no longer a class with a small group of students and a tutor, but a large class (up to 200 students) with many different tutors in one room. Even in large groups, tutorials can still be effective and valuable to students' learning as discussed by Menard, et al. (2015). However, it becomes challenging to engage large groups of students during tutorials and many methods promoting active learning become impracticable.

There are several papers exploring different styles of mathematics tutorials promoting active learning and engagement, however, only a handful make suggestions which could be applied to large tutorial classes. Seaton, King, and Sandison (2014) discuss a very exciting style of mathematics tutorials – a so called board tutorial. A board tutorial takes place in a special room where boards line all walls of the room and all students in the class work through problems on their section of the board, in pairs or on their own. The tutor is a facilitator and guides students if needed. These tutorials have great benefits: they promote active and peer learning, increase confidence, and enhance communication skills and teamwork. There is no doubt that this type of tutorial greatly increases student engagement. However, due to the special requirements on rooms and high demand on numbers of tutors, it is not possible to run these tutorials for large classes of students when resources are limited.

Shearman, Rylands, and Coady (2012) describe an intervention aimed at mathematics tutorials that improved student engagement and performance. The intervention involved the following: tutorials during which material was presented by a tutor were replaced by so called workshop tutorials, in which the tutor was a facilitator, and assisted students while they were working on problems themselves. Another study by Sharma, Mendez, and O'Byrne (2005) showed that workshop tutorials are valuable for student learning. This study provides evidence that, on average, tutorial attendance improves student performance in examinations, and shows that especially less able students benefit from attending these tutorials. Although both studies provide evidence that tutorials with active learning elements improve student engagement and performance, the tutorial groups in both studies were rather small: in the first case, there were 30 students per tutorial group and in the second, 60 students per tutorial group. Based on my own and my colleagues' experience, when this approach is applied to larger tutorial groups, student engagement suffers significantly.

Other studies suggest to split large classes into smaller tutorial groups of 20-30 students and implement collaborative learning methods (Maharaj, 2012; Oates, et al., 2005). Menard, et al. (2015) discussed that collaborative learning methods may not be very effective in large tutorial classes (70 students). Perhaps collaborative learning methods were not as effective as expected due to the learning space. Brooks (2011) and Cotner, Loper, and Brooks (2013) discuss that a suitable learning space can promote benefits of active and collaborative learning and evidence suggests that an active learning classroom (e.g. a room with round tables) has a positive impact on student performance. It is worth pointing out that in the 2013 study, active and collaborative learning strategies were successfully employed in an active learning classroom with more than 100 students.

In conclusion, it seems that the most effective tutorials, from the point of view of student engagement and learning, are those tutorials which incorporate active and collaborative learning methods, and are small in size, ideally 20 to 30 students per group, or they take place in a room suitable for active and collaborative learning.

2.3. Student engagement in large classes

Studies focusing on promoting student engagement in large classes suggest implementing certain assessment strategies (Cole and Spence, 2012; Voelkel, 2013) and/or use technology, in particular, electronic polling systems (Goff, Terpenney, and Wildman, 2007; Kappers and Cutler, 2015; King and Robinson, 2009; Sawang, O'Connor, and Ali, 2017). It should be noted that these studies consider a large class to consist of 80 to 300 students. There are two studies that discuss student engagement in

classes with more than 500 students (Exeter, et al., 2010; Jarvis, et al., 2014). In particular, Jarvis, et al. discuss a concept and interesting strategy for a flipped-classroom approach. Although these studies focus on engagement in lectures, some approaches can be easily adopted to tutorial classes. However, many of the approaches would require a significant amount of resources, facilities and time investment; e.g. splitting my class into smaller groups would create a large demand on facilities and staff. Due to pressure on these essential resources, it is not possible to apply most approaches in our context.

The most frequently suggested approach to improve student engagement in large classes is using polling systems which enable students to respond to questions anonymously in class in real time, with immediate feedback. There are many studies evaluating different types of polling systems (clickers, online polling systems using mobile phones, etc.). A comprehensive summary and literature review of different systems is provided by Florenthal (2018) and Çakır (2020). It has been consistently evidenced that polling systems are very effective for improving student engagement, (e.g. Han, 2014; Sun, 2014). The following benefits of using polling systems were identified and taken from Cubric and Jefferies (2015):

1. classroom benefits: improvements in attendance, participation, and engagement;
2. learning benefits: the increased quantity and quality of class discussion, learning performance, quality of learning and contingent teaching;
3. assessment benefits: improved feedback, effective formative assessment, the ability to compare performance with others.

It has also been shown that students perceive polling systems positively with respect to learning and classroom experience (Cubric and Jefferies, 2015), and Kappers and Cutler (2015) and Noel, Stover, and McNutt (2015) show that students consider mobile-based polling an enjoyable experience. Tobin, Lozanovski, and Haeusler (2013) use a student response system successfully in a tutorial setting. Because of this overwhelming evidence and with many online polling systems available and easily accessible, it seemed to be very appropriate and was relatively straightforward to apply this approach in my tutorial classes.

3. The Intervention

In an attempt to motivate students in the class to attend and actively engage in the tutorial, a short quiz at the beginning of the session was introduced. This quiz was facilitated by Poll Everywhere (2023) and comprised of 4 short multiple choice questions, usually with four or more distractors. The questions were designed to help revise knowledge required to solve the problems on the tutorial sheets. For less able students, the quiz was intended to give students an idea what to consider in order to solve problems; for well-prepared and more able students, the quiz was an opportunity to consolidate knowledge. The quiz also served as a self-diagnostic exercise for all students.

After introducing each multiple choice question, students used their mobile phones or other devices with Internet connection to answer the question. A summary of student answers was released, and for each distractor feedback was provided, explaining the reasons why distractors could not be a correct answer.

After the quiz was completed (the quiz took usually 10-15 minutes), the usual form of tutorial resumed: students were expected to work on problems themselves or with peers and ask the member of staff or tutor when they encountered any problems.

Poll Everywhere was chosen as the most suitable polling system because it supports LaTeX which is the most effective typesetting software for mathematical symbols and expression, and also the

University owns a full Poll Everywhere subscription. Many challenges connected to hardware and software of polling systems which were indicated in the literature (Cubric and Jefferies, 2015; Rose, 2019) were overcome thanks to development of mobile polling systems for which students use their own devices. Another challenging issue is that polling takes up some of the delivery time and careful consideration should be given when allocating time for polling to ensure that the required course material is still fully covered. These timing concerns are not an issue in the tutorial setting, since a tutorial is time for students to revise, practice and consolidate their previously gained knowledge and so there is enough time to introduce learning tools requiring time investment. An initial challenge may be for presenters to get familiar with the polling software, however this software does not have a particularly steep learning curve. Another challenge that may arise is if a student does not own a device with Internet connection, although nowadays this is highly unlikely. However, such students could still take part in the quiz by writing their answers down and they were encouraged to do so. No software or connectivity problems were encountered when presenting the quiz using Poll Everywhere.

4. Methodology and Research Questions

The purpose of this study is to investigate whether the Poll Everywhere quiz introduced at the beginning of the tutorial classes improved student engagement, and to explore student perceptions of the quiz for their engagement and learning. Two aims are proposed which were based on results of studies by Kappers and Cutler (2015) and Noel, Stover, and McNutt (2015). The difference between these two and our own study is the setting in which mobile polling system is used; our study focuses on the tutorial setting. We will discuss whether the proposed aims were achieved.

Aim 1: The Poll Everywhere quiz improved student engagement with tutorial classes.

To see whether this aim was achieved, attendance in tutorials was taken by manually counting the number of students present in the class. No official attendance monitoring was in place, since this might have increased attendance numbers artificially creating a bias in our study; when attendance monitoring is in place, many students might attend for the sake of attendance records, however the quality of their engagement may be lacking. Moreover, this kind of extrinsic motivation is linked to surface approach to student learning as opposed to intrinsic motivation promoting deep learning (Biggs and Tang, 2011), which would hopefully be encouraged by making the tutorials as effective as possible for learning.

Since tutorial classes were not officially monitored, we could argue that students who attended these classes were intrinsically motivated to do so and participated actively in the classes either by taking part in the quiz and/or solving problems on tutorial sheets. In this study, attendance may therefore be considered as a valid measure of engagement since it actually accounts for active participation, that is good quality engagement.

As the Poll Everywhere quiz was included in three tutorial classes, attendance was taken for these three tutorials.

Data from Poll Everywhere also provided information about the exact numbers of students who participated in the quiz.

Aim 2: Students felt that the Poll Everywhere quiz made tutorial classes more engaging and helped their learning.

To see if the second aim was achieved, an anonymous online survey asking students about their experiences with the Poll Everywhere quiz was conducted. The online survey was conducted using an institutional account on the Jisc Online Surveys platform. All 541 students registered on the module

were invited to fill in the online survey via an announcement on VLE, explaining all details of the study. The survey was streamed into two sections according to whether students had attended at least one tutorial or they had not attended any. For those who had not attended any tutorials, there was just one open-response question asking about the reasons for non-attendance. The part of the survey for students who attended some tutorials consisted of a question about their tutorial attendance and four five-point Likert scale items, asking details about their tutorial learning experience. The survey also contained two qualitative open-response questions. The first open-response question gave participants the opportunity to elaborate their responses to the Likert scale items and the second one asked about further comments on the Poll Everywhere quiz and participants' perceptions of the overall structure of tutorials. Additional demographic information was collected to identify whether participants were home students, students from the Chinese university or other international students.

4.1. Data analysis

To analyse the collected data, descriptive statistics were used for tutorial attendance, participation in Poll Everywhere quiz and responses to the Likert scale items. For the two open-response questions, qualitative thematic analysis was used which is a common method for analysing open-response questions in surveys (Braun and Clarke, 2006). The responses were examined for themes, and relevant themes were then linked to corresponding aims. In particular, two main themes arose; one was "*student engagement*" which can be linked to both Aim 1 and Aim 2, and the other was "*student learning*" which is linked to Aim 2.

5. Results

The tutorial attendance numbers and rates are summarised in Figure 1 and Table 1. There were three tutorial classes in this module and the intervention was conducted in all three tutorials.

The average attendance for tutorials was 39.2%. Precise tutorial attendance rates from previous years are not available, however it is estimated that average proportion of students who attended equivalent three tutorials in past was approximately 25%. (This estimate is based on past experience of the lecturer and tutors.)

Interestingly, in every tutorial a small number of students left the class as soon as the Poll Everywhere quiz was completed.

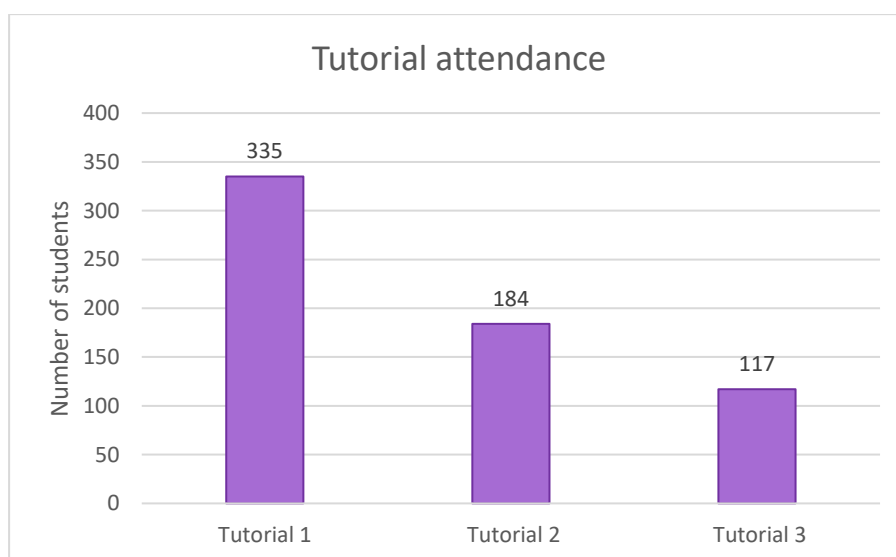


Figure 1. Tutorial attendance: number of students in each tutorial, out of total 541.

Table 1. Tutorial attendance rates: proportion out of 541 students who attended each tutorial

| Tutorial 1 | Tutorial 2 | Tutorial 3 | Average attendance |
|-------------------|-------------------|-------------------|---------------------------|
| 61.9% | 34.0% | 21.6% | 39.2% |

Table 2 summaries the proportions of students present in the class who participated in the Poll Everywhere quiz at the beginning of the tutorial.

Table 2. Engagement rates with the Poll Everywhere quiz: proportion of present students who participated in the Poll Everywhere quiz

| Tutorial 1 | Tutorial 2 | Tutorial 3 | Average engagement |
|-------------------|-------------------|-------------------|---------------------------|
| 97.0% | 73.4% | 82.9% | 84.4% |

Data suggest that a high percentage of students who attended tutorials took part in the quiz; average engagement with the quiz was 84.4%.

The online survey received n=21 responses. The survey was completed by 11 home students and 10 students from the Chinese university. Out of all respondents, 15 attended all three tutorials, 5 attended first two tutorials and one respondent attended the first tutorial only. No survey responses were received from students who did not attend any tutorials, and so reasons for non-attendance could not be identified in this case.

The results of Likert scale items are summarised in Table 3 with scale 1=strongly disagree, 2=disagree, 3=neither agree nor disagree, 4=agree, 5=strongly agree.

Table 3. Summary of the results of questionnaire Likert scale items

| | 1 | 2 | 3 | 4 | 5 | mean | st.dev. |
|---|----------|----------|----------|----------|----------|-------------|----------------|
| (1) The quiz at the beginning of the tutorial helped me to revise my knowledge of the material. | 0 | 0 | 3 | 7 | 11 | 4.38 | 0.74 |
| (2) The quiz at the beginning of the tutorial encouraged me to participate in the tutorial. | 0 | 0 | 4 | 8 | 9 | 4.24 | 0.77 |
| (3) The quiz at the beginning of the tutorial helped me to engage successfully with the tutorial sheet. | 0 | 4 | 4 | 12 | 1 | 3.48 | 0.87 |
| (4) Overall, I found that the quiz at the beginning of the tutorial was useful for my learning. | 0 | 0 | 3 | 10 | 8 | 4.24 | 0.70 |

All Likert scale items received, on average, positive responses of agree or strongly agree. Students believed that the Poll Everywhere quiz helped them to revise their knowledge (mean=4.38,

st.dev.=0.74), and encouraged their tutorial participation (mean=4.24, st.dev.=0.77). The lowest rating was received for the item asking whether the quiz helped students to engage successfully with the tutorial sheet (mean=3.48, st.dev.=0.87). Overall, students felt that the quiz was useful for their learning (mean=4.24, st.dev.=0.70).

A few open responses were received. Some open responses were relevant to both aims, and these formed the theme “*student engagement*”. For example, those were responses stating that students “*felt engaged during tutorials*”. Generally, responses stated that the quiz was “*very useful*” and “*meaningful*” and these were linked to the theme “*student learning*”. Students appreciated that the quiz was anonymous and that feedback was explained in detail. It was suggested that the quiz could have contained more questions and more challenging questions.

6. Discussion

6.1. Aim 1: The Poll Everywhere quiz improved student engagement with tutorial classes

It seems that the Poll Everywhere quiz had a positive impact on student engagement with tutorial classes; the average attendance rate was 39.2%, which was observed to be greater than in the past when the average attendance rate was around 25%. Even though the 39.2% attendance rate does not seem very high in general, it is a good result in the current setting, since the tutorial classes are optional and are not directly associated with continuous assessment, suggesting that students seemed to be intrinsically motivated to attend. Other modules in the department which adopt a similar tutorial style (but without using Poll Everywhere), and which have large numbers of students, typically report very low attendance. No attendance data is formally recorded for these modules, but in this context the average attendance rate of 39.2% is notable. One can notice that attendance rates decreased as the semester progressed. This is quite a common phenomenon which was also observed e.g. by Kassarnig, et al. (2017), however this paper does not explore the underlying reasons. In my class, the first tutorial always has the highest attendance, possibly because students want to identify whether the class is “*valuable*” to them; as discussed by Massingham & Herrington (2006), most students attend classes if they perceive them as “*valuable*”. Even though exact data are not available, it was observed that this year’s second tutorial seemed much better attended compared to previous years, suggesting that the quiz attracted more students to attend and actively engage. The reasons for the significant decrease between the first and second tutorial are unclear and deserve further investigation. This drop in attendance may be attributed to one of the reasons discussed in the introduction and it may suggest that the quiz was not a sufficient motivation for some students to attend the tutorials.

Data show that not all students who attended tutorials engaged with the quiz using a device. However, these students might have still engaged by writing their answers down. Some students did not participate in the quiz because of their late arrival. Bennett and Voelkel (2014) observed that not all students present in a class participated in polls. According to students’ views, it was because sometimes they just did not have a mobile phone with them or they just preferred to think about the question quietly without answering actively. The authors also concluded that a typical poll’s response rate dropped when questions became more complex or difficult since, for example, students required more time to answer. Although in our case the quiz questions were not complex or difficult, some students could perceive them as such, and perhaps needed more time to answer. Even if the students did not complete the quiz actively, they could still take part passively and learn from feedback provided after each quiz question. Even if some students did not participate in the quiz, they still attended the tutorial in order to engage in a different way: they attempted to solve the problems on the tutorial sheet and/or engaged in discussions with their peers, tutors or a member of staff.

Another indication that the quiz encouraged student attendance is the fact that some students left the tutorial as soon as the quiz ended. One open response in the survey also stated that students “attended

specifically for the quiz.” Although this may not be an intended result, it was observed only in a small number of cases and the quiz alone had some educational value for student learning. On the other hand, many of those students who attended only for the quiz possibly stayed after the quiz ended and worked on the tutorial problems, which is a very desirable result. However, this is an interesting phenomenon which may indicate that some students believe that they would miss out on a learning opportunity if they did not participate in the quiz, but they would not miss out on any valuable learning opportunity by avoiding the rest of the tutorial. This suggests that some students place higher value on teacher-directed activities rather than student-directed/independent learning activities. Another reason for students to leave the tutorial earlier could be that they answered all quiz questions correctly and perhaps felt that they achieved sufficient knowledge of material. Although the quiz was advertised as a learning opportunity to help students to revise some necessary knowledge for the tutorial sheet, perhaps in future it should be stressed that students should engage with the problems on the tutorial sheet as well since it provides another important and valuable opportunity to help their learning and understanding.

Some open responses in the survey also supported Aim 1; respondents stated that students “*felt engaged during tutorials*” and that “*the quiz made the tutorial for this module the most useful this year as it engaged us more than any other set of tutorials*”. This suggests that the quiz gauged some students’ interest, motivated them to attend tutorials and to engage actively.

6.2. Aim 2: Students felt that the Poll Everywhere quiz made tutorial classes more engaging and helped their learning

Likert scale survey items (2) and (3) were related to students’ perceptions of engagement. There were very favourable responses for item (2), suggesting that students felt strongly that the quiz promoted their participation in the tutorial. However, for item (3) the lowest ratings were received. There were four respondents who disagreed that the quiz helped with engagement with problems on the tutorial sheet. Even though the quiz questions were related to the problems on tutorial sheets, this indicates that they did not help some students to tackle the problems and some students could not find connections between the quiz questions and tutorial problems. Clearer communication of connections between the quiz questions and tutorial problems could be made in future, to help students engage with tutorial problems. In particular, this would be of benefit to less able students. Clearly communicated learning outcomes and constructive alignment with learning outcomes are related to increased student motivation which contributes to better student engagement (Stamov, et al., 2021).

Some open responses can be also directly linked to support Aim 2. As discussed before, respondents stated that they felt engaged and that the quiz made students engage more compared to other tutorials. Increased student engagement in classes using Poll Everywhere was also confirmed in other studies (e.g. Bennett and Voelkel, 2014).

Likert scale survey items (1) and (4) explored the effect of the quiz on students’ learning. Both items received very positive responses, indicating that students believed that the quiz helped their learning. This is also supported by open responses, stating that the quiz was a very useful and meaningful activity. Benefits of polling systems on students’ learning of mathematics were also confirmed in other studies (e.g. King and Robinson, 2009). Some respondents even suggested that the quiz questions could be more challenging. Adding more challenging questions could however demotivate less able students and the quiz could lose its intended purpose, that is to be inclusive and inviting for students of all abilities. Moreover, differentiation of abilities is achieved through problems on tutorial sheets and discussions with peers and/or tutors and therefore there is no need to make the quiz questions more challenging. There might be scope to include a few more quiz questions as suggested in some responses, however time limitations should be considered when including more questions; in my opinion, a longer quiz could introduce a risk of ‘quiz fatigue’ (cf. survey fatigue; Porter, Whitcomb and

Weitzer, 2004) and also interfere with the other important part of tutorial where students attempt to engage with tutorial problems. A careful balance between the two parts of the tutorial would have to be considered.

7. Conclusion

It seems that both aims were achieved, suggesting that the Poll Everywhere quiz increased student engagement with tutorials and that students felt that the quiz had a positive impact on their engagement and learning. This is in agreement with results of similar studies, (e.g. King and Robinson, 2009; Kappers and Cutler, 2015; Rose, 2019). However, there are some limitations to this study. There are no exact attendance data available from previous years, and respondent bias may be present; students who enjoyed the quiz may be more inclined to respond to the survey. Despite this, the study indicated that introducing the Poll Everywhere quiz was successful and a step in right direction to improve student engagement in tutorials and students seemed to enjoy it.

Although possible reasons for disengagement in tutorials were outlined earlier, it would be desirable to formally explore the reasons in the setting of tutorials. This knowledge may help to further enhance the current tutorial structure or design a new structure which could help to increase student engagement even more. However, the intervention presented in this paper seems to be an effective, innovative and easily applicable way to make tutorials more engaging and beneficial to students' learning. This approach may benefit tutorials of other large, and also small modules and is not restricted to mathematics subjects, but can be easily applied to other disciplines.

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CASE STUDY

Maths lecturers in denial about their own maths practice? A case of teaching matrix operations to undergraduate students

Alexander Partner, Department of Mathematical Sciences, University of Essex, Colchester, UK.

Email: akpart@essex.ac.uk

Alexei Vernitski, Department of Mathematical Sciences, University of Essex, Colchester, UK.

Email: asvern@essex.ac.uk

Abstract

This case study provides evidence of an apparent disparity in the way that certain mathematics topics are taught compared to the way that they are used in professional practice. In particular, we focus on the topic of matrices by comparing sources from published research articles against typical undergraduate textbooks and lecture notes. Our results show that the most important operation when using matrices in research is that of matrix multiplication, with 33 of the 40 publications which we surveyed utilising this as the most prominent operation and the remainder of the publications instead opting not to use matrix multiplication at all rather than offering weighting to alternative operations. This is in contrast to the way in which matrices are taught, with very few of these teaching sources highlighting that matrix multiplication is the most important operation for mathematicians. We discuss the implications of this discrepancy and offer an insight as to why it can be beneficial to consider the professional uses of such topics when teaching mathematics to undergraduate students.

Keywords: Matrices, higher education, research and teaching practice, educational material and media.

1. Introduction

We teach undergraduate mathematics and foundation-year mathematics, and for some time we felt vaguely uncomfortable about what seems to be an unnecessary gap between the maths used by academics and the maths taught by academics in some parts of undergraduate mathematics. Imagine a driving instructor in the UK who drives, obviously, on the left-hand side of the road. It would be preposterous to imagine that they will teach their pupils to drive on the right-hand side of the road, i.e., contrary to what they do themselves. However, our study presented below shows that this impossible example seems to illustrate what maths lecturers might sometimes do at universities; when they conduct research, they use mathematics efficiently and professionally, but simultaneously, they seem happy to teach mathematics in a way that instils a somewhat distorted view of professional mathematical practice in the students that they teach.

As we aimed to quantify our imprecise discomfort expressed in the previous paragraph, we were successful at locating one specific small area of mathematics on which we could zoom in and explore in detail, as described below.

2. Matrix operations

Matrix operations feature in a typical first-year university curriculum and in the Further Mathematics A/AS level within the UK. In a typical curriculum, matrix operations include addition and multiplication, and sometimes subtraction and ‘scalar multiplication’ (that is, multiplying a matrix by a number) also explicitly feature. For instance, Further Maths includes a section “*Add, subtract and multiply conformable matrices; multiply a matrix by a scalar*” (Cresswell, 2006). This is the area of the

curriculum on which we concentrate. Of course, there are also other operations one can apply to matrices, most notably, inverting, but these operations are nearly always introduced in other sections, not together with the operations above, and so we do not consider them here. One half of our study consisted of inspecting 40 teaching publications (that is, 20 textbooks and 20 lecture notes) to see how matrix operations are introduced in them.

Our professional experience as mathematicians has led us to believe that in applications of matrices, multiplication of matrices is used much more widely than addition. Let us present two examples from different ends of the spectrum. Firstly, in the Further Maths textbook by Cresswell (2006), there are two applications of multiplication (*“successive transformations”* and *“solve three linear simultaneous equations in three variables by use of the inverse matrix”*) but no applications of addition. Secondly, in the book on deep learning by Chollet (2017), the author states that *“deep neural networks [consist] mostly of many small matrix multiplications”*. We felt that a similar picture can be observed in mathematicians’ research outputs. As such, the second half of our study consisted of inspecting 40 recent research publications to see what place matrix multiplication occupies in them.

Thus, the questions we were asking were approximately as follows:

1. Is it true that in teaching, defining addition of matrices and multiplication of matrices are treated as topics of an equal importance?
2. Is it true that in applications of matrices (as demonstrated in research publications) multiplication of matrices is by far the most important matrix operation?
3. If the discrepancy described in the previous two questions exists, is it justified? And if it is not justified, what can be done?

3. Research outputs

Here is how we produced our data. We selected two medium-sized university mathematics departments, University of Essex and University of East Anglia, that publish research papers in a wide range of areas of mathematics. We then used staff web pages of each department to select researchers who, according to their online biography, were likely to have completed research using matrices in some way. Then we performed a search in Google Scholar using the keywords: ‘<First name> <last name> matrix’ and chose results where ‘matrix’ was highlighted in the description of the search result. We scanned the paper by eye and recorded whether we agree with the statement *“In this publication, among other matrix constructions, matrix multiplication plays the most prominent role”*. We also made a note of the total number of pages in the paper (excluding bibliography) and the approximate number of pages where matrix multiplication is used. Admittedly, due to a very wide range of mathematical research which we scanned, with various generalisations and applications, we had to treat matrix multiplication somewhat broadly, as ‘an operation that resembled matrix multiplication appeared to be used’.

We saw that in 33 publications out of 40, matrix multiplication was most prominent, and in only 7 it was not. In the latter, ‘multiplication-poor’ publications, multiplication was shunned not in favour of other matrix operations, but because no matrix operations were used.

As to the number of pages on which matrix multiplication features in a research paper, the ratio is shown in Figure 1. The horizontal axis shows the proportion of the pages in the publication which uses matrix multiplication. The vertical axis shows the number of publications. As you can see, the histogram is heavily skewed, with most of the data towards the right-hand side of the distribution, and half of research publications using matrix multiplication on at least 80% of their pages.

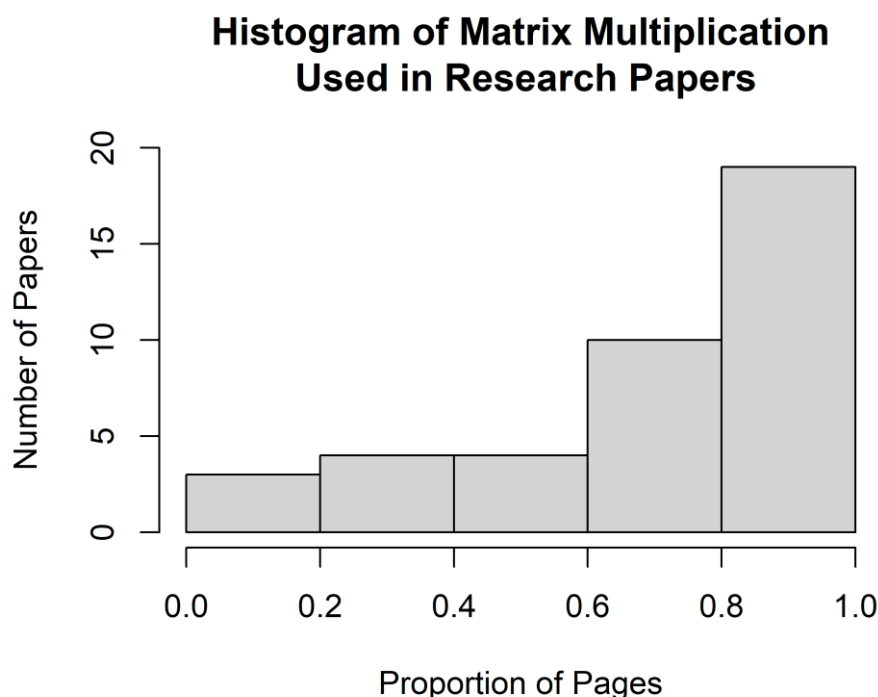


Figure 1. Proportion of pages in research papers that uses matrix multiplication as a prominent operation.

A preliminary conclusion from these observations is that among matrix constructions, matrix multiplication is of paramount importance in mathematical practice. In the next section we explore whether this fact is reflected in the way matrix operations are taught.

(Out of interest, we reflected whether not using matrix operations in some publications is a feature of these publications in particular, or of the research areas which they explore. In the publications we considered, the topics of ‘multiplication-poor’ publications are category theory, complex analysis, Markov chains, molecule imaging, and social interaction of animals. We can easily imagine that some other publications in these research areas could usefully employ matrix multiplication. Thus, not using matrix operations is a feature of specific publications.)

4. Textbooks and lecture notes

Here is how we produced our data. We selected textbooks and lecture notes from a range of years that were available as PDF documents online. We then used the contents page to locate the section where matrix operations were introduced. We scanned the section by eye and recorded whether we agree with the statement “*In the section on operations on matrices, most attention is paid to multiplication*”. We made a note of the number of pages dedicated to matrix operations in total and to matrix multiplication in particular. In most cases, the textbook or lecture notes went on to explain applications of matrix operations, however we opted to remove these pages from our count and only include pages where matrix operations were first introduced to the reader.

We saw that in 24 textbooks and lecture notes out of 40, matrix multiplication was most prominent, and in 16 it was not.

As to the number of pages on which matrix multiplication features in a section on matrix operations, the ratio is shown in Figure 2, using the same bins as in the histogram in Figure 1. The horizontal axis shows the proportion of the pages in the publication which uses matrix multiplication. The vertical axis shows the number of publications. The histogram in Figure 2 is symmetrical, with both the mean and the median just under 70%.

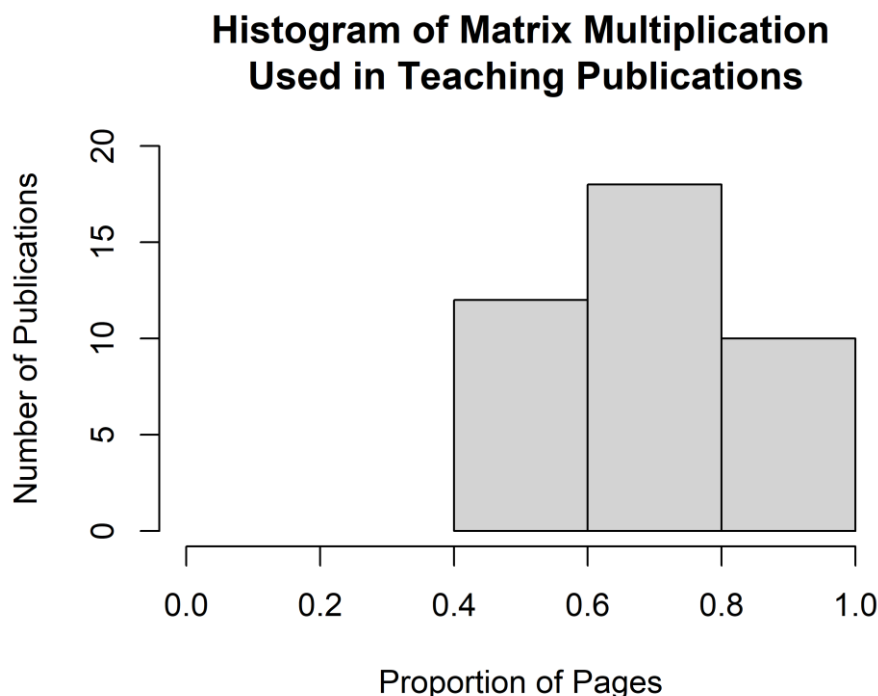


Figure 2. Proportion of pages in teaching publications that are used when introducing matrix operations and which are devoted to matrix multiplication.

In all sections on matrix operations matrix multiplication occupies more pages than other operations. If we pore over the text of these sections in more detail, the conclusions are mixed.

On the one hand, in some of the ‘multiplication-heavy’ sections, multiplication was given more attention not because it is presented as more important, but because its definition is perceived as being more complicated than those of other matrix operations. Some books explicitly suggest that out of the two operations, addition and multiplication, multiplication is the ‘uglier’ one. For example, in the textbook by Olver and Shakiban (2006), the definition of multiplication of matrices is immediately followed by saying “*Now, the bad news. Matrix multiplication is not commutative*”. In the textbook by Lang (2012), addition and scalar multiplication are parts of the definition of matrices, whereas multiplication is less so; indeed, the author defines not matrices, but “*the space of matrices*”. Similarly, for Bourbaki (1958), addition is more natural because addition of matrices can be defined for matrices over any additive group, whereas multiplication of matrices can be usefully defined only for matrices over an associative ring.

On the other hand, some textbooks and lecture notes skew the section on matrix operations towards multiplication in what seems a clear recognition of the more important role of multiplication. Out of the 40 textbooks and lecture notes, only 7 have 90% or more of the pages in the section on matrix operations dedicated to multiplication.

Hardly any textbooks or lecture notes explicitly state that matrix multiplication is more important. However, one example of a balanced solution is found in Birkhoff and MacLane (2017); first addition and scalar multiplication are introduced as “*vector operations on matrices*”, and then matrix multiplication is introduced as “*the most important combination*” of matrices.

5. Discussion and conclusions

Does the comparison presented in the previous two sections matter? Let us explain why we undertook this study. We agree with Harari (2018) saying,

the last thing a teacher needs to give her pupils is more information. They already have far too much of it. Instead, people need the ability to make sense of information, to tell the difference between what is important and what is unimportant, and above all to combine many bits of information into a broad picture of the world.

Not all undergraduate students will proceed to reading research papers and seeing which matrix operations are used or not used there. Millions of people might scan the lists of topics in A level subjects but never attempt these A levels. If we do not immediately present mathematical definitions and facts in the way in which they really are used in the practice of professional mathematicians, there might be no other opportunity. For many people, after they have read ‘add, subtract and multiply matrices’ in a mathematical curriculum, it will stay with them for life and slightly distort their mental image of mathematics. Somewhat exaggerating, to read ‘add, subtract and multiply matrices’ in a mathematical curriculum is like to read ‘Chertsey, Upminster and London’ in a geography curriculum; Chertsey and Upminster might be fine places, but they are less important than London and are not likely to feature in the same list with London and precede it.

When we teach matrices, we the authors grasp an opportunity to immediately show our own students that a clever definition of matrix multiplication makes this operation versatile and usable in many applications, and that this definition alone makes matrices usable in many applications. Reflecting on our observations presented in the previous sections, we eventually migrated towards a practice when we introduce matrix multiplication as an important construction, and in the meantime define matrices as notation which is convenient to use when one performs matrix multiplication. An example of one of activities that we use is given below:

A chelsea bun contains 45 grams of flour, 5 grams of sugar, 15 grams of milk and $\frac{1}{10}$ of an egg. A brioche bun contains 45 grams of flour, 2 grams of sugar, 2 grams of milk and $\frac{2}{10}$ of an egg. In 100 grams of flour there are 16 grams of protein, 86 grams of carbohydrate and 3 grams of fat. In 100 grams of sugar there are 100 grams of carbohydrate. In 100 grams of milk there are 3 grams of protein, 5 grams of carbohydrate and 1 gram of fat. In one egg there are 7 grams of protein and 5 grams of fat. I ate one brioche bun and two chelsea buns.

Express all the data from the previous paragraph as matrices. Multiply these matrices to calculate how much protein, carbohydrate and fat I consumed.

In addition to the example given above, you can see the first author, Alex Partner, introducing another activity with a toy food-based example in his video lecture (Partner, 2020). Our approach is similar to that of Dunn and Parberry (2002), where multiplication is described as the only interesting operation to be performed, particularly from the perspective of linear transformations. Eddie Woo uses a similar approach in one of his videos, also employing a toy food-based example (Woo, 2014).

6. References

6.1. Research outputs

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RESEARCH ARTICLE

Enhancing the Success of College Algebra Students by Incorporating Adaptive Technologies

Alina Stefanov, Department of Mathematics, University of Central Florida, Orlando, USA.

Email: alina.stefanov@ucf.edu

Barry J. Griffiths, Department of Mathematics, University of Central Florida, Orlando, USA.

Email: barry.griffiths@ucf.edu

Abstract

College students in the United States often enrol in introductory mathematics courses to acquire skills and knowledge that will enable them to solve related problems in their careers and daily activities. However, previous studies have shown limited levels of improvement that perpetuate a lack of proficiency. As a result, the gap in performance between top students and those at the bottom continues to increase. Using modern technology has been suggested as part of the solution to help students resolve their difficulties in mathematics and achieve better learning outcomes. This study assesses the longitudinal effect of redesigning College Algebra classes at a large public university, switching from a modified emporium model to a lab-based adaptive model. The results show that after redesigning the course, the pass rate increased from 68% to consistently being over 80%, while the withdrawal rate fell from 8% to 3%.

Keywords: Adaptive technology, College Algebra, ALEKS, personalised learning.

1. Introduction

Most college students in the United States today have constant and unlimited access to the internet, as well as a variety of technologies and mathematical tools that are available online. As a result, teaching and learning mathematics has been irrevocably altered in recent years to include distance learning, Massive Open Online Courses (MOOCs), and blended approaches that combine traditional lectures with online homework. Along with incorporating the various aspects of social media, these new technologies enable instructors to design courses that synchronize with the learning process and target specific needs (Engelbrecht, et al., 2020).

Included in these pedagogical innovations is the current trend of teaching College Algebra in a computer lab rather than a traditional classroom, which has prompted research educators to reassess didactic theories. A lab-based setting allows for intelligent tutoring systems to be used, which were shown in a meta-analysis of 50 studies by Kulik and Fletcher (2016) to raise test scores by 0.66 standard deviations, or from the 50th percentile to the 75th. One version that has attracted a lot of attention is active learning within an emporium model, first developed at Virginia Tech, which eliminates lectures and replaces them with a learning resource centre model featuring interactive software and on-demand personalised assistance. It allows students to choose what types of learning material to use depending on their needs, and how quickly they work through the curriculum. While requiring a significant commitment in terms of upfront cost and maintenance (Jones, 2016), the emporium model allows for multiple introductory courses to be taught using a staffing model that combines faculty, graduate teaching assistants, peer tutors and others who respond directly to the specific needs of students and direct them to the appropriate resources from which they can learn (Twigg, 2011).

Various modifications of the emporium model have been used to help researchers collect data. Most relevant to the current study is the work done by those who have explored the use of the emporium model in teaching College Algebra. Vallade (2013) found that College Algebra students in an emporium

setting had higher passing rates and overall scores, along with lower withdrawal rates, versus those taking the class in a traditional setting. The p-values indicated a statistically significant difference, though effect sizes were modest. A subsequent study by Cousins-Cooper, et al. (2017) found that students in emporium sections outperformed students in the traditional sections of College Algebra on the end of semester test by 12.63 percentage points, asserting that the results provide evidence that students at that level learn better by doing mathematics in an emporium rather than passively listening in a traditional setting.

In this study, we replace the emporium model and use adaptive learning technology to provide personalised instruction. Adaptive learning incorporates artificial intelligence to help students learn the content and assesses them on a regular or continuous basis. The concept of mastery learning is an important aspect of adaptive learning, which requires students to fully comprehend one topic or concept before moving to the next. Studies have shown that mastery learning can improve student outcomes in mathematics (Block & Burns, 1976; Kulik, et al., 1990), with Childers and Lu (2017) showing that the number of topics mastered is a significant predictor of final grades in foundational mathematics courses.

1.1. Background to the current study

In an effort to enhance the College Algebra course, various stakeholders at a large public university in the Southeastern United States, including departmental teaching staff, teaching assistants, and administrators collaborated to focus on changing the instructional methods to improve student performance, reduce the withdrawal rate, and improve student perception. Developments in the redesigned course were supported by regular conversations that took place before and during the implementation. Given that the primary aim was to increase student success, it was important to initiate and engage in discussions that specifically focused on the factors and constraints that affect attainment. These factors included the curriculum, instruction, assessment, process challenges, the development of a growth mindset, and the importance of incorporating previous research.

Expanding on these in turn, conversations involving the primary author and departmental collaborators allowed reflection on the state-mandated curriculum and how it was being enacted in the classroom. Other areas of focus included the need to address classroom diversity, promote equity, meet with students having special needs, and discuss the possible ways in which the new College Algebra format could be used to transform other courses. Regarding instruction, the discussion centred on effective pedagogical strategies, increasing student participation, and the viability and sustainability of new ideas. Assessment was considered in a manner designed to promote student learning, be attentive to the views of students regarding topics that pose particular difficulty, promote fairness, and the importance of assessment for summative, formative, and motivational purposes. Potential challenges and constraints were identified and reflected upon, along with the consideration of subsequent issues that could arise from modifying the course which could make further progress more difficult. Developing a growth mindset in students was discussed, thereby challenging the belief in a fixed mindset, where the ability of students in mathematics is predetermined (Boaler, 2013; Sun, 2018). Finally, a more general discussion took place involving the importance of utilizing recognised quantitative and qualitative techniques when conducting related research. The link between these factors is shown in the figure below.

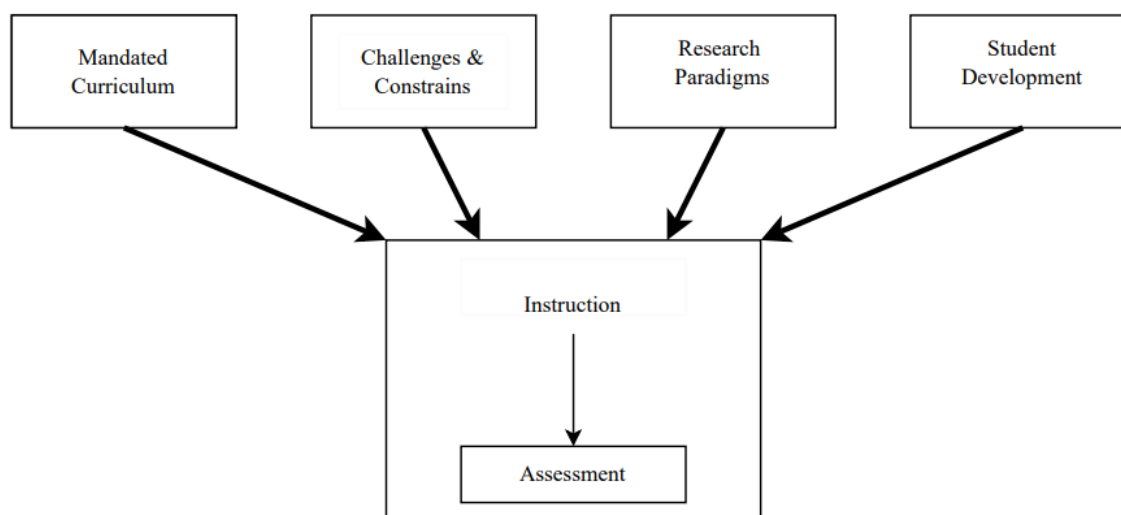


Figure 1: Factors guiding the redesigned course

1.2. Description of the course redesign

Prior to 2016, College Algebra students were taught according to a modified emporium model whereby they attended lectures for 50 minutes in a large auditorium in addition to engaging with computer-based learning for 3 hours per week in a laboratory. The online learning platform, MyLabsPlus, had no adaptive features at that time and was a one-size fits all platform consisting of weekly homework and quizzes. In the computer lab, students received on-demand assistance from the staff on duty. The staff comprised an instructor, graduate teaching assistants (GTAs), and learning assistants (LAs). By contrast, students taking the redesigned course in spring 2016 met two days per week for 75 minutes in the computer lab with their instructor, along with an instructional team comprised of GTAs and LAs ready to provide personalised help. During their class time, students used ALEKS (Assessment and Learning in Knowledge Spaces), a web-based adaptive learning system, and were involved in active learning through short lectures, class activities and problem solving. The system determined the topics that each student had mastered as well as the topics they had not, continually updating the student's knowledge map as they continued to learn new topics. The course material was divided into weekly objectives, but students could work on any course material (past or future) if they completed the current objective. At the beginning of the class period, the instructor or a graduate teaching assistant delivered a short lecture of approximately 15 to 20 minutes duration, and then during the remaining class time, students were encouraged to work on their assignments and ask the instructional team questions related to the course material. In addition, students had to take three non-adaptive midterm exams and a non-adaptive final exam, all of which were given in the computer lab. In contrast to the weekly assignments, the midterm and final exams were 'static' to avoid being affected by the adaptive nature of the ALEKS system and thus ensure that all the students were tested on the same types of problem.

2. Theoretical Framework

Keller (1967) and his colleagues noted significant improvements following the introduction of a personalised method of instruction which consisted of five features that distinguished it from the conventional method of teaching (Keller, 1968). These features are summarised below and explained in terms of how they are implemented in the current study:

1. Self-paced instruction – The student is allowed to move through the course at their own speed. Factors that determine the speed of learning include the student's ability and activities that affect the

student's availability to study. The instructor, in this case the adaptive learning system, creates a plan for the student, but the speed of learning is the student's choice.

2. Unit perfection – One important feature of the ALEKS system of personalised instruction is that the course is divided into units. The student is required to master one unit before proceeding to the next. The course material is designed prior the semester and the material is divided into units or objectives. Students who fail to master a certain unit are redirected by the adaptive systems to previously covered topics and given another test for assessment.

3. Lectures and demonstrations – A distinguishing feature of personalised instruction is the use of fewer lectures and demonstrations to motivate students, rather than them being primary sources of information. The lecture time in this instance was limited to 15-20 minutes per class meeting. During the lectures, only the topics in the current objective that many students struggle with were presented, which were identified through reports generated by the learning system.

4. Written materials – The Keller Plan emphasizes the use of written materials in teacher-student communications. Written materials allow students to study at their own pace and are portable, easy to review, and easy to annotate. The instructor's class notes were posted online prior to the class meetings in the learning management system to enable students to progress at their own pace.

5. Proctors – Humans play a vital role in the implementation of Keller's ideas. According to Keller (1968), proctors facilitate testing, tutoring, and immediate scoring. Proctors also enhance the personal-social aspect of teaching. In this instance, the instructor can permit early testing if specific criteria are met, while the instructor, undergraduate learning assistants and graduate teaching assistants are always available to address student questions and provide tutoring.

3. Research Questions

This quantitative study builds upon the ideas of Keller (1967, 1968) and seeks to answer the research questions below regarding the impact of redesigning College Algebra using the ALEKS web-based adaptive learning system.

1. What is the impact of the redesigned College Algebra course on the passing rate of students?
2. What is the impact of the redesigned College Algebra course on the withdrawal rate of students?

4. Methodology

The research design for this project was non-experimental. The final letter grades of the students were collected from the institutional database, with the aim being to explore if there were differences in the passing and the withdrawal rates before and after restructuring the course, along with an analysis to determine whether the differences were statistically significant.

The participants in the study consisted of all students enrolled in College Algebra at a large public university in the Southeastern United States during the spring semesters from 2015 to 2019. The students were placed into the course based on the scores they obtained when taking the university's placement test, along with those who had recently passed the prerequisite Intermediate Algebra course, and those retaking the course due to low grades when enrolled in the past.

The total number of students enrolled in the course during the first semester considered, Spring 2015 was 1008, while during the first semester of the redesign, Spring 2016, the number was 1250. This increased in subsequent years to 1296 in the Spring 2017 semester, 1343 in the Spring 2018 semester,

and 1491 in the Spring 2019 semester. The course was coordinated between the different sections, with all the students completing the same online homework assignments and tests.

The overall grades obtained by the students in the course were obtained from the institutional data base, compiled in an Excel spreadsheet, and then uploaded into SPSS (Version 26). The SPSS software was used to do the statistical analysis and obtain descriptive and inferential results, with the final letter grades coded as numerical values.

5. Results

To answer the first research question, final grades of A, B, and C were considered passing. The passing rates during the spring semesters from 2015 to 2019 are shown in Figure 2. As can be seen, the passing rate increased substantially from 68% in spring 2015 to a consistent figure of over 80% once the redesign took place.

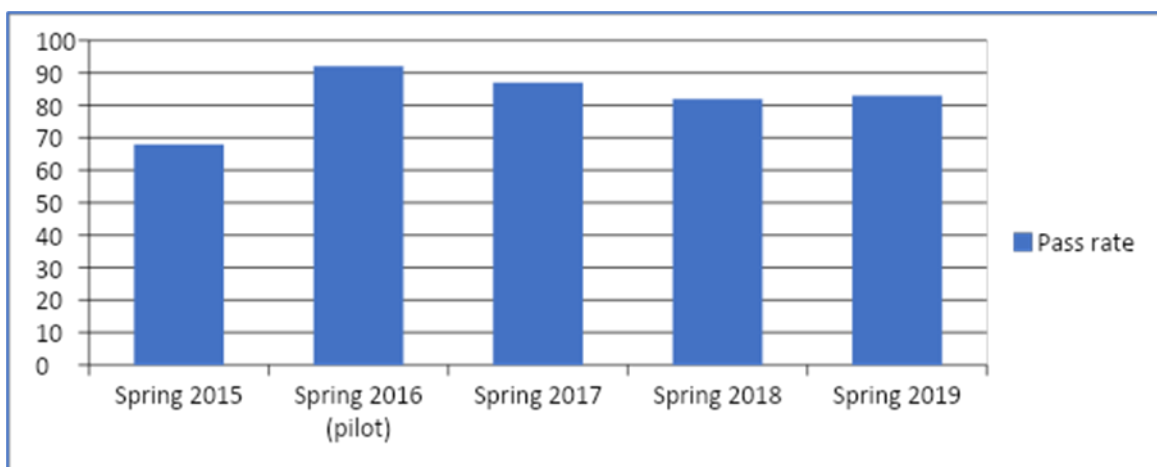


Figure 2 – Passing Rate of Students by Semester

For the second question, grades of W and WM (medical withdrawal) were considered as withdrawal. The withdrawal rate fell from 8% to a steady rate of around 3%, as can be seen in Figure 3.

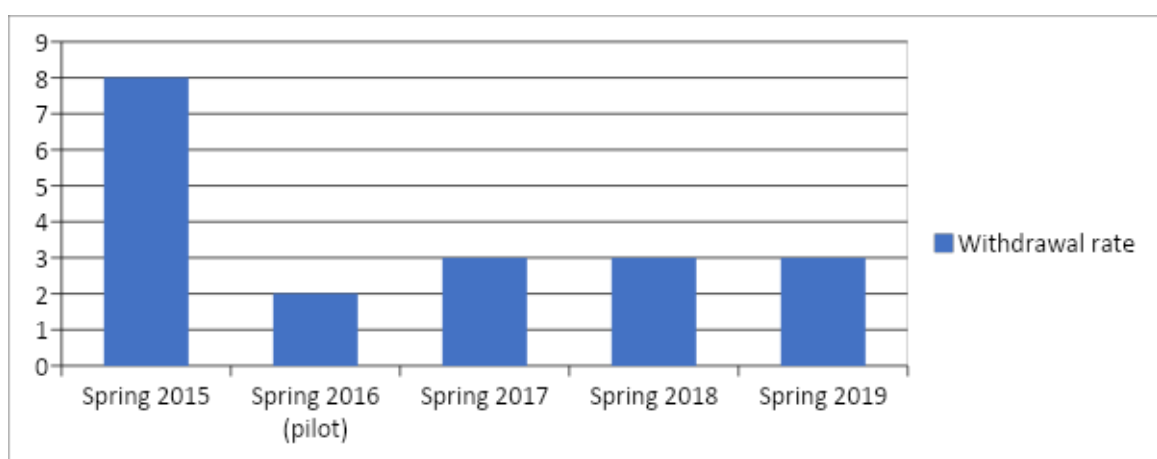


Figure 3 – Withdrawal Rate of Students by Semester

6. Discussion

This study explored student performance in College Algebra after changing the course structure and replacing the modified emporium model via the implementation of adaptive learning technology. The first semester included in the study (and used as a baseline) was the last semester when the modified emporium model was used to teach the course, while subsequent semesters included covered the period that the course redesign was adopted. Although it was intended to continue collecting data in 2020 and subsequent years, enforced changes made to the course structure and delivery caused by the Covid-19 pandemic made any further comparisons invalid.

The increase in the passing rate and the decrease in the withdrawal are in line with the studies by Hagerty, et al. (2005, 2010), Kasha (2015), Kulik & Fletcher (2016), and Boyce & O'Halloran (2020) suggesting that the redesign of College Algebra instruction methods using the ALEKS web-based adaptive learning system helps improve student success in the course. However, in this study we also considered the longitudinal effect of implementation and involved a much larger population of students. The results show that passing rates improved significantly after the changes made, from being below 70% to consistently being above 80%. At the same time, the study found that the withdrawal rate of students dropped from around 8% to a consistent rate in the following years of around 3%.

The results corroborate theoretical justifications for personalised learning advocated by Keller (1967, 1968) and more specific aspects required to implement such a tailored mastery system that consists of individual pacing, unit mastery requirements, written instructional materials, and student proctors (Thompson, 1980). It should be noted that while we did not seek student perceptions of adaptive learning in this study, one should not be take for granted that adaptive learning is necessarily popular or viewed as being beneficial, even when test scores and overall grades improve (Stuve, 2015). Careful implementation that incorporates all the stakeholders is therefore of critical importance when successfully carrying out changes to existing courses, with Sun, et al. (2021) supporting the contention of Griffiths (2015) in concluding that adaptive technologies provide a bigger boost to student learning when supplementing rather than replacing traditional instruction.

7. Conclusion

Adaptive technologies can radically alter the learning process, along with the organization of knowledge, and how this knowledge is accessed. In this study, we found that by modifying a course to incorporate the principles of personalised learning, the pass rate improved in a statistically significant manner, along with a reduction in the withdrawal rate. While the results are extremely encouraging, the current study was limited to the redesign of one course at one institution and may not generalize to a wider student population. The long-term effects of the redesign were not analysed in subsequent mathematics courses and is something that requires further study. There may also have been factors beyond the scope of this study that affected student performance, such as the instructors teaching the redesigned course. Finally, given that there are now several adaptive learning systems used in introductory college mathematics courses, it would also be natural to make quantitative comparisons of student attainment between them. In the meantime, we believe that the findings of this study can advise both higher education administrators and instructors of mathematics, especially those who teach in computer laboratories.

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RESEARCH ARTICLE

Supporting Students at the Transition to University with a Self-Paced, Online Mathematics Induction Course

Abigail Parkes, School of Engineering, Computing and Mathematical Sciences, University of Wolverhampton, Wolverhampton, United Kingdom. Email: Abigail.parkes@wlv.ac.uk
Liam Naughton, School of Engineering, Computing and Mathematical Sciences, University of Wolverhampton, Wolverhampton, United Kingdom. Email: L.naughton@wlv.ac.uk
Nazira Karodia, Edinburgh Napier University, Scotland, United Kingdom.
Email: n.karodia@napier.ac.uk

Jordan Brennan, School of Engineering, Computing and Mathematical Sciences, University of Wolverhampton, Wolverhampton, United Kingdom. Email: J.brennan@wlv.ac.uk
Emily Rodden, School of Engineering, Computing and Mathematical Sciences, University of Wolverhampton, Wolverhampton, United Kingdom. Email: E.rodgen3@wlv.ac.uk

Abstract

In this article we report on the design, development, and delivery of an online, self-paced, Mathematics Induction Course given to all incoming students in the Faculty of Science & Engineering at the University of Wolverhampton. We outline the background to the project, and we report on some preliminary findings which will be used to inform the future delivery of this course.

Keywords: Mathematics, Mathematics Education, Numeracy, Blended Learning.

1. Introduction & Background

Concerns regarding the mathematical preparedness of students entering UK Higher Education (HE) are certainly not new. As far back as June 2000, the Engineering Council recommended that all students embarking on mathematics-based degree courses should have a diagnostic test on entrance to University (Engineering Council, 2000). There have been many reports since then highlighting the need for mathematics support for students entering HE. In 2003, the LTSN MathsTEAM project carried out a survey on diagnostic provision at the time and produced a series of case studies on practices across UK HE institutions (LTSN MathsTEAM, 2003). The case studies included the delivery of paper-based diagnostic tests and computer-based diagnostic tests. Using funding provided by the Higher Education Funding Council for England (HEFCE), the Sigma Network (sigma-network.ac.uk) was created in 2005 as a collaborative Centre for Excellence in Teaching and Learning (CETL) in the provision of mathematics and statistics support. This led to the development of mathematics support centres in many UK Universities over the following fifteen or so years. See (Sigma, 2022b) for more information and see (The National HE STEM Programme, 2012) for details of the provision of mathematics learning support in UK HE institutions at the time.

In a parallel movement during the years 2000 to 2020, widening participation was becoming a priority for HE (Cox and Bidgood, 2002). This led to the development of many new routes into HE and the increasing popularity of these new routes created a very diverse cohort of students entering HE each year. In 2003, Lawson wrote (LTSN MathsTEAM, 2003)

No longer do the overwhelming majority of mathematics undergraduates have double maths and physics A-Levels or the majority of engineers have maths, physics and chemistry A-Levels. Indeed, on engineering degrees in some universities, students having A-Levels in any subject have become a minority as the numbers with vocational qualifications have steadily increased. Add to these, mature students returning to education through university foundation years, or a variety of Further Education

(FE) based access courses, not to mention a growing number of overseas students, and it becomes clear that university intakes have lost all semblance of homogeneity.

This statement is more relevant today than ever and it certainly rings true at the University of Wolverhampton. As the University of Opportunity, it *“aims to provide students, whatever their background or circumstance, the opportunity to fulfil their potential and realise their career ambitions in partnership with a supportive community”* (University of Wolverhampton, 2022).

For many years, at the University of Wolverhampton, there was a fragmented approach to diagnostic testing. Many subject areas did utilise a diagnostic tool of sorts, but these tended to be paper based and there was limited scope to follow up on findings afterwards. The adoption of the Canvas learning management system/virtual learning environment (www.instructure.com/en-gb) in 2017 provided an opportunity for a modern coordinated approach. A decision was taken to develop a cross faculty approach to be taken by all new incoming students to the Faculty of Science & Engineering (FSE) each year (approximately 900 students). The Mathematics Induction Course (MIC) was developed during summer 2017 and is now delivered to all new incoming students each year.

The Faculty of Science and Engineering offers more than 250 undergraduate programmes with minimum mathematics entry requirements of General Certificate Secondary Education (GCSE) Grade C/4 for level four students. Approximately 65% of incoming students will not have studied any mathematics post GCSE. About 25% of new students will have studied mathematics to AS-level or A-level while a further 10% of students will have other mathematics qualifications equivalent to GCSE. For this reason, it was decided that the MIC would presume a level of prior mathematics equivalent to GCSE grade 4. All incoming level four students in the faculty are automatically enrolled on the MIC on Canvas at the start of the academic year. Participation is mandatory and completion of the MIC is a compulsory element of the student Individual Learning Profile (ILP) at level four. The ILP is administered in conjunction with an academic skills coach.

This article includes analysis and insight gained from the submissions of 505 students who consented for their data to be used as part of this study.

2. The Maths Induction Course

The Maths Induction Course is an online, self-paced course aligned to a subset of the topics from the GCSE mathematics syllabus which assesses student competency in the areas shown in table 1.

The aim of the course is to encourage students to identify any of the above areas within which they may need some extra assistance and to overcome any difficulties at the outset of their studies. These content areas have been chosen based on feedback collected from staff across the faculty.

The MIC is live for the duration of the first semester, during which time students are required to complete all modules in the course. It was decided to make the course available throughout the entire semester so that students would not only have sufficient time to complete the course, but to also have the opportunity to interact with the extensive supporting materials that are available and, if required, receive any additional support.

Each module consists of a resource page, a discussion forum and an end of module quiz. The resource pages contain an introduction to the module which outlines the module requirements and the skills learned upon completion of the module. In addition to this, each resource page contains worked examples and exercises for students to engage with before completing the end of module quiz. Instructional videos are embedded at the top of each resource page where staff discuss a selection of the examples in more detail. A snapshot of one of the resource pages is shown in Figure 1. Each end

of module quiz contains five questions. The discussion forums are maintained for each module where students can post questions and receive support from the course team. Students are encouraged to make use of the supporting materials before completing each end of module quiz.

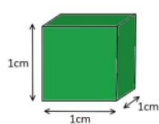
Table 1 Course modules.

| | | | |
|-----|------------------------------------|-----|------------------------------|
| 1. | Whole Numbers | 11. | Perimeter & Area |
| 2. | Fractions | 12. | Volume and Surface Area |
| 3. | Decimals | 13. | Coordinate Geometry |
| 4. | Percentages | 14. | Median & Mode |
| 5. | Ratio | 15. | Mean |
| 6. | Proportion | 16. | Interpreting Charts |
| 7. | Powers | 17. | Sets & Venn Diagrams |
| 8. | Simple & Compound Interest | 18. | Probability |
| 9. | Unit Conversion | 19. | Scatter Graphs & Correlation |
| 10. | Compound Measures (Speed, density) | 20. | Problem Solving |

Volume

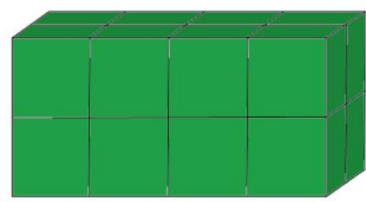
Volume is a measure of space taken up by a 3D shape. Finding the **volume** of an object can help us to determine how much space that object takes up, or how much that object can hold e.g. how much water can fill a cup.

Volume is measured in cubic units. E.g. cubic centimetres, cubic foot, cubic metre.



Volume of a cube: Length \times Width \times Height
 Volume of this cube: $1\text{cm} \times 1\text{cm} \times 1\text{cm} = 1\text{cm}^3$

Let's look at a cuboid made up of 16 of these cubes.



Volume of a cube: Length \times Width \times Height

Example:
 Let's find the volume of the 3D shape above.

Figure 1. A snapshot of one of the resource pages.

The course landing page contains a Welcome Video, informing the students of the purpose of the course, what they are required to do and how to access the course content. There is also a section dedicated to the course team where students can learn more about the members of staff who run the course.

Follow-up support is an important part of diagnostic testing and the Engineering Council also recommended that prompt and effective support should be available to students whose mathematical background is found wanting by the tests (Engineering Council, 2000). Support is built into the MIC in such a way that students are required to interact with certain supporting materials before they are able

to move on to the next module. A range of facilities are built into Canvas which enable course designers to control how users progress through the course. Each module requires students to

1. view the resource page;
2. score at least three out of five in the end of module quiz.

Students are not able to progress to the next module until the above requirements are completed. There is no limit to the number of times a student can take each of the quizzes which means that if a student fails to achieve the required score, they are able to retake the quiz as many times as required with no subsequent consequences. Each quiz is automatically marked upon submission. Question response types include multiple choice, freetext and “*arrange in the correct order*”. When creating each quiz, course designers are able to provide a range of correct answers for each question type. Upon submission of the quiz, Canvas identifies the students’ responses which match the pre-provided “*correct*” answers for each question and marks them as correct. Answers that do not match the pre-provided “*correct*” answers are marked as incorrect. This enables students to have instant feedback on their quiz submissions and allows them to immediately identify any areas that need extra attention. In this way, students can identify and address any mathematical or numerical difficulties as they progress through the course, with the aim that upon successful completion they will be equipped with the essential skills that are needed to succeed in their course. The questions within each quiz are not randomised, although this is a feature which will be added in the future. A sample End of Module Quiz is included in the appendix.

3. Methodology

Before completing the MIC, students are asked to complete a short Welcome Survey which gathers information on their previous mathematical background and the mathematics qualifications acquired before commencing their university degree programmes. The survey takes the form of an online canvas quiz accessed via the MIC homepage. A copy of the Welcome Survey is shown in table 2.

Table 2. Welcome Survey.

| # | Text | Option |
|-----------|--|---------------------------------------|
| Q1 | Please type your course of study in the box below. | Freetext |
| Q2 | When were you last in full-time education? | Less than 1 year ago |
| | | Between 1 and 3 years ago |
| | | Between 3 and 10 years ago |
| | | More than 10 years ago |
| Q3 | Have you completed a foundation year for your course of study? | Yes |
| | | No |
| Q4 | Please select the highest-level mathematics qualification you have achieved. | A Level Mathematics or equivalent |
| | | AS Level Mathematics or equivalent |
| | | GCSE Mathematics or equivalent |
| | | Core Maths |
| | | Level 2 Functional Skills Mathematics |
| | | Other mathematics qualification |

The data collected as part of the Welcome Survey is discussed in Section 4. As mentioned in Section 3, all students are also required to complete each of the twenty end of module quizzes. Students are required to score at least 3/5 in each of the end of module quizzes. If the minimum score is not met,

students are able to retake the quiz as many times as needed until they achieve the required score. There is no time restriction for each quiz and students can complete the quizzes at a time that is most convenient for them. They do not have to complete the quizzes on campus, they have the ability to do so at home, on their own devices. The scores achieved in each quiz are categorised according to School in section 4.

4. Results

In total, 505 students completed the Welcome Survey of which 499 completed the first module of the course and 434 completed the final module. Approximately one third of students who completed the MIC had previously completed a foundation year as part of their studies. Just over 25% of students were last in full time education more than one year ago. This is typical in the Faculty of Science & Engineering at the University of Wolverhampton. GCSE mathematics was the highest mathematics qualification on entry for approximately 61% of students with just over 27% having A level or equivalent qualifications. The performance of these students in the MIC modules will now be discussed.

4.1. Overall Score

Figure 2 illustrates the distribution of overall scores for all students who engaged with more than one quiz in the MIC. Here, score is taken to be the sum of the scores achieved for each of the twenty end of module quizzes. It is clear that there were few students that achieved an overall score below 60. This was to be expected as the course requirements stipulated a minimum score of three out of five for each end of module quiz in order to progress.

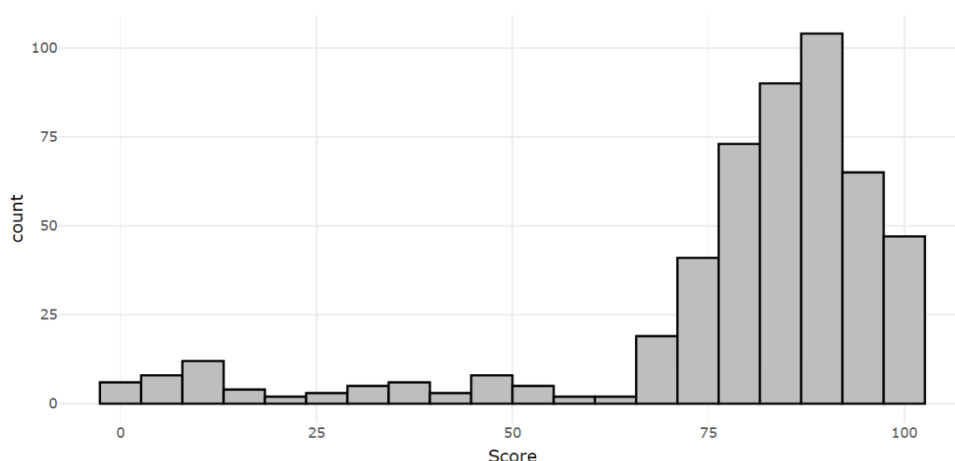


Figure 2. Overall scores for all students who engaged with more than one quiz in the Mathematics Induction Course.

It is still to be welcomed that the overwhelming majority of students scored more than 60/100 on the MIC.

The scores for each end of module quiz reveal interesting results. Figure 3 illustrates the proportion of students who achieved each of the possible scores in the end of module quizzes on their last attempt. The number of students scoring zero in the quizzes increases towards the end of the course. Again, this can be expected as not every student completed the course. Some further investigation into the reason why this is the case might be needed to determine how much of this is due to students simply self-removing from the course, or whether there was a module where students have not been able to achieve the minimum score and were consequently prevented from completing the course.

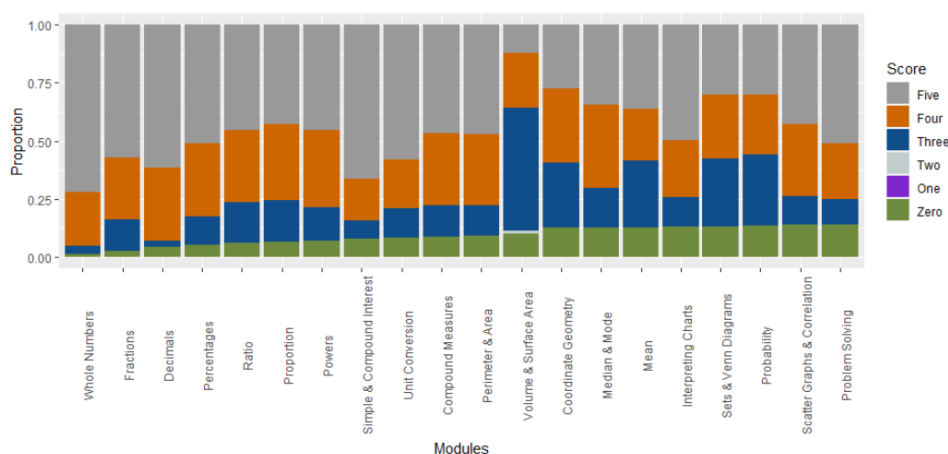


Figure 3. Proportion of students who achieved each of the possible scores in the end of module quizzes on their last attempt

The scores of zero can be discounted from the analysis at this stage as this indicates that the student did not submit the quiz. The proportion of students not submitting the quizzes grows steadily as a small number of students disengage with the MIC. A deeper analysis is ongoing to fully understand the reasons for this non-engagement and to see if it mirrors the student's course modules.

Module 12, Volume & Surface Area stands out as the first module where students seemed to experience difficulty in large numbers. The proportion achieving 5/5 in this module is much smaller than in the preceding modules. More analysis is needed to understand the reasons for this drop in level of performance.

4.2. Overall Score by School

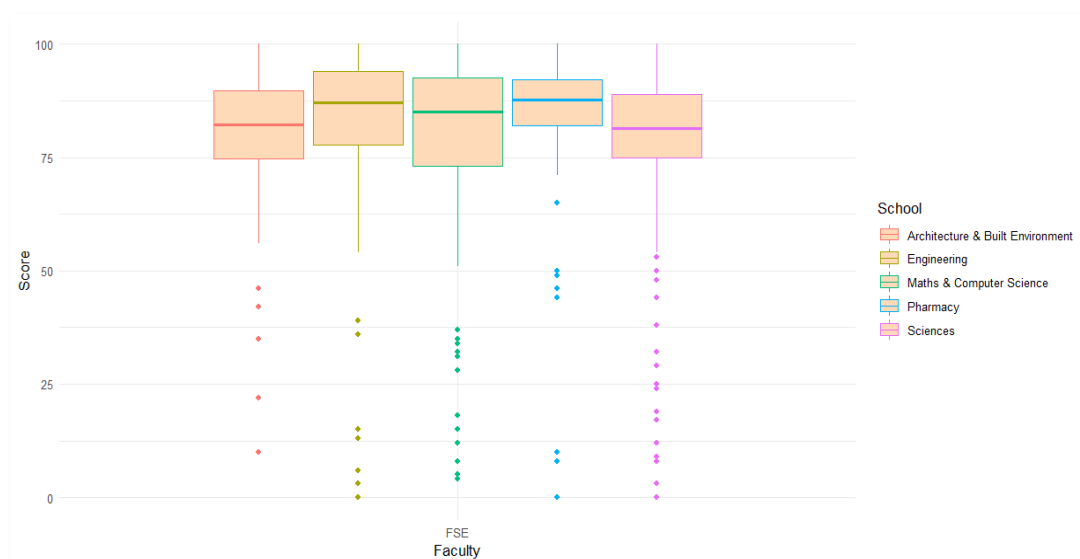


Figure 4. Distribution of scores across five academic schools.

Figure 4 illustrates the distribution of scores across each of the five schools within FSE. Students in the School of Pharmacy performed better than their counterparts and this is probably due to slightly higher average entry requirements in Pharmacy than in the other schools. The spread of scores is

widest in the School of Mathematics & Computer Science which at first glance is slightly surprising given that mathematics courses are housed within this school. However, there are a variety of courses in Computer Science including some Higher National Diploma (HND) courses where mathematics entry requirements are quite low.

The outliers in the different schools represent students who disengaged with the MIC and possibly also with their general courses although this requires more investigation.

4.3. Foundation Year

Approximately 30% of the students had completed a foundation year prior to their current course of study. Figure 5 illustrates the distribution of scores for those who have completed a foundation year compared to those who have not. Performance is similar between the two groups but there is a slight indication that students who have completed a foundation year do slightly better on the MIC than those that have not.

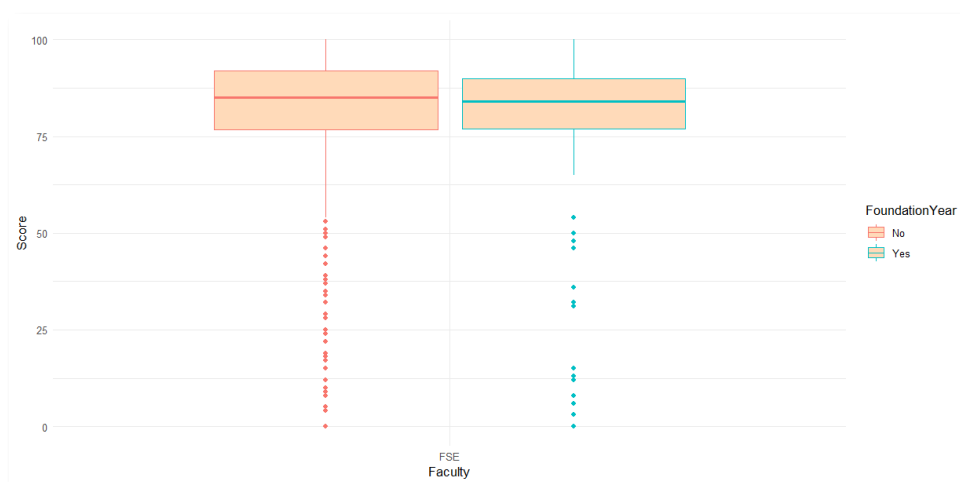


Figure 5. Distribution of scores for students who had and had not completed a foundation year prior to their current course of study.

When the scores are broken down by whether or not students have completed a foundation year within the different schools some interesting trends begin to emerge which will be of interest to learning & teaching panels with the schools. Figure 6 illustrates the distribution of scores for those who have completed a foundation compared to those who have not, broken down by School. In two of the schools, students who have completed a foundation year out-perform those who have not. In the School of Pharmacy and the School of Sciences, students who have completed a foundation year perform very slightly less well than their classmates. However, in the School of Engineering, students who have completed a foundation year perform considerably less well than their classmates. This is concerning for staff in the School of Engineering given the mathematics heavy nature of engineering courses so this indicates that additional support will be required for these students.

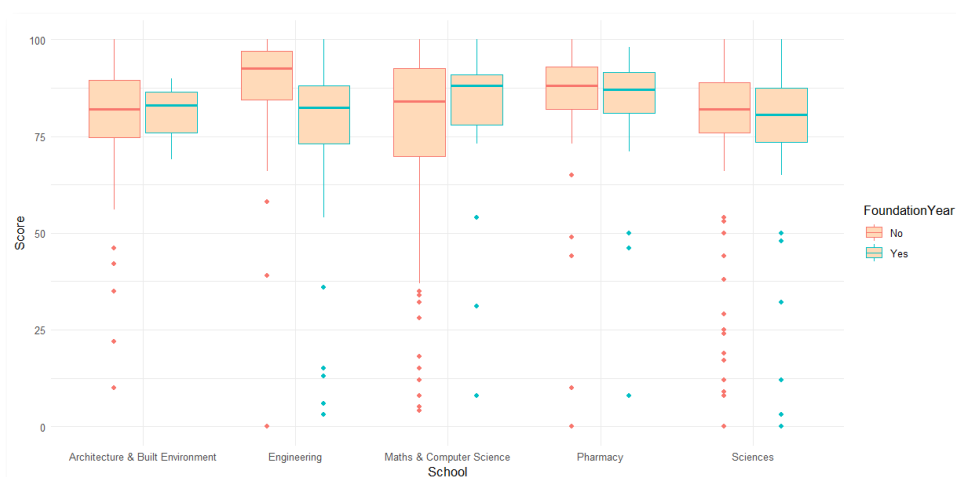


Figure 6. Distribution of scores for students who had and had not completed a foundation year prior to their current course of study, broken down by academic school.

4.4. Previous Mathematics Qualifications

Unsurprisingly, those with A level mathematics or equivalent do perform better in the MIC than students with other qualifications as illustrated in Figure 7. The performance of students with GCSE or equivalent lags slightly behind but not so far behind as to be a concern for staff. The lowest level of performance is exhibited by students with Functional Skills Level 2 Mathematics. These fourteen students represent a very small proportion of the intake, but it is certainly clear that some additional support will be needed for these students.

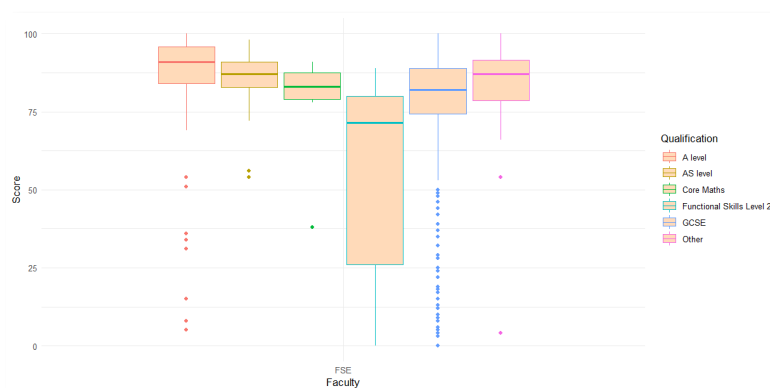


Figure 7. Distribution of scores for students by prior qualification.

5. Discussion

A common challenge with this type of research is obtaining permission to use students' data as part of the project. This was no different here. Approximately 37% of the total number of students registered on the MIC did not grant permission for the team to use their data which meant that 37% of the total data collected was omitted from the evaluation. The inclusion of this data may have made a difference to the outcomes and results.

Another limitation in this study is that not every student who began the course went on to complete all modules. Approximately 14% of those who completed the welcome survey (and granted permission to

use their data) did not complete all modules within the MIC. This data may help to understand the reason why students opt out of the MIC as well as their general studies.

The data on the number of attempts required before a student passed a module or the course as a whole would be a very interesting addition to this study. Unfortunately, this data was not available due to implementation issues with the Canvas learning management system at the university. Data from a post course survey of students' perceptions about the MIC would have also been useful and this feature will be added in future iterations of the MIC.

With the onset of the Covid-19 pandemic in the UK in March 2020, and with restrictions on social distancing still enforced at the beginning of the 2020/21 academic year, on-campus face-to-face teaching could not take place. The MIC is an online course which meant that few modifications needed to be made to the course itself in preparation for the start of the academic year. The biggest impact the restrictions had on the MIC was the initial launch of the course. For the first time, all students registered on the MIC in 2020/21 were starting their degree courses entirely online. In previous years, staff have joined face-to-face classes during the first week of the semester to introduce the MIC to incoming students. During the visits, staff would demonstrate what is required to complete the MIC whilst also explaining the purpose of the course and ensuring students were aware of the support that was available to them. Any initial questions were also addressed at this time. As an alternative, a Welcome Video was created and placed on the course landing page to outline the essential information that was usually communicated during the class visits. Additionally, on-campus support such as drop-in sessions could not take place. This meant that support was mainly provided via email, responding to comments on the discussion forums and Microsoft Teams meetings. In previous years, very few students attended the drop-in sessions, so it is unclear whether the absence of this provision has had an impact on the overall student experience.

The absence of class visits may have also had an effect on the number of students granting permission for their data to be used. Usually, any initial questions surrounding the research project and the usage of student data were addressed at the time of the on-campus class visits. It is possible that with the absence of this face-to-face interaction, initial questions and requests for clarification were not addressed before the option of granting permission was introduced and therefore may have affected their decision to grant permission of the usage of their course data.

Upon reflection, the initial design phase and the setting up of the course took the most amount of time. Extensive research took place to inform decisions taken on which numerical and mathematical topics would be included in the course, including researching topics on the GCSE Mathematics syllabus and speaking with members of staff. Developing the twenty module resources and quiz questions also took a great deal of time to prepare.

One essential development when designing this course was identifying which style of questions worked for the purpose of the MIC and which did not. Due to the number of students who take part in the MIC each year, it was essential that all quizzes could be automatically marked. The number of quizzes in the MIC, coupled with the ability of students to retake each one as many times as required, would have created an immense amount of marking that would have been impossible for the course team to manage without the facility of automatic marking. Not to mention the pace at which feedback must be given to students to be of any benefit meant that this facility was essential for the course to run successfully. Canvas provides a range of question styles when creating an online quiz. Certain question styles can be automatically marked whereas others require manual marking. One issue that quickly became apparent in the early developmental phase was the inappropriate use of certain question styles. In order to mark a question as correct, Canvas looks for a match between the pre-provided "correct" answers and the students' submissions. If the answer is not an exact match, it will

be marked as incorrect. This meant that for certain free text questions, if there was an insignificant mistake such as a typo or an incorrectly spelt word, the answer was marked as incorrect even though the answer was in fact correct. Consequently, student submissions were having to be checked by the course team to correct any mismarking. Careful consideration of the question style utilised was needed to ensure that automatic marking was successfully implemented.

The facility for the quizzes to be automatically marked enabled students to have instant feedback for each quiz. Furthermore, it enabled students to complete the course at a maintained pace that suited them without interruption caused by having to wait for the course team to mark their quiz submissions.

The next step for the course team is to look at the impact that the MIC is having on student performance at university and how the MIC can be used to make predictions about students' subsequent performance at the end of Semester one and at the end of year one. The course team also plan to investigate how the results of the course can be used to target supports for students.

The course team also plan to gather student feedback to help inform and develop the course further for future iterations of the MIC.

6. Resource availability

The resources for the Mathematics Induction Course are available from the authors on request.

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8. Appendix

8.1. Whole Numbers End of Module Quiz

| # | Text | Option |
|----|--|--|
| Q1 | Calculate $-12 - (-8)$ | <div><div>-5</div><div>-21</div><div>-4</div><div>1</div></div> |
| Q2 | A bank account is at $-\pounds 22$. $\pounds 50$ is deposited into the account. A phone bill of $\pounds 38$ is then paid from the account. How much is left in the account? | Freetext |
| Q3 | Tickets to an exhibition cost $\pounds 11$ per adult and $\pounds 6$ per child. Tickets for 3 adults and 6 children are bought. Four $\pounds 20$ notes are used to pay for these tickets. How much change should be expected? | Freetext |
| Q4 | Calculate $-14 \div -2$ | <div><div>7</div><div>-7</div><div>-16</div><div>-12</div></div> |
| Q5 | It costs $\pounds 360$ per night to rent a villa. If 12 people wish to rent the villa for 7 days, how much will each person have to pay given that the price is evenly split? | Freetext |

CASE STUDY

A Trial of the Assistive Technology EquatIO in an Irish University

Ciarán Mac an Bhaird, Department of Mathematics and Statistics, Maynooth University, Ireland. Email: ciaran.macanbhaird@mu.ie ORCID ID: <https://orcid.org/0000-0001-5971-7709>

Rachel O'Neill, Department of Mathematics and Statistics, Maynooth University, Ireland.

Email: rachel.oneill@mu.ie

Ruchi Palan, Maynooth Access Programme Office, Maynooth University, Ireland.

Email: ruchi.palan@mu.ie

Abstract

In this case study, we report on the trial of the assistive technology EquatIO with students studying mathematics and statistics at Maynooth University (MU) during the 2021-22 academic year. This software, provided by the MAP (Maynooth Access Programme) Office, was made available to support students who presented to MAP with a difficulty writing mathematics. In this paper, we give a brief outline of EquatIO and the trial. We then describe the feedback received from students who engaged with the software. Finally, we close with conclusions on our experience of providing the software for students, including observations from the tutor who assisted students as part of the trial, and end with our recommendations for possible future supports.

Keywords: Accessibility, Assistive Learning Technology, Mathematics Support.

1. Introduction and background

In 2019, MU received funding from the Higher Education Authority (HEA) in Ireland for an ICT and STEM Enhancement Project. As part of this project, the Department of Mathematics and Statistics at MU has considered different aspects and new trials of academic mathematics learning supports (MLS) provided for students with different access needs registered with the MAP Office. These include mature students and those with disabilities and additional learning needs, see for example Heraty, et al. (2021), Mac an Bhaird, et al. (2022a), and Mac an Bhaird, et al. (2022b). For further detail on accessibility issues related to MLS and service mathematics teaching see, for example, Cliffe, et al. (2020). In 2021-22, the MAP Office in collaboration with the Department of Mathematics and Statistics, piloted the software EquatIO with students who needed additional support for mathematics. For further information on the use of assistive technologies in higher education in Ireland see, for example, McNicholl, et al. (2020) and McNicholl, et al. (2021).

2. EquatIO

EquatIO (texthelp.com/products/equatio) is software offered by the company TextHelp, whose reported mission is to provide accessibility through their suite of assistive technology. EquatIO is specifically designed to be used for STEM subjects and is marketed with the description: “*Makes math both digital and accessible. Type, handwrite, or dictate any expression, with no tricky coding to master*”. When opened, EquatIO launches a toolbar which is displayed at the bottom of the screen (see Figure 1), in which, mathematical expressions can be created and then inserted into a document.

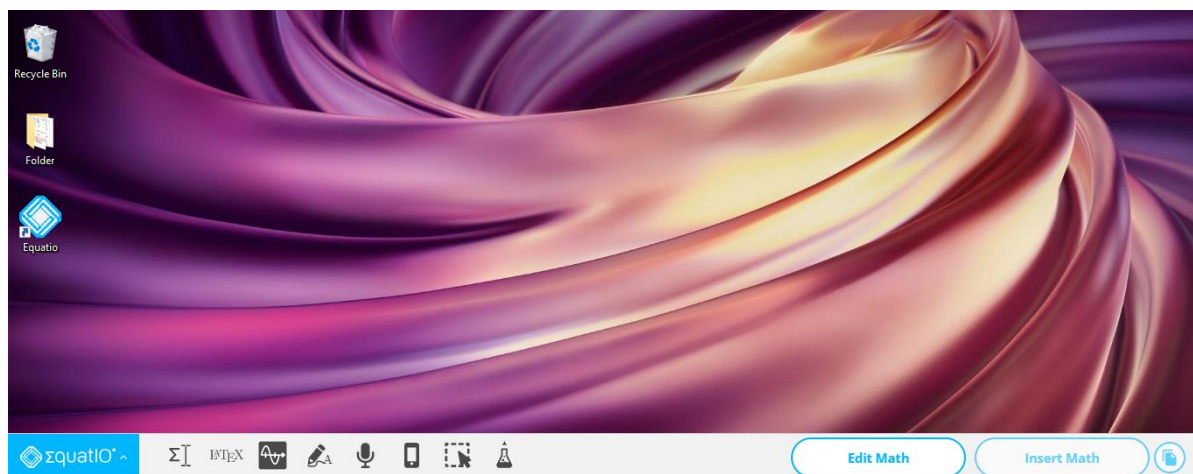


Figure 1: EquatIO Toolbar

The Windows and Mac version of EquatIO, when opened, displays over any other open application and can be configured to insert the mathematical expressions into a Microsoft Word document (on Windows and Mac) or Microsoft PowerPoint document (on Windows only). The Google Chrome version of EquatIO displays in a Google Chrome window and mathematical expressions can be inserted into a variety of Google and Microsoft online file types (such as Google Docs and Microsoft Word Online), full details are available (<https://www.texthelp.com/products/equatior/equatior-free-vs-premium/>). Other versions of EquatIO were not considered during this trial.

Users of EquatIO can create mathematical expressions in a variety of ways, for example, by typing, handwriting, speaking or through the screenshot reader. Mathematical expressions can be typed into the 'Equation Editor' where symbols are suggested to the user as they type. For example, typing the word "sum" displays three suggestions: sum +, summation $\sum_{n=}$ and sum of two cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ (see Figure 2). The mouse or the navigation and enter keys are used to choose the desired option. There is an extensive range of options available such as notation for integrals, limits and matrices, Greek letters, and many common mathematical, statistical and even physics and chemistry-related equations.

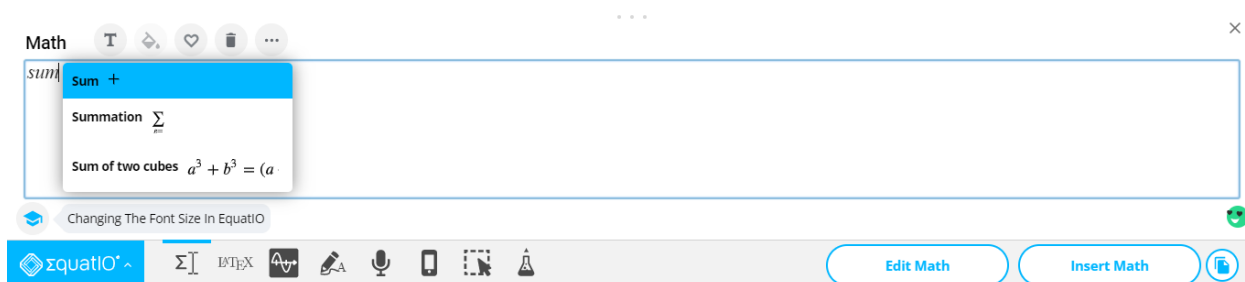


Figure 2: The Equation Editor

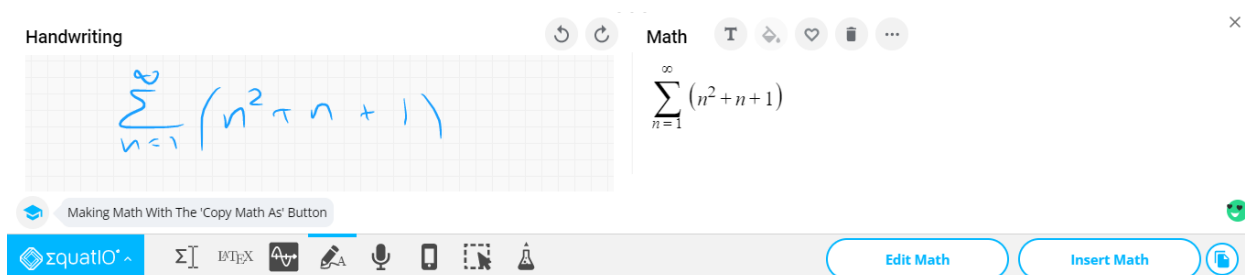


Figure 3: Handwriting Recognition

The 'Handwriting Recognition' option allows the user to write maths into a small digital whiteboard, which EquatIO then converts into text (see Figure 3).

The 'Speech Input' option recognises spoken mathematical expressions and converts them into text (see Figure 4). Note that EquatIO is designed to ignore any words that it does not consider to be mathematical expressions.

The 'Screenshot Reader' option lets the user capture any mathematical expression existing on their computer screen, in either text or image format, by drawing a box around it with a crosshair cursor. This can then be edited in EquatIO (see Figure 5). This feature also gives the user the option to hear the expression being read aloud.

While mathematical expressions are being created, using any of the previously mentioned methods, EquatIO will automatically generate the LaTeX code of the corresponding mathematics which can be viewed and edited in the 'LaTeX Editor' (see Figure 6). The user has the option to edit this LaTeX code and swap back to the Equation Editor to edit further, however, the Equation Editor is not compatible with all LaTeX. Some advanced types of LaTeX will turn the little happy green face to yellow, and warn the user that their math is "advanced". In this case, the Equation Editor can no longer be used and the user must proceed with LaTeX. We did not come across any such situation with the mathematics that our students encountered.

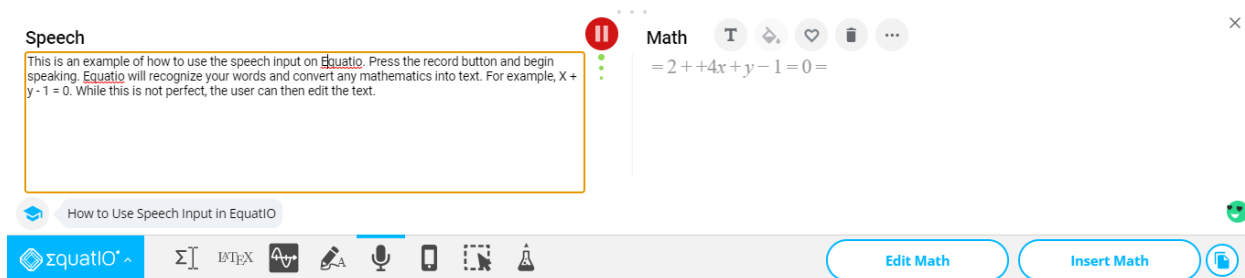


Figure 4: Speech Input

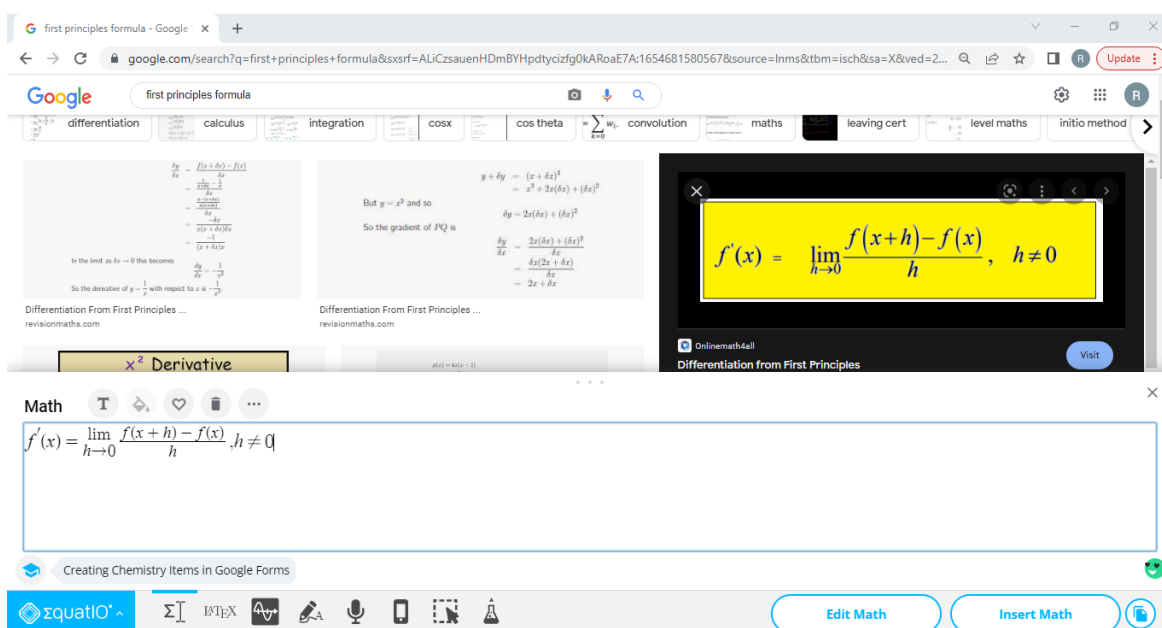


Figure 5: The Screenshot Reader

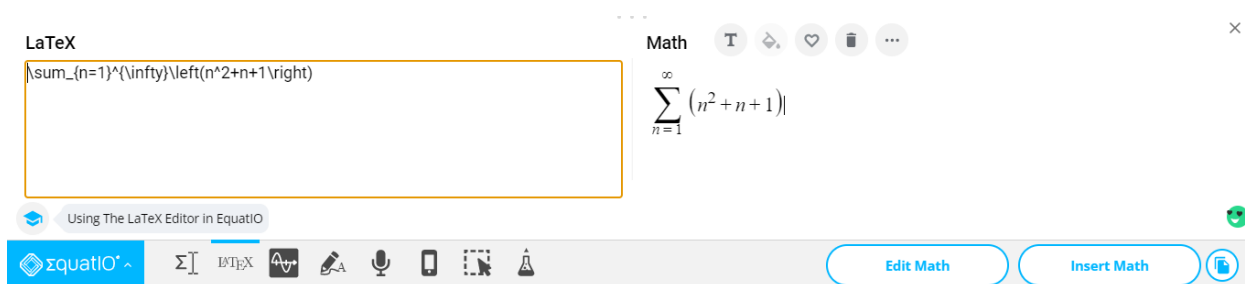


Figure 6: The LaTeX Editor

$$\sum_{n=1}^{\infty} (n^2 + n + 1)$$

Figure 7: Image of an Inserted Mathematical Expression



Figure 8: Copy Math As

Clicking 'Insert Math' inputs the mathematical expression as an image by default into the current selected compatible document type (see Figure 7). Clicking on an inserted image and then choosing 'Edit Math' returns the text of that image to the Equation Editor.

If the user wants to use an incompatible document, then the 'Insert Math' and 'Edit Math' buttons won't work. To place the mathematical expression in an incompatible document they must copy the expression using the 'Copy Math As' button, and then paste it into their document. This allows the user to copy the mathematical expression to the clipboard in a variety of formats and gives options of Image, LaTeX, MathML, Spoken Text, HTML, SVG, URL and Download PNG (see Figure 8).

EquatIO has a 'Graph Editor', which will create graphs based on user input. These graphs can also be inserted into documents. There is also an 'EquatIO Mobile' feature which allows for the use of a mobile phone for handwriting-to-text, speech-to-text and image-to-text. This creates mathematical expressions which can be inserted directly into any compatible document which the user is currently editing. This feature is accessed through scanning a QR code, and signing in on a mobile, but no download is necessary.

3. Methodology

Each Department at MU has a MAP Academic Advisor who acts as a point of contact for MAP staff and students. The first author has had that role for the Department of Mathematics and Statistics (the

Department) since 2012, see Mac an Bhaird, et al. (2022a) for further details. There is an established and successful referral process for the Department and the MAP Office in relation to MAP students and the provision of academic MLS. We decided to run the trial by embedding EquatIO as part of this process.

In the first instance, when students attended a meeting with their contact in the MAP Office, those identified with issues relating to the writing of mathematics, for example, speed of writing, clarity, organisation, etc. were referred to the Assistive Technologist (AT) specialist, the third author. The specialist would make students aware of various ATs available and where appropriate, mention the possibility of support with EquatIO and provide a link to the software along with a referral to the MAP Academic Advisor in the Department. During the 2021-22 academic year, nine students in total were referred for support using EquatIO, three of these did not respond when subsequently contacted by the MAP Academic Advisor and then contacted again by the AT specialist.

When the students met with the MAP Academic Advisor, in addition to the standard discussion about the student's progress with mathematics and the various MLS available, they were provided with additional information on EquatIO. In particular, it was clarified that a tutor (the second author) would be available to meet with the students and support their initial engagement with the software. Students were then also informed about the trial, the option of providing feedback about their experience, and that the tutor would keep notes about the sessions. It was also made clear to the students that a decision to engage or not engage with the trial would not impact on the support provided with EquatIO in any way. All six students were provided with full information on the project, including details of the ethical approval that was granted in October 2021, and all agreed to engage with the trial and provide feedback. In March and April 2022, students were issued with a short open response survey, see the appendix. Once responses were completed, the data was then anonymised for the reporting process.

4. Student Feedback

In this section, we provide an overview of the feedback collected from the six student participants in the open-response questions. Five of the participants were first-year students, two of which were mature students, and the final participant was a third year mature student who was the only one who had previous experience using LaTeX to write mathematics.

Three of the six participants indicated that, prior to being introduced to EquatIO, they had already used AT. One of these students mentioned using ATs in secondary school, but not for mathematics. The remaining two students had previously used ATs at university, but again, neither of the technologies mentioned - Read&Write (texthelp.com/products/read-and-write-education) and a Livescribe Pen (eu.livescribe.com) are specifically designed to help with mathematics. Read&Write supports students with reading and writing text whereas Livescribe Pen supports students to cope with note-taking.

All six participants indicated that, prior to this trial, they had not previously heard of or used EquatIO. In addition to the two ATs mentioned above, students described other types of assistance and technologies that they had free access to in school/university. For example, they benefited from writing courses, the note-taking software Glean (glean.co), a laptop, a Special Needs Assistant, a scribe, and exam provisions such as extra time and being allowed to type exams.

The participants reported various personal barriers to studying which they hoped EquatIO would help with when they were first informed of the technology. Saving time when writing was mentioned by four, *"I hoped it would speed up my note-taking process"*. Three participants explained having difficulties with handwriting, *"I have trouble writing, I can write, but very slowly and it's very hard to read it"*. Two participants mentioned wanting to use EquatIO specifically for taking down notes during lectures, *"Part of my difficulty is that in a typically fast-paced lecture I find it difficult to pay attention in a productive fashion to the aural content of a lecture if I am being asked to also write down a lot of content from a*

board or slides simultaneously". Finally, one student mentioned that they hoped it would make it easier to keep their notes organised.

Participants were also asked to describe how they used EquatIO, if at all, after being introduced to it and given a demonstration of how to use it. Only one student reported using EquatIO regularly, and that was to take notes during lectures. Although this student reported not having used AT before, their feedback was that *"It's a lot quicker than having to write down notes and is easier to read"*. Of the remaining five students, only two mentioned using EquatIO occasionally, and this was specifically to use the graphing function. Reasons cited for not using EquatIO included that it did not save time, *"I hoped that EquatIO would help with my time management, however when I used EquatIO it took me about the same time to capture my notes as it would have taken if I had written them"*, that it did not work with their device, *"It was frustrating. I tried to use it but it wasn't working"*, and that they didn't have time to use it *"I have not used EquatIO much yet as I was introduced to it during a busy period of study and feel that I need to spend time to get familiar and comfortable with it before I can be happy with using it on a regular basis"*. For the third-year student it wasn't the right tool for them, *"I had developed my own strategy for notetaking using LaTeX which I had grown to prefer and after trying EquatIO a couple of times I decided I didn't have the time to learn another tool when I had a perfectly good one already"*.

Participants were asked to comment further on the pros and cons of using EquatIO. One student left this question blank, one said they had no strong opinions, and one restated their frustration with using the software that did not work for them. The other three students gave pros to using EquatIO. Again, the graphing feature was mentioned, *"The graphing function was helpful, I could click into the function and output graphs into a word document easily"*. One participant, the only regular user of EquatIO, remarked that it was quicker than writing and easier to read. And, although they did not use EquatIO themselves, the ability to be able to create typed notes was praised by the third-year student, *"It is a very useful tool in terms of being able to write notes and immediately transform those written notes into a nice-looking typeset collection of notes. While typing notes with something like LaTeX is definitely effective, writing out notes particularly in Maths is beneficial for understanding and for practice. EquatIO in this regards is a powerful tool"*.

The participants were asked, if they were not using EquatIO regularly, to explain why. Five participants responded. Three of the participants cited time-related issues: two of these reported not having the time to dedicate to learning a new technology *"I haven't had time to sit and familiarise myself with it as fully as I would like"* and *"I didn't adopt it early enough in my studies and the pace of my studies are too fast to allow me to take time to integrate a new tool"*. The third of these found EquatIO time-consuming to use as it was not directly compatible with their existing study strategies, *"I find it takes longer to copy and paste and input into notes like OneNote. It ends up taking more time than writing."* Another student mentioned that a change in the format of their mathematics lectures meant they did not need to take notes in the same way and had no need for EquatIO. Finally, one student reiterated their frustration that the software would not work on their laptop. The student who did use EquatIO regularly did not respond to this question.

When asked if they thought that this software would be useful for other students, five participants agreed. They suggested that other students would find EquatIO very beneficial, even though it was not necessarily useful for them, *"This would be a fantastic tool for Maths students particularly if used in conjunction with LaTeX, if integrated from the beginning of a student's course"*.

5. Conclusion

A number of positives and negatives emerged from this trial of EquatIO. For example, the students generally found it easy to use, with download and installation normally fast and straightforward,

reducing the barrier to learning a new technology. EquatIO has a simple interface and, with some instruction, most students began using it and creating documents quickly. One caveat is that EquatIO inserts images by default, these can appear pixelated and the text in the various images can have different sizes. However, while typed documents have better appearance, the ease-of-use with EquatIO's default image insertion is a good trade-off. In our institution's Mathematics and Statistics Department, for assignments which are required to be submitted online in PDF format, students are advised to hand-write their assignment, take pictures of each page, and then convert their work to PDF format using an image to PDF conversion application. For this reason, assignments created with EquatIO have perfectly satisfactory presentation, even with some pixelation or distortion. Furthermore, the cohort of students who we would be directing to use this software would predominantly be first-year, at-risk, disabled or otherwise disadvantaged students, therefore, improving presentation beyond an already more-than acceptable level would not be a priority.

To maximize the student learning experience, an important focus of this project was to attempt to integrate EquatIO with their existing study strategies. While each student had unique strengths, they encountered different barriers when studying and they were already using a variety of supportive tools such as OneNote, Word, PowerPoint, LaTeX, Glean, Read&Write and Livescribe pens. Some of the students preferred handwriting while others typed, and this meant that, although it brought some useful features, EquatIO did not always fit smoothly into their study techniques. As a result, the second author's meetings with individual students to discuss EquatIO were very important. This facilitated greater insight into students' study techniques, the issues they were encountering within their modules and, as a consequence, the second author could offer students personal advice on various features of EquatIO and suggest ways that they could try to use it to suit their individual needs.

From these meetings and survey responses, it is clear that time management was of huge importance to the students. They each reported seeking ways to reduce the time they spent on mathematics because of the individual time-consuming barriers they encountered while studying. Learning how to use a new technology is, in itself, time-consuming and, for this reason, the meetings with the students were important to help guide them through the downloading, installation, running, and use of EquatIO. The second author also created a video, personalized to MU students, which covered these aspects of EquatIO and streamlined the process. Unfortunately, after set up, for some of our students, using EquatIO ended up being more time-consuming than their previous methods. For example, several students already used OneNote, which is not directly compatible with EquatIO. As a result, they had to copy and paste every expression they wrote. The extra time it took students to copy and paste each equation resulted in their return to handwriting, as that was faster and more accessible for them.

During our trial, we also found that EquatIO had a number of bugs, which we tried to resolve during the student tutor meetings. Many of these bugs appeared minor and had a work-around. For example, occasionally the Equation Editor would not let the user type inside it. In order to fix this issue, we discovered that first hiding and then un-hiding the Equation Editor allowed the user to type again. One of the participants also experienced a separate issue which we could not solve within a short timeframe, and this appeared to be unique to their device, which was new. When they tried to insert a mathematical expression created in EquatIO into a word document, the image appeared as a black box with no text displayed.

None of the first-year students reported having previously used ATs that were specifically designed for mathematics or STEM subjects. Prior to this trial, we had sporadically tried to introduce LaTeX to individual first-year MAP students who were struggling to write mathematics. These unstructured attempts were rarely successful and LaTeX appeared to be too advanced for these students. However, EquatIO does offer a first look at mathematics in a variety of coded formats such as HTML, MathML and LaTeX which students can build-on as they progress through their course. This means that

EquatIO would remain useful to students throughout their course, and they would not necessarily need different software as their skills improve. Developing the skills to create mathematical documents that are in coded rather than image format would be easier to edit and lend themselves better to writing documents collaboratively. Interestingly, creating documents with these more advanced skills would increase accessibility, as equations in coded format have more options for compatibility with immersive readers.

It appears, based on student survey responses, that only one student used EquatIO to any significant degree during our trial, writing up notes in lectures. However, we know from this student's comments in meetings with the second author that they also used EquatIO to complete some assignments which they were able to submit in PDF format. During the 2021-22 academic year, the majority of first-year assessments were multiple choice Moodle Quizzes, and there were only three assignments due in PDF format, which is probably why EquatIO was not a focus for most of the first-year students. It seems those few PDF assignments did not warrant the initial investment of time it takes to learn how to use a new software.

In 2022-23, almost all the first-year mathematical assessment at MU will be PDF upload, and we believe that this will incentivise more engagement from MAP students with EquatIO due to the ease with which mathematical documents can be created. The point remains, however, that it is also crucial to have a staff member available to meet with and advise students regularly on how to use EquatIO effectively for their own learning needs. Thus, for any institution considering making EquatIO available for their students, we recommend that they carefully consider both their capacity to assign staff to ensure that students receive on-going support with the software and the modes of assessment being used.

6. Acknowledgements

This study is part of the MAP Office's initiative to offer the latest AT tools for its students and assess their impact. It is also part of the ICT and STEM Enhancement Project at MU funded by the HEA. Ciarán Mac an Bhaird is the lead on the mathematics strand of this project, and Rachel O'Neill's position was fully funded by the HEA. Ruchi Palan is the assistive technology tutor at MAP and supports students with their AT needs.

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8. Appendix

1) Prior to being introduced to Equatio,

had you used any other assistive learning technologies? If so, what were they, what did you use them to help you with? Was this in school\university etc?

had you heard of Equatio or ever used it? If so, what did you know about it, what did you use it to help you with? Was this in school\university etc?

2) When you tried to use the assistive technology, what were you hoping that it would help with (e.g. don't like writing, time management, etc.....)

3) Apart from assistive learning technologies, did you get any other type of assistance with writing at school or university? If so, what was the assistance, did it help? etc.

4) Following your introduction to Equatio, and the session(s) with Rachel, what (if anything) did you use Equatio for? Was this only on one or two occasions or did you continue to use it on a regular basis?

5) How did you find Equatio to use, positives\negatives etc. What is it about Equatio that is helping you (if you are one of those using it regularly)?

6) If not using Equatio regularly, why not?

7) When compared to other technologies available\that you use, do you think Equatio is a software that should be made available to students in future years or are other existing technologies sufficient?

8) Any other comments ?

WORKSHOP REPORT

Mathematics Learning Support – Linking Practice to Research in the New Normal - A report of the IMLSN Workshop 2022

Ciaran O'Sullivan, Dept. of Mechanical Engineering (Tallaght Campus), TU Dublin, Ireland.
Email: ciaran.osullivan@tudublin.ie

Kirsten Pfeiffer, School of Mathematical and Statistical Sciences, University of Galway, Ireland.
Email: kirsten.pfeiffer@nuigalway.ie

The 13th annual Irish Mathematics Learning Support Network (IMLSN) workshop took place online via Zoom on Monday the 13th of June 2022. The aim of the event was to look forward and envisage what Maths Support might look like in the years ahead.

1. How can we capitalise on our most recent experiences during Covid-times and take what we have learned to develop Maths support for our students?
2. What other areas of opportunity for research do we see for Maths support in Ireland?

This workshop provided an opportunity to bring together researchers, tutors and coordinators of Maths support from around the country to discuss these questions. Twenty-six delegates from fourteen Irish Higher Education Institutions attended this workshop.

The chair of the IMLSN, Kirsten Pfeiffer, opened the event by emphasizing the importance of opportunities like this workshop for the community to get together and share their ideas, challenges and experiences. Attendees were invited and encouraged to get involved with the IMLSN.

The workshop consisted of two 15-minute talks and five 5-minute lightning talks on practice and/or research in progress as well as more fully formed research. All presentations are available on the IMLSN website (imlsn.ie). This talks session was followed by small group discussions in breakout rooms. The proposed themes for these discussions were:

1. Hybrid maths support / New normal,
2. Engaging non-engagers,
3. Tutor development,
4. Re-usable resources.

The workshop finished with a whole group discussion to get an overview about what has been discussed in the themed discussions and then to focus on what IMLSN can do to help its members achieve the research goals just discussed.

1. Talks Session

1.1. Patrick Browne (Technological University of the Shannon) - 'Reflections on using the WebWork platform for mathematics'

The first 15-minutes talk opened with an introduction and considerations about the WebWork online assessment tool for the teaching of mathematics to engineering students. The speaker then suggested a unique approach to getting insights into students' habits and behaviour when using the WebWork system, namely investigating log files in a server. These present a wealth of data that is not usually visible to the instructor. Some early results of this data analysis were presented to exemplify the advantage and practicality of this approach.

1.2. Anthony Brown (University College Dublin) - 'Supporting students with a weak mathematical background during their first trimester of university study using Numbas'

This 15-minutes talk first described an initiative where the online assessment system Numbas was used to support students taking two Level 0 courses at UCD. These courses were offered to undergraduate students whose mathematics results from their school leaving certificate was weak. The assessment consisted of weekly class tests through Numbas and a 'traditional' final exam. The speaker reported on a study to investigate how students' grades of both types of assessment relate to each other. The findings showed a clear correlation: students' engagement with the Numbas exams which counted towards the final grade could be compared with the engagement with the same exams which could be taken at any time by the students for revision purposes.

1.3. Fionnán Howard (Dublin City University) - 'DCU Online support – current and future'

This lightning talk described the current online support being offered by the maths learning centre in DCU and outlined what can be provided in future. With students having returned to in-person learning, the role of online support may change, but is expected to be desirable for certain students. Using attendance data from 2021-22 to inform the decision, the speaker presented their proposed online support for next year.

1.4. Claire Mullen (University College Dublin) - 'MathsFit: increasing first-year students' engagement with maths support'

This lightning talk focused on preliminary findings from the ongoing project known as MathsFit, a suite of online and in-person mathematics supports designed for in-coming first-year students of service courses at University College Dublin. MathsFit aims to increase engagement with support especially among students identified as 'at-risk' through the MathsFit proficiency quiz. The speaker described the rationale of the project, some engagement metrics, and presented results from the first year of the programme. Early results indicate that MathsFit does improve student engagement with the mathematics support services available.

1.5. Ciarán Mac an Bhaird (Maynooth University) - 'The Sigma Accessibility Special Interest Group (SIG)'

This lightning talk provided a brief overview of the work of this Special Interest Group (SIG) to-date, including a survey paper, and the development of resources for mature students and for students with dyslexia or dyspraxia were provided. These resources are for use in an MSC setting. The speaker outlined current plans for moving the SIG forward, in particular, seeking more people to get involved and splitting the focus between digital accessibility and cognitive disorders or sensory impairments.

1.6. Kirsten Pfeiffer (National University of Ireland, Galway) - 'The IMLSN Resources Website Project'

This lightning talk first described recent developments of the IMLSN resources website (imlsn.ie/index.php/resources-index). The speaker then reported about an ongoing study which aims to get insights about students' and instructors' habits and challenges with online mathematics and statistics resources. The plan is to use findings from this study for further advancement of the resources website. This is an ongoing project and the group is looking for more participants to get involved.

1.7. Surbhi Gautam (Atlantic Technological University - GMIT) - 'Using Desmos to enhance student learning in Mathematics'

This lightning talk outlined the potential for the use of Desmos to enhance student engagement and understanding. Desmos (learn.desmos.com) is a free graphing and teaching tool for mathematics.

Teachers can use prebuilt activities or can create activities themselves at teacher.desmos.com. The speaker provided interesting and novel example activities and used these to exemplify that students will learn by interacting with mathematical representations, illustrations of the world, and their classmates.

2. Group Discussion Session

Themed small group discussions in breakout rooms were held that were then followed by a whole group discussion to get an overview for the themed discussions and then to focus on what IMSLN can do to help its members deal with any issues that emerged.

2.1. Theme 1: *'Hybrid maths support/new normal'*

The group started by discussing the various approaches that participants took during the first phase of the pandemic. These included drop-in online sessions, online sessions using an appointment system, group online sessions, and support using a VLE forum.

The group then went on to talk about the advantages and disadvantages of these different sorts of support, but the only real problem they found was lack of engagement, and this applied to all the different types of support. No one had found a solution for this, but it was mentioned that mature students seemed to be more accepting of online support.

Looking forward, a real worry is that the lack of engagement might be carried through to face-to-face support when that was re-introduced, especially for first year students. It was suggested that due to the disruption, this cohort of students seems to be quite isolated and less willing to accept support. Most participants were not sure exactly what support they were going to offer in the year ahead, but there was a real eagerness to get some ideas to deal with this lack of engagement.

2.2. Theme 2: *'Engaging non-engagers'*

Participants compared their experiences with non-engagement particularly in the recent COVID-19 years. Issues were raised around students' mental health, financial pressures, and social interactions based on anecdotal knowledge and current learning analytics research. The need for a holistic and ethical approach for engaging with students deemed 'at risk' was discussed, in particular, the view that not all students wish to engage. Strategies to engage the non-engagers were desired especially as engagement seems to have decreased since the onset of the pandemic.

2.3. Theme 3: *'Tutor development'*

Due to increased workload during COVID-19, it was recognised that finding time for ongoing tutor training was difficult. Hope was expressed that more substantive training could be re-introduced this coming academic year. All members of the room highlighted the importance of quality tutor training and in this regard, the Tutor Competency Digital Badges (imlsn.ie/index.php/tutor-development) seems to tick a lot of boxes. Content templates are available on the IMSLN website which are fully customisable. The four badges were trialled at Maynooth University and the University of Galway. Feedback from the tutors that participated was extremely positive. In particular, the tutors appreciated the formal recognition that the badges provided for the training they received. It was suggested that if an institution were to run the badges in the coming year, they should consider inviting tutors from other colleges to attend. It was mentioned that completing two badges per semester is comfortable for most tutors.

3. Whole Group Summary Discussion

The workshop finished with a whole group discussion to get an overview for the themed discussions. During this whole discussion, engaging non-engagers emerged as the key challenge. The issue of engaging disaffected learners was identified as a key problem in the last academic year for many institutions. Plans were discussed to have an IMLSN event focussing on how institutions intend to address this in the coming year in early September.

Plans to promote the IMLSN Tutor Competency digital badges in as many institutions as possible in the coming academic year also emerged as a shared objective for some members of the network.

4. Insight from Feedback Forms

In feedback forms for the workshop, participants appreciated the format of the breakout rooms and highlighted the opportunity to meet people and get a sense of how things are going for them in their institution. While enjoying all of the talks, the feedback indicated that participants were very positive about the five-minute lightning talks which allowed for a lot of information to be shared in a short space of time with the possibility for follow up with the person/topic if someone was really interested.

In terms of key takeaways/learning from the workshop, the importance of collaboration across institutions was mentioned. The shared challenge of engaging students with mathematics and support both in person and online was very prevalent in the comments. In terms of potential follow-on events the feedback strongly indicated a preference for more workshop opportunities on addressing engagement in the coming year.

IMLSN would like to sincerely thank all those who contributed to this meeting – the organisers Ciarán O'Sullivan and Julie Crowley, the moderators of breakout rooms Anthony Brown, Claire Mullen and Peter Mulligan, as well as all participants and facilitators.

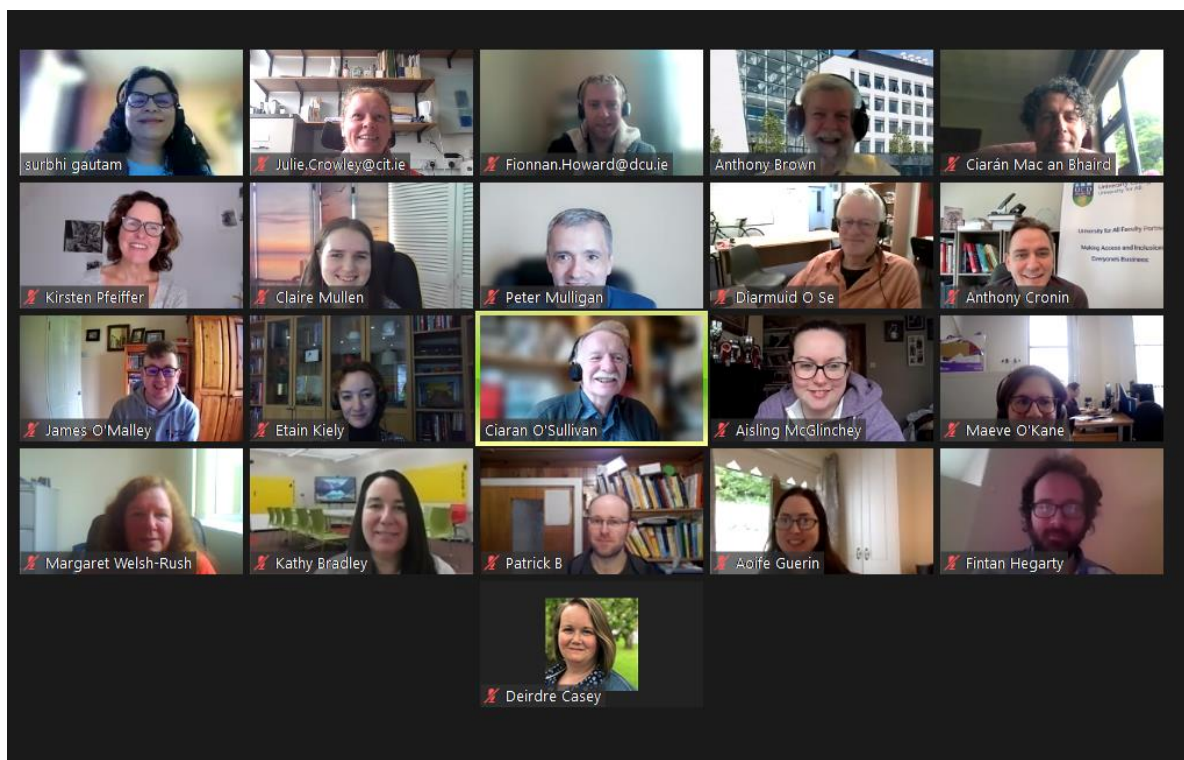


Figure 1. Some of the participants at the Annual Irish Mathematics Learning Support Network (IMLSN) Workshop 2022