MSOR Connections

Articles, case studies and opinion pieces relating to innovative learning, teaching, assessment and support in Mathematics, Statistics and Operational Research in higher education.

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Contents

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Editorial

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Welcome to the final issue of *MSOR Connections* for the academic year 2021/22. Following the two well-filled issues devoted to papers from the 2021 CETL-MSOR Conference, we are happy to be able to publish six research articles, two case studies and a workshop report in this issue.

The workshop report by Falconer et al describes a workshop of which I was one of the organisers. I am grateful to Peter Rowlett who looked after the review process for this report to avoid a conflict of interest on my part.

MSOR Connections can only function if the community it serves continues to provide content, so we strongly encourage you to consider writing research articles or case studies about your practice, accounts of your research into teaching, learning, assessment and support, and your opinions on issues you face in your work.

Another important way readers can help with the functioning of the journal is by volunteering as a peer reviewer. When you register with the journal website, there is an option to tick to register as a reviewer. It is very helpful if you write something in the 'reviewing interests' box, so that when we are selecting reviewers for a paper we can know what sorts of articles you feel comfortable reviewing. To submit an article or register as a reviewer, just go to http://journals.gre.ac.uk/ and look for *MSOR Connections*.

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RESEARCH ARTICLE

University Mathematics Assessment Practices During the Covid-19 Pandemic

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Abstract

In response to the global Covid-19 pandemic departments of mathematical sciences within the UK and Ireland needed to adapt their teaching approaches and methodologies from March 2020 to incorporate not only government social distancing requirements, but also periods of national lockdown and the fact that students were necessarily studying online. In planning for the many different and possible scenarios, universities implemented a range of emergency measures and regulation changes to provide frameworks for adapting teaching, learning and assessment approaches, and at a subject level, departments also needed to correspondingly respond to specific disciplinary needs. Here we specifically consider the changes made by mathematical sciences departments to their assessment practices in the period from March 2020 until January 2021 and their proposed adjustments for the remainder of the 2020/21 academic year. We found that departments were using a range of different approaches regarding the release of their assessments and this paper considers the implications of each for future practice. In particular we identified a concerning issue that emerged across a number of departments in relation to academic misconduct that will now require a community-wide approach if open-book online assessments are to prove a valid, reliable and fair method of assessment in the longer-term.

Keywords: Assessment, Covid-19 adjustments, Academic integrity, Open-book assessment.

1. Introduction

The Covid-19 pandemic has resulted in unprecedented challenges for every sector and industry. The education sector has been significantly impacted, with many schools, colleges and universities unable to accommodate teaching and learning in their usual way. For example, in March 2020 a UK-wide national lockdown was announced which encompassed the remainder of the 2019/20 academic year; during this period all on-campus learning activities, including in-person examinations, were affected. For the 2020/21 academic year, whilst in-person learning may have been initially allowed by government, many institutions chose to deliver their provision either in-part, or entirely online. As the pandemic evolved, further periods of lockdown were enforced throughout 2020/21, necessitating universities to constantly re-evaluate their teaching, learning, assessment and support practices.

Many universities moved at least in part to ‘online delivery’, unable to welcome their students onto their campuses as they do normally. The way in which university mathematics departments facilitated ‘online learning’ is of particular interest, since historically, “*[fully online] mathematics instruction has not been successful in comparison with traditional [face-to-face] mathematics instruction*” (Trenholm, Peschke and Chinnappan, 2019). To determine exactly how university mathematical sciences departments navigated this transition, a large-scale survey was conducted to identify the different approaches taken by departments regarding teaching and assessment.

One of the main concerns identified within the responses to the survey related to assessment. Trenholm and Peschke (2020) highlight the nature of assessment as one of the main differences between online and face-to-face mathematics courses. In face-to-face mathematics courses, the dominant form of assessment has been in-person, invigilated examinations (Iannone and Simpson, 2012). For many institutions, in-person examinations were infeasible during the pandemic and so it is of particular interest to explore the alternative arrangements used and their implications for future practice. This paper primarily focuses on the assessment arrangements used by departments and includes their experiences of implementation.

2. Research Methodology

A survey was conducted of UK mathematical sciences departments between December 2020 and February 2021. It focused upon the immediate changes in teaching and learning practices necessitated by the rapid onset of the pandemic, explored changes implemented during the first half of the 2020/21 academic year and proposed changes to examination practices for the entire 2020/21 academic year. The range of topics included teaching arrangements (in particular, arrangements for lectures, problem classes, tutorials, computer lab sessions, group work and optional modules), as well as exam arrangements, student experience and future departmental teaching and learning plans. The full questionnaire was analysed as part of an undergraduate final year project, however, this paper chooses to focus particularly on the changes to examination practices in 2019/20 and 2020/21.

The survey was targeted at Heads of Department, Directors of Teaching, or similar individuals, of all UK higher education institutions with mathematical sciences departments who would have an overview of the teaching and learning practices within their departments.

One of the main areas of concern identified through the survey was that of academic misconduct amongst students. Therefore a short follow-up survey was also conducted to investigate this aspect further asking departments to identify the extent to which they experienced instances of academic misconduct in 2020/21 and to detail any mitigating measures taken. The follow-up survey was sent, in February/March 2021, to all departments who indicated that they were happy to be contacted with further questions in the initial survey. The results from this follow-up survey are also reported here.

2.1. Research Ethics

Ethical approval for this survey and study was granted by the University of Birmingham and appropriate ethical guidelines (BERA, 2011) were followed throughout.

In particular, once all survey responses had been received, respondents' names and contact details were removed from the data set. Any references made in responses that might identify any individual or institution were also removed. Responses were then numbered such that answers to individual questions could be linked, as a department's answer to one question might provide additional context relating to another.

2.2. Data Cleansing

Two responses were received from the same university, one from its mathematics department and the other from department not classified as mathematical sciences by HESA, (the Higher Education Statistics Agency) (HESA, n.d.(a)). As this study only concerns departments of mathematical sciences, this second response was removed from the data set.

All responses were checked to ensure they had been provided by appropriate members of staff in each department with knowledge of the departmental teaching and learning practices in 2020/21. Responses were also checked for their completeness. In both instances, no modifications to the original dataset were required.

2.3. Response Rate

After data cleansing, there were 37 valid responses to the initial survey representing institutions from all nations of the UK and Ireland. Seventeen of these responses came from departments within the research-intensive Russell Group universities. Seven came from departments at Post-1992 universities, and five were received from universities that used to belong to the 1994 Group, but which are now unaligned to any mission group. As Grove, Croft and Lawson (2019) explain, the 1994 Group was an alliance of smaller research-intensive universities, but the group dissolved in 2013 after several of its members joined the Russell Group. We have decided to follow their example and categorise these 5 universities as 'unaligned*', in recognition of the fact that they are research-intensive. The remaining 8 responses came from departments unaligned with any mission group.

To ascertain the proportion of mathematical sciences students represented by the survey, data were taken from HESA. HESA provides a breakdown of the numbers of students enrolled in courses of mathematical sciences at all institutions of higher education each year in the UK. The most recent academic year for which comparable HESA data were available at the time of analysis was the 2018/19 academic year (HESA, n.d.(b)). HESA provides the individual numbers of undergraduate, postgraduate taught and postgraduate research students enrolled that year. As this study concerns adjustments to teaching (and assessment) provisions, the numbers of postgraduate research students were not of interest, and thus only the data for undergraduate and postgraduate taught students are considered.

In the 2018/19 academic year, HESA recorded a total of 42,790 undergraduate and postgraduate taught mathematical sciences students enrolled across 93 institutes of higher education in the UK. The 37 departments that responded to the survey accounted for 27,925 of these 42,790 students, which equates to approximately two thirds of the taught mathematical sciences student population within the UK.

3. Results and Analysis

The results reported here form the responses to the original survey regarding examination arrangements. Specifically departments were asked to detail their assessment mechanisms for the 2020/21 academic year, and then compare these to the emergency arrangements introduced at the end of the 2019/20 academic year. The analysis of these responses then resulted in a follow-up survey which explored in greater depth the theme of academic misconduct thereby providing a richer qualitative dataset which we report below.

3.1. Examination Arrangements

In the initial survey, detailed questioning focused upon:

1. Whether exams were, or would be, sat on-campus or remotely.
2. The form of the examination (for example, whether students needed to hand-write answers to traditional-style exam papers or problem sheets or complete an online quiz or assessment).
3. The time period given for students to complete assessments.

4. Whether students had flexibility in when to complete their assessment.

From the respondents, 36 departments answered this question. All 36 responses indicated that assessments were, or at least expected to be, completed remotely.

On the format of assessment, one response indicated that arrangements varied across all modules, with different formats and different time constraints being used. A second response again indicated that the format of the assessment differed across modules and was left to the module leader to decide, but that students would have 24 hours to complete the assessment in accordance with university policy. The remaining 34 departments all indicated that, for at least part of the assessment of some modules, students were required to complete a written assessment, that is students write and upload solutions to a traditional-style exam paper, problem sheet or similar form of coursework.

The time periods, and flexibility, in which students had to complete this written assessment varied amongst departments, but can be grouped into four broad categories:

- 16 of the 34 departments (47%) were categorised as using a ‘short release’ format, whereby students had a similar amount of time to write solutions as they would have in an invigilated, on-campus exam, plus an additional 20-60 minutes for electronic upload.

- 3 departments (9%) were categorised as using an ‘intermediate release’ format, whereby students had a period of 6-9 hours to write and upload solutions.

- 7 departments (20%) were categorised as using a ’24-hour release’ format, whereby students had 24 hours (or 23 hours in one department) in which to write and upload solutions.

- 4 departments (12%) were categorised as using a ’48-hour release’, whereby students had 48 hours in which to write and upload solutions.

Four departments (12%) could not be categorised as following any one of these approaches. Two departments were yet to decide upon the time that they would allow for their written assessments at the time of the survey. The third indicated that they were using pieces of continuous assessment and typically allowed 7-10 days for students to complete each piece. The remaining department indicated that, for undergraduate students, they were allowing 3 hours, just as they would in a normal exam, plus 90 minutes for scanning and uploading, plus a further 90 minute late penalty period; any solutions submitted within this final 90 minutes of the total 6 hour time frame would be subject to a 10 mark penalty. They added that they were following the 24-hour format for postgraduate exams.

It is also worth noting that one of the departments categorised as using a 24-hour release format indicated that they were using the short release format for Year 1 students, but the 24-hour release format for all other year groups, hence their overall categorisation.

In addition, another department following the 24-hour release format, which was the default arrangement for their whole university, requested institutional permission to use a short release format instead. However, by the time permission was granted, the department considered it too short notice to implement as doing so would be unfair on students.

Two departments using the short release format explicitly mentioned minimising the risk of academic misconduct as one of their reasons for doing so. One of these had followed the 24-hour release format at the end of the 2019/20 academic year, but reduced the timings in the current academic year “*due to grade inflation and cheating*” (Respondent 12).

Another department reported "*about a dozen cases of plagiarism*" when following the 24-hour format in the previous academic year. Interestingly, this department was undecided on whether to move to the short release format or follow the 24-hour format again but "*spend more time in designing questions where it is impossible to Google the answer*" (Respondent 4).

Additionally, one of the departments using an intermediate release format commented that in the previous academic year, some modules used a 24-hour exam, whereas others opted for 7-day coursework. Under these arrangements, the department said that "*there were clear cases of the questions being asked (and answered) on the internet*" (Respondent 3).Thus, several departments identified cases or concerns of academic misconduct when students had longer to complete written assessments.

Evidently, the most common approach used for written assessments was the short release format. One possible explanation for this could be concerns associated with the potential for academic misconduct by students. Academic integrity is of great importance for universities as Trenholm (2007) argues:

> "*Fundamentally, administrators and faculty acting as agents in society are responsible for producing a skilled and educated graduate. They are responsible to ensure that the paper certificate or degree accurately reflects the student's ability.*"

Universities must ensure that the degrees they award preserve their academic integrity and are respected by employers and society. The arrangements of departments are thus of particular interest, since on-campus, closed-book, invigilated exams are ordinarily the dominant form of assessment used within the mathematical sciences (Iannone and Simpson, 2012). Rovai (2000) argues that "*online instructors must recognize the need to design instruction appropriate to the medium*" and, in particular, Trenholm (2007) identifies that it is commonly thought that every online assessment should be regarded as being open-book.

3.2. Academic Misconduct

Given the concerns raised by several departments in relation to student academic misconduct, it was decided that further investigation would be beneficial. In the initial survey, 34 of the 37 respondents indicated that they were happy to be contacted with further questions and so these 34 respondents were sent three additional questions for response via email. Thirteen responses were received to this follow-up survey; eight came from departments at Russell group universities, two from departments at post-1992 universities, two from our categorised 'unaligned*' universities, and one from an unaligned university. For each responding department, the responses to these additional questions were then combined with those from the original survey.

Respondents were asked if they had identified any of the three following categories of academic misconduct in any of their online assessments: (1) plagiarism arising from the improper use, or referencing of, third party sources; (2) collusion (students working together and sharing answers or ideas); and, (3) contract cheating. Respondents were asked to provide qualitative details on the nature of the misconduct, as well as the number of such instances where possible.

Twelve departments responded to this question detailing the number of instances of academic misconduct that they have identified. Only one response indicated that no instances of academic misconduct had been identified, but of the remaining 11 respondents, eight indicated that they had identified cases of plagiarism between March 2020 and March 2021. One department commented that "*this was to be expected since exams are now not just 'open book' but 'open internet'*" (Respondent 21). One department said that they had identified 15 cases of plagiarism during their first semester of the 2020/21 academic year, and a second commented that around 10% of students taking one module had been identified as having submitted plagiarised work in some form during the June 2020 exam.

All 11 respondents who reported that they had identified cases of academic misconduct indicated that at least some of these were in the form of collusion. One respondent reported that 40% of students in one module were suspected of collusion. A second respondent reported 11 suspected cases of collusion during semester 1. Another department reported that 16 out of 47 students taking one module admitted to collusion. Whilst collusion was only identified in some modules by departments, and indeed no data were sought on the baseline level of collusion in pre-Covid years, the extent of it within these modules raises the question of whether it also occurred in other modules but was not detected.

With regards to contract cheating, five respondents indicated that they had found cases of their exam questions being posted on online 'study support' websites, with a sixth department commenting that they had suspicions of contract cheating but these could not be proven. However, one of these departments made the point that they could not be sure if these were cases of cheating or if students had "*sought answers after the examination simply for reassurance*" (Respondent 31). A second department commented that they had found 500 of their exercises on various 'study support' websites, and that:

> "*This term, 15 students (out of about 250) in one year cohort have been identified as posting questions…(and all of whom have now admitted to doing so).*" (Respondent 34).

Another department reported just over 80 cases of academic misconduct in their January 2021 assessment period and whilst these were still being investigated, the respondent (Respondent 5) indicated that these cases were "*probably roughly equally split*" between the three identified forms of misconduct, namely plagiarism, collusion, and contract cheating.

A fourth department commented that they identified one instance of contract cheating in their January 2021 examinations, but the issue became "*much more extensive for [their] in-course semester 2 assessments*" where instances of contract cheating were identified in all year groups (Respondent 28).

One respondent commented that academic misconduct "*appears to be getting more common*" (Respondent 7). In contrast, another department who identified cases of plagiarism and collusion in their assessments reported that these were not identified "*at a rate that is any more frequent than in non-Covid years*" (Respondent 25). However, it should be noted that this department later added:

> "*Our course is heavily weighted towards coursework, and our exams are usually open book ... so most of our assessment is already designed in a way that is fairly robust against such malpractice*." (Respondent 25).

Trenholm (2007) argues that academic misconduct is more common in online assessments:

> "*[Due to] the anonymity that the Internet affords ... students, who in a traditional classroom may never consider cheating, may find the temptation to do so in an online course too powerful to resist*".

These concerns raised by respondents coincided with the publication of a research article on contract cheating in STEM (Science, Technology, Engineering and Mathematics) subjects during the pandemic (Lancaster and Cotarlan, 2021). Contract cheating is the term used to refer to cases of students engaging someone else to provide answers on their behalf. This study by Lancaster and Cotarlan (2021) analyses the use of one 'study support' website by students for contract cheating.

Their study identifies an increase of 196% in the numbers of contract cheating requests across five STEM subjects between the time periods April 2019 to August 2019 and April 2020 to August 2020. As noted by Lancaster and Cotarlan (2021), this increase coincides with the move to online assessments as a result of the Covid-19 pandemic. Whilst it should be noted that mathematics was not one of the subjects considered, the findings within this report are nevertheless stark and of direct relevance to mathematical sciences departments.

This raises the question of how assessments can be designed to minimise the risk of academic misconduct, which we now consider.

3.3. Upholding Academic Integrity

In the initial survey, departments were asked to what extent they considered and implemented question randomisation in their assessment framework. The responses of all departments are shown in Table 1. The figures are presented according to the length of time students had to complete their written assessments. The number of departments who considered and implemented question randomisation but were not categorised as using a certain release format, or indeed as using written assessments at all, are shown in the row titled ‘unclassified’.

Release format	Extent to which departments considered and implemented question randomisation			Total
	Did not consider	Considered but not implemented	Considered and implemented	
Short release	6	3	7	16
Intermediate release	1	0	2	3
24-hour release	0	3	4	7
48-hour release	0	2	2	4
Unclassified	2	1	4	7
Total	9	9	19	37

Table 1: The extent to which departments considered and implemented question randomisation in their assessment framework (n=37).

From Table 1, it can be seen that just over half of the respondents indicated that their departments implemented question randomisation to some extent. However, departments were not explicitly asked in what context it was being used. Thus, it is not clear whether these departments were using question randomisation in their written assessments or as part of online quizzes, or whether randomisation was being used across the entire department or within only a few modules. Two departments did though indicate they were using question randomisation as part of their written assessments. One of these was following a 24-hour release format and indicated that they “*create several versions of each exam to mitigate against collusion*” (Respondent 27).

To obtain a sense of how departments were using question randomisation, departments were asked, as part of the follow-up survey, to confirm whether they were using question randomisation and if so, for what purpose. Twelve responses were received to this question. Four respondents indicated that they were only using randomisation in online quizzes. One of these four departments commented that they considered question randomisation as “*not really appropriate for an end of module exam*” (Respondent 21). In contrast, another of these four departments (Respondent 5) said that they were "*looking into*" using randomisation in written assessments if they continue to be online in the 2021/22 academic year.

Three different departments indicated that question randomisation was being used in a very small number of written assessments, as well as in some online quizzes, with one department commenting that "*most of [their] lecturers won't go through the trouble of [randomising written assessments]*" (Respondent 4). On the other hand, another of these three departments (Respondent 14), who indicated that "*a very small number of course units had a randomised take-home exam [with] a small number of variants of papers*", went on to say that:

> "*We were concerned about the workload in setting these up ... and possible confusion in marking, but it turned out to be much easier than we thought*." (Respondent 14).

Another department indicated that they were using randomisation for all questions in their stage 1 (first-year) exams, but that "*at stage 2+* [they] *don't consider randomisation so useful, as the underlying method to a problem is the same*" (Respondent 31). One other department reported they created 8-10 variants of every exam paper, "*with changes such as notation, numeric parameters* [or] *different choices of function to consider*" (Respondent 27). They noted that whilst that this did take more time to set up and check, they recommended the use of LaTeX Macros. One strategy proposed by three departments involved students using digits from their unique student ID number in numerical questions. These responses perhaps suggest that randomisation within written assessments may not be as cumbersome as departments imagine.

Question randomisation is just one of the ways of reducing the risk of misconduct identified by Clark et al. (2020) and so it is of interest to see if departments have been utilising other approaches. Therefore, the final question of the follow-up survey explored whether departments had taken other specific actions to prevent academic misconduct amongst their students. Nine responses were received and can be grouped into the following themes:

- Changes to assessment duration.
- Individualisation of assessment materials.
- Increasing student awareness of academic misconduct and its implications.
- Using vivas and oral assessments.

3.3.1. Changes to assessment duration

One department stated that an approach to minimise the risk of misconduct was using a short release format for written assessments. A second department commented that they had tried to shorten the exam duration for their summer exams from 48 hours, but were prevented by centralised university policies. However, whilst reducing the assessment duration appeared to help some departments, one respondent referenced a report made by one of their students that six or seven other students had quite an extensive plan for collusion in place for several short exams:

> "*These students were supposed to have divided up exam questions between themselves for the first hour or so of the examination, shared findings between themselves, then spent the remaining 90 minutes copying from other solutions.*" (Respondent 31).

Indeed Rovai (2000) notes that "*timed tests that reduce the opportunity to cheat also help*" in addressing misconduct, even in standard time assessments, cheating can still occur. Similarly, Lancaster and Cotarlan (2021) argue that "*there is nothing to stop students posting questions online and receiving answers within the time frame of an exam*". Indeed, one such online 'study support' website popular amongst students confirms that they seek to answer posted questions within two hours and on average, do so in well under an hour.

3.3.2. Individualised assessment materials

Only allowing standard time for online examinations in itself is not enough to eliminate the risk of contract cheating and it is therefore necessary for departments to consider other measures too. One such measure identified by Clark et al. is "*watermarking exam materials to make them more difficult to share with contract cheating providers*". One respondent indicated that they are currently "*developing technology to provide individually watermarked papers*" (Respondent 5). Unfortunately, this solution again might be compromised by students simply re-typing the question themselves to shed the watermark, though this would take more time.

3.3.3. Increasing student awareness of academic misconduct and its implications

A more common action, taken by six of the departments who responded, was simply to increase their communication with students regarding acceptable exam conduct and the penalties for cheating, highlighting both theoretical and real examples of where these had been imposed. One innovative example involved collaboration between a department and their undergraduate mathematics society. The society "*manage a discord server in which students often discuss individual modules, including their assessments*" (Respondent 34). The society had agreed to suspend the discussion areas for modules during summer assessments and to report any suspicious activity or messages to staff.

Whilst Rovai (2000) identifies several other measures that can reduce the risk of cheating, they conclude that:

> "*Arguably, the best approach is to identify the issue of plagiarism openly with the aim of affecting learner attitudes and values*."

This approach has also been shown to be somewhat effective by a survey conducted by King, Guyette and Piotrowski (2009). Here, students studying business indicated that they would be less likely to cheat if they had specifically been told that it was not allowed. Thus, a focus by departments of better educating students on the nature and implications of academic misconduct should not be overlooked.

3.3.4. Using vivas and oral assessments

One department reported that they used a significant number of viva examinations, oral examinations in which students have to defend their work, at the end of the 2019/20 academic year and would likely do so at the end of the 2020/21 academic year "*to establish authorship of the submissions received*" where there were doubts (Respondent 33).

3.3.5. Discussion: potential approaches to upholding academic integrity

Approaches for upholding academic integrity in online examinations have been suggested by Clark et al. (2020). One approach suggested, which lends itself well to statistics and some applied mathematics modules, is using unique data sets for each student. This would mean it would not only be more difficult for students with different questions to collude, but also, if questions are posted online, the student with that allocated data set can be traced. However, creating unique data sets for each student is quite extensive and unlikely to be appropriate for pure mathematics modules. But even on a lesser scale, question randomisation has been identified as an effective means for reducing the risk of academic misconduct (Rovai, 2000). Other examples of how questions might be randomised include changing coefficients in equations and expressions, or specifying different inputs for given algorithms.

Another solution identified by Lancaster and Cotarlan (2021), but which none of the respondents to either the initial or follow-up surveys indicated that they were pursuing, involves the proctoring of online assessments. In 2007, before online examinations were commonplace, Trenholm (2007) argued that proctoring was in fact the only way of eliminating the risk of cheating in online examinations. Understandably, the transition for many departments from on-campus, closed-book, invigilated exams to remote, open-book assessments was difficult enough without the added complication of organising online invigilation too. However, one department did comment that:

> "*Worryingly, we have no way of making sure they [students] are not talking to each other during the exam.*" (Respondent 21).

Whilst proctoring would provide a way of ensuring students cannot collude or cheat by any other means in an exam, it raises other concerns. A rapid review of e-proctoring by Eaton and Turner (2020) identified that some students said they felt increased levels of anxiety due to e-proctoring. One reason they identified for this is that certain behaviours, such as "*looking away from the screen for more than a few seconds ... or having another person enter the camera frame*", which might be perfectly innocent, can signal academic misconduct and result in disqualification. Furthermore, their review quotes cases of students "*vomiting into wastepaper bins on camera during the exam because they were not permitted to leave the room*". Thus, while e-proctoring has the potential to mitigate the risk of academic misconduct, it raises significant concerns for students' mental health and therefore careful consideration must be made before employing it.

Several approaches to minimising the risk of academic misconduct have been identified from our own survey and within the published literature, however none appear to eliminate the risk completely. Given the importance of academic integrity and the threats posed to it by academic misconduct in an online assessment age, the extent of the misconduct and the ways in which mathematical departments can mitigate against it are therefore worthy of further study. In addition, universities themselves must have a pivotal role in assisting individual departments in ensuring the academic integrity of their assessments.

4. Discussion

Given that using a short release format for assessments has been identified as one way of reducing students' opportunity to cheat, it is interesting to consider why departments opted for longer release formats for their assessments in the first place. Reasons that departments gave in the initial survey for using a 24-hour release include:

> "*Due to access concerns, and how tight time limits might negatively affect certain groups, we felt that it would not be right to impose significant time constraints on exams*." (Respondent 34)

> "*Students have indicated to us they [24 hour exams] reduce the pressure and better help them with their learning*." (Respondent 28)

> "*Having longer to do the exam meant the scripts were much neater, and were a better test of their true mathematical abilities*." (Respondent 27)

This raises some interesting questions that merit further study to explore the tensions that exist between short-release assessments that minimise the potential for academic misconduct, and longer-release assessments that students feel provide a better measure of their true mathematical ability and which reduce their exam stress and anxiety.

During the Covid-19 pandemic, it is likely that some international students will have studied and completed assessments in different countries and hence different time zones. Arguably a 24-hour release is a better way to accommodate these students. One department who were using a short release format commented that all exams start at 1pm UK time, with no allowances for time zone, which the respondent themselves identified as an ongoing concern. An alternative being used by several departments was to allow different start times in different time zones. This too has potential issues, particularly if the same examination papers are being used since students could share questions with those who are yet to start the assessment in their time zone. Using a 24-hour release instead means that examination papers can be released at the same time across the world without the potential problem of students in certain time zones having to stay up through the night to complete a 3-hour exam. A longer-release format is also more inclusive since it better takes into account individual student circumstances, particularly for those students who might normally receive additional time.

Whilst there are several valid reasons as to why departments might opt for a longer release format, one obvious concern is that of grade inflation. As Respondent 12 commented, using a 24-hour release format led to "*grade inflation and cheating*", however as previously stated cheating can still occur in short release exams. Conversely, Respondent 28, whose department was using the 24-hour release format, commented that "*mark profiles appear to be tracking traditional assessments*". It is though worth noting that this responded also added:

> "*We have made a transition from 'marking' to 'grading' which is more pedagogically appropriate for open-book and extended-time assessments since it considers the strengths and weaknesses of the submitted work as a whole*." (Respondent 28)

Indeed Respondent 1 whose department were using a 48-hour release format, also claimed that student attainment was similar to when using traditional assessments although they identified a slight increase in grades of the most able students. These comments are in line with a small study conducted by Phiri (1993) which sought to make a comparison between mathematics assessment by closed-book and open-book tests. The study found that whilst the two methods of assessment result in comparable grades, open-book assessments provide a better discriminator between students. In particular, the average mark in the open-book tests was slightly lower than in the closed-book tests, but the range of marks and standard deviation of the grades obtained in the open-book tests were significantly greater than in the closed-book tests.

To summarise, two departments using longer release formats indicated that student attainment was similar to that in previous years, perhaps rebutting the concern that longer exams might lead to grade inflation. However, this issue is again one meriting further investigation.

5. Conclusion

This study sought to identify the practices being used by departments of mathematical sciences for teaching and assessment purposes during the Covid-19 pandemic. Whilst teaching arrangements were considered in the survey, they are not reported here. Instead, we have focused upon assessment practices since they form an area of particular concern within the mathematical sciences during the Covid-19 pandemic. It has been identified that departments are using a range of assessment approaches, and our survey shows this diversity with no one form of assessment being identified as optimal. There are arguments in support of short-release exams, in particular that they reduce the chance for students to cheat, although evidence shows that this can still occur. On the other hand, longer release exams are more inclusive of students' individual circumstances. However, there exists evidence of an increase in reported instances of academic misconduct and in particular contract cheating by students within mathematical sciences departments. Whilst we have identified various measures that can be taken to discourage and minimise this misconduct, there does not appear to be one single method of eliminating this issue entirely. Whilst e-proctoring is one solution many universities are currently considering, it is not without its own ethical concerns. The question of how best to address academic misconduct, and hence ensure the integrity of university-level assessments, is one that requires further consideration across the entire higher education sector, particularly if online assessment practices continue to be utilised in the future.

6. Acknowledgements

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RESEARCH ARTICLE

Access, Disability and Mature Student Opinion on Academic Mathematics Supports

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Abstract

In this paper we report on the main themes which emerged from analysis of a survey of students registered with the Access, Disability and Mature Student offices at Maynooth University. The survey focussed on the students' experiences of mathematics and the mathematics academic supports available to them. The majority of student feedback was positive, for example their engagement with the Mathematics Support Centre, improved understanding of mathematics and the influence of tutors and peers. However, some issues emerged in relation to impact on learning, such as inflexible departmental structures with fixed deadlines and a lack of student awareness of the supports available. In addition to discussing the main results, we outline how the findings will guide the future provision of such supports.

Keywords: Access, Mature, Disability, mathematics support.

1. Introduction

The Maynooth Access Programme (MAP) Office at Maynooth University (MU) provides supports for 'access students', 'disability students' and 'mature students' who register with them. Access describes students who have been placed at a disadvantage on their pathway to Higher Education (HE) due to social, economic or educational reasons. Disability broadly covers students with learning or physical disabilities, and mental health or other ongoing illnesses. Further details are available from http://accesscollege.ie/hear/. Mature refers to students who are at least 23 on the 1st of January in the year of entry to HE. MU traditionally has one of the highest levels of MAP students in HE in Ireland. For example, in 2020-21 MU had approximately 11,600 undergraduate students. Of these, 24.1% were registered with MAP. There were approximately 1,600 students in the Department of Mathematics and Statistics (the Department) and, though the exact number of these registered with MAP is not available, across Irish HE, the overall number of MAP students is rising and the subject area of mathematics and statistics has one of the highest proportions of MAP students (AHEAD, 2018). While the MAP Office provides a broad range of non-academic supports (see https://www.maynoothuniversity.ie/access-office/about-map), the focus of this paper is on the academic mathematics supports provided for MAP students. Key to the success of such services is that they are research-based (Lawson et al., 2019) and having previously investigated the trial of resources to assist students with dyslexia (Heraty et al., 2021), we decided that this first local examination of all academic mathematics supports available to MAP students was an appropriate next step.

MU has a popular Mathematics Support Centre (MSC) based in the Department of Mathematics and Statistics (the Department). The MSC provides a range of supports for undergraduate students, including drop-in, on-demand workshops and online resources, the effectiveness of which has been evaluated (Berry et al., 2015). Due to the COVID-19 pandemic, these supports were adapted to an exclusively online environment and included additional supports such as 1-1 appointments and study groups (Mac an Bhaird et al., 2021). In addition to the MSC, all students have access to Department small group tutorials and lecturer office hours. There are tutorials for every module and, for service mathematics, the tutors are usually final year undergraduate or postgraduate students. For the purposes of this paper, all these supports are labelled as mathematics learning supports (MLS). Students have weekly assignments, typically due at 4pm on a Friday, which contribute to their continuous assessment. There are no extensions, but students who miss an assignment for genuine reasons (illness, bereavement etc.) are not penalised.

The MAP Office are strong supporters of the MSC and encourage MAP students to avail of its services. Furthermore, every department has an academic staff member who acts as a point of contact for MAP students. This role is called the MAP Academic Advisor and the 1st author has held this position in the Department since 2012. MAP students who are struggling with mathematics often initially contact the MAP Office. A meeting, which includes the MAP Office staff member, the student and the MAP Academic Advisor, is then arranged. Often these students have not been engaging with existing supports, so the Academic Advisor provides study advice including how to use the academic supports available. As the MAP Academic Advisor also tutors in the MSC, they arrange to meet the student there in the first instance to give them initial support, show them how to use the MSC and get them working with their peers, if appropriate.

Occasionally it emerges that MAP student engagement with academic supports is impacted by personal circumstances, e.g. poor health or particular learning needs. In these situations, the MAP Office and the Academic Advisor can approve 1-1 tuition. The MAP Office covers the cost, and the Academic Advisor normally sources an appropriate Department tutor, liaising with them in relation to the student's progress. It is an agreed policy between the MAP Office and the Academic Advisor that students are not generally made aware of the availability of these 1-1 sessions. In the first instance, students are encouraged to use existing supports, so they have the opportunity to become independent learners of mathematics.

All students are made aware of the MSC through in-class announcements, and posts on Facebook, Twitter, and via all-class emails. Additionally, MAP students are reminded of these supports and informed about MAP Academic Advisors during the MAP Office orientation events. If a MAP student first contacts a lecturer or tutor in the Department then, subject to student approval, the Academic Advisor is informed, and they arrange to speak with the student.

2. Methodology

In January 2021 we developed an anonymous survey with a mix of yes/no, multiple choice and open response questions. There were two main sections: GDPR, consent and background questions; and questions relating to their experience of mathematics and the available supports. Ethical approval was received and the survey, available from www.onlinesurveys.ac.uk, was launched at the end of March via current Department and MU Alumni emailing lists. The survey closed on the 11th of June, and a total of 33 students responded. Responses were downloaded to Microsoft Excel and we applied *Thematic Analysis* (Braun and Clarke, 2006), with each author coding the open responses separately. We then met to discuss our coding, and the main themes that emerged are reported on in Section 3. Responses were also crosschecked with background questions, for example if the students were registered with the Disability Office, or the year of study of the respondent, and any patterns are also reported.

3. Results

3.1. Reacting to difficulties

When considering what students did when they first encountered difficulties with mathematics at MU, there were three main themes: Department, Self-study and MAP Academic Advisor.

The MSC was the most frequently reported departmental support '*I went to the maths support centre, this got me through my first two years*'. Respondents also mentioned seeking help from the Department Office or their Lecturers '*I first emailed my maths lecturer. He gave me all the sources of help I could avail of such as attending maths study groups*'*.* Students also referred to making use of their tutorial when they needed assistance '*Before* [COVID-19] *I would go to the MSC but currently I would talk to my tutor during my tutorial*'.

Self-study describes students who referred to either using online resources such as Google or YouTube, or those who appeared to work on their own, '*keep on trying to understand the material*'. Most comments in relation to the MAP Academic Advisor referred to the positive student experiences. They used words like '*helpful*', '*listened*', '*advice*', and '*admiration*'. For example, '*…wonderful experience, made me feel like it wasn't because I was unintelligent*'*.*

3.2. Academic supports

When students were asked their opinions on the academic supports they availed of, four main themes emerged: Understanding, Peers, Tutors and Structures. The theme of Understanding was evident across comments on all the supports. In almost all cases, students praised these supports, they felt that they encouraged them to engage with mathematics, clarify misunderstandings they may have had, and this gave them increased confidence in their abilities. For example, '*Tutorials were very good as it forced me to do the maths and expose misunderstandings when I got questions wrong*' and '*I found* [drop-in] *to be a huge help as I was able to go through lectures that I did not understand. It was also useful to get other examples of questions I was struggling with*'. There were, however, some comments which indicated that either study groups or tutorials did not increase student understanding. Students provided different reasons, such as being too shy to ask questions or blaming the tutor, for example '*I didn't benefit much from tutorials, felt embarrassed asking questions and tutors weren't very interactive*'.

Within the theme Peers, there were two subthemes, Collaboration and Social. The Collaboration subtheme related to the sharing of ideas with peers and the ease of asking questions '*I went* [to the MSC] *with friends so we could work out our assignments and help each other*'*.* The Social subtheme covers remarks about the benefits of having an environment where students can converse while studying. For example, '*…find* [study groups] *super helpful and a way for us to have social interaction while learning online*' and '*…it was a great emotional support to have a space* [MSC] *where friends could meet to discuss hardships associated with the course…*'. A small number of students, who availed of MLS during COVID-19, acknowledged the difficulties of working with peers in an online environment and signalled a desire to return to on-campus learning '*Discussing problem sets with students online helped a lot although it hasn't been the same as meeting them in person*'.

All but one comment in the Tutor theme was positive. Students mentioned words like '*helpful*', '*patient*', '*enthusiasm*', '*advice*', and '*great explanation*'. For example, '*My tutor is so enthusiastic about the work and I loved that because it made me want to engage more with the tutorial*'. All comments which fell under the Structure theme were related to tutorials. They either referred to the tutorial format or the timing of the tutorial content in relation to assignment submission '*Tutorials are way more helpful this year as it was the content we are working on rather than what we did the week before*'.

3.3. MAP student status

From a series of questions which aimed to gain an understanding of how the students' MAP status (access, disability, mature) affected their studies, three themes emerged: Time, Academic, and Age. Time issues mainly related to comments on difficulties keeping up. Comments from disability students indicated how their disability made it difficult to maintain pace with the course content '*I need extra time to get through lecture notes because I often lose focus or just need more time to process the information so I can understand the concepts*'. Whereas mature students referred to other factors which limited their time availability '*I have other responsibilities. I care for an elderly parent*'. Some students referred to poor attendance due to illness '*If I have a panic attack it could leave me useless for a day or two*' and others suggested poor time-keeping or organisational skills. Finally, there were a couple of students who struggled with the time allotted to complete their assignments and took issue with the rigidity of the Department homework submission deadline '*I much prefer when assignments are more spaced out in case I have a bad week*'.

The theme Academic refers to comments where students identified difficulties in staying focussed, recalling information, and misreading questions. There were some comments, all from disability students about general academic issues. For example, '*Anxiety and depression symptoms mean my concentration, memory and cognitive function are majorly affected*'. Other comments were much more specific about how their status impacted their studies in mathematics. For example, students referred to their weak mathematical backgrounds. Mature students associated this with the length of time since studying mathematics at school, and disability students with either missing class or underperforming due to their disability. Several disability students also commented that they sometimes mix up mathematical details, for example '*It can be really difficult to keep all the formulas in the right order in* [my] *mind…*'. Furthermore, there were comments related to difficulties with mathematical language and mental maths '*Have difficulty with symbols and shorthand*'.

All comments in the Age theme were from mature students. Some comments identified that '*money pressures are greater for mature students*', while other comments suggested that mature students felt less connected to their peers due to age differences. In contrast, there were mature students who viewed their life experiences as an asset to them '*As a mature student I found that I could organise my study time and group sessions adequately and with good confidence*'.

3.4. Additional support

Students were asked what the Department or MSC could do to improve their experience with mathematics, and four themes emerged: Tuition, Social, Differentiated Learning, and Academic Structures. Comments which fell under Tuition expressed a desire for more contact time with tutors, more 1-1 assistance and MAP-only tutors. '*More tutors in general and …*[tutors] *specifically for MAP students*'. Under Social, students also indicated a desire for increased opportunities to work with their peers. For example, '*I can say that I wish I had made friends in first year*' and '*Have a study group with just disability students so we can understand each other*'.

Under the theme Differentiated Learning, some comments centred around a desire for staff to be more cognisant of students' knowledge level '*… need to encourage me more, but remember I'm not a pure mathematics student*'. Others suggested that more written and visual examples would be beneficial '*…as I have difficulty understanding or remembering spoken descriptions*', and several students specifically referred to increasing staff awareness of their MAP status '*to let the tutor know because they can't help if they don't know*'. Finally, under Academic Structure**,** students made a variety of recommendations that they felt would improve their situation as a MAP student. They referenced items such as: being excused from mathematical computing labs, flexible deadlines, and more drop-in late in the week. Some students suggested that there was minimal learning in traditional lectures, while also discussing the advantages of asynchronous material: '*Online learning has helped me find video examples* [which are] *an excellent resource, can rewatch and pause which really helps understanding'*. Students also expressed different opinions on the timing of Department tutorials, with some preferring tutorials on material before it was due, and others wanting tutorials with feedback on material after it had been submitted.

3.5. Further themes

Two additional themes, Anxiety and Communication, were evident across sections 3.1-3.4. Students referred to anxiety about speaking in front of classmates: '*I also struggled in the tutorials because I was too anxious to speak up in front of everyone*'. They were also anxious about their abilities in mathematics: '*I spoke to a lot of lecturers and was then told about the support centre, this changed my anxiety around learning the subject and aided me in understanding very difficult concepts in a broken down and digestible way*'. The theme of Communication captures student responses which exhibit a lack of awareness of available supports, for example '*I wasn't aware of the service* [MAP Academic Advisor] *at the time*'.

4. Discussion and Conclusion

Though the number of respondents was low, the themes which emerged from this first local look at MLS for MAP students provide useful feedback for reviewing Department services. Encouragingly, students were broadly positive about MLS and mathematics: '*I feel like I have a better understanding of maths since coming to Maynooth. I felt in school it was more about the exam but in college its more about understanding the concepts in maths*'. While positive feedback from students who are engaging is not unexpected (Lawson et al., 2003), it is reassuring for our local policy of encouraging MAP students who are struggling, in the first instance, to avail of standard MLS. The respondents were particularly positive about tutors which, again, is not unexpected. Student evaluations of MLS, almost without exception, identify the importance of tutors (O'Sullivan et al., 2014). Research also suggests potential benefits for students from engaging with peers while studying mathematics (Duah et al., 2013, Solomon et al., 2010), and this is also evident from our responses. Comments were generally positive about their experiences of working with peers and some students suggested that we facilitate increased opportunities for peer interaction. This reinforces the relevance of our study group initiative, which was launched for all undergraduates in September 2020. Initial student feedback praised, in particular, the social aspects of these communities of learning (Mac an Bhaird et al., 2021).

While there was little negative feedback, there were several issues that we can seek to work on and improve, or advise the staff responsible. Some students referred to a need for increased staff awareness of their MAP status and its impact. A survey of MSC co-ordinators and 1st year lecturers across Ireland and the UK (Cliffe et al., 2020) identified similar problems. At MU we have increased the visibility of MAP in our MSC tutor training, for example, we are involved with sigma (http://www.sigma-network.ac.uk/sigs/accessibility-sig/) in the development and trial of accessibility resources specifically for those who co-ordinate and tutor in MLS (Heraty et al., 2021). Cliffe et al. (2020) also found that staff were often unaware of how they should support MAP students academically and, furthermore, if there was collaboration between MLS and MAP services locally.

Some MAP students also felt that Department and University structures were not flexible enough for their needs, which suggests that a streamlining of policies for MAP students in required. For example '*The systems that allow me to get extensions or exam supports are spread throughout the university, and the process of getting them is so much more convoluted*'. Addressing this issue is an ongoing challenge for a Department with a large number of students who have at least one assignment due each week. The move to increased online assessment has allowed the Department increased flexibility on deadlines for MAP students through the ease of setting specific submission deadlines on Moodle for different student groups. However, it is important to recognise that online teaching and learning can introduce other barriers for students (Smith et al., 2020).

There were references in the student responses to both social and mathematics anxiety. Mathematics anxiety is a commonly reported issue (Marshall et al., 2017) and mathematics anxiety sigma resources will be trialled at MU in 2022-23 and hopefully this will allow us to further reduce barriers for these students. Interestingly, some respondents in our study commented on how availing of certain aspects of support allowed them to develop strategies that reduced their social anxiety. For example, '*Used drop-in specifically during times when* [MAP Academic] *Advisor was there as social anxiety prevented me from using it otherwise*'. However, others remarked that their social anxiety prevented them from availing of some supports altogether.

Across the survey, there were a wide variety of comments which identified an array of unique student needs. This highlights the importance of the MAP Academic Advisor who can act as a coordinator between the Department and the MAP Office to determine bespoke solutions to student learning needs. Students who did avail of the MAP Academic Advisor were extremely positive about the experience. Unfortunately, almost half the respondents reported being unaware of the MAP Academic Advisor role. Students reporting unawareness of advertised supports is not a new phenomenon (O'Sullivan et al., 2014) and we continue to work with the MAP Office staff to promote the position. In addition to continuing our development of existing services to support MAP students and following up on the issues reported in this paper, we are considering further research. MU has a large number of MAP students and only a small proportion of these responded to our survey, with a variety of responses across the different MAP cohorts. Cliffe et al. (2020, p. 196) suggested that '*the fragmented nature of the MLS work on accessibility has not impacted on general practice…*', so we would welcome further work from colleagues on the academic MLS available for MAP students in their institutions and corresponding student feedback. This would allow us to put our own work into broader context and identify how generalisable our study is, given the number of respondents. Locally, it would be interesting for us to investigate the access, disability and mature student groups individually in more detail.

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RESEARCH ARTICLE

Computer Science Students’ Perspectives on the Study of Mathematics

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Abstract

In 2019, the Department of Mathematics and Statistics at Maynooth University commenced a project which sought to address, through the provision of mathematics learning supports, the issues of poor engagement and retention of computer science students studying mathematics. In this paper, we present preliminary engagement and performance data along with interviews conducted with eight students. We discuss how the quantitative data seemed to indicate that computer science students were engaging at similar levels to their peers, but several factors, including the quality of this engagement and their mathematical backgrounds may explain their poor exam performance. It also emerged that, while students were largely negative about their experiences in large lectures and their awareness of the relevance of mathematics to computer science, they were generally positive about smaller teaching situations such as tutorials, mathematics support drop-in and opportunities to work with their peers.

Keywords: Mathematics Learning Support, Computer Science, engagement, relevance, retention, study groups.

1. Introduction

Over many years, Department of Mathematics and Statistics (Department) staff at Maynooth University (MU) identified the performance of Computer Science (CS) students with mathematics as a concern. This problem, along with the broader issue of CS non-progression and engagement rates at MU are similar to those reported by the Higher Education Authority (HEA) in Ireland (Frawley *et al.*, 2017). Research indicates that appropriate engagement with Mathematics Learning Support (MLS) can impact positively on student retention and progression (Berry *et al.*, 2015). In 2019, MU commenced an ‘ICT and STEM Enhancement’ project funded by the HEA’s Innovation and Transformation call. One strand of this project relates to the provision of MLS to target the engagement and retention of CS students taking mathematics. In this paper, we aimed to address two research questions:

1. What are the backgrounds, experiences and challenges of CS students studying mathematics at MU?

2. What, if any, additional MLS can be provided to address the issues that CS students are experiencing with their study of mathematics at MU?

To address these questions, we initially considered one year of the Department's first-year student engagement and performance data. This data includes MSC attendance records. We subsequently conducted semi-structured interviews with undergraduate CS students. In this paper, we present an overview of the mathematical programme that CS students experience at MU, and the support available to them. We discuss the main themes that emerged from the interview data analysis. We consider what answers they provide to our research questions and we briefly summarise the initiatives we established as a result.

2. Background, Literature Review and Methodology

2.1 Computer Science and Mathematics at Maynooth University

At MU, undergraduates who want to study CS modules can do so in different ways. For almost all routes, CS must be accompanied by mathematics in first year, and by at least two modules of mathematics in second year. These are large service mathematics modules and not CS-specific. All first-year service mathematics students sit a proficiency test at the start of the academic year. A passing grade is 20 or higher (out of 60) and those who fail are automatically registered for an online Mathematics Proficiency Course (MPC) which is designed to cover topics that fill knowledge gaps considered pre-requisites for mathematics in HE.

Students studying mathematics receive weekly assignments. Prior to a small-group tutorial, one question is graded by their tutor and contributes to continuous assessment (CA). MU also has a busy Mathematics Support Centre (MSC), where the main service provided is drop-in. The MSC also runs weekly student-led workshops. If students' tutorial attendance, homework submission or MPC engagement falls below acceptable levels, they are contacted by the First-Year Monitor. Monitoring considers all students taking first-year service mathematics and does not focus on any specific subgroups, e.g. CS, Finance or Biotechnology students.

2.2 Literature Review

In the past decade, several studies have featured or focused on retention and progression rates in relation to CS in HE. Research published by the HEA in 2019 considered those entering HE in Ireland in 2007-08 and whether they had graduated their institute by 2016. They found that students in computing courses had the lowest rate of completion, 55% across the combined levels 6 (higher and advanced certificates), 7 (ordinary bachelor's degrees), and 8 (higher diplomas and higher bachelor's degrees) on the national framework of qualifications (NFQ) when compared to other courses (Pigott and Frawley, 2019). This figure is 37% when we look exclusively at level 8 computing courses. In a similar study from 2018, which also included non-CS courses, the HEA considered progression rates of first-years into the second year of computer science courses across HE and all NFQ levels from the 2014-15 to 2015-16 academic years. They also compared the non-progression rates to those from 2013-14 to 2014-15, which were 22% and 21% respectively, well above the national average of 14%. When we look exclusively at level 8 degrees in universities, the rate is somewhat better at 11% (Liston *et al*., 2018). In the UK, a comparison of retention rates across disciplines in HE for 2010-11 found that CS had the lowest continuing rate of 91%, meaning 9% of students either left with a lesser degree than originally intended or did not continue their studies (Woodfield, 2014).

The factors which may have influenced these trends have been considered by researchers. Several papers examine social integration within a variety of CS courses. For example, Biggers *et al*. (2008) studied undergraduate CS students that were registered at the Institute of Technology, Atlanta, comparing survey responses of those who graduated to those that dropped out. Lack of awareness of the relevance of the course material, low exposure to real world applications, tedious boring workloads, low levels of human interaction and a perception that CS is antisocial were identified as significant contributors to course non-completion. In Ireland, the National Forum for the Enhancement of Teaching and Learning in Higher Education (National Forum) identified, in a 2015 briefing paper *'Student Non-Completion in ICT* [Information Communication Technology] *Programmes',* that high attrition rates had been associated with a range of factors including '*the limited mathematical skills and problem solving abilities of some students entering ICT programmes*' and '*poor awareness of the level of maths and computer skills required to succeed in such programmes*' (National Forum, 2015, p. 4-5).

2.3 Methodology

When this project was announced, there was a short period during which we could consider student data and use it to inform our services for the 2019-20 academic year. We began by examining all the Departmental quantitative engagement and performance records of first-year Arts students, within which we compared those students studying CS with their non-CS classmates. Some of these initial results were surprising as they appeared to contradict MU internal reports and research on CS student engagement (Colby, 2005). Therefore, we decided to investigate further by conducting semi-structured interviews. These were identified as the most effective way to gather additional data from the CS students (Sarantakos, 2012). Eleven questions were designed which targeted areas identified as being important to student academic success with mathematics, based on previous MU studies, research literature and the authors' experiences.

Ethical approval was received, and, in April 2019, details of the project were announced to students via Moodle and in the MSC. In total, eight students responded to the call for interviews, two from each of the four years of study, and interviews were conducted in May 2019. Each interview was recorded and transcribed. All identifying information was removed prior to the data analysis. The authors used Thematic Analysis to analyse the interviews (Braun and Clarke, 2006), which were read and coded independently by the authors. The authors met and discussed their findings to identify any common themes.

3. Results and Discussion

Due to the different variances in the CS and non-CS groups, it was not possible to conduct significant statistical analyses or comparisons. However, we present the initial quantitative data in Section 3.1 because we used it, in addition to the outcomes in Section 3.2, to guide our initial MSC interventions.

3.1 Quantitative Departmental and MSC Data

We began by considering proficiency test results for 2018-19 and from these, we observed that CS students, on average, appeared to enter MU with weak mathematical backgrounds. This was not unexpected and consistent with findings in other studies (National Forum, 2015). For example, if we consider first-year Arts, the mean result out of 60, for the entire class was 26.29 (n=211), for CS students it was 16.52 (n=30) and for non-CS it was 28.06 (n=181).

We then examined the Department's engagement records and found that CS students seemed to be engaging and performing at similar levels to their non-CS peers during 2018-19. See Table 1 and Table 2.

	All students n=203	**CS n=30**	**Non-CS n=173**
Mean number of tutorials attended (out of 20)	11.92	13.13	11.74
Mean number of assignments submitted (out of 20)	14.10	15.57	13.86
Mean percentage assignment grade	44.39	45.92	44.14

Table 1. First-Year Arts Tutorials and Assignment Data 2018-2019

	All students n=119	**CS n=19**	**Non-CS n=100**
Mean number of MSC visits per attendee	12.73	11.21	13.02
Mean total time spent in MSC (in minutes)	850.02	588.68	898.20

Table 2. First-Year Arts MSC Attendee Data 2018-2019

Albeit based on one year of data, these apparent similarities in engagement and assignment grade data were surprising to us. The most noticeable difference was in the category 'mean total time spent in MSC'.

Finally, we considered the final module results of these students, see Table 3.

Modules	**All students (%)**	**CS (%)**	**Non-CS (%)**
Calculus 1 (n=203)	42.13	29.43	44.34
Introduction to Statistics (n=166)	49.14	38.83	51.33
Linear Algebra 1 (n=162)	53.41	48.70	54.48

Table 3. First-Year Arts Mean Module Results 2018-2019

The differences in the module results between each group were substantial. When coupled with the similar homework grades obtained by each group, the data in the tables indicated, at least on a surface level, that while CS students appeared to be engaging appropriately with mathematics, their exam grades were well below the class average. This exam data seemed in line with the aforementioned 2018 HEA report but at odds with literature on CS student engagement (Colby, 2005).

3.2 Interview Data

One of the main themes to emerge from the interviews was the teaching approach used in different classroom situations. All eight students were negative about the traditional lecturing method of '*chalk and talk*' with one stating that you are '…*just writing notes and you're not thinking about what they're saying'.* Prior to COVID-19, lecturers in the Department did not typically upload full sets of notes online. Respondents reported different teaching methods in their other subjects: '...*they're all done up you can scroll through it with the lecturers. If you fall behind, you can scribble something down*…*you can even have it on your phone*…'.

Other negative comments referred to the fast pace of lectures, the large class size and difficulty in asking questions '*…*[even though] *every lecturer says don't be afraid to ask questions*…*you're not going to put your hand up in a class of 400 to ask something*'. Half of our respondents directly linked their lecture experience to their subsequent disengagement: '*there's no point in me coming to lectures. I'm not going to learn anything, I might as well just study*'. These experiences are similar to findings in other studies (Grehan *et al.*, 2016), although negative experiences in relation to mathematics lectures are not new or unique to MU or indeed to CS students. (Mann and Robinson, 2009, Tinto 1997).

We passed this student feedback on to the Department as lecture style and format do not fall under the remit of MLS. Nevertheless, lectures are an important part of the student experience and can influence the level of engagement with MLS. Tinto (2006, p. 4) states that '*… the classroom is, for many students, the one place, perhaps only place, where they meet each other and the faculty. If involvement* [engagement] *does not occur there, it is unlikely to occur elsewhere*'. The provision of supports such as an MSC, tutorials and assignments to complement lectures and lecture material is recommended in several studies (Macrae *et al.*, 2003), and can *'…provide students with opportunities to build and enhance academic and social skills in a positive, supportive, intentionally constructed environment.*' (Bean and Eaton, 2001, p. 86).

If we consider themes which emerged from comments in relation to tutorials, assignments and the MSC, it appears that these supports were largely successful, and the respondents were highly involved or engaged as a result. Indeed, all themes related to tutorials and the MSC were positive. The importance of small group teaching and learning in STEM is well researched (Springer *et al.*, 1999), and students who engage with MSCs tend to be very positive about their experiences (O'Sullivan *et al.*, 2014). The teaching approaches used in both tutorials and the MSC were endorsed by students who felt that material was '*explained in the tutorials in such a way that was very easy to understand'*, when compared to lectures, and that '*The MSC was incredibly useful, especially if you need a little push on some questions*'. Respondents also appreciated the smaller class sizes and found it easier to ask questions. In regard to the MSC, students also identified that the atmosphere '…*motivates you as well to do the work*'.

Group work and the opportunity to work with peers, was of particular importance to five of our interviewees: '*Whenever I work on my own I get quite frustrated*', but '*working in groups was really helpful because I could ask* [peers] *a question* [...] *and then continue on*'. While the use of social interactions by students can influence their level of engagement and how they deal with their mathematical difficulties (Grehan *et al.*, 2016), social isolation is identified as a major issue for CS students (Crenshaw *et al.*, 2008). The positive and social experience reported by interviewees corresponds to a sense of networking and community in smaller classes which is important for engagement (Crenshaw *et al.*, 2008). However, this was not the case for all students. One felt that they did not benefit from studying in groups as they '*… end up helping* [peers] *figure it out and I get nothing done…*'. Research shows that students may need direction on how to work together effectively (Oakley *et al.*, 2004).

While the majority of respondents were positive about homework, some felt like they were '*churning out assignments*' and that it was a '*big learning curve, especially in first year*…'. In Grehan *et al.* (2016), 15 of 16 students interviewed reported difficulties with their assignments at the start of the academic year, and eight of these reacted by attending the MSC.

Students were asked about the relevance or usefulness of mathematics for CS and, while all reported different levels of awareness, a number of themes emerged from their comments. In particular, students questioned the level of coordination between the departments in relation to connecting the two subjects for students: '…*I think there was a disconnect in first year like, you were just kinda doing two separate subjects*'. Baldwin *et al.* (2013, p.74) refer to '*the awkward place for mathematics in undergraduate computer science curricula*', and our interviewees identified inconsistent messaging and communication from CS staff about the importance of mathematics. One student recalled a CS lecturer saying that '… *you need a bit of maths, but we'll cover the maths in our course*'. Baldwin *et al.* (2013, p.74) also claim that '*mathematics courses align poorly with the needs of computer science*', and this is evidenced by some of our respondents who did not see how their first-year mathematics lectures were relatable to CS: '*If I was in first year and I didn't know any of this* [importance of maths] *I'd be like, why would I want to do maths!?*' In fact, only two students remembered the relevance of mathematics being explicitly communicated by teaching staff in their first year, though three students indicated that they heard via their social interactions. The two final-year students, who said that the relevance of mathematics became clearer after second year, indicated that knowing this earlier would have an impact: '*if someone from the CS department came in and showed* [us] *all the courses that you can do in final year and said you need this maths for doing that, you need to know number theory for that, calculus for doing that, I think I would have chosen subjects differently*'.

Another theme which emerged across several questions was references by students to their mathematical background. Three students, who attended MU straight from school, indicated that they did not feel prepared: '*I would have liked a week or two-week intense course for precalculus before I came into college*...*That would have helped a lot*'. While this is consistent with national reports featuring CS students (Pigott and Frawley, 2019), there was almost no engagement from interviewees with the MPC. This course was set up to tackle the mathematical deficiencies that students have entering MU, but respondents suggested that it was forgotten about or not used because of its non-compulsory nature. This suggests that the MPC needs to be better advertised and its purpose more clearly communicated to students.

Five students indicated that they did feel prepared, referencing the role of supplemental instruction prior to starting university. One student mentioned private tuition, and the other four were mature students. A study of mature students over a ten-year period at the University of Limerick, suggests that '*The initial challenges which mature students face, however, are likely to have been counteracted by their motivation to succeed*' and add that '...*mature students tend to exhibit more desirable approaches to academic learning*' (Faulkner *et al.*, 2016, p. 347). The four mature students in this study all praised the precursory summer course they attended at MU: '*If you are weak at math or anything like that then I highly recommend having that as an option for people to go…*'. The *National Strategy for Higher Education 2030* report (Department of Education and Skills, 2011) recommends such preparatory courses to ensure a positive first-year experience.

4. Conclusion and Next Stage

In this paper we considered quantitative data for one group of first-year students, and interviews from eight CS students. Thus, our preliminary findings are not necessarily representative of all CS students. Nevertheless, the data did provide interesting insights which partially answered our two research questions.

On average, CS students are entering MU with weak mathematical backgrounds, and this may be a significant contributing factor to their poor performance in mathematics examinations. Studies have shown, for example Burke *et al*. (2013), that the mathematical background of first-year students is the biggest indicator of their progression into second year. Interviewees referenced their mathematical backgrounds when describing difficulties with the subject, and while several highlighted the importance of having extra academic support in their transition to HE mathematics, none had engaged with the MPC to any extent. In an effort to increase the quality of student engagement with the MPC, the Department has made it a mandatory part of CA for all first-year students.

Interviewees reported negative experiences with large lectures, but were very positive about other teaching environments, and they described positive participation with tutorials and the MSC. The quantitative data indicated similar trends and this suggested to the authors that CS students may have focussed on getting their homework completed, rather than on gaining a fuller understanding of the material. This may also have influenced their poor performance on exams when they needed to attempt the material on their own. The Department is reconsidering its monitoring system (Burke *et al.*, 2013) to see if there are additional checks that could be put in place to measure the quality of student engagement and also whether certain subgroups, such as CS, need to be considered separately.

The interviewees also felt that communication between the Departments, and from staff to students, could be improved, especially in terms of clarifying the role of mathematics in CS. As a partial answer to the second research question the authors, in consultation with the Department of CS, drafted a document outlining connections between undergraduate service mathematics modules and CS modules at MU. These lists were distributed to lecturers and shared with students. Some of the authors also spoke at first-year CS orientation events in order to present a more collaborative image of the two Departments. Interviewees also highlighted the benefits of group work, though the quantitative data suggested that CS students may not have been using the MSC appropriately. Following a subsequent literature review, we decided that we would launch an MLS study group initiative in 2019-20 for both first and second-year CS students. At orientation, the potential benefits and pitfalls of study groups were introduced to students (Oakley *et al.*, 2004). The study groups are student-led and students are encouraged to bring questions to discuss in order to maximise the benefits of peer learning. Tutors meet with these groups once a week in the MSC to guide their learning and ensure they are working effectively. Tutors also check attendance and provide encouragement, intervening if students show any signs of disengagement. Due to their success, these study groups have continued in the MSC (Mac an Bhaird *et al*., 2021).

While this paper reports on the initial phase of this project, related research is ongoing. For example, the authors are considering a longitudinal study of Department quantitative data in relation to CS student engagement and performance. The findings from this project, which finishes in 2022, could be used by MU to consider the future structure of CS courses and the provision of mathematics for CS students.

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RESEARCH ARTICLE

An Evaluation of a Summer Mathematics Bridging Course for Mature Students

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Abstract

Each summer, the Department of Mathematics and Statistics at Maynooth University delivers a three-week bridging course for mature student applicants. This course serves a dual purpose. First, it acts as a refresher of fundamental mathematical skills, and second, for some, the summative assessments form part of the screening process to determine if their mathematics is of a sufficient level to make the move to higher education. This paper describes the results of student feedback on this course obtained through an online survey of past participants. Respondents indicate that not only is this course effective for building mathematical fluency and confidence, but that they also benefit from gaining a familiarity of the campus as well as making friends with their peers.

Keywords: Mature students, bridging course, mathematics, transition

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1. Introduction

In Ireland, a mature student is generally defined to be anyone over 23 years of age on the first of January of the year they enter Higher Education (HE). At the University of Limerick (UL), Gill (2009) identified that mature students were struggling with the transition to HE mathematics. It is also known that mature students entering HE often have a sense of inadequacy, and these feelings are '*particularly acute in the initial stage of entry*' (Kearns, 2017, p. 181). As a result, to try and ameliorate these issues, in 2007 UL introduced a week-long intensive bridging course, named *Head Start Mathematics*, taught before the start of term. During the same period, and for similar reasons, Maynooth University (MU) established the Summer Mathematics Course (SMC). The SMC has two purposes. First, it should act as a refresher course for students who, on paper, have the mathematical standard to make the transition to HE but the time elapsed since they used their mathematical skills has caused a reduction in their fluency. Second, the SMC acts as part of MU's admission process to identify applicants whose mathematical skills are not yet at a sufficient level to facilitate an immediate move to MU degree programmes. Such students can be diverted to more appropriate bridging courses, such as the year-long Certificate in Science at MU (Mulligan and Mac an Bhaird, 2017).

In a 2011 report, the Irish Department of Education and Skills (DES, 2011) when analysing enrolment levels for new entrants to HE, predicted that the proportion of mature students would almost double from 13% in 2009 to 25% in 2025. Contrary to this predicted increase, a recent study by the Irish Higher Education Authority (HEA) reported that the rate of mature student participation in HE has been steadily decreasing to 9% in 2018-19 (HEA, 2021). At MU, we found similar trends. The HEA report also added that '*The decline in participation from 2013 coincides with the reduction in unemployment, reflecting the impact of employment opportunities on the numbers of mature students*…' (HEA, 2021, p. 14). This decline in mature student applicants led, in 2020, to the MU Department of Mathematics and Statistics and the Maynooth Access Programme (MAP) Office considering the medium to long term future of the SMC. Evaluation of mathematical initiatives available to students is a key part of our research-based practices at MU and, as there had been no formal evaluation of the SMC previously, we decided that an important first step would be to survey student views about the course. As a result, in this paper we aim to answer the following research questions:

1. What is student opinion on the SMC?
2. What, if anything, can we do to improve the course?

First, we describe the SMC at MU, before providing a brief literature review of available research on similar bridging courses in Irish HE. We present the main themes which emerged from analysis of the survey responses, before discussing these findings in the context of related literature. We close by considering the answers to our research questions and the implications for the future of the SMC.

2. Summer Mathematics Course and Mature Student Entry to MU

The first two weeks of the SMC reviews topics such as arithmetic, fractions, algebra, logarithms, coordinate geometry and trigonometry. During this period, students receive five two-hour lectures, with each lecture supported by two two-hour lab sessions. In these labs, students are provided with practice problems and there are two tutors available to assist with any queries that may arise. Students also have computer access during these sessions to aid their study. At the end of week two, students sit an exam containing 20 multiple-choice questions on the topics covered. Furthermore, dispersed through the two weeks are a mix of extra events, such as guest lectures on general science topics as well as tours of the historic Russell Library (https://www.maynoothuniversity.ie/library/collections/russell-library) and the National Science and Ecclesiology Museum (https://museum.maynoothcollege.ie/), which are both situated on campus.

Week three topics include simultaneous equations, linear inequalities, coordinate geometry of the circle, polynomial division, cubic equations, the unit circle and further trigonometry. Each day, from Monday to Thursday, students receive a two-hour lecture followed by a two-hour lab session. At the end of this week, there is an exam containing 20 multiple-choice questions.

The application process for mature students to MU is similar to that at other Irish institutions (HEA, 2021). However, mature students who apply for a degree programme at MU, on which there is a mathematical component, must take an initial competency test to assess their basic mathematics knowledge. If they pass, they are then called for interview. The interview panel then make one of the following recommendations based on the candidate responses, their mathematical background, and the mathematical content of the degree they have applied for:

1. They are offered a place on a degree programme with no conditions.
2. They are referred to attend the full three weeks of the SMC.
3. They are referred to attend the first two weeks of the SMC.
4. They are referred to attend just week three of the SMC.

If a student is referred to attend the SMC, their place in MU may be conditional on passing the associated exams.

3. Literature Review

In 2021, the HEA in Ireland published a comprehensive study on mature student participation in HE. In this study, they briefly discuss the provision of what they term as foundation/bridging courses, which they state are available in many institutions in Ireland. They suggest that '…*the funding and provision of foundation/bridging courses in advance of attending a mainstream HEI* [HE Institution] *course can greatly assist students.*' (HEA, 2021, p. xix). Courses '…*can also help introduce students to the college campus, its facilities, and the daily routine of the life of a student.'* (HEA, 2021, p. 55). While this study also alludes to the success of such programmes, it does not provide direct evidence. A literature review identified publications available on year-long bridging courses in Ireland, many of which are accredited, for example O'Sullivan et al. (2017), but the authors found just four articles which relate to short non-accredited courses similar to the SMC at MU. It is not clear whether engagement with these courses plays any role in subsequent student admission to the HEI, though one mentions an end of course exam.

Gill (2009) observed that first-year mature students in UL were having difficulty catching up with the fundamentals of mathematics while simultaneously maintaining their other studies. As a result, while on secondment with sigma (http://www.sigma-network.ac.uk/), Gill developed *Head Start Mathematics* (https://ulsites.ul.ie/cemtl/head-start-maths-workbooks), a week-long course taught before the start of term to aid mature students in their transition to HE. Gill (2010) established that engagement with the course impacted on student self-concept, and in both papers student responses indicated that their participation '*…was enormously affirmative and made for a positive experience for all involved.*' (Gill, 2009, p. 37). In 2010, *Head Start Mathematics* was expanded to a two-week course and, in a follow-up study, Johnson and O'Keeffe (2016) identified an increase in the retention rates of undergraduate adult learners who participated in *Head Start Mathematics* compared to those who did not.

Cork Institute of Technology (CIT), in 2010, introduced *Maths for Matures*. O'Neill (2013), in an article which largely focusses on the structure and purpose of the course, describes how it is offered to mature students who are intending to apply for entry to a full-time science or engineering undergraduate programme. They outline 14.5 hours of instruction, delivered three evenings per week, over a three-week period and include positive and encouraging quotes from past participants, for example '*It has given me belief in knowing that I can do something if I apply myself fully. It has also given me increased confidence in maths*'.

4. Methodology

The authors designed a survey consisting of tick-box and open-response questions which were hosted on onlinesurveys.ac.uk, see Appendix A for a sample of the open-response questions completed by participants. Ethical approval was granted, and the survey was launched in the spring of 2021. Participants from 2012-2020 were invited to complete the survey via various appropriate mailing lists. In total, 32 responses from a possible 190 were gathered, a response rate of approximately 17%. The tick-box questions focussed on participant background and were intended for cross-referencing with the open responses. However, due to the homogeneous nature of participant backgrounds, this analysis revealed no further insight. All 32 respondents had proceeded to study at HE on completion of the SMC with 28 of these choosing to study at MU.

As the substance of the survey contained mostly open-response questions, the authors chose Thematic Analysis (Braun and Clarke, 2006) to analyse the collected data, as it is a '*reliable tool...*[to] *identify a set of themes that reflect the essence of textual data, and to discover recurrent patterns.'* (Sarantakos, 2012, p. 379). To this end, the authors coded the open responses individually and noted the main themes that emerged. They then met to discuss any discrepancies and agree on the final themes, which are reported in Section 5. Respondent comments often fell under more than one theme.

5. Results

In this section, we describe the five main themes that emerged from the analysis of the survey responses: mathematical preparation, confidence, transition, social and course format.

By far the most dominant theme was the 'mathematical preparation' for HE provided by the SMC. Respondents made mainly positive remarks, for example '*Prepared me for university mathematics, giving understanding of basic language and principles on which calculus was extended in modules. Without this summer course, I would have not be*[en] *able to keep up in lectures*'. Students elaborated on this point by adding that they appreciated how the basics were covered: '*I liked that things were explained from a standout of a complete beginner rather than assuming prior knowledge. This really helped when working with harder material. It's always the most basic maths that comes up very often in my studies at college, and this course helped clear things up in that regard*'.

Furthermore, several student comments placed additional emphasis on the importance of this preparation as the SMC had made them realise how much their mathematical fluency had diminished over time: '*The course opened my eyes to how out of practice I was with maths and how much I had forgotten*'. While others added that, as a result of the SMC, they also recognised the need for extra study: '…*my education gap was larger than what was taught* [on the SMC] *so a lot of self-learning had to be involved to pass the first year in mathematics and physics*'.

Three respondents felt that the SMC did not prepare them for the subsequent level of mathematics at HE: '*It is so much easier than anything actually encountered during degree studies*'.

Thematic analysis also identified increased student 'confidence' as a result of the SMC as a major theme within student comments. For example, '*It gave me the confidence to get through third level and also a taste for maths which I never would have thought possible given my horrible history with maths at second level. In fact, I changed my degree to general science in order to continue the subject*'. Others commented on an increase in their self-efficacy and how participation alleviated their initial concern about studying mathematics: '*I was initially apprehensive about returning to education at my age and trying to learn maths etc. Realising how well I did, how much I learnt, and how much I enjoyed the course, definitely reinforced my decision to return to education*'.

Another theme to emerge from student responses was one of 'transition'. Comments referred to the benefits of the SMC in easing their subsequent orientation and integration to university life: '[I was]*…much better organized when progressing to third level. I was also familiar with the buildings used by the mathematics department and even some of the tutors, which really helped*'. In a similar vein, others remarked that '*It was an intimate setting so there were plenty of opportunities to ask questions. It definitely was a good insight into what lay ahead for anyone undertaking maths as part of their degree*'. Within this theme, there were a small number of suggestions on how the SMC could be enhanced to further improve transition. For example, '*The lab session time could be better used to familiarize people with the software used in the mathematics department. I realize it might be seen as giving an unfair advantage, but the majority of people on the course are at a natural disadvantage due to not coming from a traditional education background and this could help them assimilate to university level*'.

Closely related to this theme were the 'social' supports that evolved for students as a result of attending the SMC. For example, '*I really enjoyed the social aspect and as I suffer from anxiety, knowing the campus before September was of great benefit*' and '[The lecturer] *and the other* *students were very friendly and it was great to already know some people when I started in September. One of them is a very close friend 5 years later*'. The importance of these enduring friendships made during the SMC were acknowledged by other respondents: '*I also made friends during the course which was extremely helpful when I started in the college. I'm best friends to this day with a friend I made during this course, and he encouraged me throughout. If it wasn't for this program, we may not have met*'.

The final theme identified across student responses was the 'format' of the SMC, with some comments providing extensive detail, for example '*I really benefited from all aspects of the summer maths course and I really enjoyed all of them. The lectures given by* [the lecturer] *were excellent and the follow-up lab sessions really emphasized the lecture material we were given. The guest lectures and tours etc. really complemented the course. It gave us a chance to socialise, meet new people, make new friends and become familiar with Maynooth campus*'*.*

Several respondents appreciated the timing of the different course features: '*The course structure was pleasant with morning lectures afternoon lab sessions and guests in the midst to break the day/week. Enjoyed the tour of the old library and group lunches*'. While others associated these course features with an enjoyable learning experience: '*I thought it was really nicely set up with the tour and history of maths lecture. It really opened my eyes to the wonder of maths. I liked the lab sessions because it gave you a chance to work on problems and get some one-on-one help.* [The lecturer] *is an amazing teacher and made me feel relaxed and less intimidated by the subject*'.

There were some suggestions relating to changes to the course format, focusing on either extending its duration or including more advanced material, '*Include some basic calculus, like differentiation and integration*'.

6. Discussion and Conclusion

The issue of students entering HE without the required levels of mathematical proficiency is known as *the mathematics problem* and has been widely documented (Lawson et al., 2012). The SMC was established to go some way to tackle this issue for mature students entering MU. Analysis of the survey responses, although we received no responses from students who did not continue to HE after the SMC, seems to indicate that the SMC has been broadly successful in this regard and, given the heterogeneous nature of mature students (Ryan & Fitzmaurice, 2017), it is reassuring that overall, there is a high degree of satisfaction with the SMC. The fact that all respondents to our survey continued to HE after the SMC is a possible limitation to this study as self-selection may have introduced some bias.

The five themes which emerged from student responses provide answers to our first research question. Students felt that the SMC facilitated increased mathematical preparedness and confidence in their own abilities, both of which are crucial for progression through HE (Parsons *et al*., 2009). It is also evident, from the format, social, and transition themes, that respondents identified that the SMC gave them the opportunity to familiarise themselves with campus and the facilities available, a key aspect of such courses as outlined by the Department of Education and Skills (DES, 2021). In particular, students seemed to appreciate how the course lectures and labs were punctuated by guest lectures, and tours. Furthermore, bridging courses can also '*help break the barriers of age groups and develop interactions among participants.'* (DES, 2021, p. 55), and evidence for these social interactions was very clear in our study, from lunches to establishing long-term friendships. This bonding is an essential part of a successful support network for students because '*Students learn together in class, while friends, classmates and study partners learn together outside of college campus*' (Lei, 2010, p. 156).

In relation to the second research question, a small number of responses seem to indicate that the precise function of the SMC within the pathway to HE might be better communicated. For example, suggestions that the SMC be extended and that it include some calculus. Calculus is not included in the SMC as this topic is taught from a beginner's perspective in the first year of mathematics at MU. A few respondents also suggested that the SMC did not prepare them for HE. This has caused us to reconsider the multiple-choice nature of the summative examination and the inclusion of grading of full solutions which may provide a better picture of student knowledge.

7. Future Work

Due to COVID-19 the SMC was delivered online in 2020 and 2021, and there was increased engagement with and completion of the SMC from registered students when compared to previous years. This seems at odds with general student online engagement during the pandemic (Mac an Bhaird et al., 2021). Three of the survey respondents took the SMC during this time and they provided both negative and positive comments on their online experience. However, one possible explanation for increased engagement with and completion of the SMC during this period is the flexibility that online study offers mature students as it allows them to fit education around their varied commitments (HEA, 2021). Combined with the positive feedback from Section 5 regarding the in-person SMC, we are considering a hybrid model for the medium to long-term, combining the best elements of both online and in-person SMC. We also plan to continue our research in this area, for example, a study of degree completion rates at MU for past participants of the SMC.

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Appendix A

The following are a representative subset of the open-response questions which participants answered in our survey. The main themes which emerged from the coding of responses to these questions are presented in Section 5.

1. When you applied to Maynooth University, why were you interested in returning to education?
2. What aspects of the Summer Mathematics Course did you like/dislike? Eg, lectures, lab sessions, guest lectures, library and museum tours, etc.?
3. In what ways did you find the Summer Mathematics Course useful/not useful?
4. If you could make any changes to the Summer Mathematics Course, what would they be?
5. Did your experience of the Summer Mathematics Course reinforce your decision to consider third level education? Please explain.
6. Do you feel the Summer Mathematics Course adequately prepared you for your mathematics or mathematics related studies at third level? Please explain.
7. Did the Summer Mathematics Course influence your decision to choose a course that has mathematics as a core element? Please explain.

RESEARCH ARTICLE

Diagnostic Tests: Purposes and Two Case Studies

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Abstract

It is not uncommon to use what are called diagnostic, placement, readiness or competency tests once students arrive at university to gauge their basic skills in mathematics or literacy. This paper begins by discussing diagnostic mathematics tests and identifying the key reasons for which these are run. Two such tests with repercussions for students are discussed. These two tests are for different student cohorts and are run for different reasons. We identify the purposes for which the tests were developed, and actions which eventuated. We identify any additional purposes the tests served beyond those intended. The tests had a positive impact on student learning.

Keywords: diagnostic tests, mathematics support, tertiary mathematics.

1. Introduction and background

In many countries more students than ever before can aspire to a university degree, but an increasing proportion arrives unprepared for the rigors of higher education. One method used to determine the level of basic skills of students beginning tertiary study is diagnostic testing. This paper examines two uses of diagnostic tests that are being applied to try to address the problem of students arriving at university with poor mathematical skills and knowledge. This is preceded by a discussion identifying reasons for which diagnostic tests are used.

Reports from the Organisation for Economic Co-operation and Development (2013) make clear the benefits to individuals and employers, and thus to economies, of higher levels of mathematics and numeracy, along with other basic skills. In Australia, an Australian Industry Group report (2018) states that 39 per cent of businesses are highly affected by low levels of language, literacy and numeracy. Over the last 20 years there has been a huge decrease in the proportion of students choosing calculus based mathematics in Australian secondary schools (Barrington & Brown, 2014) and in the state of New South Wales (NSW) (Nicholas & Rylands, 2015). Despite the importance of mathematics, many Australian universities do not have mathematics requirements for entry into mathematics, science, engineering and other quantitative degrees. There is justifiable concern about what skills students have when they begin university.

In Australia, lecturers rarely have access to students' records of their previous studies. One method used to gain information on the skills that students bring with them to their university studies is to run short diagnostic tests when students arrive at university.

2. Diagnostic mathematics tests

This paper considers mathematics tests which are run once students have been accepted into university. A systematic literature review of interventions that could lead to improved mathematics outcomes for first-year students (Lake, et al., 2017) noted the importance of the mathematical skills students have when beginning tertiary studies and thus the importance of diagnostic tools for determining background and appropriate interventions.

There are many examples of tests or quizzes being run because academics have concerns about the level of knowledge and skills that students bring with them to university. These tests are often referred to as diagnostic tests, placement tests, competency tests, readiness tests or skills assessments. The term diagnostic test, or test, will be used in this section.

Diagnostic tests are run before students start their studies or soon after, providing information on students' prior knowledge. Such tests usually cover basic knowledge and skills. The tests are short; usually less than an hour. These tests are usually not for credit and not all are compulsory. There is enormous variety in the many other dimensions of diagnostic tests.

A UK report (LTSN mathsTEAM, 2003) on mathematics diagnostic testing comprising 13 case studies, began by noting increasingly diverse student backgrounds, which inspired some universities to introduce such tests. The primary aims given for these tests were "to inform staff of the overall level of competence in basic mathematical skills of the cohort they are to teach" and "to inform individual students of any gaps in the level of mathematical knowledge they will be assumed to have—so that they can take action to remedy the situation." On reading the case studies one finds other purposes for which diagnostic tests were used.

At our institution we also use such tests for a variety of reasons. This inspired the first aim of this paper, which is to answer the question

• What are the purposes for which diagnostic tests are used?

A variety of purposes for diagnostic tests appears in the UK report (LTSN mathsTEAM, 2003). For example, at Queen Mary, University of London (QMUL), students were given seven attempts at a test and had to reach a score of 12 out of 15 in order to progress to second year. At the University of Strathclyde students and tutors were given test results and tutors could identify and assist students who were expected to struggle with the work. The University of York used a test to identify any remedial actions needed, and had used the same test in the same manner for 15 years. At Anglia Polytechnic University a test informed students and staff of each student's capabilities. The test gave the lecturer information on the level appropriate for teaching various topics. At the University of Bristol a test informed students what they needed to revise.

Ngo and Melguizo (2016) discuss many issues related to diagnostic tests in American community colleges, including cutoff levels and appropriate placement of students. Their context includes deciding whether or not to place students in remedial mathematics subjects.

At the University of Queensland, Australia, reports of students lacking background knowledge, student dissatisfaction and growth in attrition (Kavanagh, et al., 2009) resulted in a diagnostic test to identify gaps in students' knowledge. The 46 question test covered mathematics, chemistry, physics and thermodynamics. The authors state that the test results will inform plans to support at-risk students in the future. One of the authors' conclusions is that the value of the test may also lie in informing students of gaps in their knowledge; they also report that it was reasonably reliable for predicting success. This test was used for engineering for students at the University of Auckland, New Zealand (Shepherd, et al., 2011), however, here the main motivation was to raise students' awareness of their weaknesses and address these. They found this to be relatively successful, despite being unable to send students individual feedback. Also mentioned was using information gathered by the test to underpin support for students, especially from "Never seen it before" responses.

Wilkes and Burton (2015) report on an online test covering mathematics, among other topics. It was designed for engineering students by a team covering five Australian universities. The increasing diversity of students was a driver for the project. Students received immediate individual feedback which informed them of the skills and knowledge needed for their studies, and they were encouraged to take responsibility for learning. The test was found to be good for predicting success in mathematics even though it was run largely for applied science students.

Espey (1997) used a test to drive improvement in the basic mathematics of students by requiring them to reach a threshold of 84 per cent before the second test of the semester. Students were allowed to sit the test many times, but no more than once a day. Mathematics support was provided to students.

Carr, Bowe, and Ní Fhloinn (2013) report on a test which they refer to as a core skills assessment, run for engineering students at the Dublin Institute of Technology. It contributes 10 per cent to the final mark for first-year students. Students who do not reach 70 per cent in the test receive a contribution of 0 to their final mark, however students are allowed to sit the test many times. Immediately after the test students are given correct answers to questions they answered incorrectly. The aim is to drive learning in core skills.

For business students, Abdullah, Ujang, Ramli, Dzulkifli, and Mohamed in Malaysia (2016) use a diagnostic test to predict performance. They mention also giving teaching staff an overview of student's mathematics capabilities. Silva, Ghodsi, Hassani, and Abbasirad (2016) report on a diagnostic test run in a British university for business, accounting and finance students. The authors state that the results can be used to argue for more mathematics and statistics support and they raise questions about entry criteria.

2.1 The purposes of diagnostic mathematics tests

The previous section provided many examples of different uses of diagnostic tests.

A diagnostic test is assumed to give some evaluation of students' capabilities, but it almost always goes beyond that, as there is then some action by students or staff or both. In a few cases the action goes further, such as requesting resources in order to provide support.

Our summary of the purposes found for the use of such tests is:

(1). Predict performance.

(2). Identify at-risk students, with the aim of providing assistance.

(3). Enable students and/or staff to decide on the right level of subject for each student (in cases where there is a choice).

(4). Require students to reach a determined level of skills in order to progress.

(5). Inform teaching staff about the level of knowledge of students, perhaps enabling them to target their teaching to the level of (most) students.

(6). Inform students of any gaps in their knowledge so that they can then address these.

There is one other important purpose, which involves planning and resources. It is hinted at in some of the literature, but not often stated explicitly:

(7). Inform non-mathematicians and decision makers about the level of mathematical knowledge of students.

These purposes can be found in the papers cited in the previous section. For example, Purposes 1, 2, 5 and 6 in Kavanagh et al. (2009), Purpose 3 in Ngo and Melguizo (2016), Purpose 4 in Espey (1997) and Purpose 7 in Silva et al. (2016).

These purposes are not disjoint. For example, Purposes (2) and (6) are similar, however Purpose (6) has the focus on the student taking action, whereas Purpose (2) has the focus on the institution acting. Purposes (1) and (2) overlap as prediction of performance can be used to determine who is at risk of failing.

Though Purpose (7) is often not explicitly stated, it can be important. Poor mathematics background can lead to higher failure rates, higher attrition and lower eventual attainment. Ngo and Melguizo (2016) note the costs of misplacement and remediation. Thus decision makers and academics should be informed if students lack mathematical skills as this can have negative consequences and so should affect decisions on enrolment, support, curriculum and student advice.

As noted (LTSN mathsTEAM, 2003), a diagnostic test by itself has limited value. It is usually appropriate to follow a test by some action. If the reasons for a diagnostic test include some of Purposes (2), (3), (4) and (6) then the appropriate action is clear; for all but Purpose (3) this includes providing students with resources and support; for Purpose (7) action could include requesting resources for the provision of support (Silva, et al., 2016).

For Purposes (1)–(4) and (6) it is desirable to have every student sit the test, and for Purposes (5) and (7) a high proportion taking the test is needed. Therefore making a diagnostic test compulsory is clearly beneficial, though it could be hard to enforce.

At Western Sydney University (WSU) we found that encouraging students to use resources, or to do extra work to fill gaps in knowledge, was often unsuccessful. We felt it necessary to require students to build skills in order to progress. In the next section we present two case studies of the use of diagnostic tests in which the circumstances and actions for redressing gaps are different. For each case study three questions are viewed through the lens of the preceding discussion:

- What was or were the purpose(s) of the test?
- What actions were taken as a result of test?
- Were the actions successful in addressing the purpose(s) of the test?

3. Two tests

Western Sydney University (WSU) is a large multi-campus university with over 44,000 students in NSW, Australia.

Over the last two decades the proportion of students taking low level, or no, mathematics in the last two years of secondary school has been increasing (Nicholas & Rylands, 2015). In 2017 about two-thirds of students who completed secondary school in Australia and who were enrolled in WSU first-year mathematics subjects had inadequate mathematics backgrounds for their studies. It is therefore not surprising that academics perceive a drop in performance in first-year mathematics and find that many of our students lack very basic mathematical skills.

This situation has inspired some academics to run mathematics diagnostic tests. We report here on two tests which are administered in two mathematics subjects at or near the start of a semester in first year, and for which follow up actions have been monitored. The primary purpose of one test is to decide in which mathematics subject to place students; this will be referred to here as the placement test. The primary purpose of the other test is to inform students of where any weaknesses lie and to address these in order to progress; this will be called the diagnostic test.

3.1 A test for industrial design students

Industrial design students at WSU are often very poorly prepared mathematically, so students sit a diagnostic test early in their first-year basic mathematics subject. The test has three aims:

• to highlight to students any gaps in their basic mathematics, Purpose (6);

• to ensure that students largely address any gaps by the end of the semester, Purpose (4);

• as evidence for non-mathematics academics of the level of students' skills, Purpose (7).

In 2016 approximately 70 first-year industrial design students enrolled in their mathematics subject. As it can take considerable time to gain missing skills, and as students usually focus on assessment tasks during semester, students were required to reach a threshold of 11 out of 14 in the diagnostic test in order to pass the subject. To keep students focussed on improving their basic skills until they reached the threshold, six attempts throughout the semester were allowed. A slightly different test was used for each attempt. The test contributed 10 per cent to the final mark for the subject. Students who did not reach the threshold during the semester failed the subject, regardless of their total mark.

Before the first test was run students were given a sample test in class which they marked themselves.

The diagnostic test was a 14 question paper-based short answer test for which students were given 12 minutes. Almost all students gave an answer for each question, but for a few students the time allowed for the test was too short, so the following year the time was increased to one minute per question. Topics covered were basic fraction calculations, order of operations, multiplication and division by powers of 10, conversion of units, percentages, decimals and basic algebra. The topics were chosen based on common errors, such as errors with basic algebra and very simple calculations with fractions. A learning outcome for the subject included "specify and manipulate quantities, units and scale reliably and accurately" so change of units and proportional reasoning were included.

The use of calculators was not permitted as this made it easier to test basic fraction calculations and order of operations. Where calculations had to be done, the numbers involved were kept small. For example, to test addition of fractions students were asked to find 2/3 + 3/5 in the first test.

Marked tests were returned to students. Students were offered support including workshops to build skills, face-to-face drop-in help and online resources.

3.2 A test for engineering students

A decade ago, students enrolling in engineering were expected to have a reasonable knowledge of calculus of one variable. The subject Mathematics 1 made this assumption and reviewed calculus in the first few weeks before moving on. With students coming to university with lower levels of secondary school mathematics, this subject proved too difficult and a new subject, which we call here Preliminary Mathematics, was introduced in 2010. This new subject revises basic algebra, trigonometry and the theory of functions before introducing differential and integral calculus in the second half of the semester.

Initially, all new students were enrolled into Mathematics 1 and were encouraged to attempt a placement test comprising 50 multiple choice questions on topics from the assumed knowledge (exponents, factorisation, linear equations, surds, exponential and logarithmic equations, trigonometry, functions, graphs, differentiation and integration). The topics included in this test were felt to reflect those topics covered in high school mathematics which students who could expect to be successful in the Mathematics 1 subject should be familiar with before commencing their university studies. Students achieving less than 70 per cent in this test were recommended to switch to Preliminary Mathematics before attempting Mathematics 1. However, this did not have the desired effect as many students were reluctant to move to Preliminary Mathematics.

All new students are now enrolled in the preliminary subject and must obtain at least 70 per cent in the placement test in order to bypass it. Some aspects of this test are discussed in Rylands and Shearman (2018), although from a different point of view.

Students are given 50 minutes for the placement test and are allowed to use a calculator. As this test is essentially an aptitude test in mathematics it was felt that one minute per question should be adequate time for a student who had the required cognitive ability for Mathematics 1. This has meant that students who attempt the test without the necessary capabilities often do not complete all questions in the test. It was decided to allow the use of a calculator for this test as the focus of the test is students' mathematical reasoning capabilities and the numerical calculations are of less importance overall. In addition, the use of calculators in engineering is standard practice. The test is run in university computer laboratories, and is supervised. Running the test online means that marking is automated, so despite the large cohort, students receive their results quickly, enabling them to finalise their enrolment. The test software selects numbers from predetermined ranges for each student, minimising the possibility of cheating. As the aim is to determine students' underlying capabilities, no practice or sample tests are provided before the test is run.

The aim of the placement test is to determine which mathematics subject new engineering students will take; Purpose (3).

4. Outcomes of the tests

In this section the actions and consequences of running the tests are presented, shedding some light on the questions posed earlier about these tests.

4.1 The mathematics diagnostic test for industrial design

When the test was first run 94 per cent of enrolled students completed the test; of these, 55 per cent did not reach the threshold. The test revealed that almost a quarter of students could not change a simple measurement from metres into centimetres and over a third could not evaluate $-6 + 4 \times -5 - 3$. The easiest question, which 88 per cent of students did correctly, was to arrange from smallest to largest 0.702, 0.072, 0.72, 0.0702. The question that students performed most poorly on, with only 41 per cent giving the correct answer, was on simple proportional reasoning. This was usually the case each time the test was run; overall there was no change in what was found to be difficult by students who sat the test many times.

The test was informative for teaching staff, who did not know the extent of students' mathematical gaps, Purpose (5). The information was passed on to decision makers and other academics to increase their understanding of the level of mathematical skills of students, Purpose (7).

Special workshops were run for students who had not reached the threshold, helping them to address problems, Purpose (6). Mathematics support staff discovered that some students did not know where the decimal point belongs in an integer, further addressing Purposes (5) and (7). Discovering such aspects of students' knowledge was an indirect result of testing.

With regards to the purposes for which such tests are run, this test addressed Purposes (4), (5), (6) and (7), with the main reason for the test being Purpose (4).

Of the 61 students who were still enrolled at the end of semester, all had attempted the test at least once. Eleven never reached the threshold; none of these students would have passed the subject, even if they had reached the threshold. Of the 50 students who reached the threshold, only three completed the subject (sat the final exam) with a fail grade. Those who did not reach the threshold all had final marks less than those who did, thus the test partly addressed Purpose (1). That several students who reached the threshold failed the subject raises the question of whether the threshold should be increased.

The test was successful in its primary purpose, Purpose (4), in that a noticeable number of students spent time working on basic skills during the semester until they reached the threshold, with many more students than usual attending support workshops and staff consultations. Not all reached the threshold, but they did noticeably improve. Multiple tests have proved to be motivational in other technical disciplines (Davis, et al., 2005; McLoone, 2007).

The extra work of running and marking a test every two or three weeks was minimal. An advantage of a written test is that staff could read the working and so gain some insight into students' misunderstandings.

4.2 The mathematics placement test for engineering

Students who score less than 70 per cent in the placement test must pass the subject Preliminary Mathematics before attempting Mathematics 1. The introduction of the preliminary subject and the requirement for students to pass the placement test to gain direct entry to Mathematics 1 has resulted in a reduction in the failure rate for Mathematics 1 and a reduction in the number of students who fail this subject multiple times. The failure rate for Mathematics 1 was previously regularly above 40 per cent with occasional peaks at over 50 per cent; it is now typically about 30 per cent. Thus the test has to some extent fulfilled its aim, Purpose (3).

The failure rate for Preliminary Mathematics remains at about 40 per cent. Of this 40 per cent about half failed at least one other subject in the semester, suggesting that students who are not successful with the placement test often have other gaps in the knowledge required to complete an engineering degree.

There is interest in raising the score needed for entry to Mathematics 1, however, before that is decided, data on placement test scores and grades in Mathematics 1 needs to be analysed.

Two side effects of running the placement test are that staff have found it provides useful information about students' capabilities, Purpose (5), and it is a strong predictor of success in Preliminary Mathematics, Purpose (1).

5. Discussion and conclusion

The literature gives many reasons to use diagnostic tests. The seven purposes listed earlier cover the purposes found in the literature and reported here for conducting diagnostic tests, apart from making students feel "looked after". Information about students and student cohorts gained from running diagnostic tests can be used in a variety of ways to improve learning, as seen from the various purposes of such tests.

The weak mathematical backgrounds of students is a common concern in the literature, with some also mentioning the related problem of increasing mathematical diversity (Kavanagh, et al., 2009; LTSN mathsTEAM, 2003; Wilkes & Burton, 2015). The diagnostic and placement tests discussed here are used to improve basic skills or to direct students to subjects in which they can gain basic skills. A consequence of the tests is a reduction in the mathematical diversity of the cohorts.

Beyond the placement function of the test in engineering (Purpose (3)), students have access to their test results on a question by question basis, which could be used to guide students to resources targeting their problems; there is potential for Purpose (6). Academics teaching Preliminary Mathematics and mathematics support staff have access to the test results by question and student, making it possible to find the areas in which students have gaps. Resource shortages have not allowed this data to be used to its full advantage, improving Purposes (5) and (6). Shepherd et al. (2011) also noted an inability to make full use of information gained from test results.

The main aims of the industrial design test were Purposes (4), (6) and (7), different to those of the engineering test. Students saw where mistakes occurred, and many used resources and workshops provided to improve their test mark. Support staff were guided by the test results in the creation of workshops for these students.

Though the two WSU experiences were different, in each case purposes beyond the original could be served by the testing. Both tests were deemed to be a success, with academics finding that designing, running and marking the tests was time well spent. Success is reported elsewhere (Kavanagh, et al., 2009; Shepherd, et al., 2011), in particular with Purposes (5) and (6), and both planned to run the tests again. In the USA, the continued and entrenched use of such tests demonstrates that they are considered useful (Ngo & Melguizo, 2016).

A common feature of the two WSU tests is that both attempt to enforce action. In the past, engineering students were advised about the right choice of mathematics subject; now they do not get a choice. Failure rates improved when the choice was removed. For the industrial design students, not allowing students to progress until they have reached the threshold motivated students to improve their basic skills, and those who reached the required level mostly passed the subject. Setting a threshold was also found to be successful by Espey (1997) and at QMUL (LTSN mathsTEAM, 2003).

Support and resources for students are important when a test is run for Purposes (2), (4) and/or (6) as these enable students to take action to improve.

The levels required for each of the WSU tests discussed here are in question; perhaps they need to be raised. The difficulty of diagnostic tests or levels required are not discussed much in the literature. Analysis of relevant data is needed so that good decisions are made.

The experience of the two WSU tests and in some of the literature is that useful data can be collected when diagnostic tests are run, and that purposes other than the initial ones can be served, leading to better learning outcomes. There is scope for research and improved learning by using data related to diagnostic tests, both for mathematics and for other disciplines.

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RESEARCH ARTICLE

Gathering and Compiling Mathematical Common Student Errors in e-Assessment Questions with Taxonomical Classification

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Abstract

This article gives an overview of the interactive book called '*Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)*' which has been produced as a result of the Common Student Errors Project (CSE Project) set at the University of the West of England, (UWE Bristol). The process of creating this CSE Book is discussed in this article, namely, through the systematic collection and compilation of CSEs, and classification of them taxonomically according to a taxonomy presented in existing literature by examining first year Engineering Mathematics students' rough answer scripts, and e-Assessment-stored data. We believe that the CSEs presented in the CSE book would be useful for mathematics teachers when providing feedback to students to correct CSEs. Further, institutions can utilise it in the future development of teaching and support resources to ensure that these CSEs will be addressed to help students to acquire better understanding of mathematics. Moreover, mathematics learners can try these questions online by using the respective hyperlinks given in the CSE Book. If any of the identified CSEs are entered in the solution, then enhanced feedback is provided to correct their misconceptions instantly. Currently, the CSE Book is freely available at UWE Bristol's Repository.

Keywords: Mathematical Common Student Errors, Dewis e-Assessment system, Taxonomy

1. Introduction and Background

1.1 Common Student Errors

Students arrive at an incorrect answer when answering a mathematical question due to variety of reasons. The reasons can be listed as random errors, calculation errors or misreading the questions. These errors lead to incorrect answers or loss of accuracy marks. Many of these errors are made by just a few students. However, some of these errors are commonly made by a considerable number of students. These commonly made errors are sometimes referred to as common errors (Rushton, 2014).

Researchers express different opinions about the difference between errors and misconceptions in the literature. For Confrey (1990), the reasons for both errors and misconceptions are the rules and beliefs that students hold. They argue that the difference between errors and misconceptions is that misconceptions are attached to particular theoretical positions. However, Nesher (1987) uses the term misconceptions to describe systematic errors without reference to a theoretical position.

Rees and Barr (1984) use the term '*mal-rule*' to refer to an understandable but incorrect implementation of a process resulting from a student's misconception. For example, a classic *mal-rule* students make is to answer $a^2 + b^2$ when asked to expand $(a + b)^2$. The term '*bug*' is used by

VanLehn (1982) to refer to a systematic error resulting from wrong steps in the calculation procedure. A *Borrow Across-Zero bug* is a systematic error caused by a student having trouble with borrowing, especially in the presence of zeros (VanLehn, 1982). For example, a student answering 98 when asked to calculate $305 - 117$ would be considered as a *Borrow Across - Zero bug*. In the aforementioned calculation, the student skips the step where the zero changed to nine during borrowing across zero (VanLehn, 1982).

Research has been conducted to identify misconceptions in different areas of mathematics. For example, Brown and Burton (1978) investigated bugs (misconceptions) in high school algebra problems, and Swan (1990) focused on the misconceptions that occur in four operations (addition, subtraction, multiplication and division), and in the interpretation of graphs.

Some Mathematics Education research has explored possible causes and effects of certain mathematical misconceptions and the impact that they have on students' future learning (Booth et al., 2014; Confrey, 1990; Fischbein, 1989; Nesher, 1987; Brown and Burton, 1978). After having investigated bugs (misconceptions) in high school algebra problems, Brown and Burton (1978) discussed possible arithmetic bugs which might lead to some specific algebraic bugs. Booth et al., (2014) conducted a study to assess algebraic misconceptions that algebra students make at school. They concluded that students who make specific persistent errors due to underlying misconceptions in arithmetic may need additional intervention since misconceptions are not corrected through typical instruction. They conclude that these additional interventions can be carried out by targeting individual misconceptions or by improving conceptual understanding throughout the algebra course. The findings of Brown and Burton (1978) and then the findings of Booth et al. (2014) hold the same conclusions, that the arithmetic misconceptions held by students affect their algebraic thinking. Further, Booth et al. (2014) state that these arithmetic misconceptions can obstruct their performance and learning of algebra.

There has been recent research into theorising student errors supported by empirical studies in the topics of natural number bias (Obersteiner et al., 2013), visual saliency (Kirshner and Awtry, 2004) and over-generalisation (Knuth et al., 2006). Rushton (2014) conducted a study of common errors in Mathematics made in certain General Certificate of Secondary Education mathematics papers taken by candidates in England, including an internationally available version, as referenced by examiner reports, and errors were catalogued into themes and subthemes. More recently, Ford et al. (2018) developed a taxonomy of errors made by undergraduate mathematics students. In their study they gathered errors by firstly recalling the most obvious errors that occur and secondly by analysing students' exam scripts to categorise them in a taxonomical manner.

1.2 Assessments, e-Assessments and Feedback in Higher Education

Assessment plays a vital role in higher education. It determines the extent of students' skill and knowledge in order to ensure that they have achieved the desired learning outcomes (Stödberg, 2012). Assessment is considered an integral parts of students' learning. Not only does it promote student learning but it also allows them to receive support in order to improve their learning (JISC, 2010). Preparation and marking of traditional paper-based assessments is an expensive and long process and it also requires a significant amount of time and effort by teachers. To mitigate this situation, the use of information technologies to conduct assessment has significantly risen in higher education (Stödberg, 2012; Rolim and Isaias, 2019).

Over the past years, several e-Assessment systems, such as STACK (Sangwin, 2004), Dewis (Gwynllyw and Henderson, 2009), Math e.g. (Greenhow and Kamavi, 2012), and Numbas (Foster at al., 2012) have been developed at several universities in the UK. Easy accessibility and advantages of e-Assessment systems have led mathematics departments in many universities to conduct formative and summative assessments in the form of e-Assessments (Sangwin, 2013).

Properly performing e-Assessments are hugely beneficial for both teachers and students. Some benefits of using e-Assessment are its capability to provide instant and tailored feedback, that it can be accessed in different geographical locations at any time, and that students can undertake online tests several times to improve their learning (Sikurajapathi, Henderson and Gwynllyw, 2020; Gwynllyw and Henderson, 2009).

Dermo (2009) and Gikandi et al (2011) posit that high quality and accurate feedback delivered in a timely manner plays an important role in students' learning. In addition, by reviewing and studying this feedback, students can identify their weakness as well as their strengths in order to achieve continuous improvement in their learning. Gill and Greenhow (2008) conducted a study to find out the effectiveness of e-Assessment feedback and found that students improve their performance by engaging with the feedback provided in e-Assessments. Therefore, Greenhow (2015) suggests that e-Assessments which select questions based on pedagogic principles should be promoted as a learning tool due to its capability of providing effective feedback.

E-Assessments cannot act very flexibly like a human marker when faced with ill-posed or unanticipated student responses (Greenhow 2015). Detecting CSEs on traditional paper-based assignments compared to e-Assessments is more straightforward since the human marker has access to the students' intermediate workings and thus can spot when a CSE has been made. E-Assessment systems cannot easily point out CSEs on student answers since typically few intermediate working steps are submitted. Also, each student attempts a different but equivalent version of the question due to the use of random parameters (Walker et al, 2015).

In their paper, Walker et al (2015) states that an e-Assessment would act more like a human marker, if it could detect and report CSEs, and provide effective and tailored feedback instantly by correcting students' misconceptions. Sikurajapathi, Henderson and Gwynllyw, (2021) developed a method to detect CSEs and to provide tailored feedback in Engineering Mathematics e-Assessment questions. Sikurajapathi, Henderson and Gwynllyw, (2021) then conducted a questionnaire to find out the effectiveness of addressing CSEs in e-assessments through enhanced feedback. The questionnaire findings reveal that the majority of participants had positive feelings towards the CSE enhanced feedback. Students appreciated that the CSE enhanced feedback helped them to correct their misunderstandings and to improve their engineering mathematics learning. The highly positive perception of the enhanced feedback suggests that students find the CSE enhanced feedback valuable and that it helped them to correct conceptual understanding while improving their learning (Sikurajapathi, Henderson and Gwynllyw, 2021).

1.3 Dewis e-Assessment System

Dewis is a fully algorithmic open-source e-Assessment system, which was primarily designed and developed for numerate e-Assessments by a team of Mathematicians, Statisticians and Software Engineers at UWE Bristol (Gwynllyw and Henderson, 2009; Gwynllyw and Henderson, 2012). Dewis supports different question input types such as numerical inputs, matrices, vectors, algebraic expressions, multiple-choice, multiple-selection, graphical input, and computer programs. It has a lossless data collection feature and a number of student-friendly features, such as shutdown recovery and pre-processing checks on student input.

Over the past decade, Dewis has been used very successfully to facilitate both formative and summative e-Assessments across a number of modules, delivered to students in a wide range of fields, e.g. Business, Computer Science, Nursing, Software Engineering, Engineering, Mathematics and Statistics. One aim of the CSE project is to enhance the full potential of Dewis, by developing and using additional features allowing Dewis to detect CSEs and to provide instant tailored feedback.

1.4 The Common Student Errors Project at UWE Bristol

The CSE project at UWE began in 2017 with an aim of developing a technique to detect CSEs and to provide tailored feedback in Dewis e-Assessment questions, used in a first year Engineering Mathematics module (CSE Project at UWE, 2019; Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021). We started the project with the aim of answering the following research questions:

- What CSEs do first year Engineering Mathematics students make in e-Assessment questions?
- How to detect CSEs and improve Dewis feedback to address these CSEs?

There are several benefits to answering these research questions. Even though this research has been done in a particular context using the Dewis e-Assessment system, the research outcomes contribute to the knowledge to inform more general practice in assessment and learning. For example, the collection of mathematical CSEs collected during this research is not only beneficial for first year Engineering mathematics students and lecturers, but also it is equally beneficial for secondary, and first year university level mathematics students and teachers. The CSE collection presented in Sikurajapathi, Henderson and Gwynllyw (2022) can be used to correct students' mathematical misconceptions either in hand-written assessments or e-assessment questions.

Further, this CSE detecting technique will be beneficial to several disciplines and organisations that either use Dewis or any other e-assessment system which has features to give dynamic feedback based on a student answer. The new knowledge raised from this research can be used in any e-assessment system so that it emulates a human marker to provide instant enhanced feedback highlighting possible CSEs. This will help students to correct their mathematical misconceptions. Also, teachers can use the findings to identify areas in which more help is needed in student learning. Integrating the research outcomes from the CSE project into other e-assessment systems will be beneficial to generations to come (Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021; Sikurajapathi, Henderson and Gwynllyw, 2022).

The CSE Project involves five stages (Stage One: Data (CSEs) Collection; Stage Two: CSE code Development; Stage Three: CSE code Trial Phase; Stage Four: Students' Perceptions on CSE Feedback and Stage Five: Impact of CSE Project). Detailed information about these five stages and other findings can be found in CSE Project at UWE Bristol (2019), Sikurajapathi, Henderson and Gwynllyw (2020) and Sikurajapathi, Henderson and Gwynllyw (2021).

2. Creating Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)

2.1. Gathering Mathematical CSEs in in e-Assessments Questions

The main aim of the CSE Project at UWE Bristol was to identify CSEs made in First Year Engineering Mathematics e-Assessment questions. The CSEs presented in the CSE Book were collected by examining the 2017-2018 and 2018-2019 e-examination data on the Dewis e-Assessment system and from students' rough work scripts. These e-examinations were run using the Dewis e-Assessment system and were held under controlled conditions. The e-examinations were held in two sessions (morning and afternoon) to mitigate logistic issues. In each session, all of the students received the same, fixed parameter questions. During the e-examination, students were given booklets to use for their rough work. These booklets were used by students to work through the mathematical questions before submitting their final answers on Dewis.

All of the CSEs that students made are documented in the CSE Book, regardless of whether they are mal-rules, bugs, slips, misconceptions, systematic errors etc. The reason for this is that all of these CSEs can be useful for educators, institutions, assessment makers, and most importantly for mathematics learners. Altogether 65 CSEs were identified in the following different topics areas of Engineering Mathematics: Algebra, Unit-step functions, Wave forms, Trigonometric functions, Differentiation, Implicit differentiation, Partial differentiation, Mean Value Theorem, Complex numbers, Geometric series, Maclaurin Expansion, Centre of Mass, Integration by parts, Volume of revolution and Dimensions.

This CSE Book (Sikurajapathi, Henderson and Gwynllyw, 2022) can be freely access at UWE Bristol's Repository on Public URL: https://uwe-repository.worktribe.com/output/9303961

2.2. Compiling Mathematical Common Student Errors in e-Assessment Questions with Taxonomical Classification

All of the CSEs found in the course of the CSE project are documented in a systematic order in the CSE book together with their mathematical taxonomy coding. Here we adapted the general taxonomy proposed by Ford et al. (2018) to select and categorise only those CSEs which are relevant to e-assessment.

The theoretical study of classification, including its bases, principles, procedures and rules is called a taxonomy (Ford et al., 2018; Simpson, 1961, p.11). The entities in a successful taxonomy can be verifiable by observation and will offer both an appropriate and suitable class for each entity (Ford et al., 2018; Bailey, 1994, p.3). The taxonomy of cognitive mechanisms and the phenomenological taxonomy can be considered as the two main styles that can be used to categorise mathematical errors (Senders and Moray, 1991, Ford et al., 2018).

The taxonomy introduced by Ford et al. (2018) was developed to categorise the errors which undergraduate mathematics students make. Ford et al. (2018) identified six main error categories by firstly recalling obvious mathematical errors that occur among mathematics undergraduates and secondly by analysing a selection of students' paper-based exam scripts from first year undergraduate mathematics courses. These main categories were named as Errors of slips of action (S), Errors of understanding (U), Errors in choice of method (CM), Errors in the use of a method (UM), Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C). Here we sought to use the same Main Categories, Codes and Errors given in the taxonomy introduced by Ford et al. (2018) to categorise mathematical CSEs in the e-Assessment questions.

The CSEs that we have found during the CSE project only fall into four of the error categories (S, U, CM and UM) from the Ford et al. (2018) taxonomy. Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C) were not found among the CSEs made by the Engineering Mathematics students, due to the nature of the questions asked and the nature of the system used to deliver the questions. None of the e-Assessment questions delivered by Dewis involve mathematical theorems and proofs and hence Errors related to proof (P) were not viable in this CSE collection. Further, the e-examination did not contain questions that required student's communication of their mathematical solutions, correct use of notation or labelling and qualitative judgements on clarity of expression. Therefore, errors in student's communication of their mathematical solutions (C) were not found in this CSE collection. Further, a few of the CSEs found fall into two categories due to the mix of misconceptions made by the students as they arrived at their incorrect answer.

Under the category Errors of slip of action (S), three main errors, namely copying error, careless errors on simple calculations, and incorrect algebraic manipulation were identified. A total of 13 out of 65 CSEs were found to fall into the Errors of slip of action category (S).

Seven main errors were identified under the Errors of understanding (U) category, such as confusing different mathematical structures, incorrect argument, lack of consideration of potential indeterminate forms, proposed solution is not viable, definition/method/theorem not recalled correctly, partial solution given and Incorrect assumptions. In total 43 CSEs are in the Errors of understanding category.

Only one main error was found in each of the Errors in choice of method (CM) and Errors in use of method (UM) categories. Five CSEs were grouped into the main error of applying an inappropriate formula/method/theorem in CM. There were 9 CSEs which fell into Error in use of an appropriate definition/method/theorem in the UM category. Table 1 shows how we categorised the CSEs we found related to e-Assessment questions into Main Categories, relevant Codes and Errors using the taxonomy introduced in Ford et al. (2018) together with examples from an e-Assessment context.

Table 1: Taxonomy of Mathematical Common Student Errors in e-Assessments

Main Category	**Code**	**Error**	**Examples**
Slip of action	S1	Copying error	Incorrect copying of the question
			Mistake copying/ submitting answer into e-assessment
			Incorrect interpretation of the question
	S2	Careless errors on simple calculations	Overlooking negative signs
			Omission of denominator
	S3	Incorrect algebraic manipulation	Incorrect division of two complex numbers
			Sum of product is split as a product of two sums
			Incorrect handling of powers

Errors of understanding	U1	Confusing different mathematical structures	Confusing the structure of completing the square and the quadratic equation
			Stating that a unit step function is a number
	U2	Incorrect argument	Incorrectly assuming the derivative of the product of two functions is equal to the product of the individual derivatives
			Taking the integration of the product of two functions as the product of individual integrals
	U3	Lack of consideration of potential indeterminate forms	Taking the square of a negative number to be negative
	U4	Proposed solution is not viable	Angle is not within the given range
	U5	Definition/method/ theorem not recalled correctly	Method of completing the square is not recalled correctly
			Definition of waveform properties not recalled correctly
			Method of differentiating a standard function is not recalled correctly
			Method of solving trigonometry equation is not recalled correctly
			Chain rule is not recalled correctly
			Method of Partial differentiation not recalled correctly
			Method of differentiating implicit functions is not recalled correctly
			Mean value theorem is not recalled correctly
			Method of calculating the argument of a complex number is not recalled correctly
			Binomial theorem is incorrectly followed
			Definition of Centre of Mass is not recalled correctly

			Method of finding the principal value of the argument of a complex number is not recalled correctly
			Method of integrating not recalled correctly
			Definition of volume of revolution is not recalled correctly
	U6	Partial solution given	Correct workings but unfinished solution
	U7	Incorrect assumptions	Incorrect assumptions on the mean value theorem
			Taking dimension of velocity is $[v] = [MT^{-1}]$
Errors in choice of method	CM1	Applying an inappropriate formula/ method/ theorem	Uses a method which is not relevant in the situation
			Uses a formula which is not relevant in the situation
Errors in use of method	UM2	Error in use of an appropriate definition/ method/ theorem	Error in the use of the chain rule
			Error in use of partial differentiation method
			Incorrect units applied
			Method finding the volume of revolution is incorrectly followed

2.3. Guide to the CSE Recording Template

Each CSE found to date has been recorded using the template as shown in Table 2 below. The template contains seven areas and each area and its contents are described in detail below.

① The link to the online Dewis e-assessment question is available here. The reader may access the online question by clicking the Question hyper-link. By attempting the question and answering with a relevant CSE response, it is possible to see how Dewis detects the CSE and provides instant tailored feedback to address the CSE made in the solution.

② In this area, a screenshot of the Dewis question is given.

③ The correct solution to the question is presented in brief here.

④ The taxonomy code of the CSE, which is presented in ⑤ **,** is given here.

⑤ A sample of the CSE and the incorrect answer(s) that led from it is presented here. At the top of this area, the CSE error is summarised by a statement which is presented in red text. Then the detailed steps of the exact way the CSE is made and the solution as written by students in their rough work booklets is presented. We use tilde (~) on the CSE answer to differentiate it from the correct answer. For example, in Table 2, the CSE answer for this question is denoted as, $\tilde{f}(2) = 55$ in red text.

⑥ In this section, the number of CSE answers made, the total incorrect answers made in the question and the CSE percentage for each year are presented as No. of CSEs /No. incorrect answers **(CSE %)**. For example, in Table 2, in the 2017-18 exam, this particular CSE was made by 35 out of the 86 students who gave an incorrect answer to this question; therefore, the CSE percentage is **41%**. This data is presented in this area as 35/86 (**41%**). Similarly, the data for 2018-19 is presented as 32/100 (**32%**).

⑦ The exam year that data was collected from is presented here. Table 2 shows that 35/86 (**41%**) and 32/100 (**32%**) presented in ⑥ relate to the years 2017-18 and 2018-19 presented in ⑦ respectively.

Table 2: CSE Recording Template

<table>
<tr><td colspan="4"><u>Question</u> 1</td></tr>
<tr><td colspan="4">The function $f(t) = 7u(t+5) - 3u(t-4)$
where $u(t)$ represents the unit step function.
Calculate the value of $f(2)$.
Enter $f(2)$: 2</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$$f(t) = 7u(t+5) - 3u(t-4)$$ $$\begin{aligned} f(2) &= 7u(2+5) - 3u(2-4) \\ &= 7u(7) - 3u(-2) \\ &= 7 \times 1 - 3 \times 0 \\ f(2) &= 7 \end{aligned}$$ 3</td></tr>
<tr><td>CSE 1 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>U1 4</td></tr>
<tr><td colspan="4">Answer was derived by assuming $u = 1$ and not a function.
$$f(t) = 7u(t+5) - 3u(t-4)$$ $$f(2) = 7u(7) - 3u(-2)$$ $$\tilde{f}(2) = 7(7)u - 3(-2)u$$ $$\tilde{f}(2) = 49u + 6u$$ $$\tilde{f}(2) = 55u \quad \text{since } u = 1$$ $$\tilde{f}(2) = 55$$ 5</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>35/86 (41%) 6
32/100 (32%)</td><td>Date collected</td><td>2017-18 7
2018-19</td></tr>
</table>

3. Common Student Error Examples

In this section we present examples of CSEs in each taxonomical category (Slip of action, Errors of understanding, Errors in choice of method, and Errors in use of method). These and the rest of the CSEs we found in the CSE Project can be found in UWE Bristol's Repository (Sikurajapathi, Henderson and Gwynllyw, 2022).

3.1. Common Student Errors due to Slip of Action

Table 3 shows a CSE related to a question in Algebra (Completing the Square) (see Section 2.1.1. Sikurajapathi, Henderson and Gwynllyw, (2022)). Students' answer scripts indicated that even though students had solved the question correctly, they submitted incorrect answers for b which corresponded to the negative of the correct value of b. Therefore, this CSE can be considered as copying error when submitting answer into e-assessment. In 2017-2018, 28 students, out of the 56 who answered this question incorrectly (50%) made this CSE. In 2018-2019, 33 students from 57 who answered this question incorrectly (58%) made the same mistake.

Table 3: CSE in Algebra (Completing the Square) Question due to Slip of action in algebra

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">The expression
$t^2 - 12t + 40$
can be expressed in the form:
$a(t-b)^2 + c$
where a, b and c are constants.
Calculate the values of these constants - note that all these solutions are integers:
Enter the value of a [ ]
Enter the value of b [ ]
Enter the value of c [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$$\begin{aligned} t^2 - 12t + 40 &= (t-6)^2 - 36 + 40 \\ &= (t-6)^2 + 4 \end{aligned}$$
$$a = 1, \quad b = 6 \text{ and } c = 4$$</td></tr>
<tr><td colspan="2">CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>S1</td></tr>
<tr><td colspan="4">Give answer $\tilde{b}$ which corresponds to the negative of the correct value of b.
$$\begin{aligned} t^2 - 12t + 40 &= (t-6)^2 - 36 + 40 \\ &= (t-6)^2 + 4 \end{aligned}$$
$$\tilde{b} = -6 \text{ and } c = 4$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>28/56 (50%)
33/57 (58%)</td><td>Date collected</td><td>2017-18
2018-19</td></tr>
</table>

3.2 Common Student Errors due to Errors of Understanding

Table 4 shows a CSE related to a question on complex numbers (rectangular form) (see Section 3.3.1. Sikurajapathi, Henderson and Gwynllyw, (2022)) Students' answer scripts indicated that the square of a negative number was taken to be negative. Therefore, this CSE can be considered as lack of consideration of potential indeterminate forms. In 2017-2018, 40 students, out of the 57 who answered this question incorrectly **(70%)** triggered this CSE.

Table 4: CSE in Complex Number (Rectangular Form) Question due to Error of understanding

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Find the modulus $|z|$ of the complex number $z = -2 + 5j$, correct to <u>two</u> decimal places.

Enter $|z|$ correct to 2 decimal places: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$z = -2 + 5j$
$|z| = \sqrt{(-2)^2 + 5^2}$
$= \sqrt{4 + 25}$
$= \sqrt{29}$
$|z| = 5.39$</td></tr>
<tr><td colspan="2">CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>U3</td></tr>
<tr><td colspan="4">Taking $(-n)^2 = -n^2$

$z = -2 + 5j$
$|z| = \sqrt{(-2)^2 + 5^2}$
$\widetilde{|z|} = \sqrt{-4 + 25}$
$= \sqrt{21}$
$\widetilde{|z|} = 4.58$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>40/57(70%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

3.3. Common Student Errors due to Errors in Choice of Method

Table 5 shows a CSE related to a question on infinite geometric series (see Section 4.1.2. Sikurajapathi, Henderson and Gwynllyw, (2022)). Students' answer scripts indicated that 34 students out of 67 who answered this question incorrect (51%) just summed the first four terms instead of using the formula to find the sum of the infinite series. Therefore, this CSE can be considered as applying an inappropriate formula in Error in Choice of Method.

Table 5: CSE in Infinite Geometric Series Question due to Errors in Choice of Method

Question			
Consider the following geometic series, S, where: $S = 2 + 2(0.7) + 2(0.7)^2 + 2(0.7)^3 \cdots.$ Write down the first term, a and the common ratio, r in the boxes below. Enter a: [ ] Enter r: [ ] Hence calculate the sum, S and enter your result in the box below. Enter S (to three decimal places) here: [ ]			
Correct Solution			
The first term $a = 2$. The common ratio $r = 0.7$ The sum of an infinite series (S) exists, provided $\lvert r \rvert < 1$ $S = \frac{a}{1-r} = 6.667$			
CSE 1 related to this question	**CSE Taxonomy Code:**	**CM1**	
Finding the sum of first four terms instead of the sum of the infinite series. $\tilde{S} = \frac{a(1-r^n)}{1-r}$ $\tilde{S} = \frac{2(1-0.7^4)}{1-0.7}$ $\tilde{S} = 5.066$			
No. of CSEs /No. incorrect answers (CSE %)	34/67**(51%)**	**Date collected**	**2017-18**

3.4. Common Student Errors due to Errors in Use of Method

Table 6 shows a CSE related to differentiating $f(x) = cos^4(3x)$ (See Section 5.1.2. Sikurajapathi, Henderson and Gwynllyw, (2022)). 22 students out of 73 **(30%)** incorrectly answered that the differentiation of $f(x)$ is $-12\sin^3(3x)$ due to an error in the use of the Chain Rule. Therefore, this CSE can be considered as an error in use of an appropriate method.

Table 6: CSE in Differentiation (Chain Rule) Question due to Errors in Use of method

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Select the most appropriate method to use in order to find the derivative of $f(x) = \cos^4(3x)$.
Select
Hence find $\frac{df}{dx}$ as a function of x.
Enter the answer as a function of x:</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$f(x) = cos^4(3x)$
$f'(x) = -4 \times \cos^3(3x) \times \sin(3x) \times 3$
$f'(x) = -12\sin(3x)\cos^3(3x)$</td></tr>
<tr><td>CSE 2 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>UM2</td></tr>
<tr><td colspan="4">Taking $\frac{d}{dx}(cos^n(ax)) = -a \times sin^{n-1}(ax) \times a = -a^2 sin^{n-1}(ax)$
$f(x) = cos^4(3x)$
$\tilde{f}'(x) = -4 \times \sin^3(3x) \times 3$
$\tilde{f}'(x) = -12\sin^3(3x)$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>22/73(30%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

4. Discussion, conclusion and future work

This article presents an overview of the CSE Book created by collecting and compiling CSEs systematically by examining First Year Engineering Mathematics students' rough answer scripts, and Dewis e-Assessment-stored data. All of the CSEs found in this process have been categorised taxonomically. One of the special features of this book is that it provides hyperlinks to each question on the Dewis e-Assessment system in order to facilitate the reader to try these questions online. If any of the identified CSEs are submitted as answers, then enhanced feedback will be provided, which aims to correct any misconceptions in a timely manner.

The information in this book may be used to inform teachers so that they can provide students with a better understanding of the mathematical skills and knowledge while teaching the subject. It may also be useful for institutions as they can utilise it in the future development of teaching materials to ensure that these CSEs will be addressed. Further, the content of this book can be used to develop support materials and resources to address CSEs which will help students to acquire better understanding of mathematics. In addition, students who learn mathematics at university level or in secondary school can refer to this booklet to address their misconceptions and can try the Dewis questions several times. Since, in each attempt, Dewis produces questions with random parameters, student can use this facility to correct their misconceptions by practicing the same question but with different parameters.

We anticipate that this book will be useful to identify and address some misconceptions that students have in mathematics. We plan to continue with this research and will update the book if we find new CSEs in the future. Currently, the CSE Book is freely available at UWE Bristol's Repository.

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CASE STUDY

Can the same statistics module be used for service teaching by tailoring the support based on the student's chosen qualification?

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Abstract

At the Open University in the UK, students taking undergraduate degree qualifications in areas such as Data Science and Economics are now the largest cohort of students on a second-year undergraduate statistics module, originally written for specialist mathematics and statistics students. This paper outlines an ongoing project to identify how more targeted support could be provided to students who are studying non-mathematics and statistics qualifications. This has involved engaging all the tutors who provided support in the project to create a new way of adapting their teaching styles and tutorial content. By grouping students who had similar qualification goals together and linking these groups to individual tutors, the new way of working created an atmosphere where students feel able to share their own misunderstandings and see how statistics is useful within their chosen qualifications.

Keywords: Service teaching, student support, action research

1. Introduction

1.1 Teaching statistics to non-mathematics and statistics students

Many university qualifications include a proportion of statistical training for students who are studying non-mathematics and statistics undergraduate degrees. How students are taught statistics can differ depending on the qualification and the university in which they are studying. It can range from a few afternoons of statistical training, modules which focus on statistical learning through to subject-focussed modules with elements of statistical learning embedded within them. It is also the case that many of these students show a lack of confidence about mathematics and statistics which can be problematic for their studies (Hodgen et al, 2014).

At the Open University (OU) many of the statistics modules serve multiple qualifications, with students on a single module studying a wide range of different qualifications. To meet the differing needs of these students the module material is usually written with a range of examples and scenarios to which all students can relate (Hilliam and Vines, 2021). However, there are some modules which are written to serve students on a specific qualification, this was the case for the module *Analysing Data (M248)*; the subject of this case study. This module was written for students studying specialist Mathematics and Statistics (M&S) qualifications. As new qualifications have been developed, one example being the Data Science degree, existing modules have been used to fulfil learning outcomes for these qualifications. Whilst the fundamental statistical methods which students need are the same regardless of their qualification, students are more likely to understand the relevance of the material if it is embedded within their chosen qualification subject area (Mustafa, 1996; Tishkovskaya and Lancaster, 2012 and MacDougall et al, 2020). Of course, due to resourcing issues it is not always possible to create multiple versions of the same module to be used across multiple different qualifications, and indeed this was not possible for *M248.* Nor was it possible, given resource and time constraints, to re-write the module to include examples specifically chosen to appeal to multiple audiences. Therefore, through the action research outlined in this paper, a different approach has been taken which focussed on providing dedicated qualification-based support for groupings of students on similar qualifications (qualification-based groups).

1.2 Supporting students at the OU

Academic support is provided at the OU by a network of about 6,000 tutors officially designated Associate Lecturers (ALs). Several ALs are contracted to each module and each support a group of students, usually 20, through their study of that individual module. This enables students to engage with distance learning through a combination of high-quality teaching material (both printed and online) and receive correspondence tuition from their designated AL. The ALs provide tutorials (online versions are usually recorded for later viewing), correspondence tuition (via feedback on continuous assessment) and one-to-one academic support via email amd telephone. For historical reasons, mainly to ensure UK wide coverage of face-to-face tutorials, ALs usually support a group of 20 students within a given geographical area. Online tutorials have been provided by these ALs since the early 2000s, and up until March 2020 a combination of face-to-face and online tutorials were provided. The attendance at face-to-face tutorials on M248 had been decreasing prior to March 2020 when the Covid19 pandemic forced all tutorials online. Decreasing the number of face-to-face tutorials and replacing these with online tutorials, meant there were a much larger number of online tutorials which students could attend. This provided an opportunity to think about the best way of using the increased number of online tutorial hours and linking these to specific qualifications. The case study in this paper outlines some of the steps taken to move from geographically based support to a qualification-based support which included a range of different types of online tutorials.

1.3 M248 students

Alongside the lack of attendance at face-to-face tutorials there was a growing realisation that the cohort of students who studied *M248* was changing. Mathematics and statistics modules at the OU are reviewed every year. Whilst the modules are usually refreshed, re-written or replaced every 5-10 years, the content is unlikely to substantially change unless there is a compelling business case to do so. The last re-write of *M248* was completed for students to start studying in Oct 2017. It was expected that the material would remain substantially unchanged for at least 10 years. At that time the module was written primarily for students studying for qualifications in mathematics and statistics with the module split into 13 units:

- Unit 1: Exploring and interpreting data
- Unit 2: Modelling variation
- Unit 3: Models for discrete data
- Unit 4: Population means and variances
- Unit 5: Events occurring at random and population quantiles
- Unit 6: Normal distributions
- Unit 7: Point estimation
- Unit 8: Interval estimation
- Unit 9: Testing hypotheses
- Unit 10: Nonparametric and goodness-of-fit tests
- Unit 11: Regression
- Unit 12: Transformations and the modelling process
- Unit 13: Applications.

Each unit covers about 16-18 hours of student learning which students are expected to complete in roughly 2 weeks. Unit 13 is a consolidation unit and contains no new material. It is assumed that students possess a basic knowledge of calculus and a reasonable level of mathematical maturity with the ability to manipulate mathematical equations, which is true for all students studying M&S qualifications.

Table 1 shows the distribution of which qualifications *M248* students are studying. In Figure 1, to make comparisons easier, the qualifications have been split into four main groups: Mathematics & Statistics (M&S), Economics, Data Science & Computing (Data Science) and Other qualifications and standalone study (Open). In 2017 and 2018 over 45% of students on M248 were studying for M&S qualifications (Table 1 and Figure 1). In addition, there were a reasonable number of students studying various other qualifications including the Open degree (where students can choose any modules to study for their degree providing the total number of credits at each level is satisfied). Students can also study individual modules without linking them to any qualification, known as standalone study, there have always been students who studied OU statistics modules in this way as part of their ongoing continual professional development.

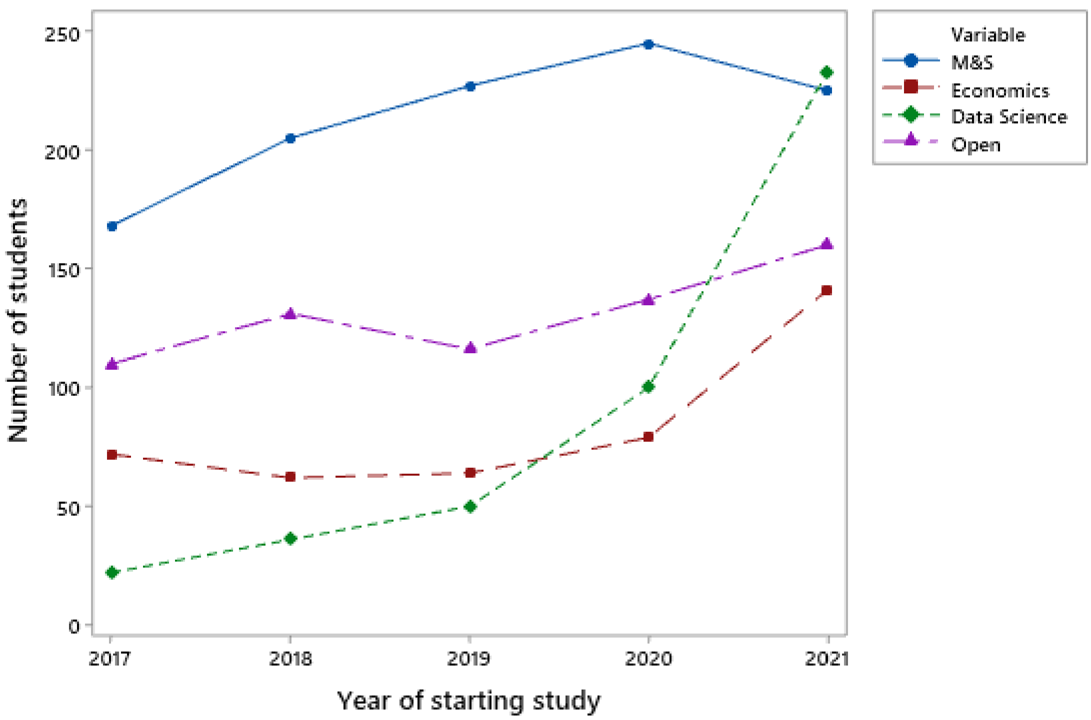

Figure 1. Number of students on qualification types students who have studied M248 between 2017 and 2021.

		Year in which study of M248 commenced (Oct)				
Qualification-based group	**Qualification studied**	2017	2018	2019	2020	2021
M&S	BSc (Hons) Mathematics	61	111	108	137	113
	BSc (Hons) Mathematics & Statistics	107	94	119	108	112
Economics	BSc (Hons) Economics & mathematical sciences	71	59	53	53	66
	BSc (Hons) Economics	1	3	11	26	75
Data Science	BSc (Hons) Computing & IT with statistics	17	32	32	33	33
	BSc (Hons) Data Science	5	4	18	67	200
Open	Standalone module study	23	31	29	36	30
	BSc (Hons) Open	61	49	38	43	40
	BSc (Hons) Combined STEM	12	28	33	36	45
	Other qualifications	14	23	16	22	45
Total		372	434	457	561	759

Table 1. Distribution of students studying *M248* between 2017 and 2021 split by qualification.

In 2019 two new degrees were introduced at the OU, BSc (Hons) Economics and BSc (Hons) Data Science, both of which include *M248* as a compulsory module. This led to an increase in students on M248 and changed the distribution of the type of student studying on *M248* (Table 1). This change in the student distribution meant that by 2021 *M248* was primarily a service module for non-M&S students, even though the module material was very much written for mathematically competent students.

The increase since 2019 in the non-M&S qualifications can be clearly seen in Figure 1 (note that it is possible for students to retrospectively link a module to a qualification which explains the Data Science and Economics student numbers prior to 2019). These four qualification-based groups formed the basis for deciding how to tailor support for students dependent on their qualification goal. This paper outlines the steps taken in the action research starting in 2018 to address this issue.

2. Methodology

In 2018 it was evident that students on non-M&S qualifications tended to have a lower pass rate than their M&S counterparts. To address this several interventions were put in place over a period of three years. An action research approach was taken. Action research is practitioner research and follows a cyclical pattern of identifying a problem and designing an intervention, acting on the intervention, evaluating the result and then modifying and acting all over again (McNiff and Whitehead, 2005). At each phase of the study different methodology was used to answer the new question or questions generated by the previous phase of the action research. Figure 2 shows the timeline of the project and a summary of the methodology used at each phase. The interventions, evaluations and results for each phase are outlined in Section 3.

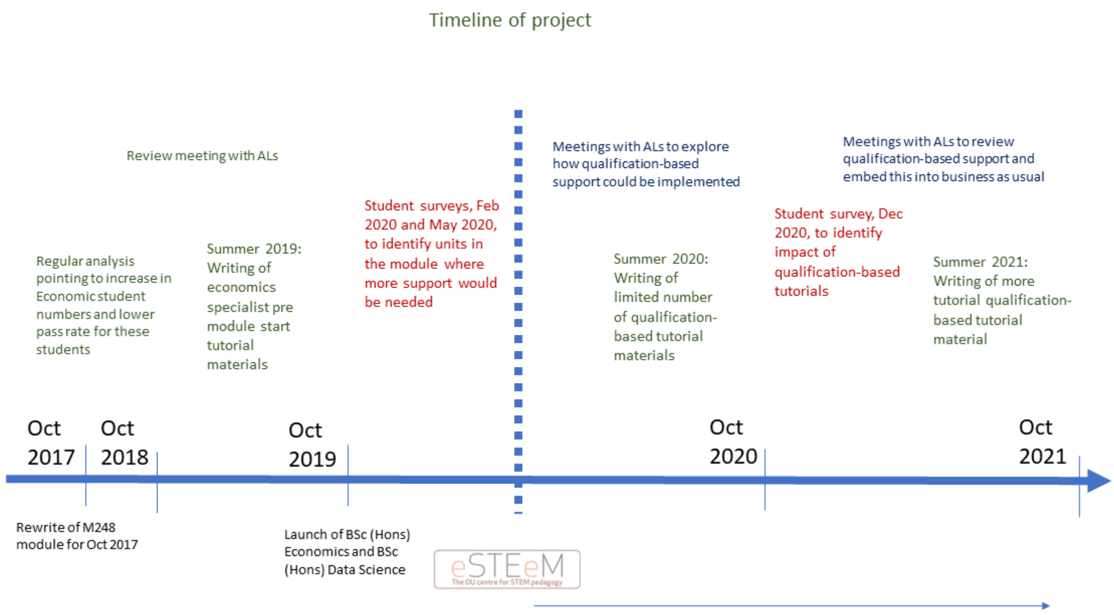

Figure 2. Timeline of the qualification-based support project on M248.

To facilitate the interventions, it was necessary to ensure the ALs were involved in the project. In 2017 there were 18 ALs on *M248* who were essentially recruited with the aim of tutoring M&S students. By 2021 this had grown to 40 ALs and because of this project some of ALs were recruited specifically to tutor explicit groups of non-M&S students. The transformation in focus for these ALs has been considerable and a large amount of work has gone into ensuring this change has been a team effort. The first stage was to ensure the ALs understood the reason change was needed, initially due to the differential pass rate and later due to the large increase of non-M&S students. März and Ketch (2003) highlight the need for an individual to understand the problem and the social process needed for the acceptance of change. The emphasis on shared understanding was exactly the approach taken with the ALs on *M248*, many meetings were held to talk through the issues and explore possible solutions and barriers.

3. Results

3.1. Phase 1: Economics students Oct 2019 – June 2020

In 2017 and 2018 Economics students who completed the module tended to have a lower pass rate (92% in 2017 and 84% in 2018), compared to the BSc (hons) Mathematics (98% in 2017 and 99% in 2018) and BSc (Hons) Mathematics and Statistics students (93% in 2017 and 95% in 2018). Whilst this difference was not large the pass rate based on students who started the module was much more stark as students on the M&S qualifications have a greater module retention rate. One contributing factor was thought to be the limited prior knowledge of calculus possessed by non-M&S students. Whilst the calculus in *M248* is not difficult, and the ideas are presented from scratch, the concepts are challenging if this is a student's first exposure to calculus.

To address this issue study materials, covering the basics of calculus and explaining how these concepts were used by economists, were written by an *M248* AL who was an economist. The aim was for students to use this material during the summer prior to the module starting. Like most OU modules, students' study *M248* from October to the following June, enabling the summer months to be used to revise and refresh concepts. In mathematics and statistics all this material is embedded into one website, the Mathematics and Statistics Study Site, which students use as a one-stop-shop throughout their study (Hilliam et al, 2021). The study materials for the economists were embedded in the Mathematics and Statistics Study Site in May 2019 ready for students to use prior to starting *M248* in Oct 2019. The material included a handbook of techniques and supplementary example questions which were used in online tutorials for the economics students between June and September 2019. These tutorials were recorded for students who could not attend the sessions synchronously. Feedback from students, was generally positive with comments such as:

"I found the recording helpful to do a general refresher of calculus from a different angle."

It seemed that the provision of dedicated study materials may have helped to improve outcomes for the Economics students although it did not completely solve the problem, as economics students still had a lower pass rate 95% compared to the 98% for the BSc (Hons) Mathematics and Statistics students. However qualitative evaluation of the economics support suggested that students appreciated having dedicated support which linked the statistics to their qualification. This led to exploring whether it would be possible to provide dedicated tutorials for the four qualification-based groups shown in Figure 1.

3.2. Phase 2: Pilot of qualification-based tutorials for Oct 2020 – June 2021

Qualitative feedback from the pre-module economics sessions suggested these had created a feeling of community amongst the economics students. To attempt to recreate this same community feel for all the four qualification-based groups during the module bespoke tutorials for each of the four groups were designed. Each AL had historically provided 10 hours of tutorials, each tutorial covering the material in a particular unit. It was agreed that the overall time allocated for ALs to deliver tutorials would be rearranged/split into time for core tutorials – which is like the existing provision – and dedicated qualification-based tutorials. Furthermore each AL's group of 20 students were from the same qualification-based group. The project team felt this change in support would become increasingly important as the numbers of non-M&S students were set to increase. This increase was due to the introduction in 2019 of the new qualifications in Economics and Data Science. It was likely that larger numbers of students on these two qualifications would start to study *M248* from 2020 onwards.

Murphy (2016) explain that resistance in any long-term change project can appear at any point, however it is most likely to be apparent during the phase before change takes place. Therefore, to ensure the ALs were part of this process, regular online meetings between project team and ALs for the sharing and discussion of information were introduced. The changing distribution of students on *M248* were not widely acknowledged by the ALs. Hence data was shared in the online meetings regarding the two new degrees and the changing distribution of the students. The sessions enabled, the idea of each AL having a group of 20 students who were in the same qualification-based group to be slowly introduced. Until this point ALs had been used to supporting a group of students in a particular geographical area who could be studying 20 different qualifications. These sessions took place over a year, giving ALs several opportunities to voice their views before a consensus was reached. It was agreed that each tutor group would consist of students on one of the four qualification-based groups: M&S, Data Science, Economics, and Open, as in Table 1. This would allow each AL to focus on how best to offer correspondence tuition to a particular set of students.

The ALs raised two specific concerns with the proposal regarding competency and fairness of workload. Each of the ALs had originally been recruited to teach primarily M&S students and had limited expertise in data science, economics, or other STEM areas. Therefore, several ALs were concerned they would be unable to find suitable examples to construct qualification-based tutorials. It was therefore agreed that tutorial material, could be written for each of the qualification-based groups, by small number of ALs. This team of ALs all had experience of either working or teaching in Economics, Data Science, or specialising in explaining statistics to non-statisticians. This followed the same pattern which had been used in Phase 1 of the project to write pre-module material for economics students. The intention was to provide a suite of materials that all ALs could use depending on the type of tutorial they were timetabled to provide. The second issue of fairness of workload centred around a concern that the non-M&S students would generally be weaker students and require more support. This was addressed by reducing the number of tutorials that ALs with non-M&S students would be required to give so that they could use the extra time to support their group.

To pay the group of ALs who were creating the new qualification-based tutorial material and to evaluate the effectiveness of this new way of working, funding was acquired from the OU centre for STEM pedagogy, eSTEeM. The students were surveyed in February and May 2019 in order to identify which areas of the module they found tricky, Unit 7 (which covers likelihood) was found to be particularly problematic, followed by Units 5, 10 and 11. Synchronous attendance, and viewings of the recordings of tutorials, for the Oct 2019-Jun 2020 students, also indicated these were areas were students sort more support.

Due to time-constraints qualification-based tutorial material was produced for only a subset of the 12 units which could be used for Oct 2020 – June 2021. And based on the results of the questionnaires the ALs focussed on providing material for those units which had been identified as more challenging. In total the students had access to 47 core tutorials (covering all 12 units), 3 M&S qualification-based tutorials, 6 Economics qualification-based tutorials, 6 Data Science qualification-based tutorials and 8 Open qualification-based tutorials. In core tutorials the ALs were free to identify key elements in the printed materials to expand on, explain and provide examples for students to tackle. The core tutorials for each Unit were usually 90 minutes in length. The qualification-based tutorials used the material which had been written by the specialist ALs and were typically shorter in length, usually 1 hour. These focused on examples from one of the four qualification-based groups. This distinction resulted in the qualification-based tutorials having a more informal feel, with an emphasis on students tackling questions in the session.

In December 2020 students were invited to feedback on the tutorial provision through an anonymous questionnaire. This consisted of open-ended questions which were analysed using text-based analysis. As OU students tend to provide positive comments to such questionnaires, the questions for the *M248* students were phrased in a way to elicit negative responses. For example, one question asked students to complete the sentence: “One thing that really irritates me about *M248* tutorials is…”. Figure 3 shows how the responses were split between the different qualification groups, together with the relative number of positive and negative responses to questions from qualification-based group. The Economics qualification-based group represented 9% of the responses to the questionnaire but contributed 15% of the positive words and 15% of the negative words. They therefore represented a more vocal group than their numbers suggested, but they were equally positive as negative. The least vocal group were the M&S qualification-based group who represented 39% of the respondents, like the economists these students were equally likely to respond positively or negatively (33% compared to 32%). The Data Science qualification-based group contributed 16% of the responses, but 27% of the positive words and were therefore the most positive group. The least positive group were the Open qualification-based group.

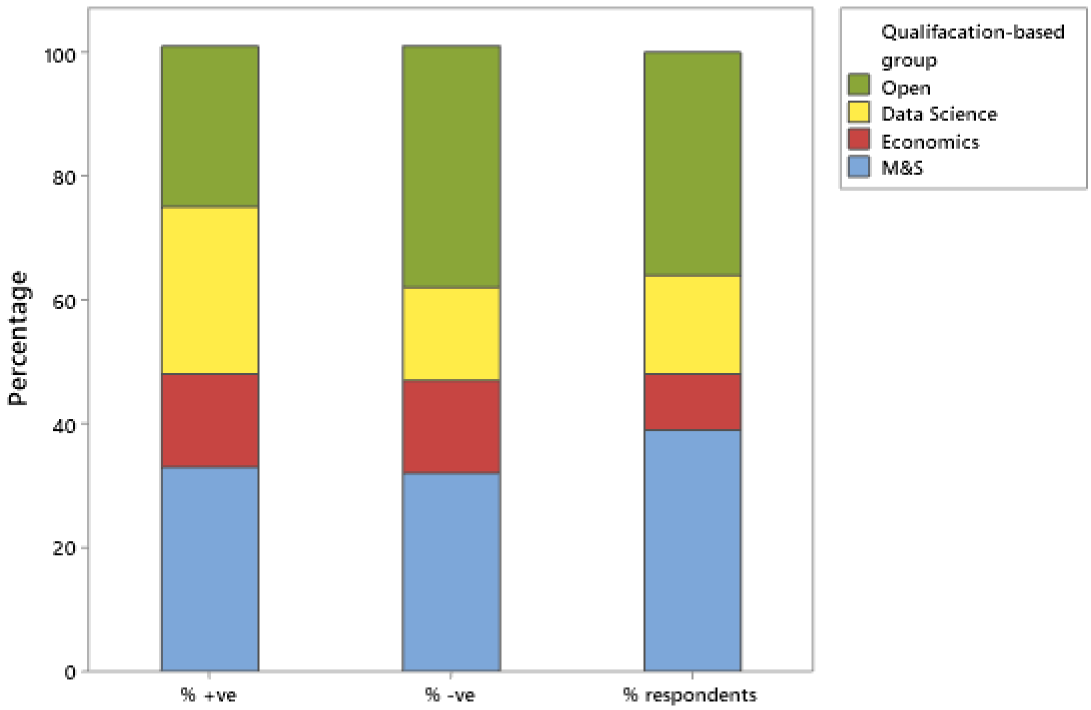

Figure 3. Percentage of positive and negative free text word responses by qualification group.

All tutorials, both core and qualification-based, were open to every student, however there was no evidence that, for example, a student from the Economics qualification-based group would attend a tutorial targeted at the M&S qualification-based group. Whilst there were only 3 M&S qualification-based tutorials it was assumed these students would attend the core tutorials. However, the M&S students expressed regret that they did not have more qualification-based tutorials, this was therefore rectified ready for Oct 2021. Table 2 shows the synchronous attendance and viewing of the recording for the different types of tutorials. Rather than absolute numbers these numbers are scaled by per hundred students or viewings to aid with comparison. The figures suggest that if a student was going to attend a tutorial synchronously, they were more likely to attend a qualification-based tutorial. Whereas, if a student was going to view a recording of a tutorial, they were more likely to view a core tutorial. This makes sense as the qualification-based tutorials had a more informal atmosphere and tended to have more student participation.

Tutorial type	Synchronous attendance per 100 students	Viewing per 100 students	Divisor
Core	4.2	697.0	Number of students on M248
Qualification -based M&S	6.1	54.5	Number of students in the named qualification-based group
Qualification-based Economics	9.4	16.9	
Qualification-based Data Science	11.4	224.0	
Qualification-based Open	8.2	331.0	

Table 2. Attendence and viewing figures for the 2020 cohort.

The Covid19 pandemic meant that exam conditions for each of the cohorts of students from 2018-2020 were very different and therefore comparison of exam results is not entirely useful. Students did however report that they felt more able to express their misunderstandings in the more supportive and inclusive environment which qualification-based tutorials provided.

3.3. Phase 3: Qualification-based tutorials for Oct 2021-June 2022

Based on the positive feedback from the Oct 2020-June 2021 students, more qualification-based tutorials were written ready for the Oct 2021 cohort. This has resulted in every unit having a qualification-based tutorial, for each of the four qualification-based tutorial groups, in addition to the core tutorials.

As student numbers had increased, 18 new ALs, including 7 of whom were new to the OU, were recruited ready for Oct 2021. Unlike previous years, ALs were recruited who were qualified and willing to teach a statistics module to non-M&S students. Whilst finding people with expertise in economics and data science remains problematic, the existing ALs were far more enthusiastic about taking a non-M&S group. This was a huge change from Phase 2 where only half the ALs expressed a willingness to take any qualification-based group. In Phase 3 no AL wanted to move from their allocated qualification-based group to a M&S qualification-based group. Furthermore, ALs expressed how much they had enjoyed the challenge of explaining statistics to non-specialists. The high level of discussion that had taken place during Phase 2 between the ALs has continued into Phase 3. There are now 40 ALs and the new ALs are very enthusiastic about building on the changes introduced with the original cohort of 22 ALs.

Both students and ALs feel that the qualification-based tutorials provide an environment where students can express their misunderstanding more openly as they feel they are amongst peers. This has increased the level of synchronous engagement in these tutorials. In addition, students can see the benefit of statistics within their own qualification.

4. Summary

The project grew out of a realisation that there were increased numbers of students taking the statistics module as part of non-M&S qualifications, many of whom were lacking some essential mathematical pre-requisite knowledge. As the actual module material could not be altered, changing the way in which students were supported was suggested as one possible solution. This involved highlighting issues and helping ALs who provided the support to understand there was a problem. The inclusive approach allowed the project team to collaborate with the ALs to create a new way of supporting students on different qualifications. As the project has evolved the team have worked with ALs to increase the level of interaction in qualification-based tutorials and use these tutorials to provide places where students feel comfortable expressing their misunderstandings. In addition, the type of comments provided to students who have differing qualification goals through correspondence tuition has been encouraged.

Due to the differences in exam arrangements over the period of the project it has not been possible to evaluate whether the new support has had any effect on the pass rate of different qualification-based groups. However, the surveys and feedback from ALs suggest that the qualification-based tutorials offer a more relaxed online environment for students. This enables them to feel more comfortable amongst their peers to discuss problems and ask for help. In addition, they feel they are not alone in their difficulties and see how statistics is used within their chosen discipline. The project team believe this method of qualification-based support could help students who suffer from statistics anxiety. It should be noted that statistics anxiety, is different – albeit related to – mathematics anxiety and various scales such as the Statistics Anxiety Rating Scales (STARS) were developed to address this difference (Cruise, Cash and Bolton, 1985). During the next two years the project team will use statistics anxiety rating scales such as STARS to evaluate whether students in the different qualification groups suffer from different types of statistics anxieties and whether interventions before the start of the module and during the qualification-based tutorials could be delivered to alleviate these issues.

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CASE STUDY

A Blind Spot in Undergraduate Mathematics: the Circular Definition of the Length of the Circle, and How It Can Be Turned into an Enlightening Example

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Abstract

We highlight the fact that in undergraduate calculus, the number pi is defined via the length of the circle, the length of the circle is defined as a certain value of an inverse trigonometric function, and this value is defined via pi, thus forming a circular definition. We present a way in which this error can be rectified. We explain that this error is instructive and can be used as an enlightening topic for discussing different approaches to mathematics with undergraduate students.

Keywords: calculus, engineering mathematics, definition, integral, arc length

1. Context

What I describe is borne out of many years of my experience of teaching first-year undergraduate mathematics. Also my approach benefits from my research interests. After a period of conducting research in pure mathematics, I diversified in two directions. On the one hand, I train artificial intelligence to produce proofs of simple mathematical results. On the other hand, I explore how producing and noticing shapes and patterns helps the brain to learn mathematics. In the former context, mathematics is "more formal" than usual, and in the latter context, mathematics is much "less formal" than usual. This dual perspective gives me a unique vantage point to appreciate the interplay between different approaches to mathematics. The purpose of this note is to highlight one example from first-year university mathematics which, as I will show, can become a topic for an eye-opening discussion on how "more formal" and "less formal" approaches to mathematics are both important for learning and understanding.

This duality of mathematics is perceived by many maths lecturers and, unfortunately, is felt as something which must be, at best, overcome or, at worst, reluctantly tolerated. A passionately written example of it is Allenby's book for first-year undergraduates *Numbers and Proofs*. Allenby writes about "good old days", when "one learnt to construct proofs", whereas "in recent years" "finding of strongly suggestive 'patterns' seems to have replaced *real* mathematical activity". Having said this, Allenby seems grudgingly to recognise persuasive limitations of "real" mathematics, saying that at the basic level, learners of mathematics should "be happy to accept certain easily believed, simple assertions as being unquestionably true" and at the advanced level, mathematicians "often have to read over proofs of results they 'know' (that is, 'feel in their bones') are true, just to see if the argument given is sufficient to establish a result they, in any case, believe".

However, as we discuss below, it is perfectly possible to appreciate the importance of both processes of "constructing proofs" and "feeling in one's bones that a result is true", and I believe it is possible and desirable to talk about this to students, using suitable examples like the one below.

2. The vicious circle

How do you define number π? If you are like me, you define it as a half of the length of the unit circle (and this definition, or an equivalent definition, is given in mathematical dictionaries). The next question is, how do we define the length of the unit circle? Undergraduate calculus (or engineering mathematics) tells us that the length of a curve represented by a function $y = f(x)$ is defined as $\int_a^b \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$. Consider the unit circle defined by the equation $x^2 + y^2 = 1$. Let us use the definition of the length of the curve as expressed by the integral above to find the length of the top-right quarter of this circle. The function is $f(x) = \sqrt{1 - x^2}$, and the limits or integration are $a = 0$, $b = 1$. Accordingly, the length of the quarter-circle is expressed as

$\int_0^1 \sqrt{1 + \frac{x^2}{1-x^2}}\,\mathrm{d}x$, which can be simplified to $\int_0^1 \frac{\mathrm{d}x}{\sqrt{1-x^2}}$. We can look up this integral in the table of indefinite integrals; it is an inverse trigonometric function $\sin^{-1}(x)$. To finish finding the value of the definite integral, we will need the value of $\sin^{-1}(x)$ at $x = 1$, so we look up the definition of $\sin^{-1}(x)$ and find out that the value of $\sin^{-1}(1)$ is defined to be equal to $\frac{\pi}{2}$. We have come full circle: unexpectedly, in undergraduate mathematics the value of number π is defined via the value of number π.

One can attempt to bypass this circular definition by representing the circle in the parametric form. As we will see now, this does not help. Let us start from the beginning again; we want to define number π; it is defined as a half of the length of the unit circle; but what is the length of the unit circle? If a curve is given by a parametric representation $x = x(t)$, $y = y(t)$ then its length is defined as $\int_a^b \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$. For the unit circle defined by the equation $x^2 + y^2 = 1$, according to undergraduate calculus (or engineering mathematics), we have $x = \cos t$, $y = \sin t$, and t varies from $a = 0$ to $b = 2\pi$. Therefore, the integral turns into $\int_0^{2\pi} 1\,\mathrm{d}t$ and is equal to 2π. If we knew what the value of π is, this would be a perfectly satisfactory answer. However, we do not know yet what π is; we were taking this integral hoping to use the answer as a definition of π, and it does not help us, because it only says, basically, that the value of 2π is defined as being equal to 2π.

Why is it a problem? This is a circular definition. Perhaps the value of π actually does not exist. Perhaps function $\sin^{-1}(x)$ is not defined at $x = 1$. Perhaps the unit circle does not have length. By defining these concepts one via another we might be creating an illusion of a watertight mathematical exposition, but what if none of these things exist?

The circular definitions in this section are not artificially constructed by me to bewilder you; as you can check by perusing textbooks on calculus or engineering mathematics, they really form a part of undergraduate courses in mathematics. Here are examples from books which are perfectly good in other respects. *Calculus* by Varberg et al. contains an exercise asking to find the circumference of the circle, with a solution equivalent to the integral $\int_0^{2\pi} 1\,\mathrm{d}t = 2\pi$ above, whereas some 250 pages earlier in the book, the circumference of the circle has already been said to be 2π (and this is where 2π in the limits of integration comes from). The same approach is used in *Thomas' Calculus* by Thomas et al.; an exercise asks to find the circumference of a circle using the parametric representation, and some 400 pages earlier in the book, the definition of radians implicitly defines the length of the unit circle as 2π. In *Calculus* by Sullivan and Miranda, inverse trigonometric functions are defined, including the fact that $\sin^{-1}(x) = \frac{\pi}{2}$; then, some 400 pages later, an exercise similar to $\int_0^1 \frac{\mathrm{d}x}{\sqrt{1-x^2}}$ above is presented, but not solved; some 600 pages later, in the appendix, the length of the unit circle is said to be 2π; at last, some 100 pages later, in another appendix, a table of integrals instructs us to use $\sin^{-1}(x)$ to take the integral. As you can see, this circular definition naturally involves referring to a number of chapters in undergraduate mathematics, separated by hundreds of pages in textbooks, and this is why it is likely to remain unnoticed by students and, perhaps, some lecturers.

Thus, we have to conclude that undergraduate mathematics leaves the question of the existence of π open. Perhaps π, and the length of the circle, and the inverse trigonometric functions exist, or perhaps they do not.

3. How can an undergraduate student define π?

A good news is that π, and $\sin^{-1}(x)$, and the length of the circle exist. Here is how we could define them without circular definitions in a way which a determined undergraduate student could follow.

As we consider integral $\int_0^1 \frac{\mathrm{d}x}{\sqrt{1-x^2}}$, of course we cannot say that it is $\sin^{-1}(x)$ because we don't know yet if function $\sin^{-1}(x)$ exists. However, we can formally consider a function f such that $f(0) = 0$ and $f' = \frac{1}{\sqrt{1-x^2}}$, and with some effort, we can write the Maclaurin series of f; let us denote this series by $M(x)$. Unfortunately, it is difficult to prove that $M(x)$ converges, and it is especially difficult to prove that $M(1)$ converges, and I would not expect a first-year student to be much interested in all details of these proofs. But fortunately, and importantly, these proofs involve only real analysis and number theory and don't depend on the existence of π. (See Section 6 for a plan of the proof.)

After we have satisfied ourselves that one can prove that $M(x)$ converges, recall that $\int_0^1 \frac{\mathrm{d}x}{\sqrt{1-x^2}} = M(1)$ is the length of a quarter of the unit circle; therefore, the length of the unit circle exists and is equal to $4\int_0^1 \frac{\mathrm{d}x}{\sqrt{1-x^2}} = 4M(1)$. Number π is a defined as a half of this length, that is, π exists and is equal to $2M(1)$, or, to be more specific, $\pi = 2 + \frac{1}{3} + \frac{3}{20} + \cdots$.

At last, we can define $\sin^{-1}(x)$ to be equal to $M(x)$. Note that function $\sin x$ is still not defined. Indeed, $\sin x$ is defined in undergraduate calculus (or engineering mathematics) either using angles in triangles or using arc lengths in the unit circle. While we did not know what π is, we could not measure angles in radians, and while we did not know that the unit circle has length, we could not measure arc lengths. Now that we know that function $\sin^{-1}(x)$ exists, we can define $\sin x$ as the inverse function of $\sin^{-1}(x)$.

4. Other constructions

After searching literature and asking colleagues, it seems that the only non-circular definition of π available in books is as twice the smallest positive solution of the equation $\cos x = 0$, where function $\cos x$ is defined as a Maclaurin series. This definition is known as the definition of π from Edmund Landau's textbook *Differential and integral calculus* (see Definitions 60 and 61). Remmert, in an interesting article "What is π?", discussing the history of this definition, states that "the definition of $\frac{1}{2}\pi$ as the smallest positive zero of $\cos x$ is now commonplace". However, in my experience, no undergraduate textbook on calculus or engineering mathematics attempts to give a non-circular definition of π.

When I compare Landau's definition of π with that proposed in the previous section, I feel that the latter is slightly more practical in an undergraduate course because, firstly, it is explicitly motivated by the aim of finding the length of the circle and, secondly, immediately proposes an explicit way of calculating the value of π.

When I discussed these ideas with colleagues, they naturally asked if series $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$, also known in another notation as $\zeta(2)=\frac{\pi^2}{6}$, can be used as a definition of π. The answer is no, we cannot use this equality to define π. Indeed, there are many known proofs of this equality, and each one of them requires good knowledge both of trigonometric functions and of the role π plays in trigonometry (for example, it helps to know that the period of $\sin x$ is 2π, or that the zeros of $\sin x$ are exactly the multiples of π). Now suppose we consider series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ without having defined π. We can prove that the series converges, but then we reach a dead end, in the sense that without knowing enough about trigonometric functions and the length of the circle we will not be able to show that the sum of the series has something to do with either. (There are also other formulas involving π, for example, the Wallis product and Stirling's formula, and the same argument applies to them, too.)

5. Turning it into an enriching experience

Now that I know that the definition of π and the length of the circle in undergraduate mathematics is circular and, therefore, invalid, what am I supposed to do about it?

One approach is to ignore this fact and to continue teaching in the usual way, without telling my students about it. However, if I do this, I miss an opportunity to talk to my students about interesting and important details of the way mathematics is developed and used.

Another approach is to stress the validity of the intuitive understanding of mathematics. If we equip ourselves with a measuring tape and examples of circles (for examples, some pots and pans), we can convince ourselves that circles have length, and π exists. Then I can tell students that the formal definition of the circle length and π in undergraduate mathematics is flawed, but it does not matter, because from our experience we know that circles have length, and π exists. Although this approach is practical, I believe it sends a wrong message to students; we want to feel that mathematics is robust and logical.

Yet another approach is to shun the intuitive understanding of π as invalid, and follow the formal approach thoroughly, defining the value of π to be $2+\frac{1}{3}+\frac{3}{20}+\cdots$, as I described in Section 3. Although there is some appeal in this approach, I believe it is unnatural. For instance, we want to be able to use the usual geometric definition of π. The series $2+\frac{1}{3}+\frac{3}{20}+\cdots$ looks contrived, and speaking practically, it is not even a good way to approximate the value of π. We want to be able to use a geometric definition of $\sin x$, rather than defining it as an inverse function of a certain Maclaurin series.

I believe that the best solution is to combine the above approaches and to explain to students different facets of mathematical practice, approximately as follows.

"It is true that our experiments with pots and pans do enable us to produce perfectly workable definitions of the circle length and π. It is an inspiring example of ingenuity of human mind that after some experiments with circles, in one stroke of genius we can conceive of a range of useful mathematical concepts.

“It is true that in comparison with this intuitive ingenuity, formal mathematics feels like a blunt tool; a mathematician needs to write hundreds of pages defining limits, series, integrals etc. before she is able to define the circle length and π. However, on the positive side, we must recognise that the mathematician can achieve the result, and the circle length and π can be defined formally. Therefore, separately from our intuitive ingenuity at conceiving mathematical concepts, we should also celebrate the mastery of mathematicians at wielding formal mathematical tools. As we do this, we should also appreciate the surprising reach of formal mathematics; formal mathematical arguments might feel circuitous, but they eventually catch up with our expectations, enriching our understanding on the way.

“What I described is a rich multifaceted picture of mathematics; however, this is not yet the whole story. Formal mathematics is intentionally moulded to fit in with our informal understanding of mathematics. Indeed, mathematicians actively wanted circles to be defined in such a way that they have length, and π to be defined so that its value matches the value measured in experiments. This is why mathematicians spent two thousand years perfecting definitions of limits, series, integrals etc. until these definitions, whereas satisfactorily formal and logical, also matched our informal concepts of the circle length and π.”

Understanding and untangling the circular definition described in this note, and making it formal, involves the use of a wide range of topics of undergraduate mathematics, as we saw in Section 2 and 3. This, in itself, is enough to make this circular definition worth discussing with students. However, what makes this circular definition a special example to me is how disengaged our intuitive understanding of the circle length and π, on the one hand, and the formal definitions of them, on the other hand, are from each other. This gap is the reason why this circular definition appears repeatedly in calculus (or engineering mathematics) textbooks and remains unnoticed by students. This is where an opportunity arises; this circular definition is a perfect material on which the role of different approaches to mathematics can be explored and discussed.

6. Appendix: a plan of the proof

We consider a function f such that $f(0) = 0$ and $f' = \frac{1}{\sqrt{1-x^2}}$. We want to build the Maclaurin series of f, which we denote by $M(x)$, and then we want to show that $M(1)$ converges.

There are many proofs showing that $M(x) = \sum_{n=1}^{\infty}\left(\frac{1}{2n+1}\cdot\binom{2n}{n}\cdot\frac{1}{4^n}\cdot x^{2n+1}\right)$; none of these proofs is very short, but an undergraduate student should be able to understand some of them.

Therefore, $M(1) = \sum_{n=1}^{\infty}\left(\frac{1}{2n+1}\cdot\binom{2n}{n}\cdot\frac{1}{4^n}\right)$.

Naturally, the next step is to approximate $M(1)$ and then show that it converges using the p-series test. The part on which we need to concentrate is $\binom{2n}{n}$. The standard tool for approximating the central binomial coefficient is Stirling’s approximation, but we may not use it, because it employs π. Fortunately, there are other formulas which can be usefully applied here, for example, the inequality $\binom{2n}{n} \leq 4^n \cdot \frac{3}{4} \cdot \frac{1}{\sqrt{n+1}}$.. Proofs of these formulas are technical, but they are short and can be understood by an undergraduate student.

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WORKSHOP REPORT

Is Mathematics Inclusive or Exclusive? Putting Colour, Culture and Context in the Curriculum

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Keywords: Decolonising, awarding gap, inclusive pedagogy, race, intersectionality.

Abstract: The workshop, "Is Mathematics Inclusive or Exclusive? Putting colour, culture and context in the curriculum" was held in January 2022 with the goal of supporting a national discussion around race and the mathematics curriculum in UK higher education. This report summarises the talks and discussion, which related to racial and ethnic inclusion in the history of mathematics, race and culture in mathematics education, and ethics and inclusion in mathematics. It concludes with a proposal of actions for individuals, departments and institutions and the mathematics community in UK higher education to move work on this area forward.

1. Overview

The workshop "Is Mathematics Inclusive or Exclusive? Putting colour, culture and context in the curriculum" was held online on 25-26 January 2022. It was hosted by the International Centre for Mathematical Sciences (ICMS, Edinburgh, UK), and organised by a committee representing several UK professional societies in mathematical sciences[1]. The goal of the workshop was to begin a discussion within the UK higher education mathematics community about how to best to improve the inclusion of ethnic and racial perspectives in mathematics teaching in UK higher education, and how to ensure that students from all ethnic and cultural backgrounds feel included and supported in their mathematical studies. This is set in the context of broad calls for decolonisation in higher education curricula, which means recognising the cultural dominance of Eurocentric knowledge systems and working to create spaces in which all global cultures and their contributions are valued. These calls, however, have most generally focussed on disciplines, such as history, where cultural considerations are more obviously central to the subject itself. So, one of the fundamental questions the workshop aimed to investigate was, what could or should racially and culturally inclusive curriculum and pedagogy mean in the context of mathematical sciences? What steps can be taken immediately at the level of individual instructor or department, and what work does the broader UK mathematical sciences community need to undertake moving forward to incorporate these ideas into mathematics teaching at university?

[1] Supported by the British Society for the History of Mathematics, the Institute of Mathematics and its Applications, the London Mathematical Society, the Operational Research Society, the Royal Statistical Society, and the **sigma** Network.

Speakers at the workshop came from the UK and abroad and covered three main areas: history of mathematics, race and culture in mathematics education, and ethics and inclusion in mathematics. In addition, break-out discussion groups permitted participants to share thoughts and raise questions, with a chance to feed back to the full group afterwards. The workshop concluded with a panel discussion aimed at understanding the similarities and differences between the US and UK contexts. Over 200 participants attended the workshop, from 14 countries on five continents and from 70 UK universities. Recordings from the workshop and other resources are available online through the workshop website (ICMS 2022).

2. Colour and Context: Race, Culture and Mathematics

The workshop featured two prominent speakers from the US mathematics community who are advocates for both research and practice that recognise the importance of culture and race in mathematics education. Danny Martin is an internationally recognised expert in mathematics education, known particularly for articulating and leading a research programme focussed on race and identity in the mathematical education of Black Americans. Aris Winger is a mathematician who has been active in improving racial inclusivity in university mathematics through his talks, podcast and writings, including two books with mathematician Pamela Harris.

Aris Winger opened the workshop with a talk entitled 'Mathematics, Race and Belonging'. In it, he led the participants in an interactive exploration (via anonymous padlet) of their beliefs and attitudes about mathematics and the relevance of race to the discipline. He began by setting out the major problem, belonging ("*Mathematics is the greatest subject in the world, but it may not be the most welcoming.*") and the cultural challenges in solving this: political division ("*Supporting all students in mathematics should not be political, but it can be made political.*") and time pressures ("*We have a lot of things on our plates. This is not something extra--it is something that has been on our plates, but we haven't been paying attention to it.*"). He then gave an overview of principles needed for work on racial inclusion in mathematics: expect and embrace discomfort, be fully present in discussions and listen actively and respectfully to all points of view and keep students of colour at the centre of the discussion.

Winger then presented two perspectives on mathematics that influence how we think about belonging: mathematics as a body of knowledge, and mathematics as a human activity. The first of these leads to a view of mathematics as a discipline that belongs to the experts, like medicine. The second leads to a view of mathematics as something everyone can and should do, like sport or games. These views are often set in an unnecessary opposition—rigour versus inclusion—and lead to the question, do we need gatekeeping in mathematics, or do we need to ensure broad access to a rigorous mathematical education?

Finally, Winger asked, what does race have to do with mathematics? This brought out a wide range of views. Concerns were raised that relating race to mathematics can lead to a view of mathematics as a 'European tradition' and a 'western way of thinking'. Other comments put forward that Mathematics is not value neutral in terms of what topics we choose to study and whose contributions are recognised. Winger summarised with three questions: who do we imagine as mathematically adept? How do we teach and advise different students? What mathematics do we do? "*Social inequality is a math problem*," he pointed out, "*but we are not working on this problem with the same intensity and fervour in mathematics as in other disciplines, and this relates to race*." Winger concluded with a particular message to white mathematicians: "B*eing white in the UK means something. The future of the discipline is tied to how we understand whiteness and being white, to white supremacy and fighting it. Examining the discomfort associated with these ideas is where we need to start in order to make the discipline better*."

Danny Martin gave the final talk of the workshop, 'Rethinking Equity and Inclusion as Racial Justice Models in Mathematics (Education)'. He posed three questions:

1. What do white supremacy and antiblackness have to do with mathematics education?

2. What are some of the limitations of equity- and inclusion-oriented justice projects, especially in relation to Black learners and mathematics education in the US?

3. Beyond equity and inclusion, what are some justice projects that can respond to the material realities, needs and desires of Black people inside and outside of mathematics and mathematics education?

Note that the phrase “white supremacy” is used here to refer not to individual racists or extremist groups advocating a white ethnostate such as the KKK, but more broadly to social systems which uphold the privilege and power of white people and centre and normalise them and their experiences. He then proposed that white supremacy and antiblackness are adaptive social systems that self-correct, so that work that contributes to inclusion at some levels and in some contexts does not preclude exclusion at other levels and in other contexts. In this way, they can uphold and entrench inequality under the pretext of social justice. He mentioned for example that desegregation in the US, which was supposed to lead to inclusion for students, led to the firing of large numbers of Black teachers as Black schools closed. "*This idea is challenging and difficult for individuals engaged in diversity work,*" Martin noted. “*The vision is that if we engage in this work, it will lead to the success of Black students.*” He discussed the decades of mathematics education reform in the US, particularly with the equity-themed discourse of “Mathematics for All” that began in the mid-1980s. This has not led in the intervening years to any greater proportion of Black Americans among mathematics majors and has in fact been accompanied by a substantial decline (from about 8% to about 4%) since the mid 1990s. “*It is very difficult to dismantle systems that benefit so many people individually and collectively. We need to rethink mathematics reform from this viewpoint.*” To do this, we need to be aware of the social issues and the social science research around race and identity. This research indicates the impact of, for instance, microaggressions and the assertion of the neutrality of knowledge and knowledge production in the discipline. He discussed the concept of “white institutional spaces” in the US and how these lead to inclusion efforts resulting in marginalisation and assimilation, and which do not change the nature of the space.

Martin recognised that different geopolitical contexts have different racialised social systems, and these effects play out differently in different locations, but that the existence of the racialised system is the commonality. One aspect of this is the concept of identity, and in mathematics in particular, how do students from different ethnic backgrounds construct a mathematical identity? And, how do their racial and mathematical identities intersect and interact? “*What we are finding from our research [in the US] is that being Black matters to Black people who do maths,*” he recounted, “*Over the past 20 years we have found that across educational levels, what works is to open up a space for a non-deficit approach, which focuses on identity, socialisation, resilience and success, where success is put in a broader context of humanity, and mathematical success can be actualised in different ways, not only through obtaining a PhD.*”

3. Visibility and Culture in the History of Mathematics

Three short presentations and a longer plenary demonstrated the contribution that history of mathematics can make to inclusive mathematics curricula, by emphasising the contributions of many different cultural and occupational groupings to ways of doing mathematics, and promoting diverse role models The importance of such role models, and unfamiliar perspectives on mathematics, in enhancing inclusion came out strongly in breakout discussions (section 6) June Barrow-Green, Chris Hollings, and Edmund Robertson outlined resources being developed at their respective universities. In ‘Diversifying the Curriculum through History of Mathematics’, Barrow-Green talked about an online resource that she and Brigitte Stenhouse are currently developing at the Open University. It will contain original and secondary source material to exemplify the rich diversity of contributions to mathematical development. It is aimed at students but will be openly licensed and freely available to all on the Open Learn platform (Open University, 2022). Hollings’ talk on ‘Diversifying Mathematics in Oxford’ gave three mini case studies of interventions he has been involved with:

1. The Diversifying STEM Curriculum Project worked with student interns to co-create resources for lecturers in STEM subjects to help them present a more diverse image of their subjects (Oxford University, 2022).

2. Using examples drawn from the mathematical practices of various cultures (ethnomathematics) around the world, as exercises in mathematics tutorials. Hollins gave a specific example of using Australian Aboriginal kinship relationships as an exercise in group theory.

3. Designing a series of posters to portray a more diverse image of mathematics, making the environment within which the curriculum is taught more inclusive (Oxford Mathematical Institute, 2022).

Robertson’s ‘MacTutor: My own Personal Journey’ chronicled the development of the MacTutor history of mathematics website (St. Andrews, 2022), which contains biographies of around 3000 historical mathematicians and 2000 pages of related material. The biographical subjects come from 94 different countries, with deliberate attempts to increase the coverage of under-represented groups such as women (currently around 1250) and Africans (over 400 men and 200 women with PhDs in mathematics). MacTutor is openly licensed and aimed at a general audience.

All three speakers are developing resources that fulfil the need for powerful role models of diverse mathematicians – a need expressed strongly by participants in subsequent breakout discussions – but vary in the amount of pedagogical support they provide for staff wanting to diversify their curricula. They range from the Open University’s stand-alone lessons that students could study independently, through resources designed for lecturers at Oxford, to MacTutor which provides a far greater variety of potential models but leaves it to staff or students to work out how to use them.

A tension between complexity and simplicity came across strongly in Barrow-Green and Hollings’ talks and in ensuing discussions. One of the main messages of history is that mathematics is a human activity shaped by a complex web of interacting historical and cultural factors. This understanding can challenge many of the persistent myths about mathematics that may deter students: ‘mathematicians are born not made’, ‘black people/girls are no good at maths’, ‘mathematicians burn out at 30’. It can provide insight into the process of practicing mathematics in many contexts and levels and understanding of past (and present) barriers to education, knowledge, resources, and recognition. But this very complexity can be overwhelming, preventing students and staff from engaging or finding space for it within the curriculum. A challenge for historians is to work with educators to develop ways of organising and presenting material so that it is attractive and engaging without losing the benefits of insight.

Hollings made two further important points. First, that if we, as historians, are to be more inclusive, we need to broaden views of what counts as mathematics, recognising mathematical cultural and social practices (including Western ones) whose aim is not just to develop new mathematics. This recognition is pursued in the discipline of ethnomathematics. Hollings’ second point is that to inspire diverse students we not only need to show that mathematics was a global activity in the past, but also the mathematics that is going on all over the world today. Each in his poster series include current as well as past mathematics. This is an area where MacTutor can help, with a number of its biographies covering living mathematicians from minority backgrounds.

In her plenary, Karine Chemla first highlighted two ways in which diversity is commonly approached in history of science, and in particular, history of mathematics, that she considers deeply problematic. She illustrated her points from writings on Chinese mathematics. The first is a crude partitioning of the world into nations, cultures, and civilizations. She contrasted Lucien Febvre's 1949 argument that historical research should focus on "*everything that circulated from one group to the other*" and that "*the partitioning of the World is nothing but a fiction*" (Febvre 1953) to Joseph Needham's 1950 aim to "*outline the various patterns of the great cultures and civilisations, their particular world-outlooks which were characteristic of them,*" (Petitjean 2006). Drawing on an example of Geneviève Guitel's comparative study of written numeration systems (Guitel 1975), she showed how a methodology based on written evidence brings with it a hidden assumption that language is the primary categorisation for number systems; the conclusion that number systems are best characterised by language culture follows naturally but is only warranted in so far as the initial assumption is warranted. Research by Chemla and colleagues challenges this assumption, evidencing the existence of non-written and often non-verbal numeration and calculation practices that differ from one specialist group to another within a single language culture but are common to similar specialist groups in different language cultures – suggesting a connection and circulation of practice among the groups. This is diversity of a different and more complex kind than a simple partition of nations, cultures and civilizations.

Chemla's second concern is that historians of science and mathematics have tended to partition peoples into groups whose styles of thought and intellectual activity are characterised by different parts of the world. They assume that knowledge activity can be characterised by specific features that are then used to contrast these parts of the world with each other, and that these features are enduring. Drawing on the example of the Chinese mathematician Wu Wenjun (1919-2017) who turned to automated theorem proving and history of mathematics during the cultural revolution, she shows the consequences of this type of history in constructing national narratives. On the basis of allegedly differing but enduring styles of thought in the ancient mathematics of East and West, Wenjun was able to present his research on automated theorem proving as a return to a specifically Chinese tradition that has modern significance. This narrative was adopted by President Jiang Zemin in 2001 when awarding Wenjun the highest distinction in science and technology in China. Similar examples can be found in all parts of the world and all disciplines.

Chemla's essential claim, evidenced by a case study of numeration systems, is that diversity exists, but it is not to be found within the boundaries of people, cultures and civilisations. Such crude categorisations may prevent us from understanding the actual nature of mathematical activity and the collectives that are meaningful in mathematics. There are other ways to approach diversity that may be more useful in the classroom, for example, for pupils to consider *practices* of numeration and computation and to think about diversity in more fluid terms.

4. Ethics and Inclusion in Mathematics

Two talks from Tarik Aougab and Kutoma Wakunuma focused on how ethics and inclusion can be addressed through changes to curriculum, either through innovative approach to curriculum design, or discussion of ethics in mathematical modules.

Tarik Aougab gave an overview of a module Ethics and the Use of Mathematics introduced in 2020-21 in Haverford College. The course was a response to the concerns expressed by students involved in the Black Lives Matter movement. Unlike most modules in mathematics, it is run as a seminar: each week students are given pre-reading and then meet to discuss a specific topic. In the first two weeks, a set of ground rules is agreed by all students and staff involved, then, seminar-style, different topics of mathematics and ethics are discussed in weeks 3-8. The topics discussed in the modules covered: use and abuse of mathematical methods such as predictive policing, facial recognition and surveillance, controversial publications such as those related to the Variability Hypothesis, which claims that males generally display greater variability in traits than females do; mathematical perspectives on economics, hierarchies and the role of imperialism in science. In the final weeks of the module, students pick a specific topic from the list already discussed and prepare a project (either individually or as a group) in the form of pedagogical material, which can then be used in other courses. Resources created in the module will form a library from which all lecturers can draw materials to integrate into their modules.

Kutoma Wakunuma from Centre for Computing and Social Responsibility, De Montfort University, discussed design of mathematical curriculum from personal perspective of a student in primary and secondary education in Zambia, and from a professional perspective of theory of curriculum design. Wakunuma's personal experiences were marked by the lack of role models, poor resources and lack of encouragement, which lead her to see mathematics as a subject not worth or appropriate to pursue. She argued that some of these issues can be eliminated by a better curriculum design, one following the theory of curriculum design which uses the four AREA dimensions:

- Anticipating: consider what the curriculum might mean and for whom it might be meaningful;
- Reflecting: if the above questions suggest that the curriculum might not be inclusive, consider what can be done to improve it,
- Engaging: consider who needs to be engaged to ensure that the curriculum works for all,
- Acting: make changes following the first three steps.

This approach enables us to provide a more relevant and inclusive curriculum, which reflects values of wider range of stakeholders.

5. Panel Discussion: Mathematics, Race and Belonging

The panel consisted of Aris Winger, Danny Martin and Vijay Teeluck. The first two are internationally recognised experts from the US mathematics community, and the latter works on projects aiming to support BAME (Black, Asian and Minority Ethnic) students through her role as a senior lecturer and Maths Support Tutor in the UK. The main points of the discussion centred on pertinent questions posed by the chair which included racialised systems, a sense of belonging, success in the mathematical realm, closing the awarding gap in the UK, white supremacy, maths as a collaborative space, and how racism influences the way mathematics is shaped.

Although it was accepted that the racialised systems in the UK and USA were broadly different, there were similarities with schools playing a part in social reproduction which is guarded by white supremacy and anti-blackness. Martin voiced that schools should be independent institutions but are an arm of the state and carry out the objectives of the state in different ways through policies and legislation which has an absence of local control, therefore, reproducing hierarchies that exist in society. Teeluck echoed that these inequalities within the UK system play out with higher numbers of Black students being disproportionately excluded, and Winger reiterated that this is the way structures work i.e., we should not expect the education system to be different especially with respect to the disparities that go on in society.

Winger advocated developing an analogous understanding through listening to marginalised people and providing an authentic space in which their experiences could be heard - otherwise the white space becomes the dominant space. On a macro scale, Martin suggested that racism relies on capitalism and that researching the interconnectedness of racial exploitation and capital gain would allow for further understanding of racial stratification. The panel discussed elements of success in tackling the marginalisation of non-white students, however, Martin stressed that it was important to allow Black people to self-determine what they do with mathematics and that comparisons to white success (e.g., the awarding gap) was only normalising the white students and reinforcing that Black students are inferior. Hence, it was vital to examine the nature of the gap discourse and not only what it says ideologically but what it implies.

When aiming to move away from white supremacy, Winger was clear that one of the main markers was 'Who is upset?' when actions are taken to challenge the current systems. Reference was made to a joint publication, 'Asked and Answered: Dialogues on advocating for students of color in mathematics' [Harris and Winger, 2020]. Martin touched on the topic of reparation; not only would this mean surrendering privileges that we have and that continue to benefit us at the exclusion or detriment of someone else but also paying the debt that is owed to the descendants and beneficiaries who were enslaved in the US.

When considering factors which exacerbated inequalities and racism in maths, Martin stated that mathematics was no different than any other social enterprise and that it was a particular type of political project with relationships to militarism, the economy, the war on terror and international competitiveness. Teeluck referred to lack of BAME staff as role models and the methods of student assessment that perpetuate these inequalities. Moreover, Winger strongly advocated that the nature of the discipline is determined by the people, and that new directions would mean re-thinking who is going to be taking part in these conversations.

Figure 2. Panellists from left to right: Danny Martin, Aris Winger, Vijay Teeluck

6. Issues Raised in and from Discussions

There were several opportunities for discussion and participant contribution at the workshop, and they were quite actively used, indicating the benefit to the community of providing such fora for discussion of the topics of the workshop. This was echoed in participant comments, such as "*A really engaging and emotive workshop*".

The prompts used by Aris Winger in his talk demonstrated a range of experience and perspectives. For example, the responses to the prompt about if members of our community view mathematics more as a body of knowledge, or as a natural human activity got responses from "*You can't access modern mathematics in a creative sense if you don't first master a vast body of knowledge*" to "*Framing it as something everyone can do is helpful in outreach and making people feel like they can get involved*". There was a clear tension between the notion of higher mathematics as an elite, gatekeeping body of knowledge and mathematical thinking as something everyone needs to do in order to be the best they can be, but also with some indication that engagement at different levels is seen differently in this regard. This suggests a refinement: is gatekeeping required anywhere in mathematics, and if so where and why, and how can this be achieved while also ensuring equal access?

The prompt ‘What does Race have to do with Math?’ also generated substantial discussion, with comments ranging from “*nothing*” to “*everything*”. Notable comments included that “*mathematics is something done by people who call themselves mathematicians*” who are perceived as predominantly white, males, and results in (unintentional) exclusion and that ‘race’ is one of [the] lenses by which we construct the mathematician, which is inseparable from the maths. Other comments were about the Euro-centric nature of mathematical terminology and that the experience of mathematics has links to one's identity and thus the way it is taught can make a big difference in terms of getting our students to understand how maths has influenced our world and whether they feel included in mathematical thinking. Again, this suggests that mathematics as done by researchers is seen to some extent as a separate and different activity from mathematics as done by students, with race relating in the case of researchers to how the identity ‘mathematician’ is constructed by society, whereas for students to how it is related to the construction of their own identity as related to mathematics, with these being linked through the concept of role models. It was not clear where in this distinction individuals who use mathematics in a non-research career fit.

Discussion break-out groups were asked to consider how to put history and stories into the curriculum in UK. What came through strongly in the discussion groups was the importance of diversity in role models, with mathematics in non-research careers specifically mentioned: “*It would be really great to have some short videos of diverse mathematicians from industry talking about some mathematics idea that they found really inspiring when they were students, or that they struggled with, or that they use in their work*.” Several groups referred to drawing on existing graduate students as speakers and as sources of information. The Algebra group offered a useful example: In which respect is the elimination of Ancient China different from Gaussian elimination? - discussion of this question was found to be a way to develop understanding. Other members proposed that adding in something about how a mathematical concept got its name doesn’t take up much time, for instance, lecturers could give a link to a debate, and it does help generate interest. There were also suggestions regarding links between arts and maths in particular “*African textiles, Aboriginal art and Native American Indian textiles (symmetry, rotation, reflection, pattern, sequence etc.)*”, and the importance of including applications to topics learners are concerned about such as climate and justice. The role of project work in giving room to such aspects in the curriculum was proposed. Another theme was the possibility to reach out to students to come up with diverse examples of applications**.** Overall, there was interest in the development and dissemination of more resources to help with incorporation of history and stories in teaching, with a discussion about how we pool resources, as items developed in one place might be useful for folks in another. A platform for sharing these types of resources was suggested.

Regarding the inclusion of ethics into the curriculum, one chat theme started with the question How do you ensure that a Math Ethics course isn’t just a different form of indoctrination? A contributor introduced the notion of decolonial practice as helping with this, in which “*Student voices are heard as opposed to a transmission model; students are co-creators of knowledge, teachers are encouraging deep thinking and providing a wide range of resources*”. The model proposed by Tarik Aougab was seen as “*courageous*”, and was positive about this, but less positive about the perceived barriers to curriculum change. One role for the community could therefore be in lowering those barriers, or even considering how course accreditation standards could be adapted to include ethics.

Finally, from all discussion, there was evidence of technical vocabulary of relevance to these topics that is not necessarily widely understood, with terms such as ‘brave spaces’, ‘intersectionality’, ‘solidarity’, ‘decolonial’. The term ‘BAME’, though commonly used in the UK, is also one that deserves unpacking, in terms of combining in one acronym a collection of very different groups. This suggests the utility of a resource that lays out these terms with definitions and examples of their relevance in mathematics and mathematics education.

7. Conclusions and Next Steps

This workshop was just the beginning of a larger discussion and project in the UK to improve racial inclusion in HE mathematics education and more broadly. It suggested several ways forward.

Actions for individuals include:

- Read relevant literature on race and education from social sciences—Danny Martin's talk and references can serve as a starting point,

- Integrate existing materials showing diverse history and case studies into your teaching, either linked to lecture material or just as a showcase at the start of lectures —see multiple linked resources on the workshop website,

- Co-design ethnic diversity, ethics, social justice related resources to integrate into existing modules with students through projects or a seminar – Tariq Aougab's talk and syllabus or Chris Hollings' talk and example projects can serve as a starting point.

Actions for departments and schools include:

- Initiate discussions—embrace difficult and uncomfortable conversations about race and belonging in your departments, as it is through discomfort that we move forward. Aris Winger's talk recording can serve as a starting point, framework and set of guidelines for discussions,

- Analyse your curricular design for inclusivity—Kutoma Wakunuma's talk on the AREA framework (see also the UKRI Framework for responsible innovation (UKRI 2022)) can serve as a starting point.

Actions for the community and professional societies include:

- Undertake qualitative research at the national level on the experiences of ethnic minority students,
- Provide support for projects to develop resources for incorporating ethics and race in teaching,
- Develop an easily searchable and accessible resource bank for sharing such resources,
- Work on changing the culture and accreditation standards to include ethics and inclusion in UK mathematics courses,
- Organise follow up/follow on events to provide further opportunities for collaboration and discussion.

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